



Munich Personal RePEc Archive

# **An Input-Output Approach to the Estimation of the Maximum Attainable Economic Dependency Ratio in four European Economies**

Mariolis, Theodore and Soklis, George and Groza, Heleni

Department of Public Administration, Panteion University, Athens,  
Greece

12 May 2010

Online at <https://mpra.ub.uni-muenchen.de/22661/>  
MPRA Paper No. 22661, posted 14 May 2010 00:15 UTC

# **An Input-Output Approach to the Estimation of the Maximum Attainable Economic Dependency Ratio in four European Economies \***

**THEODORE MARIOLIS\*\* , GEORGE SOKLIS \*\* & HELENI GROZA \*\***

*\*\*Department of Public Administration, Panteion University, Athens, Greece*

## **ABSTRACT**

*The purpose of this paper is to explore, in terms of input-output models, the proximate determinants of the maximum attainable Economic Dependency Ratio and to provide estimates of that ratio in four European economies (Finnish, German, Greek, Spanish). The evaluation of the results reveals certain central socio-technical features of the actual economies under consideration.*

**KEY WORDS:** Austrian rate of surplus labour, net labour saving from trade, economic dependency ratio-consumptions-growth frontier

**JEL CLASSIFICATION:** C67, D57, E24, H55

## **1. Introduction**

The so-called Economic Dependency Ratio or Labour Market Adjusted Dependency Ratio, defined as the number of persons not employed (children under the age of 15, students, home duties, unable to work, retired, unemployed, first time job seekers) per person employed, reflects basic relationships between the productive and the unproductive parts of the socio-economic system, and constitutes one of the most

---

*Correspondence Address:* Theodore Mariolis, Department of Public Administration, Panteion University, 136, Syngrou Ave, Athens 17671, Greece; Email: mariolis@hotmail.gr

\* A first draft of this paper was presented at a Workshop of the ‘Study Group on Sraffian Economics’ at the Panteion University, in September 2007: We are indebted to Renetta Louca, Eleftheria Rodousaki and Nikolaos Rodousakis for their helpful comments. We thank Menia Marioli for her inspiration and encouragement. Furthermore, we are indebted to Bart Los for extremely helpful comments and suggestions on an earlier version of the paper. The usual disclaimer applies.

important variables for the social security system.<sup>1</sup> This paper, first, explores the *proximate* determinants of the *maximum attainable* economic dependency ratio (MEDR hereafter), defined as the economic dependency ratio compatible with the ruling (i) technical conditions of production; and (ii) sizes and compositions of the final consumption expenditures of the household sector, investments and net exports, and, second, provides estimates of this ratio in actual economies. For this purpose we use linear models, which have a modern ‘classical’ flavour (in the sense of Kurz and Salvadori, 1998, Essays 1 and 2), but focus attention on the *quantity* side of the system, and input-output data from the Finnish, German, Greek and Spanish economies. This data selection is based on the guesstimate that between the said European economies there will be remarkable differences and similarities in the relative strength of the proximate determinants of the MEDR (*e.g.*, Greek versus German economy and Greek versus Spanish economy, respectively).

The remainder of the paper is structured as follows. Section 2 expounds the models.<sup>2</sup> Section 3 presents and critically evaluates the results of the empirical analysis. Section 4 concludes and makes some remarks about the direction of future research efforts.

---

<sup>1</sup> For alternative (demographic and labour market adjusted) measures of ‘dependency’, see Foot (1989). Many empirical studies find that over the next 50 years the *old-age* dependency ratio (the number of persons aged 65 and over divided by the number of persons of working age, namely 15-64 years old) will increase substantially in most countries of the world (see, *e.g.*, United Nations, 2006, ch. 2). According to Eurostat’s latest population projection scenario (EUROPOP2008 – convergence scenario), for the EU-27 this ratio is expected to increase substantially from its current levels of 25.4% to 53.5% in 2060, whilst the *young-age* dependency ratio (the number of younger persons of an age when they are generally economically inactive, namely 0-14, divided by the number of persons aged 15-64) is projected to rise moderately from its current levels of 23.3% to 25.1% in 2060.

<sup>2</sup> This section is based on Mariolis (2006).

## 2. The Analytic Framework

We begin with a closed, linear system with only single-product industries, circulating capital, homogeneous labour, which is not an input to the household sector, and without ‘self-reproducing non-basic commodities’ (in the sense of Sraffa, 1960, §6 and Appendix B). The system (i) is viable, *i.e.*, the Perron-Frobenius (P-F hereafter) eigenvalue,  $\lambda_A$ , of the  $n \times n$  matrix of input-output coefficients,  $\mathbf{A}$ , is less than 1; and (ii) follows a balanced, steady path of expansion at rate  $g$ . The net product is distributed to gross profits and wages: gross profits split into income of the capitalists (net profits) and income of the non-employed (transfer income), whilst wages are paid at the end of the common production period and there are no savings out of this income. There is a uniform consumption pattern, *i.e.*, the composition of the vectors of consumption out of wages, net profits and transfer income are identical and rigid, and the givens in our analysis are (i) the technical conditions of production, *i.e.*, the pair  $(\mathbf{A}, \mathbf{a})$ , where  $\mathbf{a}^T$  is the  $1 \times n$  vector of direct labour inputs (‘ $\tau$ ’ is the sign for transpose); and (ii) the real wage rate, which is represented by the  $n \times 1$  vector  $\mathbf{b}$ . Finally, we suppose that all commodities enter, directly or indirectly, into the production of wage goods, *i.e.*, the matrix of the ‘augmented’ input-output coefficients,  $\mathbf{A} + \mathbf{b}\mathbf{a}^T$ , is irreducible.

On the basis of these assumptions, the quantity side of the system may be described by the following relation:

$$\mathbf{I}\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{c} + \mathbf{i} \quad (1)$$

where

$$\mathbf{c} = L\mathbf{b} + L\mathbf{b}_{np} + N\mathbf{b}_{ne} \quad (2)$$

$$\mathbf{b}_{np} = c_{np}\mathbf{b}, \mathbf{b}_{ne} = c_{ne}\mathbf{b} \quad (3)$$

$$\mathbf{i} = g\mathbf{A}\mathbf{x} \quad (4)$$

$$L \equiv \mathbf{a}^T \mathbf{x} \quad (5)$$

$\mathbf{I}$  denotes the  $n \times n$  identity matrix,  $\mathbf{x}$  the  $n \times 1$  activity level vector,  $\mathbf{c}$  the total consumption vector,  $\mathbf{i}$  the net investment vector,  $L$  the total employment,  $\mathbf{b}_{np}$  the vector of consumption out of net profits per employed,  $\mathbf{b}_{ne}$  the vector of consumption of the non-employed per non-employed,  $N$  the number of non-employed,  $c_{np}$  ( $\geq 0$ ) the index of consumption out of net profits, and  $c_{ne}$  ( $> 0$ ) the index of consumption of the non-employed. Substituting (2), (3) and (4) in (1) and solving for  $\mathbf{x}$  we obtain

$$\mathbf{x} = [(1 + c_{np})L + c_{ne}N]\mathbf{B}(g)\mathbf{b} \quad (6)$$

where each element in  $\mathbf{B}(g) \equiv [\mathbf{I} - (1 + g)\mathbf{A}]^{-1}$  is positive and increases (without limit) as  $g$  increases from  $-1$  to its finite maximum value,  $g = G \equiv (1/\lambda_A) - 1$ .<sup>3</sup> Pre-multiplying (6) by  $\mathbf{a}^T$ , and by invoking (5), we get:

$$L = [(1 + c_{np})L + c_{ne}N]\mathbf{v}^T(g)\mathbf{b}$$

or

$$1 = [(1 + c_{np}) + c_{ne}(N/L)]\mathbf{v}^T(g)\mathbf{b}$$

---

<sup>3</sup> For  $\mathbf{b} = \mathbf{0}$  we get  $\mathbf{x}_A = (1 + g)\mathbf{A}\mathbf{x}_A$ . Since a non-positive activity level vector is economically insignificant, it follows that  $1/(1 + g)$  is the P-F eigenvalue of  $\mathbf{A}$  (or  $g = G \equiv (1/\lambda_A) - 1$ ) and  $\mathbf{x}_A$  is the corresponding right-hand side eigenvector or, alternatively, the activity level vector of Sraffa's (1960, ch. 4) 'Standard system'. Thus, the 'Standard ratio' (*ibid.*, §28), defined as the capital productivity in the Standard system,  $\boldsymbol{\pi}^T[\mathbf{I} - \mathbf{A}]\mathbf{x}_A / \boldsymbol{\pi}^T\mathbf{A}\mathbf{x}_A$ , equals  $G$  for each vector of commodity prices,  $\boldsymbol{\pi}$  (for a detailed exposition, see, *e.g.*, Kurz and Salvadori, 1995, ch. 4). On the other hand, if the price vector is the left-hand side P-F eigenvector of  $\mathbf{A}$  or, alternatively, the 'pure capital theory of value' (Pasinetti, 1977, pp. 76-78) holds, *i.e.*,  $\lambda_A \boldsymbol{\pi}_A^T = \boldsymbol{\pi}_A^T \mathbf{A}$ , then the capital productivity in the actual system,  $\boldsymbol{\pi}_A^T[\mathbf{I} - \mathbf{A}]\mathbf{x} / \boldsymbol{\pi}_A^T \mathbf{A}\mathbf{x}$ , equals  $G$ . On this basis, it has been argued that  $1/G$  can be viewed as an indicator of the aggregate intensity of the demand for intermediate goods, which reflects the structural characteristics of the productive system (see Marengo, 1992).

or

$$R_0 \equiv N/L = (e(g) - c_{np})/c_{ne} \quad (7)$$

and substituting (7) in (6) yields

$$\mathbf{x}/L = (1 + e(g))\mathbf{B}(g)\mathbf{b} \quad (8)$$

where  $R_0$  denotes the maximum attainable number of non-employed per employed or MEDR,  $\mathbf{v}^T(g) \equiv \mathbf{a}^T\mathbf{B}(g)$  the vector of the ‘synchronized labour costs or Austrian socially necessary labour’ (Samuelson and v. Weizsäcker, 1971; Wolfstetter, 1973, pp. 793-794) and  $e(g) \equiv (1/\mathbf{v}^T(g)\mathbf{b}) - 1$  the ‘Austrian rate of surplus labour’, which constitutes a strictly decreasing function of every element of  $(\mathbf{A}, \mathbf{a}, \mathbf{b})$ . Relation (7) defines the ‘ $R_0 - c_{np} - c_{ne} - g$  frontier’ for this economy, in which each variable is *inversely* related to each of the others.<sup>4</sup> Thus, we may derive the following conclusions: (i) to any exogenously given value of  $(g, c_{np}, c_{ne})$  there corresponds a particular value of  $R_0$ ,<sup>5</sup> whilst the structure of outputs is independent of  $(c_{np}, c_{ne})$  (see (8)),<sup>6</sup> and, as is well known, can change in a complicated way as  $g$  changes (see, *e.g.*, Pasinetti, 1992); (ii)  $R_0$  is positive iff  $c_{np} < e(g)$ , whilst for  $g = 0$ ,  $c_{np} = 0$  and  $c_{ne} = 1$ , the MEDR equals the ‘Marxian rate of surplus labour’, *i.e.*,  $R_0 = e(0)$ ; (iii) if  $\varepsilon_i$ ,  $i = 1, 2, 3$ , represents the elasticity of  $R_0$  with respect to  $g$ ,  $c_{np}$  and  $c_{ne}$ ,

---

<sup>4</sup> It should be stressed that in the case of joint production, which is of great empirical importance (see Steedman, 1984; Bidard and Erreygers, 1998; Faber *et al.*, 1998), each element in  $\mathbf{B}(g)$  is not necessarily a positive increasing function of  $g$  (see Steedman, 1985, pp. 135-138; Kurz and Salvadori, 1995, ch. 8). This entails that the existence of a *positive* correlation between  $R_0$  and  $g$  is entirely possible.

<sup>5</sup> It may be noted that if we take into account the saving-investment mechanism, then  $g$  and  $c_{np}$  cannot be treated as independent variables, *i.e.*, given from outside the system (see Appendix 1).

<sup>6</sup> This statement does not hold true when reducible systems are allowed for (see Appendix 2).

respectively, it is then easy to see that, for  $R_0 > 0$ ,  $\varepsilon_1$  equals the ratio of net investment to consumption of the non-employed in terms of Austrian socially necessary labour,<sup>7</sup> and  $\varepsilon_2 (= c_{np}/c_{ne}R_0) > \varepsilon_3 (= 1)$  for  $c_{np} > e(g)/2$ ; and (iv) technical changes that fulfil the cost-minimizing criterion do not necessarily imply a rise in  $e(g)$  (see Okishio, 1961) and, therefore, have ambiguous effects on  $R_0$ .<sup>8</sup>

It need hardly be said that government expenditure can be introduced into the model by assuming, for example, that it is maintained as a constant fraction of the capital stocks, *i.e.*,  $d\mathbf{Ax}$ , or, alternatively, of the gross outputs, *i.e.*,  $d\mathbf{x}$  (clearly, these relations are special cases of  $\mathbf{Dx}$ , where  $\mathbf{D}$  denotes an exogenously given  $n \times n$  matrix). In the former case, (7) still holds, provided only that  $g$  is replaced by  $g + d$ , whilst in the latter, (7) becomes

$$R_0 = [e(g, d) - (d / \mathbf{v}^T(g, d)\mathbf{b}) - c_{np}] / c_{ne} \quad (7a)$$

where  $\mathbf{v}^T(g, d) \equiv \mathbf{a}^T[\mathbf{I} - [(1 + g)/(1 - d)]\mathbf{A}]^{-1}$  and  $e(g, d) \equiv (1 / \mathbf{v}^T(g, d)\mathbf{b}) - 1$ . On the other hand, by assuming that the non-employed are divided into  $k$  groups, characterized by different consumption indices, (7) becomes

$$\left( \sum_{i=1}^k (c_{ne})_i N_i \right) / L = e(g) - c_{np}, \quad \sum_{i=1}^k N_i = N \quad (7b)$$

or

---

<sup>7</sup> Differentiation of (7) with respect to the rate of growth gives

$$\partial R_0 / \partial g = -\mathbf{v}^T(g)\mathbf{AB}(g)\mathbf{b} / [c_{ne}(\mathbf{v}^T(g)\mathbf{b})^2]$$

and recalling (8),  $(1 + e(g))\mathbf{v}^T(g)\mathbf{b} = 1$  and the definition of  $R_0$  it follows that

$$\varepsilon_1 \equiv -(\partial R_0 / \partial g)(g / R_0) = \mathbf{v}^T(g)\mathbf{i} / (c_{ne}N\mathbf{v}^T(g)\mathbf{b})$$

<sup>8</sup> For a theoretical analysis of different forms of technical change within the framework of static input-output models, see Seyfried (1988). For a one-commodity model, which includes, however, fixed capital and the degrees of its utilization, depreciation, supplementary or 'overhead' labour and investment function(s), see Kurz (1990, pp. 226-235). For relevant empirical analyses, in terms of dynamic input-output models, see Leontief and Duchin (1986) and Kalmbach and Kurz (1990).

$$(R_0)_i \equiv N_i / L = [e(g) - c_{np} - (\sum_{\substack{j=1 \\ j \neq i}}^k (c_{ne})_j (R_0)_j)] / (c_{ne})_i \quad (7c)$$

where  $(R_0)_i$ ,  $(c_{ne})_i$ ,  $N_i$  denote the MEDR, consumption index and population of the  $i$ th group, respectively.

Now, consider the *more* realistic case of a non-proportionally growing and open economy. Then (1) becomes

$$\mathbf{I}\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{c} + \mathbf{i} + \mathbf{e} \quad (9)$$

where  $\mathbf{i}$  ( $\neq g\mathbf{A}\mathbf{x}$ ) is now exogenously given and  $\mathbf{e}$  denotes the exogenously given net export vector. Substituting (2) and (3) in (9), and solving for  $\mathbf{x}$ , leads to

$$\mathbf{x} = [(1 + c_{np})L + Nc_{ne}]\mathbf{B}(0)\mathbf{b} + \mathbf{B}(0)(\mathbf{i} + \mathbf{e}) \quad (10)$$

Pre-multiplying (10) by  $\mathbf{a}^T$ , and by invoking (5), we get:

$$L = [(1 + c_{np})L + Nc_{ne}]\mathbf{v}^T(0)\mathbf{b} + \mathbf{v}^T(0)(\mathbf{i} + \mathbf{e})$$

or

$$R_1 \equiv N/L = \{e(0) - c_{np} - [\mathbf{v}^T(0)(\mathbf{i}' + \mathbf{e}') / \mathbf{v}^T(0)\mathbf{b}]\} / c_{ne} \quad (11)$$

where  $\mathbf{i}'$  ( $\equiv \mathbf{i}/L$ ) denotes the vector of net investments per employed,  $\mathbf{e}'$  ( $\equiv \mathbf{e}/L$ ) the vector of net exports per employed, and  $-\mathbf{v}^T(0)\mathbf{e}'$  may be conceived as the 'net labour saving from trade' (see Erdilek and Schive, 1976, pp. 318-319). As is well known, international trade dictated by the cost-minimizing criterion do not necessarily imply a positive net labour saving from trade (see *ibid.*, p. 320; Steedman, 1979, Essays 4, 9 and 12) and, therefore, has ambiguous effects on the MEDR (precisely like technical changes).

An alternative, but rather different, determination of  $R_1$  is obtained by setting  $\mathbf{i} = \mathbf{A}\hat{\mathbf{G}}\mathbf{x}$ , where  $\hat{\mathbf{G}} \equiv [g_j]$  denotes the diagonal matrix of the sectoral rates of growth,

and  $\mathbf{e} = \mathbf{e}_x - \mathbf{M}\mathbf{x}$ , where  $\mathbf{e}_x$  denotes the export vector and  $\mathbf{M} \equiv [m_{ij}]$  the matrix of imports per unit activity level, giving

$$R_1 = [e(g_j, m_{ij}) - c_{np} - (\mathbf{v}^T(g_j, m_{ij})\mathbf{e}_x' / \mathbf{v}^T(g_j, m_{ij})\mathbf{b})] / c_{ne} \quad (11a)$$

where  $\mathbf{v}^T(g_j, m_{ij}) \equiv \mathbf{a}^T[\mathbf{I} - \mathbf{A}[\mathbf{I} + \hat{\mathbf{G}}] + \mathbf{M}]^{-1}$ ,  $e(g_j, m_{ij}) \equiv (1 / \mathbf{v}^T(g_j, m_{ij})\mathbf{b}) - 1$  and  $\mathbf{e}_x' \equiv \mathbf{e}_x / L$ .

In what follows we shall estimate the MEDR in actual economies from (i) the relation (7), with  $c_{np} = 0$ ,  $c_{ne} = 1$  and  $0 \leq g < g^*$ , where  $g^*$  denotes the economically significant value of the rate of growth that corresponds to  $e(g^*) = 0$ , i.e.,  $R_0(g, 0, 1)$ ; and (ii) the relation (11), with  $c_{np} = 0$  and  $c_{ne} = 1$ , i.e.,  $R_1(0, 1)$ .<sup>9</sup>

### 3. Results and their Evaluation

The results from the application of the previous analysis to the input-output tables of the Finnish (for the years 1997 and 1998), German (for the year 2000), Greek (for the years 1997 and 1998) and Spanish (for the year 2000) economies are displayed in Tables 1 through 3.<sup>10</sup>

Table 1 gives the upper bounds for the uniform rates of growth, which are determined by the technical and the socio-technical conditions of production, respectively: the second column gives the maximum possible uniform rates of growth or Standard ratios,  $G$ , the third column gives the uniform rates of growth,  $g^*$ , that correspond to  $e(g) = 0$ , and the fourth column gives the ‘maximum relative rates of

<sup>9</sup> Thus, the interested reader can easily calculate every other value of the MEDR. See Appendix 3 for the available input-output data as well as the construction of relevant variables.

<sup>10</sup> *Mathematica 5.0* is used in the calculations. The analytical results are available on request from the authors.

growth', defined as the ratios of  $g^*$  to  $G$ , *i.e.*,  $\gamma \equiv g^*/G$  (it goes without saying that  $G$ ,  $g^*$  are strictly decreasing functions of every element of  $\mathbf{A}$  and  $(\mathbf{A}, \mathbf{a}, \mathbf{b})$ , respectively).

**Table 1.** Upper bounds for the uniform rates of growth

	$G$		$g^*$		$\gamma$	
<b>FIN</b>	<b>1997</b>	<b>1998</b>	<b>1997</b>	<b>1998</b>	<b>1997</b>	<b>1998</b>
	0.712	0.703	0.467	0.457	0.656	0.650
<b>GE</b>	<b>2000</b>		<b>2000</b>		<b>2000</b>	
	1.005		0.512		0.509	
<b>GR</b>	<b>1997</b>	<b>1998</b>	<b>1997</b>	<b>1998</b>	<b>1997</b>	<b>1998</b>
	0.608	0.492	0.528	0.440	0.868	0.894
<b>SP</b>	<b>2000</b>		<b>2000</b>		<b>2000</b>	
	0.664		0.392		0.590	

Table 2 presents  $R_0(g, 0, 1)$  or the Austrian rates of surplus labour (see relation (7)) as functions of the uniform rate of growth.

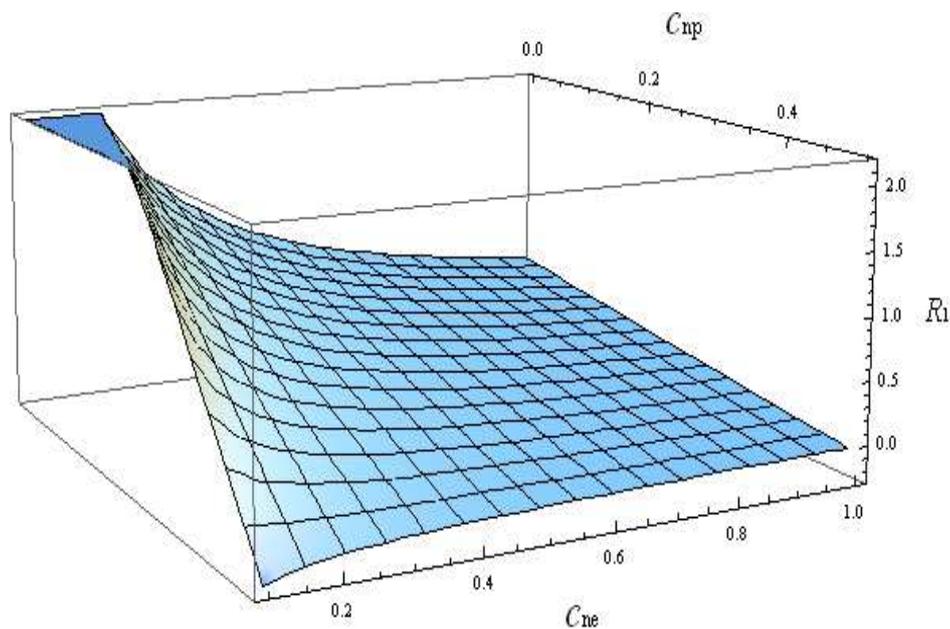
**Table 2.** The Austrian rates of surplus labour as functions of the uniform rate of growth

	<b>FIN</b>		<b>GE</b>	<b>GR</b>		<b>SP</b>
$g$	<b>1997</b>	<b>1998</b>	<b>2000</b>	<b>1997</b>	<b>1998</b>	<b>2000</b>
0	1.239	1.217	1.030	2.332	2.257	1.070
0.1	1.002	0.979	0.832	2.003	1.920	0.827
0.2	0.754	0.729	0.632	1.651	1.544	0.566
0.3	0.491	0.464	0.431	1.264	1.090	0.284
0.4	0.208	0.177	0.229	0.814	0.434	<0
0.5	<0	<0	0.026	0.226	<0	<0

Finally, Table 3 is associated with relation (11): it presents estimates of  $R_1(0,1)$ , and of their constituent components. Furthermore, and in order to obtain an idea of the changes induced by changes in the indices of consumptions, Figure 1 displays  $R_1$  as function of the indices of consumptions ( $0 \leq c_{np} \leq 0.5$  and  $0.1 \leq c_{ne} \leq 1$ ) for the German economy.

**Table 3.** Decomposition of the MEDR in the case of non-proportionally growing and open economy

	FIN		GE	GR		SP
	1997	1998	2000	1997	1998	2000
$R_1(0,1)$	0.619	0.583	0.476	2.118	2.043	0.678
$\mathbf{v}^T(0)\mathbf{b}$	0.447	0.451	0.493	0.300	0.307	0.483
$e(0)$	1.239	1.217	1.030	2.332	2.257	1.070
$\mathbf{v}^T(0)\mathbf{i}' / \mathbf{v}^T(0)\mathbf{b}$	0.446	0.463	0.468	0.777	0.794	0.572
$\mathbf{v}^T(0)\mathbf{e}' / \mathbf{v}^T(0)\mathbf{b}$	0.174	0.171	0.086	-0.563	-0.580	-0.180



**Figure 1.**  $R_1$  as a function of the indices of consumptions; German economy

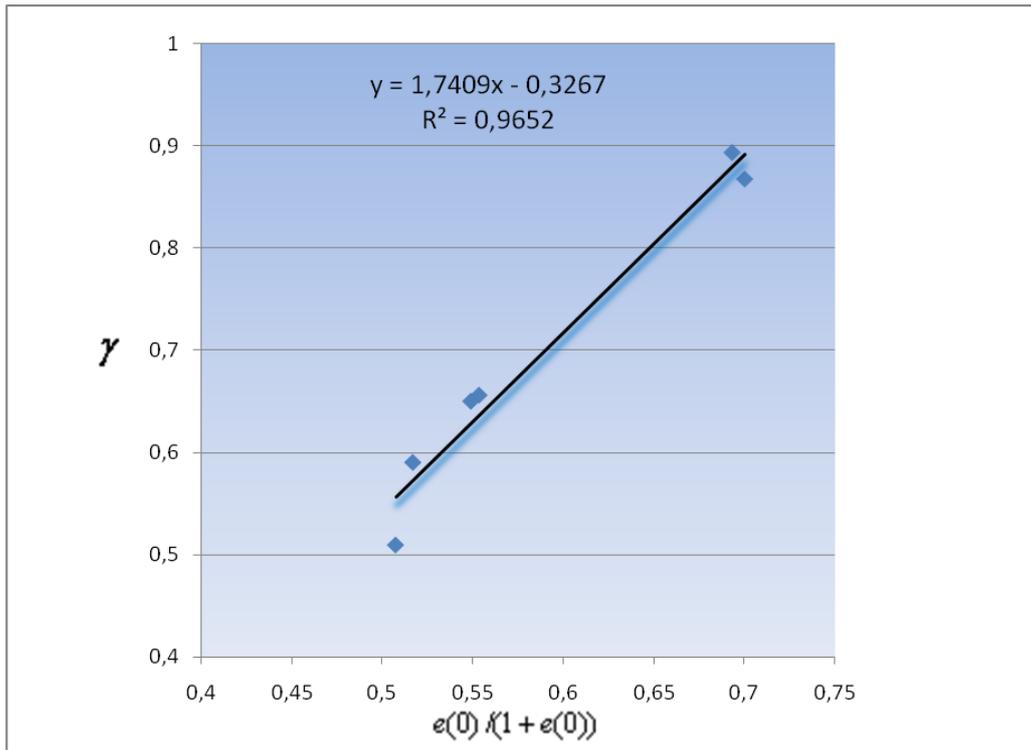
From these tables, the associated numerical results and the hitherto analysis we arrive at the following conclusions:

(i). Table 1 indicates that the German (Greek) economy presents the largest (smallest) Standard ratio,  $G$ , and the smallest (largest) maximum relative rate of growth,  $\gamma$ . Speaking somewhat loosely, one may say that the former implies that this economy is characterized by the largest (smallest) capital productivity,<sup>11</sup> whilst the latter is due to the fact that this economy is characterized by the smallest (largest) Marxian rate of surplus labour,  $e(0)$  (see the fourth row of Table 2). However, only if we set aside the Greek economy (in which  $\gamma_{1997} < \gamma_{1998}$  and  $e(0)_{1997} > e(0)_{1998}$ ), the ranking of the economies according to  $\gamma$  coincides with their ranking according to  $e(0)$ . Moreover, it is worth noting that, in the context of the economies under consideration,  $\gamma$  is always greater than the share of gross profits in net income expressed in terms of Marxian socially necessary labour,  $1 - \mathbf{v}^T(0)\mathbf{b} = e(0)/(1 + e(0))$ , and the relative errors in the approximation  $\gamma \approx e(0)/(1 + e(0))$  are as follows: 0.4% (GE), 12.4% (SP), 15.5% (FIN, 1998), 15.7% (FIN, 1997), 19.4% (GR, 1997), and 22.5% (GR, 1998) (see also Figure 2 that displays in a scatter diagram the relationship between  $e(0)/(1 + e(0))$  and  $\gamma$ : we observe a positive relationship and an  $R$ -square of 96.5%).<sup>12</sup>

---

<sup>11</sup> See footnote 3.

<sup>12</sup> For the theoretical relationships between  $\gamma$  and  $e(0)/(1 + e(0))$ , see Appendix 4 (which builds upon an idea presented in Mariolis, 2010).



**Figure 2.** Maximum relative rates of growth versus shares of gross profits in net income expressed in terms of Marxian socially necessary labour

(ii). Table 2 indicates that, for each  $g$ , the Greek economy presents the largest Austrian rate of surplus labour, whilst the German (Spanish) economy presents the smallest Austrian rate of surplus labour for  $0 < g < g' \cong 0.091$  (for  $g > g'$ ; see also Figure 3 below).<sup>13</sup> If, however, we want to focus attention on the relative impact of

<sup>13</sup> Calculations are performed by varying the rate of growth from zero to  $g^*$  with the step equal to 0.01. The numerical examination of the results reveals that both curves are *concave* to the origin but very close to the linear trends: for the German and Spanish economies we obtain

$$e(g) \cong 1.033 - 2.010g \quad , \quad R\text{-square} \cong 99.9985\%$$

and

$$e(g) \cong 1.096 - 2.718g \quad , \quad R\text{-square} \cong 99.8426\%$$

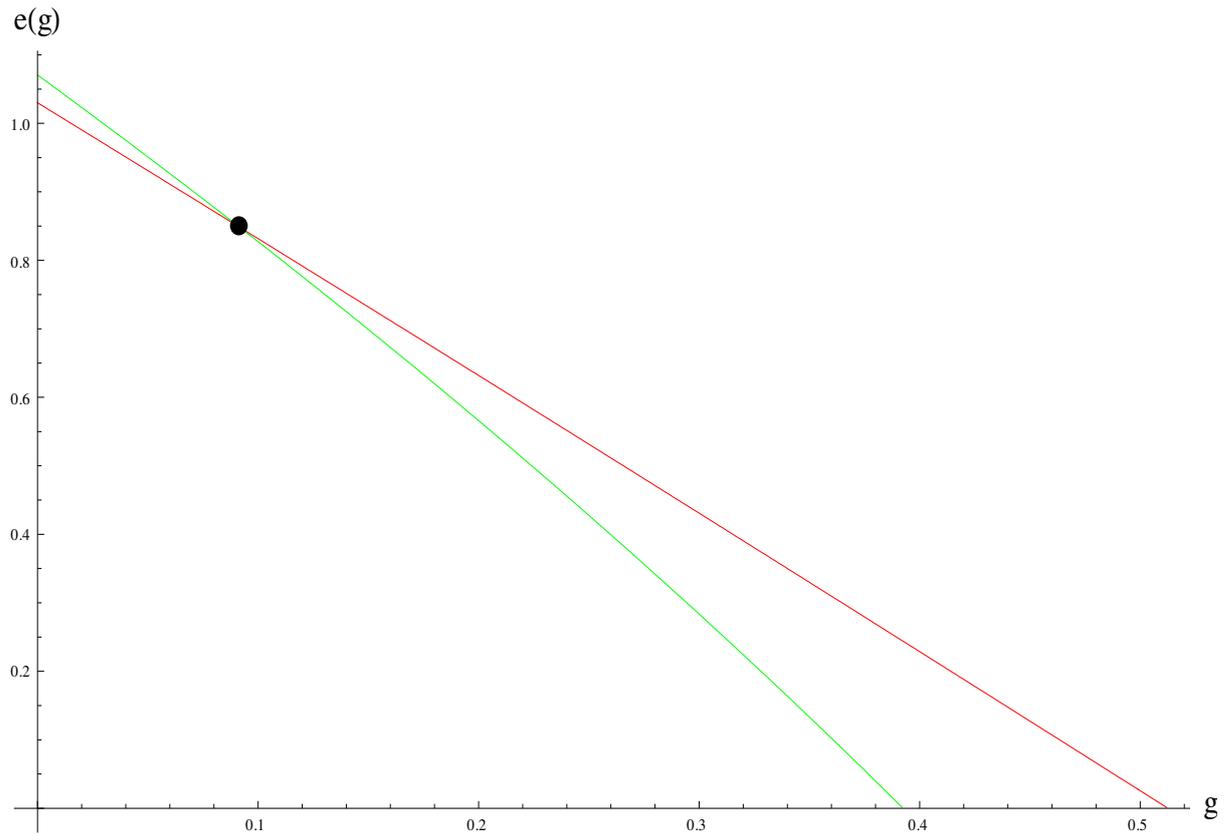
respectively. It is quite clear that these findings (i) are in line with the already mentioned relative errors in the approximation  $\gamma \approx e(0)/(1+e(0))$  (see also Appendix 4); and (ii) might be of some interest to those researchers who investigate the empirical relevance of the so-called Cambridge controversy (see

the national technical conditions of production, it seems to be appropriate to assume an *internationally* uniform real wage rate. Thus, by assuming, for example, that the real wage equals that of the German economy, it follows that the Marxian rates of surplus labour in the remaining economies are *negative*, whilst by assuming that the real wage equals that of the Greek economy, 1997, the ranking of the economies according to the Marxian rate of surplus labour is as follows: GE ( $e(0) \cong 22.184$ ), SP ( $\cong 7.863$ ), FIN, 1998 ( $\cong 7.060$ ), FIN, 1997 ( $\cong 6.279$ ), GR, 1998 ( $\cong 2.223$ ).<sup>14</sup> Hence, taking into account also the ranking according to the Standard ratio (see Table 1), it might be argued that Germany (Greece) has the most (less) technologically powerful economy.

---

Han and Schefold, 2006 and Schefold, 2008, for a powerful contribution to the debate), in the sense that the curve  $e(g) + 1$  may be conceived as a ‘consumption-growth (or wage-profit) frontier’ (see, *e.g.*, Kurz and Salvadori, 1995, ch. 4).

<sup>14</sup> It may be noted that the distance between  $\mathbf{b}_{GE}$  and  $\mathbf{b}_{GR,1997}$ , measured by the ‘Root-mean-square-error’, equals 28.8% (setting aside the element associated with the product ‘Fish and other fishing products; services incidental of fishing’,  $\mathbf{b}_{GE} \geq \mathbf{b}_{GR,1997}$ ). Moreover,  $\mathbf{v}(0)_{GE} < \mathbf{v}(0)_{GR,1997}$  and the distance between them, measured by the ‘Root-mean-square-error’, equals 64.0%. As is well known, the reciprocal of the elements of  $\mathbf{v}$  can be viewed as indicators of the sectoral productivity of labour (see, *e.g.*, Okishio, 1963).



**Figure 3.** The Austrian rate of surplus labour as a function of the uniform rate of growth; German and Spanish economies

(iii). Table 3 indicates that, first, the ranking of the economies according to  $R_1(0,1)$  does *not* coincide with the ranking according to  $e(0)$  and, second, the economies are *divided* into those which are characterized by a negative net labour saving from trade and those which are characterized by a positive one. The ranking of the former according to the ratio of  $c_{ne}R_1(0, c_{ne})$  to  $e(0)$ , which equals the share of consumption of the non-employed in gross profits (in terms of Marxian socially necessary labour), is as follows (and coincides with the ranking according to  $e(0)$  or  $R_1(0,1)$ ): FIN, 1997 ( $\cong 50\%$ ), FIN, 1998 ( $\cong 47.9\%$ ) and GE ( $\cong 46.2\%$ ), whilst the ranking of the

latter is as follows (and coincides with the ranking according to  $e(0)$  or  $R_1(0,1)$ ): GR, 1997 ( $\cong 90.8\%$ ), GR, 1998 ( $\cong 90.5\%$ ), and SP ( $\cong 63.4\%$ ).<sup>15</sup>

Finally, it may be concluded that the main determinants of the relatively high level of  $R_1(0,1)$  in the Greek economy are the Marxian rate of surplus labour and the net labour saving from trade, where the former (the most important) is not related to the technological strength of the system but rather to the relatively low level of the real wage rate, whilst the latter offsets, to a great extent (*i.e.*, about  $0.563(0.580)/0.777(0.794) \cong 73\%$ ), the negative impact of investment. By contrast, the net labour saving from trade of the Spanish economy offsets to a less extent (*i.e.*, about  $0.180/0.572 \cong 31.5\%$ ) the negative impact of investment.

(iv). Since there exist *statistical estimates* of the actual growth rates (of the real gross domestic product), total populations (and their age distribution), employed persons, unemployment rates and number of pensioners, we may compute the ‘actual’ Austrian rates of surplus labour as well as the following three ‘actual’ economic dependency ratios:

---

<sup>15</sup> Stein (2009) argues that ‘the countries facing the greatest deterioration in their [old-age] dependency ratio are generally also those with current account surpluses (*i.e.*, excess savings). There are exceptions to this rule: Spain is a deficit country, yet one with the prospect of a substantial demographic deterioration over the next 45 years. So, to a lesser extent, are Italy and France. These countries are likely to face substantial difficulties as their populations age – especially Italy, where the population and the labor force are already shrinking and aging fast. As households increase their spending, some other sector will have to save more. This must be either companies (*i.e.*, they must become less profitable), or governments (by moving into deficit, or further into deficit), or foreigners (*i.e.*, countries whose households are increasing their spending must move further into current account deficit). In other, saving countries – for example, Germany, Korea, Japan, Singapore, to take those with the worst demographic profile – the switch from household saving to spending can be more easily accommodated, since it will simply require a smaller current account surplus – which, as it so happens, is exactly what is needed for these countries from a global economic perspective.’. For an empirical study of the relationships between demographic dependency ratios and current account balances, see Chin and Prasad (2003).

- the ‘total economic dependency ratio’ (TEDR), defined as

$$\text{TEDR} \equiv (\text{TP} - \text{EP}) / \text{EP} \quad (12)$$

where TP denotes the total population and EP the employed persons;

- the ‘needs weighted (or expenditure) economic dependency ratio’ (WEDR; see, *e.g.*, Foot, 1989, pp. 104-109; Osterkamp, 2003, pp. 69-70), defined as

$$\text{WEDR} \equiv (w_1 N_1 + w_2 N_2) / \text{EP} (< \text{TEDR}) \quad (13)$$

where  $N_1$  denotes the number of people aged 0-14,  $N_2 = \text{TP} - \text{EP} - N_1$ ,  $w_1 = 0.25$  and  $w_2 = 0.75$ , *i.e.*,  $w_2 / w_1 = 3$  (see also relation (7b));<sup>16</sup> and, finally,

- the ‘unweighted effective economic dependency ratio’ (UEEDR), defined as

$$\text{UEEDR} \equiv (\text{UP} + \text{P}) / \text{EP} (< \text{TEDR})$$

or, since  $\text{UP} = [u / (1 - u)] \text{EP}$ ,

$$\text{UEEDR} = [u / (1 - u)] + (\text{P} / \text{EP}) \quad (14)$$

where UP denotes the unemployed persons, P the pensioners, and  $u$  the unemployment rate. Given that, in the real world, there are non-employed who do not receive *any* income transfer, it follows that the UEEDR corresponds much more closely to our notions of the MEDR.

Thus, we can compare the ‘actual’  $R_0(g, 0, 1)$  (or ‘actual’ Austrian rates of surplus labour) and  $R_1(0, 1)$  with the aforesaid ‘actual’ ratios: Table 4 presents data on the actual growth rates,  $g^a$ , population (1000 persons) and unemployment rates for the considered economies. Table 5 presents  $R_0(g^a, 0, 1)$ , the three ‘actual’ ratios

---

<sup>16</sup> It need hardly be said that the actual weights are not internationally uniform and the chosen weights are therefore *only* representative, to give some indication of the WEDR (see also Clark *et al.*, 1978, pp.921-923; Gee, 2002, pp. 751-752).

(TEDR, WEDR and UEEDR), the percent differences between  $R_0(g^a, 0, 1)$ ,  $R_1(0, 1)$  and the UEEDR, defined as

$$(RD)_i \equiv 100 \times |R_i(\bullet) - UEEDR| / [(R_i(\bullet) + UEEDR) / 2], \quad i = 0, 1$$

and, finally, the values of  $(c_{np}, c_{ne})$  for which  $R_1(c_{np}, c_{ne})$  (see relation (11)) coincide with the UEEDR.

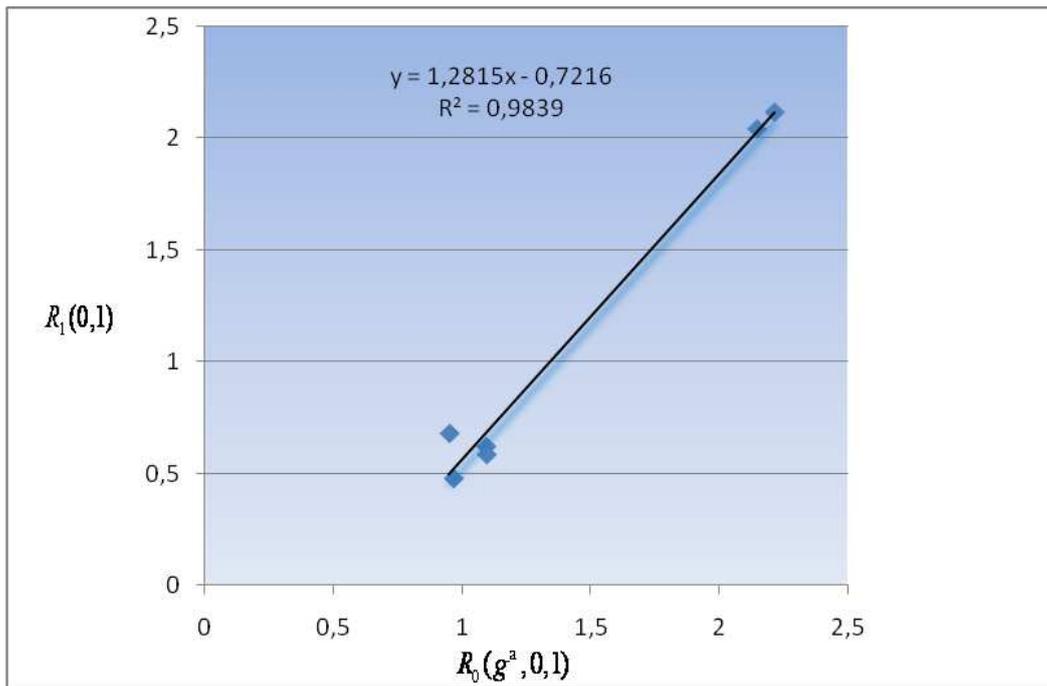
**Table 4.** Data on the actual growth rates, populations (1000 persons) and unemployment rates

	FIN		GE	GR		SP
	1997	1998	2000	1997	1998	2000
$g^a$	0.062	0.052	0.032	0.037	0.034	0.050
TP	5132	5147	82163	10745	10808	40050
EP	2150	2192	39144	3784	3940	15221
$N_1 / TP$ (%)	19	19	16	16	16	15
$N_1$	975	978	13146	1719	1729	6007
$N_2$	2007	1977	29873	5241	5139	18823
P	1117	1131	19007	1943	1981	7649
$u$ (%)	12.7	11.4	7.9	9.8	10.9	14.1

Sources: Eurostat, National Statistical Services, and authors' compilation

**Table 5.** MEDR versus ‘actual’ economic dependency ratios

	FIN		GE	GR		SP
	1997	1998	2000	1997	1998	2000
$R_0(g^a, 0, 1)$	1.093	1.095	0.967	2.213	2.145	0.951
TEDR	1.387	1.348	1.099	1.839	1.743	1.631
WEDR	0.814	0.788	0.656	1.152	1.089	1.026
UEEDR	0.665	0.645	0.571	0.621	0.625	0.667
$(RD)_0$	48.7	51.7	51.5	112.4	109.7	35.1
$(RD)_1$	7.2	10.1	18.1	109.3	106.3	1.6
$c_{ne}$	$0.93 - 1.50c_{np}$	$0.90 - 1.55c_{np}$	$0.83 - 1.75c_{np}$	$3.41 - 1.61c_{np}$	$3.27 - 1.60c_{np}$	$1.02 - 1.50c_{np}$



**Figure 4.**  $R_0(g^a, 0, 1)$  versus  $R_1(0, 1)$

Figure 4 displays in a scatter diagram the relationship between  $R_0(g^a, 0, 1)$  and  $R_1(0, 1)$ . Thus, we observe a positive relationship and an  $R$ -square of 98.4%, results

that indicate the cohesion between our two estimates of the MEDR. Furthermore, Table 5 shows that  $R_0(g^a, 0, 1)$  is considerably greater than the UEEDR (their percent difference is in the range of 35.1% (SP)-112.4% (GR, 1997)), whilst the ranking of the economies according to  $R_1(0, 1)$  coincides with the ranking according to the TEDR or the WEDR. Moreover, setting aside the Greek economy, first,  $R_1(0, 1)$  and  $R_0(g^a, 0, 1)$  are both lower than the TEDR, second,  $R_1(0, 1)$  is lower than the WEDR, and, third,  $R_1(0, 1)$  and UEEDR are quite close to each other or, more specifically, their percent difference is in the range of 1.6% (SP)-18.1% (GER), whilst the values of  $(c_{np}, c_{ne})$  for which they coincide seem to be reasonable: for example, by setting  $c_{np} = 0$ , it follows that  $c_{ne}$  is in the range of 0.83-1.02, a result which is consistent with the available evidence on the relevant unemployment benefit and pension replacement rates (see, *e.g.*, Nickell, 2006, pp. 4-5, OECD, 2006, and OECD, 2009a, respectively).<sup>17</sup> It will be clear that, within our framework, it remains impossible to

---

<sup>17</sup> Cichon *et al.* (2003, p. 11) estimate that, during the decade 1991-2000, the TEDR in the EU-15 fluctuated between 1.34 (2000) and 1.48 (1994). Moreover, it may be noted that, according to Statistics Finland (2007), the TEDR in Finland, for the year 2000 (2004), is almost 1.32 (1.31), *i.e.*, 2952 (2973) [non-employed; 1000 persons]/2229 (2262) [employed; 1000 persons], where Non-employed = 936 (914) [Aged 0-14] + 579 (594) [Students, others] + 318 (299) [Unemployed] + 1119 (1166) [Pensioners], and ‘shows great regional variation. In rural municipalities with migration loss the ratio can be over 2.00, while in the best performing municipalities around Helsinki it is lower than 1.00.’. Finally, in more general terms, Parjanne and Sirén (2003, pp. 3-4) stress: ‘The drop in the employment rate and widespread unemployment weakened the [total] economic dependency ratio in Finland during the economic recession in the early 1990s. The situation has improved in recent years, but it is predicted that the economic dependency ratio will take a turn for the worse again in about 2010, after which it will remain at a higher level permanently. The development of the economic dependency ratio depends on many factors. Firstly, it is affected by population age structure. Demographic forecasts suggest the proportion of old age pensioners will grow rapidly after 2010 as the baby-boom generation born in 1945-1955 will reach retirement age and life expectancy continues to rise. Population ageing in Finland will occur sooner and more rapidly than in most other OECD countries. The low birth rate cannot compensate for the ageing of the population. Finland’s fertility rate is still quite high compared

determine whether the detected peculiarities of the Greek economy are due to the so-called ‘generosity’ (Bank of Greece, 1999, p. 182; see also OECD, 2009b, pp. 71-78) of its social security system.<sup>18</sup> Nevertheless, it may be argued that, *in general*, the deviations between estimated and ‘actual’ values will be reduced by taking into account relations (7a), (7b) and (11a), integrating the quantity and the price sides of the considered systems and<sup>19</sup> allowing for differentiated consumption patterns (provided that the necessary data can be compiled). Therefore, in order to arrive at valid conclusions, these two lines of research should be combined.

---

with many European countries; in 2000 it was 1.73, compared with the EU average of 1.53. There is a risk, however, that it will fall closer to the EU average. Consequently, within the next few years, the new workforce entering the labour market will already be smaller than the numbers leaving the labour market. The fall in labour supply will jeopardize economic growth and tax base, while increases in pensioners raises pension expenditures.’

<sup>18</sup> According to the Annual Report of the Bank of Greece for the year 1998, ‘[i]n view of *international experience*, the pension system in Greece gives the impression of being comparatively generous. This impression is based not only on the level of pensions in relation to earnings in active service, but also on the ‘ease’ with which the right to receive a pension is established [...]. The generosity of the system is also reflected in the amount of accumulated claims of those insured by pension funds. According to OECD data, insurance funds’ liabilities in Greece exceed 150 per cent of GDP and are among the highest in the OECD area. Not only benefits, but also contributions to social security funds are very high in Greece, particularly after the 1990-1992 reform. This fact, combined with the *very low competitiveness* of the Greek economy and the heavy competition it will face after joining EMU, and with the globalisation of economic activity, leaves no room for a further increase in social security contributions (with the exception of certain isolated cases). Besides, it should be taken into account that, after the entry of Greece into EMU, the Stability and Growth Pact provides essentially for an effectively balanced budget, while efforts aimed at a further reduction in the debt-to-GDP ratio must continue. In this context, the scope, if any, for financing insurance funds out of the ordinary budget would be extremely limited. From the above it is obvious that in the medium and long term it is difficult to maintain the present situation and immediate reforms are required. These reforms would necessarily be oriented towards reducing benefits and introducing stricter eligibility criteria.’ (Bank of Greece, 1999, pp. 182-183; emphasis added).

<sup>19</sup> See Appendix 1 and 2, respectively.

#### 4. Concluding Remarks

In this paper we have proposed, in terms of linear models, which have a modern ‘classical’ flavour, robust ways to estimate the maximum attainable economic dependency ratio (MEDR) and we have applied the theoretical analysis to the input-output tables of the Finnish, German, Greek and Spanish economies. It has been found that although Greece has the less technologically powerful economy (in terms of both labour and capital productivity), it presents the largest MEDR (it is almost 2.00). This is attributed, primarily, to the relatively low level of the real wage rate and, secondarily, to the current account deficit, which offsets, to a great extent (*i.e.*, about 73%), the negative impact of investment. By contrast, although Germany has the most technologically powerful economy, it presents the smallest MEDR (it is in the range of 0.476-0.967), and this is attributed primarily to the relatively high level of the real wage rate and, secondarily, to the current account surplus. Setting aside Spain’s current account deficit, which offsets, to relatively small extent (*i.e.*, 31.5%), the negative impact of investment on the MEDR, Finnish and Spanish economies *tend* to share similar features with the German economy. Furthermore, it has been indicated that our alternative estimates of the MEDR are consistent with each other as well as with the available empirical evidence. Since there are, all over the world, heated debates about the future of the social security systems, this line of enquiry would seem to be of some interest.

Future work should, first, carry the analysis at a more concrete level by including the presence of differentiated consumption patterns, fixed capital and the degrees of its utilization, differential depreciation, imported inputs, ‘overhead’ labour and pure joint products, second, estimate the effects of expected technical changes (and/or changes in income distribution and consumptions) on the MEDR and, finally,

integrate income distribution, pricing, capital accumulation and government fiscal activity considerations into a 'two-country' model.

## References

- Bank of Greece (1999) *Annual Report 1998* (Athens: Bank of Greece).
- Bidard, C. and Erreygers, G. (1998) Sraffa and Leontief on joint production, *Review of Political Economy*, 10, pp. 427-446.
- Chinn, M. and Prasad, E. (2003) Medium-term determinants of current accounts in industrial and developing countries: an empirical exploration, *Journal of International Economics*, 59, pp. 47-76.
- Cichon, M., Knop, R. and Léger, F. (2003) White or Prosperous: How much migration does the ageing European Union need to maintain its standard of living in the twenty-first century?, paper presented at the 4th International Research Conference on Social Security, 'Social security in a long-life society', Antwerp, Belgium, 5-7 May 2003 (Geneva: International Labour Office).
- Clark, R., Kreps, J. and Spengler, J. (1978) Economics of aging: a survey, *Journal of Economic Literature*, 16, pp. 919-962.
- Erdilek, A. and Schive, C. (1976) Can trade be harmful for a growing capitalist economy?, *Kyklos*, 29, pp. 317-320.
- Faber, M., Proops, J. L. R., Baumgärtner, S. (1998) All production is joint production. A thermodynamic analysis, in: S. Faucheux, J. Gowdy, I. Nicolai (Eds) (1998) *Sustainability and Firms: Technological Change and the Changing Regulatory Environment*, pp. 131-158 (Cheltenham: Edward Elgar).
- Foot, D. K. (1989) Public expenditures, population aging and economic dependency in Canada, 1921-2021, *Population Research and Policy Review*, 8, pp. 97-117.

- Gee, E. M. (2002) Misconceptions and misapprehensions about population ageing, *International Journal of Epidemiology*, 31, pp. 750-753.
- Han, Z. and Schefold, B. (2006) An empirical investigation of paradoxes: reswitching and reverse capital deepening in capital theory, *Cambridge Journal of Economics*, 30, pp. 737-765.
- Horn, R. A and Johnson, C. R. (1990) *Matrix Analysis* (Cambridge: Cambridge University Press).
- Kalmbach, P. and Kurz, H. D. (1990) Micro-electronics and employment: a dynamic input-output study of the West German economy, *Structural Change and Economic Dynamics*, 1, pp. 371-386.
- Kurz, H. D. (1990) Technical change, growth and distribution: a steady-state approach to 'unsteady' growth, in: H. D. Kurz (1990) *Capital, Distribution and Effective Demand. Studies in the 'Classical' Approach to Economic Theory*, pp. 211-239 (Cambridge: Polity Press).
- Kurz, H. D. and Salvadori, N. (1995) *Theory of Production. A Long-Period Analysis* (Cambridge: Cambridge University Press).
- Kurz H. D. and Salvadori N. (Eds) (1998) *Understanding 'Classical' Economics: Studies in Long-period Theory* (London and New York: Routledge).
- Leontief, W. and Duchin, F. (1986) *The Future Impact of Automation on Workers* (New York and Oxford: Oxford University Press).
- Marengo, L. (1992) The demand for intermediate goods in an input-output framework: a methodological note, *Economic Systems Research*, 4, pp. 49-52.
- Mariolis, T. (2006) *Input-output Models for Estimating the Maximum Attainable Economic Dependency Ratio* (in Greek), Internal Report of the 'Study Group on

- Sraffian Economics', June 2006, Department of Public Administration, Panteion University.
- Mariolis, T. (2010) Norm bounds for a transformed price vector in Sraffian systems, *Applied Mathematical Sciences*, 4, pp. 551-574.
- Miller, R. E. and Blair, P. D. (1985) *Input-Output Analysis: Foundations and Extensions* (New Jersey: Prentice Hall).
- Nickell, W. (2006) The CEP-OECD institutions data set (1960-2004), November 2006, Centre for Economic Performance, Discussion Paper No 759.
- OECD (2006) Benefits and Wages: gross/net replacement rates, country specific files and tax/benefit models, March 2006.  
([http://www.oecd.org/document/0/0,3343,en\\_2649\\_34637\\_34053248\\_1\\_1\\_1\\_1,00.html#statistics](http://www.oecd.org/document/0/0,3343,en_2649_34637_34053248_1_1_1_1,00.html#statistics))
- OECD (2009a) Pensions at a glance 2009: Retirement-income systems in OECD countries. Online country Profiles, including personal income tax and social security contributions.  
(<http://www.oecd.org/els/social/pensions/PAG>).
- OECD (2009b) *OECD Economic Surveys: Greece* (Paris: OECD).
- Okishio, N. (1961) Technical change and the rate of profit, *Kobe University Economic Review*, 7, pp. 85-99.
- Okishio, N. (1963) A mathematical note on Marxian theorems, *Weltwirtschaftliches Archiv*, 91, pp. 287-299.
- Okishio, N. and Nakatani, T. (1985) A measurement of the rate of surplus value, in: M. Krüger and P. Flaschel (Eds) (1993) *Nobuo Okishio-Essays on Political Economy*, pp. 61-73 (Frankfurt am Main: Peter Lang).
- Osterkamp, R. (2003) Demographic dependency ratios, *Dice Report. Journal for Institutional Comparisons*, 1, pp. 69-71.

Parjanne, M. L. and Sirén, P. (2003) Social protection expenditure and its financing in an ageing society: estimations on Finnish data, paper presented at the 4th International Research Conference on Social Security, ‘Social security in a long-life society’, Antwerp, Belgium, 5-7 May 2003.

<http://www.iesf.es/fot/Social-Protection-AgeingSociety-2003.pdf>

Pasinetti, L. (1973) The notion of vertical integration in economic analysis, *Metroeconomica*, 25, pp. 1-29.

Pasinetti, L. (1977) *Lectures on the Theory of Production* (New York: Columbia University Press).

Pasinetti, L. (1992) ‘Standard prices’ and a linear consumption/growth-rate relation, in: L. Pasinetti (Ed.) (1992) *Italian Economic Papers*, vol. 1, pp. 265-277 (Oxford: Oxford University Press).

Samuelson, P. A. and v. Weizsäcker C. C. (1971) A new labor theory of value for rational planning through use of the bourgeois profit rate, *Proceedings of the National Academy of Science*, 68, pp. 1192-1194.

Schefold, B. (2008) Families of strongly curved and of nearly linear wage curves: a contribution to the debate about the surrogate production function, *Bulletin of Political Economy*, 2, pp. 1-24.

Seyfried, M. (1988) Productivity growth and technical change, in: M. Ciaschini (Ed.) *Input-Output Analysis*, pp. 167-178 (London: Chapman and Hall).

Sraffa, P. (1960) *Production of Commodities by Means of Commodities. Prelude to a Critique of Economic Theory* (Cambridge: Cambridge University Press).

Statistics Finland (2007) Finland 1917-2007: From slash-and-burn fields to post-industrial society – 90 years of change in industrial structure, February 2007 [http://www.stat.fi/tup/suomi90/helmikuu\\_en.html](http://www.stat.fi/tup/suomi90/helmikuu_en.html).

- Steedman, I. (Ed.) (1979) *Fundamental Issues in Trade Theory* (London: Macmillan).
- Steedman, I. (1984) L' importance empirique de la production jointe, in: C. Bidard (1984) (Ed.) *La Production Jointe*, pp. 5-20 (Paris: Economica).
- Steedman, I. (1985) Joint production and technical progress, *Political Economy. Studies in the Surplus Approach*, 1, pp.127-138.
- Stein, G. (2009) The impact of demographics on business and the world economy, in: *Q Finance: The ultimate resource*, pp. 780-784 (London: A&C Black).
- United Nations (2006) *World Population Prospects: The 2004 Revision*, vol. 3 (New York: United Nations).
- Wolfstetter, E. (1973) Surplus labour, synchronised labour costs and Marx's labour theory of value, *Economic Journal*, 83, pp. 787-809.

### Appendix 1: Closing the System

For simplicity and brevity, assume that (i) there is a uniform rate of profit,  $r$ ; (ii) the vector of commodity prices,  $\mathbf{p}$ , is normalized by setting  $\mathbf{p}^T \mathbf{b} = 1$ ; and (iii) non-employed do not save. Then we may write

$$\mathbf{p}^T = (1+r)\mathbf{p}^T \mathbf{A} + w\mathbf{a}^T, \quad w = \mathbf{p}^T \mathbf{b}$$

or

$$\mathbf{p}^T = (1+r)\mathbf{p}^T \mathbf{C} \tag{A1.1}$$

and

$$g\mathbf{p}^T \mathbf{A} \mathbf{x} = s_{np} (r\mathbf{p}^T \mathbf{A} \mathbf{x} - c_{ne} N \mathbf{p}^T \mathbf{b})$$

or, recalling (7), (8) and the normalization equation,

$$c_{np} = e(g) - (1+e(g))[r - (g/s_{np})]\mathbf{p}^T \mathbf{A} \mathbf{B}(g)\mathbf{b} \tag{A1.2}$$

where  $\mathbf{C} \equiv \mathbf{A}[\mathbf{I} - \mathbf{b}\mathbf{a}^T]^{-1}$  and  $s_{np}$  denotes the propensity to save out of net profits ( $0 < s_{np} < 1$ ). The normalization equation and the relation (A1.1) determine a unique, positive solution for  $(r, \mathbf{p})$ , provided only that the P-F eigenvalue of  $\mathbf{C}$  is less than 1. Thus, if  $g$  is given from outside the system, relation (A1.2) determines a unique value of  $c_{np}$ .

## Appendix 2: Reducibility

Suppose that  $(\mathbf{A}, \mathbf{a}, \mathbf{b}, \mathbf{b}_{np}, \mathbf{b}_{ne})$  can be partitioned as follows

$$\mathbf{A} \equiv \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{0} & \mathbf{A}_{22} \end{bmatrix}, \mathbf{a}^T \equiv [\mathbf{a}_1^T, \mathbf{a}_2^T], \mathbf{b} \equiv \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{0} \end{bmatrix}, \mathbf{b}_{np} \equiv c_{np} \begin{bmatrix} \mathbf{0} \\ \mathbf{b}_2 \end{bmatrix}, \mathbf{b}_{ne} \equiv c_{ne} \mathbf{b}$$

where 1 refers to basic and 2 to non-basic commodities and matrices  $\mathbf{A}_{ii}$  are assumed to be irreducible. So, the matrix of the ‘augmented’ input-output coefficients is reducible. In an obvious notation, the proportions of total labour which are allocated to each system are given by

$$L_2 \equiv \mathbf{a}_2^T \mathbf{x}_2 / L = c_{np} \mathbf{a}_2^T \mathbf{B}_{22}(g) \mathbf{b}_2 \quad (\text{A2.1})$$

$$L_1 \equiv \mathbf{a}_1^T \mathbf{x}_1 / L = c_{np} \mathbf{a}_1^T \mathbf{B}_{12}(g) \mathbf{b}_2 + [1 + c_{ne}(N/L)] \mathbf{v}_1^T(g) \mathbf{b}_1 \quad (\text{A2.2})$$

which imply that

$$L_1 = 1 - c_{np} \mathbf{a}_2^T \mathbf{B}_{22}(g) \mathbf{b}_2 \quad (\text{A2.3})$$

and

$$R_0 \equiv N/L = [e(g) - c_{np}(\mathbf{v}_2^T(g) \mathbf{b}_2 / \mathbf{v}_1^T(g) \mathbf{b}_1)] / c_{ne} \quad (\text{A2.4})$$

where  $\mathbf{B}_{12}(g) \equiv (1 + g)\mathbf{B}_{11}(g)\mathbf{A}_{12}\mathbf{B}_{22}(g)$ ,  $\mathbf{v}_1^T(g) \equiv \mathbf{a}_1^T \mathbf{B}_{11}(g)$ ,

$$\mathbf{v}_2^T(g) \equiv \mathbf{a}_1^T \mathbf{B}_{12}(g) + \mathbf{a}_2^T \mathbf{B}_{22}(g)$$

or

$$\mathbf{v}_2^T(g) = [(1+g)\mathbf{v}_1^T(g)\mathbf{A}_{12} + \mathbf{a}_2^T]\mathbf{B}_{22}(g)$$

and  $e(g)$  is now equal to  $(1/\mathbf{v}_1^T(g)\mathbf{b}_1) - 1$ . Thus, it may be concluded that (i) the structure of outputs depends on  $c_{np}$  or, more specifically,  $L_1$  is a strictly decreasing function of  $c_{np}$ ; (ii)  $R_0$  is positive iff

$$c_{np}\mathbf{v}_2^T(g)\mathbf{b}_2 < 1 - \mathbf{v}_1^T(g)\mathbf{b}_1 = e(g)/(1+e(g)) \quad (\text{A2.5})$$

where the term on the right-hand side equals the share of consumption out of net profits and of the non-employed in total consumption (in terms of Austrian socially necessary labour); and (iii)  $R_0$  depends on the technical conditions of production in *both* systems, and it is worth noting that technical progress in the non-basic system, *i.e.*, a reduction in any element of  $(\mathbf{A}_{12}, \mathbf{A}_{22}, \mathbf{a}_2)$ , leads to a decrease in the term on the left-hand side of (A2.5) (by reducing the elements of  $\mathbf{v}_2(g)$ ) and, therefore, to an increase in  $R_0$ , whilst technical progress in the basic system leads to an increase (a decrease) in the term on the right-hand (left-hand) side of (A2.5) (by reducing the elements of  $\mathbf{v}_1(g)$  and, therefore, the elements of  $\mathbf{v}_2(g)$ ).

### **Appendix 3: A Note on the Data**

The symmetric input-output tables (SIOT) of the Finnish (for the years 1995 through 2004), German (for the year 1995, and 2000 through 2002), Greek (for the years 1997 and 1998) and Spanish (for the years 1995 and 2000) economies are available via the Eurostat website (<http://ec.europa.eu/eurostat>). We have chosen to apply our analysis to the SIOT of the Finnish and Greek economies for the years 1997 and 1998, and of the German and Spanish economies for the year 2000. The purpose of this choice was to maximize the chronological comparability of the results among the four economies.

The SIOT describe 59 products, which are classified according to CPA (Classification of Product by Activity). However, in the case of the Spanish, German and Greek economies, all the elements associated with the product with code 12 (Uranium and thorium ores) equal zero and, therefore, we remove them from our analysis. So, the SIOT of Germany, Spain and Greece have dimensions  $58 \times 58$ , whilst those of Finland have dimensions  $59 \times 59$ . The levels of sectoral employment of Finland (1997 and 1998) and Germany (2000) are included in the input-output tables, whilst those of Greece are included in the input-output table (1997) or provided by the National Statistical Service of Greece (1998). Finally, the levels of sectoral employment of Spain (2000) are provided via the website of the National Statistics Institute of Spain (<http://www.ine.es/>).

The market prices of all products are taken to be equal to 1; that is to say, the physical unit of measurement of each product is that unit which is worth of a monetary unit (see, *e.g.*, Miller and Blair, 1985, p. 356). Thus, the matrix of input-output coefficients,  $\mathbf{A}$ , is obtained by dividing element-by-element the inputs of each sector by its gross output. Furthermore, wage differentials are used to homogenize the sectoral employment (see, *e.g.*, Sraffa, 1960, §10, and Kurz and Salvadori, 1995, pp. 322-325), *i.e.*, the vector of inputs in direct homogeneous labour,  $\mathbf{a} \equiv [a_j]$ , is determined as follows:  $a_j = (L_j / x_j)(w_j^m / w_{\min}^m)$ , where  $L_j, x_j, w_j^m$  denote the total employment, gross output and money wage rate, in terms of market prices, of the  $j$ th sector, respectively, and  $w_{\min}^m$  the minimum sectoral money wage rate in terms of market prices. By assuming that there are no savings out of wages and that consumption out of wages has the same composition as the vector of the final consumption expenditures of the household sector,  $\mathbf{h}_{ce}$ , directly obtained from the

input-output tables, the vector of the real wage rate,  $\mathbf{b}$ , is determined as follows:  $\mathbf{b} = (w_{\min}^m / \mathbf{s}^T \mathbf{h}_{ce}) \mathbf{h}_{ce}$ , where  $\mathbf{s}^T \equiv [1, 1, \dots, 1]$  denotes the row summation vector identified with the vector of market prices (see also, *e.g.*, Okishio and Nakatani, 1985, pp. 66-67). Thus, the estimates of the MEDR are independent of the choice of the unit of measurement of the quantity of labour. Finally, it must be noted that the available input-output tables do not include inter-industry data on fixed capital stocks and on imported inputs. As a result, our investigation is restricted to a circulating capital model and, regarding the estimations associated with the case of non-proportionally growing and open economy, (i) we replace  $\mathbf{i}$  by the ‘gross capital formation’ vector, which is obtained from the tables (and includes ‘gross fixed capital formation, changes in inventories and acquisitions less disposal of valuables’); and (ii) we use the net export vector, which is also obtained from the tables.

#### **Appendix 4: Theoretical Relationships between the Maximum Relative Rate of Growth and the Marxian Rate of Surplus Labour**

From  $\mathbf{B}(g^*)[\mathbf{I} - (1 + g^*)\mathbf{A}] = \mathbf{I}$ , it follows that if  $\gamma \equiv g^*/G < 1$ , then

$$\mathbf{B}(g^*) = [\mathbf{I} - \mathbf{A}]^{-1} [\mathbf{I} - g^* \mathbf{H}]^{-1} = [\mathbf{I} - \mathbf{A}]^{-1} [\mathbf{I} - \gamma \mathbf{J}]^{-1} = [\mathbf{I} - \mathbf{A}]^{-1} [\mathbf{I} + \gamma \mathbf{J} + \gamma^2 \mathbf{J}^2 + \dots]$$

or

$$\mathbf{B}(g^*) = [\mathbf{I} - \mathbf{A}]^{-1} [\mathbf{I} + \gamma \mathbf{J} [\mathbf{I} - \gamma \mathbf{J}]^{-1}] \quad (\text{A4.1})$$

where  $\mathbf{H} \equiv \mathbf{A}[\mathbf{I} - \mathbf{A}]^{-1}$  denotes Pasinetti’s (1973) matrix of the vertically integrated technical coefficients of production and the P-F eigenvalue of  $\mathbf{J} \equiv \mathbf{G}\mathbf{H}$  equals 1. Thus,  $e(g^*) = 0$  or  $\mathbf{a}^T \mathbf{B}(g^*) \mathbf{b} = 1$  can be restated as

$$\mathbf{v}^T(0) \mathbf{b} + \gamma \mathbf{v}^T(0) \mathbf{J} [\mathbf{I} - \gamma \mathbf{J}]^{-1} \mathbf{b} = 1$$

or

$$\gamma \mathbf{v}^T(0) \mathbf{J} [\mathbf{I} - \gamma \mathbf{J}]^{-1} \mathbf{b} = 1 - \mathbf{v}^T(0) \mathbf{b} = e(0)/(1 + e(0)) \quad (\text{A4.2})$$

In the *trivial* case in which  $\mathbf{v}^T(0)$  or  $\mathbf{b}$  is the P-F eigenvector of  $\mathbf{J}$  (or, equivalently, of  $\mathbf{A}$ ), (i) relation (A4.2) implies that

$$[\gamma/(1-\gamma)] \mathbf{v}^T(0) \mathbf{b} = e(0)/(1+e(0))$$

or, since  $e(0) = [(1 - \lambda_A)/\mathbf{a}^T \mathbf{b}] - 1$  and  $\lambda_A = 1/(1+G)$ ,

$$\gamma = e(0)/(1+e(0)) = 1 - \mathbf{a}^T \mathbf{b} [(1+G)/G]$$

*i.e.*, the system constitutes a quasi-one-commodity economy and, therefore,  $\gamma$  equals the share of gross profits in net income expressed in terms of Marxian socially necessary labour (see also Sraffa, 1960, § 29); and (ii)  $e(g)$  is a *linear* function of  $g$ , *i.e.*,

$$e(g) = \{[1 - \lambda_A(1+g)]/\mathbf{a}^T \mathbf{b}\} - 1$$

or

$$e(g) = e(0) - (1+e(0))(g/G) \quad (\text{A4.3})$$

Now, consider the *general* case: let  $\mathbf{y}_j$  be the positive left-hand side P-F eigenvector of  $\mathbf{J}$  (or, equivalently, of  $\mathbf{A}$ ) and let  $\hat{\mathbf{y}}_j$  be the diagonal matrix formed from the elements of  $\mathbf{y}_j$ . Given that

$$\mathbf{s}^T [\hat{\mathbf{y}}_j \mathbf{J} \hat{\mathbf{y}}_j^{-1}] = \mathbf{y}_j^T \mathbf{J} \hat{\mathbf{y}}_j^{-1} = \mathbf{y}_j^T \hat{\mathbf{y}}_j^{-1} = \mathbf{s}^T$$

it follows that  $\mathbf{J}$  is similar to the *column stochastic* matrix  $\mathbf{K} \equiv \hat{\mathbf{y}}_j \mathbf{J} \hat{\mathbf{y}}_j^{-1}$ , the elements of which are independent of the choice of physical measurement units (and the normalization of  $\mathbf{y}$ ). Substituting  $\mathbf{J} = \hat{\mathbf{y}}_j^{-1} \mathbf{K} \hat{\mathbf{y}}_j$  in (A4.2) yields

$$\gamma \boldsymbol{\omega}^T(0) \mathbf{K} [\mathbf{I} - \gamma \mathbf{K}]^{-1} \boldsymbol{\beta} = e(0)/(1+e(0)) \quad (\text{A4.4})$$

where  $\boldsymbol{\omega}^T(0) \equiv \mathbf{v}^T(0)\hat{\mathbf{y}}_J^{-1}$ ,  $\boldsymbol{\beta} \equiv \hat{\mathbf{y}}_J \mathbf{b}$  and  $\mathbf{s}^T[\mathbf{I} - \gamma \mathbf{K}]^{-1} = [1/(1-\gamma)]\mathbf{s}^T$ , which implies that

$\|\mathbf{I} - \gamma \mathbf{K}\|^{-1} = 1/(1-\gamma)$ , where  $\|\bullet\|$  denotes the ‘maximum column sum matrix norm’.

Thus, taking norms of (A4.4), and using the well-known Hölder’s inequality (see, *e.g.*, Horn and Johnson, 1990, p. 536), we obtain

$$[\gamma/(1-\gamma)]f \geq e(0)/(1+e(0))$$

or

$$\gamma \geq F$$

where  $F \equiv e(0)/[f(1+e(0))+e(0)]$  and  $f \equiv \|\boldsymbol{\omega}^T(0)\|\|\boldsymbol{\beta}\|$ . Since  $\boldsymbol{\omega}^T(0)\boldsymbol{\beta} = \mathbf{v}^T(0)\mathbf{b}$  and

$\|\boldsymbol{\omega}^T(0)\boldsymbol{\beta}\| = \boldsymbol{\omega}^T(0)\boldsymbol{\beta} \leq f$ , where the equality holds iff  $\mathbf{v}^T(0)$  is the P-F eigenvector of

$\mathbf{A}$ , it follows that  $f(1+e(0)) \geq 1$ . So, we conclude that  $F$ , a lower bound for  $\gamma$ , is no

greater than  $e(0)/(1+e(0))$ . Finally,  $e(g)$  can be expressed as

$$e(g) = \{1/[\boldsymbol{\omega}^T(0)\boldsymbol{\beta} + (g/G)\boldsymbol{\omega}^T(0)\mathbf{K}[\mathbf{I} - (g/G)\mathbf{K}]^{-1}\boldsymbol{\beta}]\} - 1$$

Thus, taking norms, we obtain

$$e(g) \geq [(1-f)/f] - (1/f)(g/G)$$

where the term on the right-hand side is no greater than the term on the right-hand

side of (A4.3), since  $f(1+e(0)) \geq 1$  and  $g/G \leq 1$ .