Two examples to break through classical theorems on Nash implementation with two agents

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[E. Maskin, *Rev. Econom. Stud.* **66** (1999) 23-38] is a seminal paper in the field of mechanism design and implementation theory. [J. Moore and R. Repullo, *Econometrica* **58** (1990) 1083-1099] and [B. Dutta and A. Sen, *Rev. Econom. Stud.* **58** (1991) 121-128] are two fundamental papers on two-player Nash implementation. Recently, [H. Wu, http://arxiv.org/pdf/1004.5327v1] proposed a classical algorithm to break through Maskin's theorem for the case of many agents. In this paper, we will give two examples to break through the aforementioned results on two-agent Nash implementation by virtue of Wu's algorithm. There are two main contributions of this paper: 1) A two-player social choice rule (SCR) that satisfies Condition $\mu 2$ cannot be Nash implemented if an additional Condition λ' is satisfied. 2) A non-dictatorial two-player weakly pareto-optimal SCR is Nash implementable if Condition λ' is satisfied. Although the former is negative for the economic society, the latter is just positive. Put in other words, some SCRs which are traditionally viewed as not be Nash implementable may be Nash implemented now.

Keywords: Quantum games; Mechanism design; Implementation theory; Nash implementation; Maskin monotonicity.

1. Introduction

Mechanism design is an important branch of economics. Compared with game theory, the theory of mechanism design just concerns a *reverse* question: given some desirable outcomes, can we design a game that produces them? Ref. [1] is a fundamental work in the field of mechanism design. It provides an almost complete characterization of social choice rules that are Nash implementable. In 1990, Moore and Repullo [2] gave a necessary and sufficient condition for Nash implementation with two agents and many agents. Dutta and Sen [3] also independently gave an equivalent result for two-agent Nash implementation. In 2009, Busetto and Codognato [4] gave an amended necessary and sufficient condition for Nash implementation with two agents. These papers together construct a framework for two-agent Nash implementation.

In 2010, Wu [5] claimed that quantum strategies dramatically change the theory of mechanism design when the number of agents is larger than three, i.e., by virtue of a quantum mechanism, agents who satisfy Condition λ can combat Pareto-inefficient social choice rules instead of being restricted by the traditional mechanism design theory. Although current experimental technologies restrict the quantum mechanism to be commercially available, Wu [6] proposed an algorithm that helps agents benefit from the quantum mechanism immediately when the number of agents is not very large (e.g., less than 20). Following the aforementioned results, it is naturally to ask what will happen if quantum strategies are considered in the field of Nash implementation with two agents. This paper just concerns this question.

The rest of this paper is organized as follows: Section 2 recalls preliminaries of two-agent Nash implementation published in Ref. [4]. Section 3 is the main part of this paper, two examples that break through Moore and Repullo (1990) and Maskin (1999) are given in detail.

2. Preliminaries

Consider an environment with a finite set $I = \{1, 2\}$ of agents, and a (possibly infinite) set A of feasible outcomes. Each agent $i \in I$ has a complete and transitive preference relation on A, which is denoted by R_i. For each $i \in I$, $P(R_i)$ denotes the strict preference relation corresponding to R_i. An ordered pair of preference relations $R = (R_1, R_2)$ is called a preference profile. The unrestricted domain of preferences, denoted by $\mathcal{R}_A$, is the set of all preference profiles on A. The unrestricted domain of strict preferences, denoted by $\mathcal{P}_A$, is the set of all profiles of linear orderings on A. A domain of preference is a set $\mathcal{R} \subseteq \mathcal{R}_A$ of preference profiles.

For any $i \in I$, $R \in \mathcal{R}$ and $a \in A$, let $L_i(a, R) \equiv \{c \in A : aR_ic\}$, and $M_i(C, R) \equiv \{a \in C : aR_ic, \text{ for all} c \in C\}$, for any $C \subseteq A$.

Given a domain of preferences $\mathcal{R}$, a social choice rule (SCR) is a correspondence $f : \mathcal{R} \to A$, which associates a nonempty set $f(R) \subseteq A$ with each preference profile $R \in \mathcal{R}$. An SCR f is dictatorial if there exists $i \in I$ for whom $f(R) = M_i(A, R)$, for all $R \in \mathcal{R}$. An SCR f is weakly Pareto optimal if for all $R \in \mathcal{R}$ and $a \in f(R)$, there is no $b \in A$ such that $bP(R_i)a$, for all $i \in I$.

A mechanism is a function $g : S \to A$, which associates an outcome $g(s) \in A$ with each pair of strategies $s = (s_1, s_2) \in S = S_1 \times S_2$, where S_i denotes the strategy space of agent $i \in I$. For each $R \in \mathcal{R}$, the pair (g, R) defines a game in normal form. Let $NE(g, R) \subseteq S$ denote the set of pure strategy Nash equilibria of the game (g, R). A mechanism is said to implement the SCR f if for all $R \in \mathcal{R}$, $\{g(s) : s \in NE(g, R)\} = f(R)$.

Definition 1 An SCR f satisfies Condition $\mu2$ if there is a set B and, for each $i \in I$, $R \in \mathcal{R}$, and $a \in f(R)$, there is a set $C_i(a, R) \subset B$, with $a \in M_i(C_i(a, R), R)$; moreover, for each 4-tuple $(a, R, a', R') \in A \times \mathcal{R} \times A \times \mathcal{R}$, with $a \in f(R)$, $a' \in f(R')$, there is $e = e(a, R, a', R') \in C_1(a, R) \cap C_2(a', R')$; finally, for each $R^* \in \mathcal{R}$, we have:

(i) if $a \in M_1(C_1(a, R), R^*) \cap M_2(C_2(a, R), R^*)$, then $a \in f(R^*)$;

(ii) if $c \in M_i(C_i(a, R), R^*) \cap M_j(B, R^*)$, for $i, j \in I$, $i \neq j$, then $c \in f(R^*)$;

(iii) if $d \in M_1(B, R^*) \cap M_2(B, R^*)$, then $d \in f(R^*)$;

(iv) if $e = e(a, R, a', R') \in M_1(C_1(a, R), R^*) \cap M_2(C_2(a', R'), R^*)$, then $e \in f(R^*)$.

Theorem 1 (Moore-Repullo, 1990): A two-player social choice function is Nash implementable if and only if it satisfies Condition $\mu 2$.

Theorem 2 (Maskin, 1999): A two-player weakly pareto-optimal SCR is Nash implementable if and only if it is dictatorial.

3. Main results

3.1. *Breaking through Moore-Repullo's theorem*

Table 1. SCR1: a two-player SCR that satisfies condition $\mu 2$. Hence, it can be Nash implemented traditionally. However, Wu's algorithm makes SCR1 not Nash implementable. As a result, the Moore-Repullo's theorem is broken through.

State R_1		State R_2		State R_3	
*agent*1	*agent*2	*agent*1	*agent*2	*agent*1	*agent*2
a_3	a_2	a_4	a_3	a_2	a_2
a_1	a_1	a_1	a_1	a_1	a_3
a_2	a_4	a_2	a_2	a_3	a_4
a_4	a_3	a_3	a_4	a_4	a_1
$f(R_1)=\{a_1\}$		$f(R_2)=\{a_2\}$		$f(R_3)=\{a_2\}$	

Table 2. SCR2: a two-player Pareto-optimal non-dictatorial SCR. According to Maskin's impossibility theorem, it can not be Nash implemented. However, Wu's algorithm makes it Nash implementable (see Table 1). As a result, the Maskin's impossibility theorem on Nash implementation with two agents is broken through.

State R_1		State R_2		State R_3	
*agent*1	*agent*2	*agent*1	*agent*2	*agent*1	*agent*2
a_3	a_2	a_4	a_3	a_2	a_2
a_1	a_1	a_1	a_1	a_1	a_3
a_2	a_4	a_2	a_2	a_3	a_4
a_4	a_3	a_3	a_4	a_4	a_1
$f(R_1)=\{a_1\}$		$f(R_2)=\{a_1\}$		$f(R_3)=\{a_2\}$	

Consider the SCR1 specified by Table 1. $A=\{a_1,a_2,a_3,a_4\}$, $\mathcal{R}=\{R_1,R_2\}$. Let $B=A$, $C_i(a,R)=L_i(a,R)$ for $i\in I$, $R\in\mathcal{R}$ and $a\in f(R)$, i.e.,

$$
\begin{aligned}
C_1(a_1,R_1)&=L_1(a_1,R_1)=\{a_1,a_2,a_4\},\\
C_2(a_1,R_1)&=L_2(a_1,R_1)=\{a_1,a_3,a_4\},\\
C_1(a_2,R_2)&=L_1(a_2,R_2)=\{a_2,a_3\},\\
C_2(a_2,R_2)&=L_2(a_2,R_2)=\{a_2,a_4\},\\
C_1(a_2,R_3)&=L_1(a_2,R_3)=\{a_1,a_2,a_3,a_4\},\\
C_2(a_2,R_3)&=L_2(a_2,R_3)=\{a_1,a_2,a_3,a_4\}.
\end{aligned}
$$

Obviously,

$$\begin{aligned}
a_1 &\in M_1(C_1(a_1,R_1),R_1)=\{a_1\},\\
a_1 &\in M_2(C_2(a_1,R_1),R_1)=\{a_1\},\\
a_2 &\in M_1(C_1(a_2,R_2),R_2)=\{a_2\},\\
a_2 &\in M_2(C_2(a_2,R_2),R_2)=\{a_2\},\\
a_2 &\in M_1(C_1(a_2,R_3),R_3)=\{a_2\},\\
a_2 &\in M_2(C_2(a_2,R_3),R_3)=\{a_2\}.
\end{aligned}$$

For each 4-tuple $(a,R,a',R')\in A\times\mathcal{R}\times A\times\mathcal{R}$, let

$$\begin{aligned}
e(a_1,R_1,a_1,R_1) &= a_1\in C_1(a_1,R_1)\cap C_2(a_1,R_1)=\{a_1,a_4\},\\
e(a_1,R_1,a_2,R_2) &= a_2\in C_1(a_1,R_1)\cap C_2(a_2,R_2)=\{a_2,a_4\},\\
e(a_1,R_1,a_2,R_3) &= a_2\in C_1(a_1,R_1)\cap C_2(a_2,R_3)=\{a_1,a_2,a_4\},\\
e(a_2,R_2,a_1,R_1) &= a_3\in C_1(a_2,R_2)\cap C_2(a_1,R_1)=\{a_3\},\\
e(a_2,R_2,a_2,R_2) &= a_2\in C_1(a_2,R_2)\cap C_2(a_2,R_2)=\{a_2\},\\
e(a_2,R_2,a_2,R_3) &= a_2\in C_1(a_2,R_2)\cap C_2(a_2,R_3)=\{a_2,a_3\},\\
e(a_2,R_3,a_1,R_1) &= a_1\in C_1(a_2,R_3)\cap C_2(a_1,R_1)=\{a_1,a_3,a_4\},\\
e(a_2,R_3,a_2,R_2) &= a_2\in C_1(a_2,R_3)\cap C_2(a_2,R_2)=\{a_2,a_4\},\\
e(a_2,R_3,a_2,R_3) &= a_2\in C_1(a_2,R_3)\cap C_2(a_2,R_3)=\{a_1,a_2,a_3,a_4\}.
\end{aligned}$$

Case 1): Consider $R^*=R_1,\ f(R^*)=\{a_1\}$.
For rule (i):

$$\begin{aligned}
M_1(C_1(a_1,R_1),R^*)\cap M_2(C_2(a_1,R_1),R^*) &= \{a_1\}\cap\{a_1\}=\{a_1\},\\
M_1(C_1(a_2,R_2),R^*)\cap M_2(C_2(a_2,R_2),R^*) &= \{a_3\}\cap\{a_2\}=\phi,\\
M_1(C_1(a_2,R_3),R^*)\cap M_2(C_2(a_2,R_3),R^*) &= \{a_3\}\cap\{a_2\}=\phi.
\end{aligned}$$

Hence, rule (i) is satisfied.
For rule (ii):

$$\begin{aligned}
M_1(C_1(a_1,R_1),R^*)\cap M_2(B,R^*) &= \{a_1\}\cap\{a_2\}=\phi,\\
M_1(C_1(a_2,R_2),R^*)\cap M_2(B,R^*) &= \{a_3\}\cap\{a_2\}=\phi,\\
M_1(C_1(a_2,R_3),R^*)\cap M_2(B,R^*) &= \{a_3\}\cap\{a_2\}=\phi,\\
M_2(C_2(a_1,R_1),R^*)\cap M_1(B,R^*) &= \{a_1\}\cap\{a_3\}=\phi,\\
M_2(C_2(a_2,R_2),R^*)\cap M_1(B,R^*) &= \{a_2\}\cap\{a_3\}=\phi,\\
M_2(C_2(a_2,R_3),R^*)\cap M_1(B,R^*) &= \{a_2\}\cap\{a_3\}=\phi.
\end{aligned}$$

Hence, rule (ii) is satisfied.
For rule (iii):

$$M_1(B,R^*)\cap M_2(B,R^*)=\{a_3\}\cap\{a_2\}=\phi.$$

Hence, rule (iii) is satisfied.

For rule (iv):

$$\begin{aligned}
&e(a_1,R_1,a_1,R_1)=a_1,\ M_1(C_1(a_1,R_1),R^*)\cap M_2(C_2(a_1,R_1),R^*)=\{a_1\}\cap\{a_1\}=\{a_1\},\\
&e(a_1,R_1,a_2,R_2)=a_2,\ M_1(C_1(a_1,R_1),R^*)\cap M_2(C_2(a_2,R_2),R^*)=\{a_1\}\cap\{a_2\}=\phi,\\
&e(a_1,R_1,a_2,R_3)=a_2,\ M_1(C_1(a_1,R_1),R^*)\cap M_2(C_2(a_2,R_3),R^*)=\{a_1\}\cap\{a_2\}=\phi,\\
&e(a_2,R_2,a_1,R_1)=a_3,\ M_1(C_1(a_2,R_2),R^*)\cap M_2(C_2(a_1,R_1),R^*)=\{a_3\}\cap\{a_1\}=\phi,\\
&e(a_2,R_2,a_2,R_2)=a_2,\ M_1(C_1(a_2,R_2),R^*)\cap M_2(C_2(a_2,R_2),R^*)=\{a_3\}\cap\{a_2\}=\phi,\\
&e(a_2,R_2,a_2,R_3)=a_2,\ M_1(C_1(a_2,R_2),R^*)\cap M_2(C_2(a_2,R_3),R^*)=\{a_3\}\cap\{a_2\}=\phi,\\
&e(a_2,R_3,a_1,R_1)=a_1,\ M_1(C_1(a_2,R_3),R^*)\cap M_2(C_2(a_1,R_1),R^*)=\{a_3\}\cap\{a_1\}=\phi,\\
&e(a_2,R_3,a_2,R_2)=a_2,\ M_1(C_1(a_2,R_3),R^*)\cap M_2(C_2(a_2,R_2),R^*)=\{a_3\}\cap\{a_2\}=\phi,\\
&e(a_2,R_3,a_2,R_3)=a_2,\ M_1(C_1(a_2,R_3),R^*)\cap M_2(C_2(a_2,R_3),R^*)=\{a_3\}\cap\{a_2\}=\phi.
\end{aligned}$$

Hence, rule (iv) is satisfied.

Case 2): Consider $R^*=R_2,\ f(R^*)=\{a_2\}$.

For rule (i):

$$\begin{aligned}
&M_1(C_1(a_1,R_1),R^*)\cap M_2(C_2(a_1,R_1),R^*)=\{a_4\}\cap\{a_3\}=\phi,\\
&M_1(C_1(a_2,R_2),R^*)\cap M_2(C_2(a_2,R_2),R^*)=\{a_2\}\cap\{a_2\}=\{a_2\},\\
&M_1(C_1(a_2,R_3),R^*)\cap M_2(C_2(a_2,R_3),R^*)=\{a_4\}\cap\{a_3\}=\phi.
\end{aligned}$$

Hence, rule (i) is satisfied.

For rule (ii):

$$\begin{aligned}
&M_1(C_1(a_1,R_1),R^*)\cap M_2(B,R^*)=\{a_4\}\cap\{a_3\}=\phi,\\
&M_1(C_1(a_2,R_2),R^*)\cap M_2(B,R^*)=\{a_2\}\cap\{a_3\}=\phi,\\
&M_1(C_1(a_2,R_3),R^*)\cap M_2(B,R^*)=\{a_4\}\cap\{a_3\}=\phi,\\
&M_2(C_2(a_1,R_1),R^*)\cap M_1(B,R^*)=\{a_3\}\cap\{a_4\}=\phi,\\
&M_2(C_2(a_2,R_2),R^*)\cap M_1(B,R^*)=\{a_2\}\cap\{a_4\}=\phi,\\
&M_2(C_2(a_2,R_3),R^*)\cap M_1(B,R^*)=\{a_3\}\cap\{a_4\}=\phi.
\end{aligned}$$

Hence, rule (ii) is satisfied.

For rule (iii):

$$M_1(B,R^*)\cap M_2(B,R^*)=\{a_4\}\cap\{a_3\}=\phi.$$

Hence, rule (iii) is satisfied.

For rule (iv):

$$
\begin{aligned}
&e(a_1,R_1,a_1,R_1)=a_1,\ M_1(C_1(a_1,R_1),R^*)\cap M_2(C_2(a_1,R_1),R^*)=\{a_4\}\cap\{a_3\}=\phi,\\
&e(a_1,R_1,a_2,R_2)=a_2,\ M_1(C_1(a_1,R_1),R^*)\cap M_2(C_2(a_2,R_2),R^*)=\{a_4\}\cap\{a_2\}=\phi,\\
&e(a_1,R_1,a_2,R_3)=a_2,\ M_1(C_1(a_1,R_1),R^*)\cap M_2(C_2(a_2,R_3),R^*)=\{a_4\}\cap\{a_3\}=\phi,\\
&e(a_2,R_2,a_1,R_1)=a_3,\ M_1(C_1(a_2,R_2),R^*)\cap M_2(C_2(a_1,R_1),R^*)=\{a_2\}\cap\{a_3\}=\phi,\\
&e(a_2,R_2,a_2,R_2)=a_2,\ M_1(C_1(a_2,R_2),R^*)\cap M_2(C_2(a_2,R_2),R^*)=\{a_2\}\cap\{a_2\}=\{a_2\},\\
&e(a_2,R_2,a_2,R_3)=a_2,\ M_1(C_1(a_2,R_2),R^*)\cap M_2(C_2(a_2,R_3),R^*)=\{a_2\}\cap\{a_3\}=\phi,\\
&e(a_2,R_3,a_1,R_1)=a_1,\ M_1(C_1(a_2,R_3),R^*)\cap M_2(C_2(a_1,R_1),R^*)=\{a_4\}\cap\{a_3\}=\phi,\\
&e(a_2,R_3,a_2,R_2)=a_2,\ M_1(C_1(a_2,R_3),R^*)\cap M_2(C_2(a_2,R_2),R^*)=\{a_4\}\cap\{a_2\}=\phi,\\
&e(a_2,R_3,a_2,R_3)=a_2,\ M_1(C_1(a_2,R_3),R^*)\cap M_2(C_2(a_2,R_3),R^*)=\{a_4\}\cap\{a_3\}=\phi.
\end{aligned}
$$

Hence, rule (iv) is satisfied.

Case 3): Consider $R^*=R_3,\ f(R^*)=\{a_2\}$.

For rule (i):

$$
\begin{aligned}
M_1(C_1(a_1,R_1),R^*)\cap M_2(C_2(a_1,R_1),R^*)&=\{a_2\}\cap\{a_3\}=\phi,\\
M_1(C_1(a_2,R_2),R^*)\cap M_2(C_2(a_2,R_2),R^*)&=\{a_2\}\cap\{a_2\}=\{a_2\},\\
M_1(C_1(a_2,R_3),R^*)\cap M_2(C_2(a_2,R_3),R^*)&=\{a_2\}\cap\{a_2\}=\{a_2\}.
\end{aligned}
$$

Hence, rule (i) is satisfied.

For rule (ii):

$$
\begin{aligned}
M_1(C_1(a_1,R_1),R^*)\cap M_2(B,R^*)&=\{a_2\}\cap\{a_2\}=\{a_2\},\\
M_1(C_1(a_2,R_2),R^*)\cap M_2(B,R^*)&=\{a_2\}\cap\{a_2\}=\{a_2\},\\
M_1(C_1(a_2,R_3),R^*)\cap M_2(B,R^*)&=\{a_2\}\cap\{a_2\}=\{a_2\},\\
M_2(C_2(a_1,R_1),R^*)\cap M_1(B,R^*)&=\{a_3\}\cap\{a_2\}=\phi,\\
M_2(C_2(a_2,R_2),R^*)\cap M_1(B,R^*)&=\{a_2\}\cap\{a_2\}=\{a_2\},\\
M_2(C_2(a_2,R_3),R^*)\cap M_1(B,R^*)&=\{a_2\}\cap\{a_2\}=\{a_2\}.
\end{aligned}
$$

Hence, rule (ii) is satisfied.

For rule (iii):

$$M_1(B,R^*)\cap M_2(B,R^*)=\{a_2\}\cap\{a_2\}=\{a_2\}.$$

Hence, rule (iii) is satisfied.

For rule (iv):

$$e(a_1,R_1,a_1,R_1)=a_1,\ M_1(C_1(a_1,R_1),R^*)\cap M_2(C_2(a_1,R_1),R^*)=\{a_2\}\cap\{a_3\}=\phi,$$
$$e(a_1,R_1,a_2,R_2)=a_2,\ M_1(C_1(a_1,R_1),R^*)\cap M_2(C_2(a_2,R_2),R^*)=\{a_2\}\cap\{a_2\}=\{a_2\},$$
$$e(a_1,R_1,a_2,R_3)=a_2,\ M_1(C_1(a_1,R_1),R^*)\cap M_2(C_2(a_2,R_3),R^*)=\{a_2\}\cap\{a_2\}=\{a_2\},$$
$$e(a_2,R_2,a_1,R_1)=a_3,\ M_1(C_1(a_2,R_2),R^*)\cap M_2(C_2(a_1,R_1),R^*)=\{a_2\}\cap\{a_3\}=\phi,$$
$$e(a_2,R_2,a_2,R_2)=a_2,\ M_1(C_1(a_2,R_2),R^*)\cap M_2(C_2(a_2,R_2),R^*)=\{a_2\}\cap\{a_2\}=\{a_2\},$$
$$e(a_2,R_2,a_2,R_3)=a_2,\ M_1(C_1(a_2,R_2),R^*)\cap M_2(C_2(a_2,R_3),R^*)=\{a_2\}\cap\{a_2\}=\{a_2\},$$
$$e(a_2,R_3,a_1,R_1)=a_1,\ M_1(C_1(a_2,R_3),R^*)\cap M_2(C_2(a_1,R_1),R^*)=\{a_2\}\cap\{a_3\}=\phi,$$
$$e(a_2,R_3,a_2,R_2)=a_2,\ M_1(C_1(a_2,R_3),R^*)\cap M_2(C_2(a_2,R_2),R^*)=\{a_2\}\cap\{a_2\}=\{a_2\},$$
$$e(a_2,R_3,a_2,R_3)=a_2,\ M_1(C_1(a_2,R_3),R^*)\cap M_2(C_2(a_2,R_3),R^*)=\{a_2\}\cap\{a_2\}=\{a_2\}.$$

Hence, rule (iv) is satisfied.

To sum up, the SCR1 given in Table 1 satisfies Condition $\mu 2$. Therefore, according to Moore-Repullo's theorem, the SCR is Nash implementable.

However, according to Wu [6], we can design a classical algorithm by which the Moore-Repullo's theorem will be broken through if the following Condition λ' is satisfied.

1) λ'_1: Given an SCR f, a preference profile $R\in\mathcal{R}$ and $a\in f(R)$, if there exists $R'\in\mathcal{R}$, $R'\neq R$, $a'\in f(R')$ such that $a'R_ia$ for each agent $i\in N$, and $a'P_ja$ for at least one $j\in N$, then in going from R' to R, both of two agents encounter a preference change around a'.

2) λ'_2: Consider the payoff to the second agent, $\$_{CC}>\$_{DD}$, i.e., he/she prefers the expected payoff of a certain outcome (generated by rule 1) to the expected payoff of an uncertain outcome (generated by rule 3).

3) λ'_3: Consider the payoff to the second agent, $\$_{CC}>\$_{DC}$.

The *Matlab* program in Ref. [6] is directly available for two agents by simply setting $n=2$.

3.2. *Breaking through Maskin's impossibility theorem with two agents*

Maskin [1] showed that a two-agent weakly Pareto optimal SCR, defined on the unrestricted domain of preferences, is Nash implementable if and only if it is dictatorial. However, according to the aforementioned discussion, the non-dictatorial SCR2 specified by Table 2 is weakly Pareto optimal and can be Nash implemented by using the Wu's algorithm. Therefore, the Maskin's impossibility theorem on Nash implementation with two agents is broken through. In this sense, the quantum mechanism is beneficial for both the designer and the agents.

References

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