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SOME RESULTS ON THE STOCHASTIC SIMULATION OF A NONLINEAR MODEL OF THE ITALIAN ECONOMY

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Experiments of stochastic simulation on a nonlinear macroeconometric model are described in this paper. The results are used both for improving the validation of a model of the Italian economy and for revisiting the heuristic value of the stochastic simulation methodology.

1. WHY STOCHASTIC SIMULATION ?

An important matter for consideration, when dealing with simulation of econometric models, is whether simulation should be deterministic or stochastic [20]. We call deterministic simulation the simultaneous solution of an econometric model obtained by replacing the structural disturbances with their expected values, which are all zero. We call stochastic simulation the simultaneous solution of an econometric model obtained after adding to the intercept of each behavioural equation a pseudo-random shock with specified stochastic properties (which are in general related to those of the structural disturbances of the estimation phase).

The reasons for performing stochastic simulation of macroeconometric models have been underlined by several authors [7], [14], [21]. Without entering into details about these specific reasons, we want to emphasize the result by Howrey and Kelejian that the properties of dynamic nonlinear models should not be studied in terms of nonstochastic simulation procedures [14,p.309]. This statement is very important, even if, as far as the hypothesis of systematic divergences between the deterministic and the mean stochastic solution is concerned, the authors [2] have shown that, for the model used in the experiments that will be here described, the relevance of the bias seems to be negligible and, in any case, is not such as to justify the very high number of replications performed to put it into evidence. In spite of this result and of the little use which up to now has "been made of the statistical information embodied in replicated simulation data", only by means of repeated experiments it will be possible to perform "statistical testing of inferences from simulation ... and ex-ante evaluation of alternative policies on the basis of formal significance tests" [21,p.214].

Before passing to the description of the stochastic simulation technique (section 2), let us introduce the main subjects of the various sections of this paper. In section 3, after a short description of the model, some preliminary considerations on the performed experiments are drawn. In sections 4 and 5 the standard errors of the reduced-form equations and the mechanism of transmission of errors are respectively analyzed in detail. In section 6 some general conclusions are derived. In appendix, finally, the main computational aspects of the experiments are discussed.

2. THE STOCHASTIC SIMULATION TECHNIQUE

An econometric model can generally be represented, in its structural form, as:

$$Y_t = F(Y_t, Y_{t-k}, X_t) + U_t$$

where Y_t is the vector of the current jointly dependent endogenous variables, Y_{t-k} are the lagged endogenous variables (k is the related lag), X_t is the vector of the exogenous variables at time t, U_t is the vector of the random structural disturbances which are supposed to have a multivariate normal distribution with zero mean, finite constant covariance matrix and, in our case, no serial correlation.

The generation of the pseudo-random disturbances involves three different steps:

(a) Generation of independent pseudo-random numbers uniformly distributed in the open interval (0,1). In this application the power residue method has

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been used, with prime modulus $2^{31}-1$ and its primitive root 7^5 as a multiplier [18].
(b) Transformation of the previously generated numbers into standard normal deviates. The logarithmic-trigonometric procedure by Box and Muller [5] has been applied, after an intermediate phase of shuffling [6].
(c) Transformation of the standard normal deviates into the required pseudo-random disturbances "with mean zero and covariances equal to those of the equations in question" [10,p.24]. The algorithm by McCarthy [19] has been chosen in this application.

The pseudo-random disturbances are added on the right hand side of each stochastic equation, then the model is solved. The process is repeated several times, so that a distribution of outcomes in each period is available for each endogenous variable. The number of replications which is sometimes "decided on purely ad hoc grounds" [21,p.210], could be fixed on a rational base, using the nonparametric tolerance interval. Of course, from a practical point of view, the number of replications should be as little as possible for time and cost problems. For this reason, in most of the applications described in the literature [7], [20], [21], the number of replications is no greater than 20-30. The results we shall present hereunder are referred to a much larger number (1000). In fact, even if with a small number of replications it is in general possible to perform validation experiments with a sufficient reliability, other experiments that will be hereunder discussed require more precise results, that can be obtained only by increasing the number of replications.

3. PRELIMINARY CONSIDERATIONS ON THE EXPERIMENTS

The nonlinear model of the Italian economy analyzed in this paper has been originally developed at the University of Ancona by a team led by Professor Giorgio Fua' [11]. The results of its revision, updating and reestimation, presented in [4], have been used in this study. The model, which is dynamic for the presence of lagged endogenous variables, consists of 38 equations, 16 of which are behavioural. It is based on yearly data: the initial year of the sample period varies from 1953 to 1959; the final year is always 1973. The involved variables are mainly related to the real and fiscal sectors.

For brevity's sake, only some of the endogenous variables, whose results will be analyzed in the paper, are listed below; the complete list of the variables, as well as the list of the equations of the model, can be found in [4].

- CPR -private consumption.
- HCS -social security contributions.
- HVAP -private production at factor prices.
- HYW -contractual earnings (including government allowances).
- HYWI -contractual earnings in the industrial sector.
- HYWTA-contractual earnings in the agricultural and tertiary sectors.
- LIDC -number of dependent workers in the industrial sector.
- LICH -total man hours worked in the industrial sector.
- MM -imports of goods.
- MS -imports of services.
- MMA -imports of goods of the agricultural and manufacturing sectors.
- PA -index of agricultural wholesale prices (1966=100).
- PCV -cost of living index (1963=100).
- PI -index of wholesale prices of non-agricultural products (1966=100).
- PING -index of wholesale prices (1963=100).
- PVAI -deflator of gross product of the industrial sector at factor prices (1963=100).
- PVAP -deflator of private production at factor prices (1963=100).
- S -inventory investment.
- SHCS -share of earnings paid for social security, i.e. HCS/(HYWI+HYWTA).
- VAI -gross product of the industrial sector.
- VAT -gross product of the tertiary sector (building services excepted).
- WM -earnings for eight hours work in the industrial sector (less family allowances) at market prices and at factors cost.
- WMO -average cost of work for eight man hours (including social security costs).
- YD -disposable income.

Other symbols are the following:

- Δ -first difference.
- $\dot{\Delta}$ -percentage change.
- ln -natural logarithm.

The model can be divided into various blocks. As far as the stochastic equations are concerned, the decomposition into blocks is the following:

DD - Domestic Demand (1 equation).
FT - Foreign Trade (2 equations).
DP - Domestic Production (2 equations).
LM - Labour Market (2 equations).
WP - Wages and Prices (6 equations).
ID - Income Distribution (3 equations).

The interactions among these blocks will be analyzed in detail in section 5.

In spite of the dynamic characteristics of the model, all the results which will be displayed in the following sections have been obtained by means of one-step simulation. In fact in these experiments we do not want to superimpose the dynamic effects on the analysis of the "reduced-form" of the model [16]. Our comments will be focussed on some results that can be correctly obtained, in practice, only by means of stochastic simulation, with two major points of interest (both from an econometric and an economic policy point of view):

(a) Computation of the standard errors of the reduced-form equations and feasibility of economic policy targets; this has been accomplished by means of stochastic simulation with disturbances in all the behavioural equations and allows to analyze some properties of the endogenous variables (section 4).
(b) Analysis of the mechanism of transmission of errors across the model, identifying the blocks with the largest responsibility of error transmission; this has been accomplished by means of stochastic simulation with disturbances introduced into single blocks of the model (section 5).

Before entering into details about the above mentioned points, we briefly recall some experiments that have been preliminarily performed for the validation of the model. They are not treated in detail here because some of them have already been discussed in [2], while the others, even if performed also by means of stochastic simulation, strictly speaking are not all tipical experiments of stochastic simulation. The main conclusion that was drawn from these experiments was that this model, in spite of its apparently strong nonlinearity in the structural form, is mildly nonlinear from the simulation point of view. The mean stochastic solutions, in fact, were always quite close to the deterministic solution values. Furthermore several measurements of goodness of fit (suggested, for example, in [7], [9], [16], [20] and [21]), such as the means over the simulation period, the Mean Absolute Percentage Errors (MAPE), the Root Mean Squared Errors (RMSE) and the Inequality Coefficients by Theil were practically equal when computed on the deterministic solution and on the sample mean of the replicated stochastic simulation results. The same held for the regression coefficients of the observed values on the deterministic and on the mean stochastic simulation results. The same conclusion of mild nonlinearity was also derived from the symmetrical dispersion of the stochastic simulation results around the mean.

4. STANDARD ERRORS OF THE REDUCED-FORM EQUATIONS

It is well known that, when dealing with systems of simultaneous relationships, in order to "bring out the explicit dependence of the dependent variables on the predetermined variables and the disturbances, we should solve the structural form into the reduced form" [12,pp.297-298]; to be more precise, in order to avoid misunderstanding with the directly estimated reduced-form, we are here referring to the so called "derived reduced-form". This derivation is always analytically feasible in the case of linear relationships. By means of the reduced-form we analytically transform the structural disturbances into reduced-form disturbances and the covariance matrix of these disturbances provides an estimate of the system-wide dispersion of the dependent variables. As an alternative computational method, in static linear systems or, that is the same, in one-step simulation of dynamic linear systems, the reduced-form disturbances can be directly computed by using difference between the observed and the deterministic simultaneous solution values [16,p.265]. In this case, the reduced-form variance is, for each variable, equal to the MSE (Mean Squared Error). It is clear that, in linear models, the stochastic simulation of the system would supply results, in terms of variance, that would be equal to those obtained by means of the two previous methods. In the case of nonlinear systems, however,the only way, at the same time practical and correct even if with some approximations, to derive the reduced-form equation variances is stochastic simulation. In fact, [14,p.309] there are difficulties involved in obtaining analytical solutions to a system of nonlinear equations and ... the reduced-form equations ..., generally, will be unknown". On the other hand [14,p.300]

Table 1

YEAR	MEAN	ST.DEV.	MEAN	ST.DEV.
	CPR		MMA	
61	17463.	454.31	2801.5	217.95
62	18494.	457.64	3197.4	224.87
63	19539.	442.75	3662.3	237.35
64	20824.	454.27	3604.5	250.75
65	21720.	473.69	3580.3	243.21
66	22053.	449.36	3785.6	249.65
67	23450.	456.89	4437.7	282.21
68	25876.	467.04	5509.1	317.88
69	26914.	470.34	5987.7	342.16
70	27952.	475.59	6364.1	356.16
71	30361.	511.45	7034.1	374.10
72	31131.	517.08	7567.4	412.80
73	32667.	518.63	8451.3	443.24
	VAI		VAT	
61	9266.3	198.66	7480.2	131.16
62	10124.	200.71	7947.6	133.76
63	10828.	200.54	8385.0	132.45
64	11163.	196.91	8693.0	128.37
65	11412.	206.44	9234.6	137.87
66	11888.	195.32	9581.9	128.98
67	12864.	200.75	10084.	126.54
68	14262.	208.83	11119.	135.26
69	15289.	205.05	11896.	134.99
70	15857.	206.93	12256.	137.69
71	16590.	222.77	13140.	150.41
72	17220.	224.62	13854.	150.89
73	18523.	239.05	14823.	155.99
	LICH		S	
61	12414.	394.59	477.75	146.41
62	12740.	379.35	457.80	139.71
63	12549.	379.11	592.34	148.08
64	12636.	369.10	273.32	141.75
65	12184.	373.17	258.28	148.65
66	11439.	326.57	350.39	138.67
67	11649.	320.90	550.74	151.38
68	12295.	329.20	87.808	162.50
69	12201.	327.87	390.87	176.74
70	11753.	325.20	447.40	179.31
71	11858.	329.97	397.07	191.48
72	11884.	314.03	292.53	200.04
73	11651.	305.31	821.17	212.33
	MS		lnMMA	
61	324.09	45.191	7.9349	.07770
62	424.85	45.990	8.0676	.07055
63	481.27	47.635	8.2037	.06532
64	587.93	46.891	8.1875	.06925
65	686.05	46.575	8.1809	.06787
66	700.60	45.530	8.2368	.06619
67	772.84	45.730	8.3959	.06379
68	904.48	47.223	8.6125	.05776
69	1195.9	46.500	8.6958	.05751
70	1484.2	45.375	8.7569	.05592
71	1746.0	45.995	8.8571	.05317
72	2044.0	45.853	8.9301	.05475
73	2424.4	48.417	9.0407	.05249

"the application of nonstochastic simulation ... yields results that are not consistent with the properties of the reduced-form of the model", and then, in order to obtain the reduced-form variances, we cannot use the MSE of residuals obtained via non-stochastic simulation. Last but not least, it must be recalled that, being the reduced-form disturbances heteroschedastic, even if the structural disturbances are homoschedastic, "the properties" (in our case the variance) "of reduced-form disturbances should not be inferred from those of the structural disturbances" [14,p.308].

Table 1 presents mean and standard deviation, computed across 1000 replicated runs of stochastic simulation over all the simulation period (1961-1973), for some variables. The first consideration that can be drawn from this table regards the presence, for most of the endogenous variables, of an induced effect of heteroschedasticity. This phenomenon can essentially be explained by the nonlinear structure of the model. More exactly, an accurate analysis of the results shows that two different sources of heteroschedasticity can be identified:

(a) direct effect induced by the simultaneous presence of the same endogenous variable in different functional forms (levels, logarithms, percentage changes and other nonlinear transformations);
(b) indirect effect induced by the simultaneous solution of the nonlinear system.

To clarify these points, avoiding at the same time any possible confusion with the problem of heteroschedasticity in the estimation phase, let us consider the case of the variable MMA, whose values, in levels, have a positive trend. In the structural form of the model, the logarithmic transformation (lnMMA) is estimated, which is supposed to be homoschedastic. The pseudo-structural disturbances which are added in the stochastic simulation are homoschedastic as well. If we look, in Table 1, at the simulation results for MMA, in levels, a standard deviation increasing with time can be observed, which is induced by the exponential transformation which allows to pass from lnMMA to MMA. MMA has, in fact, a multiplicative error structure and is, consequently, heteroschedastic for the above mentioned presence of positive trend in its levels. On the other hand, this effect of heteroschedasticity is not as strong as one could expect; it must be noted, in fact, that the variable lnMMA shows an heteroschedastic pattern with a standard deviation decreasing with time for some effects induced by the whole model (Table 1). Analogous considerations can be drawn for other variables, like LICH (included in Table 1), LIDC, WM, etc.

The effect described in (b) can be directly remarked if we look at some variables whose equations have been estimated in levels and at some variables which are defined by nonstochastic (definition) equations. The largest effects can be observed for CPR, VAI and VAT, which show a standard deviation monotonically increasing of approximately 15% from 1961 to 1973.

Another point we want to discuss in this section is the comparison between the standard errors of the structural form and those of the reduced-form. We have already seen that for most of the variables the standard errors of the reduced-form change over time, so that a correct comparison should be made year by year. Nevertheless, in Table 2 in addition to the standard errors of structural and reduced form at 1973, the RMSE and the quadratic mean of the standard errors of the reduced-form over the simulation period are displayed. In the table, only the endogenous variables defined by the respective stochastic equations are included; moreover, in order to make homogeneous comments on the results, the involved variables are presented in the same functional form in which they appear on the left hand side of the related structural equations.

Table 2

Variab.	Struct. Form St.Err.	Red.Form St.Err. (1973)	Quad.Mean St.Err.	RMSE
CPR	302.	518.	473.	528.
MS	53.8	48.4	46.3	51.0
lnMMA	0.06	0.05	0.06	0.06
VAI	135.	239.	208.	234.
VAT	88.2	156.	137.	141.
ΔLICH	2.32	2.67	2.84	3.07
ΔLIDC	1.50	1.51	1.63	1.73
ΔWM	1.82	1.87	1.95	1.82
ΔPI	1.18	1.10	1.15	1.24
ΔPVAI	1.30	1.35	1.37	1.36
lnPA	0.03	.027	.028	.027
ΔPCV	0.94	1.16	1.09	0.85
ΔPVAP	0.70	1.08	1.13	1.15
ΔHYWI	1.41	3.47	3.83	4.26
ΔHYWTA	1.21	1.19	1.18	1.34
SHCS	.009	.009	.009	.009

The following considerations can be drawn from Table 2:

(a) The quadratic means over the simulation period of the reduced-form standard errors are close to the RMSE values. This means, as already mentioned in section 3, that the nonlinearities of the model are not very strong and that, from a practical point of view, also RMSE could be used as a reliable average indicator of the dispersion over time.
(b) When passing from the standard errors of the structural form to those of the reduced-form (both 1973 values and quadratic mean) an increase for most of the variables can be observed.
(c) As far as a comparison between the standard errors of the reduced-form at 1973 and mean over the simulation period is concerned, we can notice that the variables for which there are significant differences are the same for which the presence of high heteroschedasticity has been previously underlined.

The last consideration of this section is based on a comparison, in terms of mean values and standard errors, among the endogenous variables which could be considered as targets in economic policy experiments performed by means of this model. In other words, we try to rank some target variables (VAI, LIDC, PCV, PI, VAT, MM+MS, WM) according to their relative dispersion. A measure of absolute dispersion, which is the standard deviation (displayed, for some of these endogenous variables, in Table 1), is already available, but we have to consider that the mean values of the target variables are quite different from each other, so that a measure of relative dispersion appears to be more appropriate. This relative dispersion can be defined by [22]:

Relat.Dispers. = Absol.Dispers./Average

If, as in our case, the absolute dispersion is the standard deviation and the average is the mean value, the previous ratio is called Karl Pearson's coefficient of variation [15]. It is independent of units used and is generally expressed as a percentage. The coefficients of variation relative to the above mentioned target variables are presented in Table 3 for the years from 1970 to 1973, which, being the final years of the simulation period, are the most interesting, as far as the purposes for which the original model was reestimated are concerned. For each variable, the values are quite constant when passing from one year to the other (this behaviour could be observed also for some other endogenous variables not included in the table, but not for all). Significant differences can be, on the contrary, observed in the same year for different variables; economic policy experiments whose target is a price variable (PCV or PI) seem to be, therefore, much more reliable than experiments whose target are imports (MM+MS).

Table 3

Coefficients of variation

Variab.	1970	1971	1972	1973
VAI	1.30	1.24	1.29	1.30
LIDC	1.55	1.57	1.61	1.49
PCV	0.99	1.02	1.04	1.05
PI	1.09	1.12	1.06	0.95
VAT	1.12	1.14	1.08	1.05
MM+MS	3.75	3.60	3.66	3.52
WM	1.54	1.61	1.64	1.49

5. THE MECHANISM OF TRANSMISSION OF ERRORS

Further experiments have been carried out in order to put into evidence the mechanism of transmission of errors across the model.

Table 4

Variab.	Blocks of the Model					
	DD	FT	DP	LM	WP	ID
CPR	284.	146.	122.	237.	216.	112.
MS	6.	54.	2.5	4.8	4.4	2.3
lnMMA	0.01	0.04	0.01	0.01	0.01	.005
VAI	56.	154.	122.	47.	43.	22.
VAT	87.	45.	102.	72.	66.	34.
$\dot{\Delta}$LICH	0.3	0.8	0.6	2.2	0.2	0.1
$\dot{\Delta}$LIDC	0.1	0.4	0.3	1.5	0.1	0.06
$\dot{\Delta}$WM	0.2	0.4	0.3	0.2	1.6	0.08
$\dot{\Delta}$PI	0.03	0.06	0.05	0.03	0.9	0.01
$\dot{\Delta}$PVAI	0.07	0.1	0.1	0.06	1.3	0.03
lnPA	.005	.002	.002	.004	.02	.002
$\dot{\Delta}$PCV	.07	.003	.02	.06	1.07	.03
$\dot{\Delta}$PVAP	0.47	.13	.10	.09	0.97	.05
$\dot{\Delta}$HYWI	0.5	1.2	0.9	2.4	1.4	1.
$\dot{\Delta}$HYWTA	0.1	0.2	0.2	0.08	0.8	1.
SHCS	.1E-6	.2E-6	.2E-6	.2E-6	.2E-6	.008

Instead of introducing the pseudo-random disturbances into all the behavioural equations simultaneously, they have been introduced into a single sector (made up of one or more equations), while no disturbances have been added to the others even if, to avoid sampling errors, disturbances have been generated for all the stochastic equations.

For year 1973 and for the same variables as in Table 2, Table 4 presents the so obtained results. The values in each column are the standard errors of each variable computed after the introduction of disturbances only in the block corresponding to the column itself, indicated on the top. The diagonal block values are, therefore, the standard errors induced, into a block, by the introduction of pseudo-structural disturbances only into the block itself.

Several considerations can be drawn from a comparison of the results in Tables 2 and 4. First of all, it must be noted that for almost every block the largest standard error is observed when shocking the same block, so that we can infer that the main quantitative source of variability in the block is due to the presence of the disturbance terms in the block itself. Among the standard errors caused by disturbance terms in other blocks, the largest are those due to foreign trade and labour market blocks. This result suggests that foreign trade and labour market are the main system wide sources of errors and confirms the observations in Crivellini [8,pp.34,41,74] about the non-satisfactory structural specifications of the related equations. It can also be observed that in the two cases in which the standard errors induced by other blocks are larger than those due to the blocks under observation (equations of VAI and HYWI), the largest errors are again induced respectively by the blocks of foreign trade and labour market.

A comparison of Tables 2 and 4 also shows that the block of wages and prices, while inducing errors into the other blocks, is practically unaffected by them, once again confirming the hypothesis of the model builders [8,p.75] about the prevalence of a mark-up effect. The wages-prices block has been further on analyzed by introducing pseudo-random disturbances only into the wages equation and by investigating the effects on the prices. These effects (Table 5) are more or less relevant according to the presence, in the behavioural equations, of current WMO (equations of PVAI, PI, PVAP), lagged WMO (equation of PCV) or absence of WMO (equation of PA).

Table 5

	Wages-Prices	Wages
$\dot{\Delta}$WM	1.6	1.4
$\dot{\Delta}$PI	0.9	0.2
$\dot{\Delta}$PVAI	1.3	0.47
lnPA	0.02	0.0005
$\dot{\Delta}$PCV	1.07	0.05
$\dot{\Delta}$PVAP	0.97	0.32

The search of the variables scarcely influenced by shocks in the other blocks can be extended also to those variables which do not appear on the left hand side of behavioural equations. The most interesting is the variable S (inventory investment, Table 6) which is practically insensitive to shocks given to the domestic demand sector as if there were an immediate adjustment of supply to demand modifications (Keynesian hypothesis).

Table 6

Blocks of the Model

	Red.Form St.Err.	DD	FT	DP	LM	WP	ID
S	212.	.5	215.	192.	.32	16.	.12

The last consideration regards the quasi-independence of disturbance effects due to the interaction of blocks. If we square the values in Table 4 and sum for every endogenous variable the variances related to shocks into the single blocks, the result is always not too far from the variance of the reduced-form, whose square root is displayed in Table 2 (both the tables are referred to the year 1973), thus suggesting an approximate null total effect of covariances.

6. CONCLUDING REMARKS

The use of stochastic simulation techniques is suggested in the literature mainly by theoretical considerations, but the performed experiments allow to conclude that also from a practical point of view this methodology could be profitably applied so as to become a powerful instrument either when validating or when using a nonlinear model for economic policy experiments.

By means of stochastic simulation it is possible to compute the reduced-form variances and decompose them into the contributions of each stochastic equation or of each block of equations (analysis of the mechanism of trasmission of errors), so that it is possible to discover the blocks or the equations responsible for the main sources of errors. The reduced-form standard error of each endogenous variable can be regarded as a component of the standard error of forecast, more exactly the component related to the presence of the structural disturbances. In this way, when performing stochastic simulation experiments, information could be obtained for evaluating, at least in part, the risk of forecast for some target variables or the reliability of the model for achieving some economic policy targets.

The experiments reported in this study do not intend to exhaust the area of the possible applications of the stochastic simulation methodology. For example, in recent years (see the several contributions in Hickman [13]), following the suggestions of Adelman and Adelman [1] for the analysis of the business cycles, the cyclical properties of a nonlinear model are frequently investigated by applying spectral analysis to stochastic simulation results.

Finally it must be pointed out that in all the experiments here described only one source of errors has been taken into account, that is the set of the structural disturbances (this is what is properly called "stochastic simulation"); therefore the results should be interpreted as conditional on the exogenous variables and on the estimated parameters.

APPENDIX. THE COMPUTATIONAL ASPECTS

The displayed results have been obtained by means of a package [3] developed at the IBM Scientific Center of Pisa. The package has been written in FORTRAN-G language and works under the operating system VM-370/CMS on the computer IBM/370 model 168 installed at CNUCE (Centro Nazionale Universitario di Calcolo Elettronico) in Pisa. The package consists of approximately one thousand statements, in addition to the statements (nearly one hundred for the model considered) necessary to formalize the model according to the Gauss-Seidel solution method [17]. The required storage for the program is approximately sixty kilobytes; an additional large work area is required for the intermediate results of computation. For the model considered, each replication for each year took about 0.035 seconds of CPU time. Two different methods are available in the program for generating standard normal deviates (direct or inverse method [6]) and two different methods (Nagar [20] or McCarthy [19] algorithms) are available to generate the pseudo-random disturbances.

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