The unemployment volatility puzzle: the role of the underground economy

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Abstract

Relying on the non-negligible role played by the underground economy in the labour market fluctuations, this paper extends the standard matching model à la Mortensen-Pissarides by introducing an underground sector along with an endogenous sector choice for both entrepreneurs and workers. These modifications improve the quantitative properties of the standard matching model, thus providing a possible explanation for the unemployment volatility puzzle.

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1. Introduction

In an influential paper, Shimer (2005) shows that the standard matching model fails to reproduce the large volatility in unemployment and vacancies during the business cycle. In fact, these variables are much more volatile in the U.S. data than in the calibrated model subject to productivity shocks of a realistic magnitude.

This is a very important setback for the profession: the reason is that the Mortensen-Pissarides model (1994) is the "workhorse" model used by academic and government economists to evaluate various economic policies and to study the problem of unemployment (Hagedorn and Manovskii, 2008). Hence, many and influential improvements have been offered to enrich the quantitative properties of the standard matching model and to challenge the unemployment volatility puzzle (see e.g. Shimer, 2004, 2005; Hall, 2005; Garibaldi, 2006; Uren, 2007; Hall & Milgrom, 2008; Hagedorn & Manovskii, 2008).[1]

The current paper belongs to that literature that points out the need to focus on the underground economy. There is in fact substantial economic activity in the underground sector in many developing countries as well as in many transition countries, and the underground sector is also important in some developed economies (Albrecht et al., 2009). Since these underground activities make a sizeable contribution to the GDP, it is difficult to understand the business cycles without some knowledge of the underground economy fluctuations. Indeed, introducing an underground sector in a *Real Business Cycle* framework improves the fit of the model to the data (Busato and Chiarini, 2004). Furthermore, Bosch and Esteban-Pretel (2009) build a search and matching model extended to the underground sector which satisfactorily explains most of the cyclical properties found in the Brazil data.

This paper presents a two-sector search and matching model which addresses the lack of amplification in unemployment and vacancy volatility by introducing the sector choice of individuals. Precisely, the model assumes the exogenous existence of firms (posting vacancies) and workers (searching for jobs); however, heterogeneous entrepreneurs sort themselves into official or unofficial statuses

[1] See Hornstein, Krusell, and Violante (2005), Yashiv (2006) and Mortensen and Nagypál (2007), for exhaustive surveys of this huge and recent literature.

according to their ex-ante entrepreneurial ability levels, and heterogeneous workers direct their searches towards one of the two types of firms according to their ex-ante skill levels. Furthermore, entrepreneurs and workers can move from one sector to the other in reply to exogenous changes in policy parameters.[2]

We then calibrate and simulate the model to try to account for the Shimer puzzle. As in Uren (2007), we compare different steady-state equilibria in which the value of the productivity of the match and the job destruction rate are stochastic and are drawn from a bivariate normal distribution constructed to match the stylised business cycle facts of the U.S. economy.[3] The labour market tightness in both sectors are given at the microeconomic level and endogenously determined in the aggregate economy. Indeed, once the aggregate value of vacancies and unemployment has been calculated in reply to productivity and job destruction shocks, the value of labour market tightness in each sector can be obtained. This process triggers off two extra effects on vacancies and unemployment: the first works on the probabilities to find a job and fill a vacancy (*direct effect*), and the second works on the sector choice of individuals, thus modifying the share of entrepreneurs and workers in the two sectors (*indirect effect*).

Furthermore, since in this model the zero-profit condition is no longer used to determine the labour market tightness, the incidence of wage on job creation is mediate and it is thus possible to maintain the Nash bargaining rule.[4]

The main aim of this paper is to show that, once incidence of the underground economy on the business cycle (i.e. the sector choice of individuals) is accounted for, a matching model is able to replicate most of the fluctuations in unemployment and job vacancies caused by productivity shocks of plausible magnitude.

The rest of the paper is organized as follows: section 2 contains a brief review of the related literature; sections 3 and 4 present a two-sector matching model with an endogenous sector choice for both entrepreneurs and workers; section 5

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[2] Bosch and Esteban-Pretel (2009) consider only direct transitions from informal to formal jobs.

[3] The labour productivity is the key parameter and represents the driving force in business cycle analysis in most macro-models of the labour market. Fluctuations in the separation rate are considered a further potential driving source (Garibaldi, 2006). Indeed, according to the Brazil data, the volatility of unemployment is explained in large part by changes in the job separation rate (Bosch and Esteban-Pretel, 2009).

[4] Recall that in the standard framework (see Pissarides, 2000), the *Job Creation Condition* depicts a negative relationship between labour market tightness and wage, since the zero-profit (or free entry) condition directly relates the two variables.

provides a quantitative evaluation of an economy subject to shocks to productivity and job destruction rate; section 6 concludes.

2. Related Literature

The literature has mainly responded to the Shimer puzzle by suggesting that the wage setting mechanism in the standard model has to be altered in order to break the close link between wages and productivity (Shimer, 2004, 2005; Hall, 2005; Hall & Milgrom, 2008). Indeed, according to the literature, a principal reason for the lack of explanatory power of the standard matching model (in which the labour market tightness are obtained by the zero-profit condition) is that the wage implied by the Nash bargaining rule responds to offset all the effects of productivity shocks on job creation. Hence, following Binmore, Rubinstein and Wolinsky (1986), Hall and Milgrom (2008) propose a strategic bargaining game in which the default option, i.e. the threat point, is the delay rather than the search for an alternative job. This modification limits the influence of labour market conditions on wages.[5] The insight is the following: if the delay is the only outside option, then the wage agreement will not reflect the value of unemployment and, consequently, the wage is more rigid (i.e. less pro-cyclical) than that implied by the standard sharing rule, i.e. the Nash rule. Nevertheless, as claimed by Mortensen and Nagypál (2007), this solution cannot be sufficient to solve the Shimer puzzle by itself.

Several papers take a different route. In particular, there is a piece of the literature which suggests that the problem lies not in the model itself, but in the way the model is typically calibrated. Hence, Hagedorn and Manovskii (2008) propose a new calibration strategy for the two central parameters of the Mortensen-Pissarides model: the opportunity cost of employment and the worker's bargaining power. Indeed, in Hagedorn and Manovskii (2008), the standard matching model may generate the observed volatility of unemployment and vacancies. However, as claimed by Mortensen and Nagypál (2007), they make use of unrealistic calibration values (i.e. a huge opportunity cost of employment and a tiny workers' bargaining power) to solve the puzzle.

[5] Using a simple rigid wage, the response in the vacancy-unemployment ratio to productivity shocks is still too small. Furthermore, the rigid wage assumption is difficult to justify (Mortensen and Nagypál, 2007).

Other recent papers introduce new and interesting features in the standard Mortensen-Pissarides model. Garibaldi (2006) shows that a calibration of the model that explicitly considers hiring freeze and bankruptcy of firms can account for 20 to 35 percent of the variability displayed by the data. Mortensen and Nagypál (2007) find that an augmented matching model, which includes capital costs, countercyclical job destruction rate and a less procyclical wage, explains about 40% of the observed volatility of the vacancy-unemployment ratio. Finally, Uren (2007) displays that the simple introduction of the endogenous decision of an individual to either become entrepreneur or worker can substantially amplify the impact of productivity shocks upon the level of unemployment and vacancies; such a model in fact generates fluctuations in unemployment and vacancies that are roughly 3 and 5 times greater than those produced by the standard model as evaluated by Shimer (2005).

3. The economy

The economy is populated by a continuum of entrepreneurs and a continuum of workers.[6] Each entrepreneur is born with a specific entrepreneurial ability *x*, which is drawn from a known distribution $F:[x_{min},\ x_{max}]\rightarrow[0,1]$. Similarly, each worker is endowed with a different level of skill *q*, with known distribution $G:[q_{min},\ q_{max}]\rightarrow[0,1]$. The economic environment is characterised by a labour market with search frictions and wage bargaining. Each entrepreneur can either operate regularly or against the tax regulations. Further, our notion of underground employment is one of low productivity jobs (see e.g. Boeri and Garibaldi, 2002, 2006).[7]

The matching of vacancies (v) and unemployed (u) per unit of time is regulated by an aggregate matching function: $m_i = m(v_i, u_i)$ with the restrictions $m(v_i, 0) = m(0, u_i) = 0$ and $m(v_i, u_i) \leq min(v_i, u_i)$, where $i \in \{r, s\}$ denotes the sector (with *r* = *regular*, *s* = *shadow*). As usual, the matching function is non-negative, increasing, concave in both arguments and homogenous of degree one

6 Time is continuous, and individuals are risk neutral and infinitely lived. We neglect the endogenous decision of individuals to either become entrepreneurs or workers because it is widely discussed in the matching literature (see Fonseca, Lopez-Garcia and Pissarides, 2001; Pissarides, 2002, and Uren, 2007). This framework can thus be seen as a following step.

7 We neglect possibilities of moonlighting, so workers can perform only one activity at a time.

(Pissarides, 2000; Petrongolo and Pissarides, 2001), so that the vacancy-filling rate, $f(\theta_i) \equiv m(v_i, u_i)/v_i = m(1, \theta_i^{-1})$, and the job-finding rate, $g(\theta_i) \equiv m(v_i, u_i)/u_i = m(\theta_i, 1)$, are both functions of the vacancy-unemployment ratio, $\theta_i = v_i / u_i$. Standard technical assumptions are assumed:

$f'(\theta_i) < 0$, $f''(\theta_i) > 0$, $g'(\theta_i) > 0$, $g''(\theta_i) < 0$;

$\lim_{\theta_i \to \infty} f(\theta_i) = \lim_{\theta_i \to 0} g(\theta_i) = 0$, $\lim_{\theta_i \to 0} f(\theta_i) = \lim_{\theta_i \to \infty} g(\theta_i) = \infty$, with $i \in \{r, s\}$.

Following Garibaldi (2006), each individual takes as given θ_i, and in the aggregate economy the labour market tightness depends on the aggregate number of vacancies and the stock of unemployed.

The Bellman equations used to find infinite horizon steady-state solutions are:

$$rJ_r = p - w_r - \tau + \delta \cdot [V_r - J_r], \qquad rJ_s = \kappa \cdot p - w_s + (\delta + \rho) \cdot [V_s - J_s]$$

$$rW_r = w_r + \delta \cdot [U_r - W_r], \qquad rW_s = w_s + (\delta + \rho) \cdot [U_s - W_s]$$

$$rV_i = -c_i + f(\theta_i) \cdot [J_i - V_i], \qquad rU_i = z_i + g(\theta_i) \cdot [W_i - U_i] \qquad \text{with } i \in \{r, s\}$$

where J_i is the value of a filled job; W_i is the value for being employed; V_i is the value of a vacancy; U_i is the value for seeking a job; r is the discount rate; p is the productivity of the match; w_i is the wage; δ is the job destruction rate; c_i is the vacant job cost and z_i is the opportunity cost of employment. In the regular sector firms pay a production tax τ, while in the underground sector this tax is evaded and there is a monitoring rate ρ. Conditional on being monitored in the underground sector, the irregular job is destroyed. Furthermore, as in Bosch and Esteban-Pretel (2009), evading taxation implies that irregular firms can only take advantage of a fraction $\kappa \in (0,1)$ of the productivity of the match. The key payoffs for the entry into the labour market, i.e. the value of a vacancy and the value for seeking a job, can be expressed as single-valued functions of tightness θ_i, with $\partial U_i / \partial \theta_i > 0$ and $\partial V_i / \partial \theta_i < 0 \;\; \forall i$ (see *Appendix A*).[8] Finally, wages are assumed to be the outcome of a Nash bargaining problem:

$$w_i = arg\,max\{(W_i - U_i)^{\beta_i} \cdot (J_i - V_i)^{1-\beta_i}\} \Rightarrow (W_i - U_i) = \frac{\beta_i}{(1-\beta_i)} \cdot (J_i - V_i) \qquad \text{with } i \in \{r, s\}$$

[8] Intuitively, this is straightforward to understand since the greater θ_i the smaller the probability of filling a vacancy for the firm, and the greater θ_i the higher the probability of finding a job for the worker.

where $\beta_i \in (0,1)$ is the surplus share for labour. Simple manipulations thus yield the formulae of wages:

$$w_r = (1-\beta_r)\cdot rU_r(\theta_r) + \beta_r \cdot (p \cdot y_r - \tau - rV_r(\theta_r))$$

$$w_s = (1-\beta_s)\cdot rU_s(\theta_s) + \beta_s \cdot (p \cdot y_s - rV_s(\theta_s))$$

with $w_i'(\theta_i) > 0 \;\; \forall i$, since $V_i'(\theta_i) < 0$, and $U_i'(\theta_i) > 0 \;\; \forall i$.

4. Entrepreneurs and workers

To start up a regular business an entrepreneur has to pay an extra cost h, which measures the burden of overall constraints in the official sector (the so-called *barriers to entrepreneurship*).[9] Entrepreneurial ability x influences this start-up cost, but as in Pissarides (2002) it does not affect the job productivity. Therefore entrepreneurs whose x satisfies the following inequality enter the regular sector:

$$V_r(\theta_r) - x \cdot h \geq V_s(\theta_s) \qquad [3]$$

hence, regular (good) entrepreneurs have a low x, irregular (poor) entrepreneurs have a high x. It follows that there is a *reservation entrepreneurial ability* $x = R$, with $R'(\theta_r) < 0$ and $R'(\theta_s) > 0$, such that an entrepreneur enters the official sector if $x \leq R$, otherwise he/she starts a business in the underground sector. Therefore, a fraction $F(R)$ of the entrepreneurs (either posting a vacancy or producing) are regular, while the complementary fraction $1 - F(R)$ are irregular. The evolution of vacancies in each sector is thus given by:

$$\dot{v}_r = \underbrace{[F(R) - v_r]\cdot \delta}_{inflow} - \underbrace{v_s \cdot f(\theta_r)}_{outflow}$$

$$\dot{v}_s = \underbrace{[1 - F(R) - v_s]\cdot (\delta + \rho)}_{inflow} - \underbrace{v_s \cdot f(\theta_s)}_{outflow}$$

steady-state implies $\dot{v}_r = \dot{v}_s = 0$, so that:

$$v_r = \frac{F(R)\cdot \delta}{f(\theta_r) + \delta} \qquad [4]$$

$$v_s = \frac{[1 - F(R)]\cdot(\delta + \rho)}{f(\theta_s) + (\delta + \rho)} \qquad [5]$$

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[9] This can be explained by higher entrance barriers into the official sector or access costs to legality associated with excessive regulations, administrative burdens, licence fees, bribery (see e.g. Bouev, 2005), but also money protection if the firm copes with a context where organized crime operates.

From the supply side, workers can be either high-skilled or low-skilled, thus achieving two different levels of productivity, depending on whether they choose to invest in education properly. Indeed, formal education enhances the worker's skill (Laing et al., 1995). As in Dulleck et al. (2006), workers with higher ability (i.e. in the specific instance with lower *q*) have lower costs of higher education *e*. Therefore workers whose *q* satisfies the following inequality enter the regular sector: [10]

$$U_r(\theta_r) - q \cdot e \geq U_s(\theta_s) \qquad [6]$$

as a result, there will be a *cut-off* value Q, with $Q'(\theta_r) > 0$ and $Q'(\theta_s) < 0$, below/equal which workers are regular/high-skilled and above which they are irregular/low-skilled. Then, $G(Q)$ and $1 - G(Q)$ are the share of high-skilled and low-skilled workers, respectively. Under these assumptions, the unemployment rate adjusts according to the following *law of motion*:

$$\dot{u}_r = [G(Q) - u_r] \cdot \delta - u_r \cdot g(\theta_r)$$

$$\dot{u}_s = [1 - G(Q) - u_s] \cdot (\delta + \rho) - u_s \cdot g(\theta_s)$$

in steady-state, unemployment is thus given by:

$$u_r = \frac{G(Q) \cdot \delta}{g(\theta_r) + \delta} \qquad [7]$$

$$u_s = \frac{[1 - G(Q)] \cdot (\delta + \rho)}{g(\theta_s) + (\delta + \rho)} \qquad [8]$$

In the aggregate economy, the equilibrium value of θ_i must be consistent with the value assumed at the microeconomic level. Formally, the aggregate definition of θ_i is:

$$\theta_r = \frac{v_r}{u_r} \Rightarrow \theta_r = \frac{F(R) \cdot \delta}{f(\theta_r) + \delta} \cdot \frac{g(\theta_r) + \delta}{G(Q) \cdot \delta} \qquad [9]$$

$$\theta_s = \frac{v_s}{u_s} \Rightarrow \theta_s = \frac{[1 - F(R)] \cdot (\delta + \rho)}{f(\theta_s) + (\delta + \rho)} \cdot \frac{g(\theta_s) + (\delta + \rho)}{[1 - G(Q)] \cdot (\delta + \rho)} \qquad [10]$$

which are non-negative, by construction, since $v_i, u_i \in [0,1]$. Rearranging them, the equilibrium value of θ_r and θ_s must satisfy the following equations:

$$[g(\theta_r) + \theta_r \cdot \delta] \cdot G(Q) - F(R) \cdot [g(\theta_r) + \delta] = 0 \qquad [9']$$

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[10] Indeed, there is a strong negative correlation between informal-sector employment and education level within countries (cf. Albrecht et. al., 2009).

$$[g(\theta_s)+\theta_s \cdot (\delta+\rho)]\cdot[1-G(Q)]-[1-F(R)]\cdot[g(\theta_s)+(\delta+\rho)]=0 \qquad [10']$$

since $g(\theta_i)=\theta_i \cdot f(\theta_i)$. A unique equilibrium exists if there is a unique value of θ_r and θ_s that solves equations [9′] and [10′], respectively. It is possible to verify that,

Proposition 1. *A steady state equilibrium exists and is unique (see Appendix C for the proof).* Furthermore,

Proposition 2. *Given the assumption of search frictions in both sectors, the equilibrium is characterized by an interior solution (see Appendix D).* [11]

This model differs from the standard Mortensen-Pissarides framework because the effects of exogenous changes to parameters on vacancy-unemployment ratio depend on the variation of the threshold values. Indeed, in this model the labour market tightness in each sector is given by the aggregate level of vacancies and unemployment. The comparative static results are generally intuitive and straightforward, since the effects of higher monitoring, taxes and labour market regulations are common in the literature:

$$\frac{\partial R}{\partial \rho}>0;\ \frac{\partial R}{\partial \tau}<0;\ \frac{\partial R}{\partial c_r}<0;\ \frac{\partial R}{\partial p}>0;$$

furthermore, this last result implies that in boom regular firms take more advantage of the increase in productivity (irregular jobs are less productive than regular ones since $\kappa<1$).

5. Quantitative evaluation of the model

In order to evaluate the quantitative properties of the model, a numerical simulation is performed. The model calibration is reported in table 1 (see end). To take the sectoral differences into account, we make use of Boeri and Garibaldi's (2006) calibrations (see table 2).

As in Uren (2007), we compare different steady states as the value of the productivity (*p*) and the job destruction rate (δ) vary. Consistent with the data presented in Shimer (2005), the productivity (*p*) and the job destruction rate (δ)

[11] This is anything but a trivial result. Indeed, as claimed by Bouev (2005), the most important weakness in the underground economy theory is represented by the restrictive assumptions required to find an interior equilibrium in which the underground sector coexists with the regular one in the long run.

are stochastic and are drawn from a bivariate normal distribution constructed to match the stylised business cycle facts of the U.S. economy. Hence, the mean value of p is normalised to 1 and the standard deviation is set to 0.019, whereas the mean of δ is set to 0.1 and the standard deviation is set to 0.075. Finally, the correlation between these variables is -0.592, by construction.

The sector choice is also solved analytically by the variation of the threshold values R and Q. Both the cumulative distribution functions are assumed to be negative exponential.[12]

The simulation works as follows: given p and δ the corresponding values of w_r, w_s, R, Q, v_r, v_s, u_r and u_s are calculated. Furthermore, once the aggregate value of vacancies and unemployment has been calculated, we get the "updated" value of labour market tightness in each sector. In short, this process triggers off three effects on vacancies and unemployment, thus amplifying their volatility. First, the productivity and job destruction shocks work on the sector choice of individuals, thus modifying the share of entrepreneurs and workers in the two sectors (*starting effect*). Second, the aggregate value of labour market tightness in each sector works by modifying the probabilities to find a job and fill a vacancy (*extra direct effect*). Finally, the aggregate value of labour market tightness in each sector also affects the sector choice of individuals (*extra indirect effect*).

This process is repeated 10,000 times and the correlations and the standard deviations of the key variables are calculated. The simulation results are shown in table 3. We can summarize them as follows: [13]

- *Aggregate Unemployment*. The standard deviation of aggregate unemployment ($u = u_r + u_s$) explains about 70% of the unemployment volatility observed in the data by Shimer (2005);
- *Aggregate Vacancies*. A two-sector matching model explains more than 70% of the observed volatility of job vacancies;
- *Beveridge Curve*. The model succeeds in correctly predicting a downward sloping vacancy-unemployment relationship in both sectors;

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[12] A negative exponential distribution is used by Boeri and Garibaldi (2006) for the distribution of labour productivity.

[13] The statistics for job vacancies and unemployment are those reported in Shimer (2005) for an Hodrick-Prescott (HP) filter with a smoothing parameter of 10^5 instead of 1600 as followed by Andolfatto (1996) and Merz (1995). However, as claimed by Hornstein et al. (2005), the choice of smoothing parameter has no impact on the unemployment and vacancy statistics.

- *Cyclical Behaviour*. The counter-cyclical behaviour of the underground economy is caught by the negative correlation between p and v_s.

6. Conclusions

Relying on the non-negligible role played by the underground economy in the labour market fluctuations, this paper extends the standard matching model *à la* Mortensen-Pissarides by introducing an underground sector along with an endogenous sector choice for both entrepreneurs and workers. These modifications improve the model's implications for the amplification of shocks, thus providing a possible explanation for the unemployment volatility puzzle. Indeed, a two-sector matching model explains about 70% of the unemployment volatility observed in the data by Shimer (2005), and more than 70% of the observed volatility of job vacancies.

Appendixes

Appendix A: Value functions

From the Bellman equations very simple algebra gives:

$$[J_r - V_r] = \frac{p \cdot y_r - w_r - \tau + c_r}{r + \delta + f(\theta_r)}; \qquad [J_s - V_s] = \frac{p \cdot y_s - w_s + c_s}{r + \delta + \rho + f(\theta_s)}$$

$$[W_r - U_r] = \frac{w_r - z_r}{r + \delta + g(\theta_r)}; \qquad [W_s - U_s] = \frac{w_s - z_s}{r + \delta + \rho + g(\theta_s)}$$

Hence, it is straightforward to get:

$$rV_r = \frac{f(\theta_r) \cdot (p \cdot y_r - w_r - \tau) - c_r \cdot (r + \delta)}{r + \delta + f(\theta_r)} \qquad \text{[A.1]}$$

$$rV_s = \frac{f(\theta_s) \cdot (p \cdot y_s - w_s) - c_s \cdot (r + \delta + \rho)}{r + \delta + \rho + f(\theta_s)} \qquad \text{[A.2]}$$

$$rU_r = \frac{g(\theta_r) \cdot w_r + z_r \cdot (r + \delta)}{r + \delta + g(\theta_r)} \qquad \text{[A.3]}$$

$$rU_s = \frac{g(\theta_s) \cdot w_s + z_s \cdot (r + \delta + \rho)}{r + \delta + \rho + g(\theta_s)} \qquad \text{[A.4]}$$

with $\partial V_i / \partial \theta_i < 0$ and $\partial U_i / \partial \theta_i > 0$, $i \in \{r, s\}$. Further,

$lim_{\theta_r \to 0} rV_r = py_r - w_r - \tau \qquad lim_{\theta_s \to 0} rV_s = py_s - w_s; \qquad lim_{\theta_i \to 0} rU_i = z_i$

$lim_{\theta_i \to \infty} rV_i = -c_i; \qquad lim_{\theta_i \to \infty} rU_i = w_i \qquad$ with $i \in \{r, s\}$

Appendix B: Existence and uniqueness of the equilibrium

To prove the existence and uniqueness of the equilibrium, let us define:

$$\Gamma(\theta_r) = [g(\theta_r) + \theta_r \cdot \delta] \cdot G(Q) - F(R) \cdot [g(\theta_r) + \delta] \qquad \text{[B.1]}$$

$$\Gamma(\theta_s) = [g(\theta_s) + \theta_s \cdot (\delta + \rho)] \cdot [1 - G(Q)] - [1 - F(R)] \cdot [g(\theta_s) + (\delta + \rho)] \qquad \text{[B.2]}$$

and note that,

$lim_{\theta_r \to 0} \Gamma(\theta_r) = -F(R) \cdot \delta; \qquad lim_{\theta_s \to 0} \Gamma(\theta_s) = -[1 - F(R)] \cdot (\delta + \rho);$

$lim_{\theta_r \to \infty} \Gamma(\theta_r) = +\infty$ and $lim_{\theta_s \to \infty} \Gamma(\theta_s) = +\infty$, by the *l' Hôpital rule*.[14]

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[14] Note that, in order to apply the *l' Hôpital rule*, we can rewrite [B.1] and [B.2] as follows:

$$[g(\theta_r) + \delta \cdot \theta_r] \cdot G(Q) \cdot \left[1 - \frac{[g(\theta_r) + \delta] \cdot F(R)}{[g(\theta_r) + \delta \cdot \theta_r] \cdot G(Q)}\right];$$

$$[g(\theta_s) + (\delta + \rho) \cdot \theta_s] \cdot [1 - G(Q)] \cdot \left[1 - \frac{[g(\theta_s) + \delta + \rho] \cdot [1 - F(R)]}{[g(\theta_s) + (\delta + \rho) \cdot \theta_s] \cdot [1 - G(Q)]}\right].$$

Furthermore, for given R and Q, we get:

$$\Gamma'(\theta_r) = [g'(\theta_r) + \delta] \cdot G(Q) - F(R) \cdot g'(\theta_r) > 0,$$

$$\text{if } G(Q) \cdot \left[1 + \frac{\delta}{g'(\theta_r)}\right] > F(R);$$

$$\Gamma'(\theta_s) = [g'(\theta_s) + \delta + \rho] \cdot [1 - G(Q)] - [1 - F(R)] \cdot g'(\theta_s) > 0$$

$$\text{if } [1 - G(Q)] \cdot \left[1 + \frac{\delta + \rho}{g'(\theta_r)}\right] > [1 - F(R)].$$

More importantly, when R and Q are handled as variables the previous result is obtained without any restriction. Indeed, since $R'(\theta_r) < 0$, $R'(\theta_s) > 0$, $Q'(\theta_r) > 0$ and $Q'(\theta_s) < 0$, when θ_r rises, $F(R)$ decreases and $G(Q)$ increases. Similarly, when θ_s rises, $[1 - F(R)]$ decreases and $[1 - G(Q)]$ increases. Furthermore, according to the model, the labour market tightness in the two sectors cannot move in the same direction. Hence, their effects on R and Q cumulate and strengthen. As a result, equations [B.1]–[B.2] are strictly increasing and continuous functions of θ_r and θ_s, respectively. Therefore, the *intermediate value theorem* implies existence of a solution to [9′]-[10′] and the *monotone nature* of $\Gamma(\theta_r)$ and $\Gamma(\theta_s)$ guarantees uniqueness.

Appendix C: Interior solution

Note that we can rewrite the aggregate definition of θ_r and θ_s as the ratio of entrepreneurs-workers in each sector:

$$\frac{\theta_r \cdot [f(\theta_r) + \delta]}{g(\theta_r) + \delta} = \frac{F(R)}{G(Q)} \quad \text{[C.1]}$$

$$\frac{\theta_s \cdot [f(\theta_s) + (\delta + \rho)]}{g(\theta_s) + (\delta + \rho)} = \frac{[1 - F(R)]}{[1 - G(Q)]} \quad \text{[C.2]}$$

it is straightforward to prove that equations [C.1] and [C.2] are equal to zero (or infinite) if θ_i is equal to zero (or infinite). But as shown in *Appendix A*, when $\theta_i = 0$ or $\theta_i = \infty$ the value functions rV_i and rU_i are no longer functions of labour market tightness. Indeed, when $\theta_i = 0$ the vacancies are immediately filled and when $\theta_i = \infty$ the job-seekers are immediately employed. As a result, a corner solution can exist only in absence of search frictions in one of the two sectors.

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Tables

Table 1. Model calibration

Matching function	$m_i = v_i^{\eta} \cdot u_i^{1-\eta}$		Petrongolo & Pissarides (2001)
Parameter	***Notation***	***Value***	***Source / Target***
job-finding rate elasticity with respect to market tightness	η	0.4	Uren (2007)
matching function elasticity with respect to unemployment rate	$1 - \eta$	0.6	Uren (2007)
productivity loss in the irregular sector	κ	0.81	Bosch and Esteban-Pretel (2009)
discount rate	r	0.012	Shimer (2005); Uren (2007)
bargaining power of workers	β	0.50	Uren (2007); Boeri & Garibaldi (2006)
barriers to entrepreneurship (U.S. index)	h	1.236	OECD (2009)
opportunity cost of employment	z	0.4	Shimer (2005)
higher education cost (U.S. index on OECD average)	e	1.896	OECD (2009)
vacant job cost	c	0.215	Uren (2007)
exponential distribution parameter	λ	1	Boeri & Garibaldi (2006)
Variable			
vacancies	v	endogenous	*law of motion* (steady state solution)
unemployment	u	endogenous	*law of motion* (steady state solution)
wage	w	endogenous	Nash bargaining rule
entrepreneurial ability threshold	R	endogenous	*cut-off condition*
worker's ability threshold	Q	endogenous	*cut-off condition*
Key parameter			
productivity of the match	p	stochastic	simulation
job destruction rate	δ	stochastic	simulation

Table 2. Sector calibration[*]
(source: *Boeri & Garibaldi*, 2006)

	notation	***regular***	***shadow***
production tax	τ	0.20	–
monitoring rate	ρ	–	0.06
labour market tightness	θ	2.70	0.16
job-finding rate	$g(\theta)$	0.82	0.28
vacancy-filling rate	$f(\theta)$	0.30	1.75

Table 3. Results

Standard deviation	0.019	0.075	0.112	0.103	0.098	0.084
Correlations	p	δ	v_r	v_s	u_r	u_s
p	1					
δ	-0.596	1				
v_r	0.259	0.293	1			
v_s	-0.215	0.341	-0.014	1		
u_r	-0.219	0.352	-0.095	0.165	1	
u_s	0.186	0.323	0.185	-0.149	-0.221	1

Aggregate Statistics (standard deviation)	σ_u	σ_v
our calibration	0.128	0.145
Uren's calibration (2007)	0.124	0.087
standard calibration (Shimer, 2005)	0.031	0.011
volatility observed in the data by Shimer	0.190	0.202

[*] As in Boeri and Garibaldi (2006), we assume that β and η are identical in the two sectors. Note that the values of labour market tightness, job-finding rate and vacancy-filling rate reported in table 2 are the starting exogenous values of these variables.