Axiom of Monotonicity: An Experimental Test

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Abstract

The Axiom of Monotonicity (AM) is a necessary condition for a number of expected utility representations, including those obtained by de Finetti (1930), von Neumann and Morgenstern (1944) and Savage (1954). The paper reports on experiments that directly test AM by eliminating strategic uncertainty, context, and peer effects. In this sterile and simple environment we do not observe AM violations under uncertainty but we do observe violations under ambiguity.
JEL codes: D9, C7, C9.
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1 Introduction

Ellsberg (1961) asserted that under ambiguity, decision makers violate either Savage's Sure Thing Principle (Postulate 2 in Savage) or complete and transitive preferences over acts. Since then, Savage's axioms have been relaxed to derive, amongst others, the Choquet Expected Utility representation in Schmeidler (1989), the Maxmin Expected Utility representation in Gilboa and Schmeidler (1989) and the representation of Klibanoff, Marinacci and Mukerji (2005). A weaker postulate, implied by the Sure Thing Principle (STP), is the Axiom of Monotonicity (AM). This axiom states that if some act is preferred to another in all states of the world, then the same preference should hold under uncertainty over the state space.[1] Various versions of AM, act as *necessary* conditions for all the above mentioned expected utility representations (see Gilboa 2009).

Apart from these representations, the intuitively appealing Axiom of Monotonicity is a *necessary* condition for the earlier expected utility representation obtained by Savage (1954). Appropriate modifications of the axiom act as necessary conditions for representations obtained in de Finetti (1930), von Neumann and Morgenstern (1944), and Anscombe and Aumann (1963). In game-theoretic settings, AM provides an epistemic foundation for the choice of dominant strategies.

In this paper we present an experimental test of AM. To our knowledge this is the first test of AM in a salient and non-strategic setting. Our results indicate that AM is significantly violated under ambiguity but not under uncertainty.[2] Thus, under ambiguity, our results favor expected utility

[1]Savage used the term Sure Thing Principle to motivate Postulate 2. Ellsberg calls postulate 2, the Sure Thing Principle and so do Gilboa (2009) and others. Shafir and Tversky (1992a) and others, however, call AM, the Sure Thing Principle.

[2]The literature is not consistent in the use of the terms ambiguity and uncertainty. We are also loose in using these terms. By uncertainty we mean a situation where people are likely to agree on probabilities. By ambiguity we mean a situation where people need not agree on probabilities or that probability is a correct metric of measurement. Our notion of ambiguity is consistent with a Knightian notion of uncertainty.

representations which do not assume AM. Examples being the "case based qualitative beliefs" of Gilboa and Schmeidler (1995, 2001) and the state-space free approach of Ahn (2008).

2 The Axiom of Monotonicity

In the world of Savage a decision maker is aware of a set of actions A, a set of states of nature S and a set of consequences Z. Acts are functions from S to Z and A is the set of all such functions. An event in E is a subset of S and E^c is the complement of E in S. Formally, the decision maker has complete and transitive preference, $\succeq$ $(\succ)$, over all acts in A. Define conditional preference of f over g in E as; $f \succ_E g$ if and only if $f' \succ g'$ where $f = f'$ and $g = g'$ in E and $f' = g'$ in E^c (Gilboa, 2009). Then, AM is stated as :

The axiom of monotonicity (AM): If $f \succ_E g$ and $f \succ_{E^c} g$ then $f \succ g$.

AM should not be confused with Savage's Sure Thing Principle (STP) which is defined as follows. Let f, f', g' and g be four acts and E be a subset of S such that: (i) $f(s) = f'(s)$ and $g(s) = g'(s)$ for all s belonging to E and (ii) $f(s) = g(s)$ and $f'(s) = g'(s)$ for all s not belonging to E. Then, STP states that $f \succ g$ if and only if $f' \succ g'$. As mentioned earlier, there are several papers, both theoretical and experimental, which follow from Ellsberg's criticism of STP. Halevy (2007) provides a nice experimental evaluation of these theories. The paper also provides an adequate discussion of the literature.

AM, however, is a weaker condition. It is implied by STP along with completeness and transitivity (see Gilboa 2009, pp. 141). However, AM may hold even when STP is violated. To see this, consider the following example where STP is violated but AM is not.

	s_1	s_2
f	a	b
f'	a	a
g	b	b
g'	b	a

The table above depicts a state space $S = \{s_1, s_2\}$, two consequences $Z = \{a, b\}$ and the set of all acts $A = \{f, f', g.g'\}$. Let preferences be given by $f \succ g$, $f' \succ g$, $g' \succ f'$, $g' \succ g$, $f' \succ f$, $g' \succ f$. These preferences are complete and do not violate transitivity. Now using $f \succ g$ and the definition of conditional preferences we get $f' \succ_{s_1} g$. Using $g' \succ f$, we get $f' \succ_{s_2} g$. We already have by assumption that $f' \succ g$. So AM is satisfied for the pair of actions f' and g. The reader can verify that the antecedent of AM is not satisfied for any other pair of acts. This implies that AM holds.[3] It is easy to see that $f \succ g$ and $g' \succ f'$ violate STP.

AM is a fundamental axiom for most of decision theory and has natural implications in game theoretic settings as well. In a game, player i's pure strategy can be thought of as an act which maps from opponents' pure strategy profiles (the state space) to her own payoffs. If for all such profiles (states), a strategy s_i is strictly preferred to any other strategy then s_i is said to be a dominant strategy. When i knows the game, and knows that strategy choice is independent across players, she knows that she can never be better off by choosing a strategy different from s_i.

Shafir and Tversky (1992a) were perhaps the first to question the validity of AM. They termed the violations in their experiment, the "disjunction effect." They ran several hypothetical experiments. In one of them, subjects were asked if they would choose to go for a vacation to Hawaii, conditional on knowing that they have "passed" or "failed" an exam. Around 55% of the subjects opted for the vacation under either contingency. Another set of

[3]Notice that although $\succ$ is complete, conditional preferences, $\succ_{s_1}$, may not be. For example, preference $\succ_{s_1}$is not defined between f and f'. A resolution could then be to say that f is indifferent to f' in s_1. Since the completeness of conditional preferences do not matter for the purpose of this paper, we shall not deal with this issue any further.

subjects were asked to chose before they knew the result. Only 32% opted for the vacation. These figures are quite remarkable and thought-provoking. However, because these experiments were non-salient and placed in a strong context it is not clear how to interpret the results.

Our main objective was to design a simple and direct test of AM. In our experiment, subjects first made a sequence of two choices, each between two different monetary amounts. These choices were very simple. Each boiled down to essentially choosing between a higher and a lower amount of money. Then, we presented each subject with a choice between two lotteries. The first, "dominant" lottery, used prizes that were revealed preferred, i.e., equal to the amounts chosen earlier, and the "dominated" lottery used prizes that were revealed inferior. With this structure a violation of AM would appear as the choice of the dominated lottery.

The nature of uncertainty was at the heart of our experiment. To obtain the most direct test of AM we first implemented choice under simple uncertainty. We were very explicit about all the details of the randomization process, used a physical randomization device - a bingo cage, and set the probability for resolving the chosen lottery at 50%[4]. But upholding AM under these simple conditions does not necessarily imply that AM holds. A tougher test of AM is one under ambiguity. For example, the disjunction effect, which is at present the strongest evidence suggesting violation of AM, has been found exclusively under conditions of ambiguity[5]. Therefore, in the second treatment, we induced ambiguity by giving subjects no information about the number of balls in the bingo cage. Under these conditions subjects

[4]Notice that although AM is independent of beliefs we wanted to present subjects with very simple probabilities that they are familiar with.

[5]There is a great leap from the simple uncertainty environment where the subject is familiar with the randomization device and understands the chances over outcomes, and the ambiguity environment where the chances are obscure. Under ambiguity, computing the set of relevant states, as Savage referred to them, may be quite complex. This higher complexity of the state-space may then lead the subject to apply an alternative heuristic for choosing between the lotteries, e.g., see the case-based approach by Gilboa and Schmeidler (1995).

could disagree on what the objective probabilities are.

In other experiments by Shafir and Tversky (1992b), subjects played a sequence of various prisoner's dilemma games with randomly chosen opponents. At times, subjects saw their opponents move before choosing their own actions and at times they did not. Again more than 30% of subjects who defected whenever they observed the opponent's choice, cooperated when the opponents's choice was unobservable. Croson (1999) used across-subject design to support the disjunction effect hypothesis. In her experiment, subjects often cooperated in the simple prisoner's dilemma. But when asked to state their belief about the opponent's play before choosing their own strategy, they defected at much higher rate.

Because our experiment is within-subject it could also be viewed as a screening procedure for subjects' behavioral types. We could identify each subject as being behaviorally consistent or inconsistent with AM. This property allows us to ask how inconsistency with AM translates into play in the prisoner's dilemma (PD) game. After we screened the subjects for their types we had them play a one-shot prisoner's dilemma with a randomly chosen opponent. A naive conjecture could be that those who have violated AM should cooperate at higher rate than the rest. However, as we will argue later, this conjecture is far from clear because the PD game is a much more complicated environment. It presents subjects with strategic ambiguity and could invoke social or other regarding preferences (e.g., Andreoni and Samuelson 2006, Fehr and Schmidt 2000, Bolton and Ockenfels 1999, Rabin 1993) that are absent in our screening procedure and are typically hard to measure or control[6].

[6] These are also the reasons why games are not an ideal environment for testing AM.

3 The experiment and hypotheses

3.1 The experiment

The objective was to create a simple, nonstrategic environment in which violation of AM can be observed directly. For this purpose we designed an experiment in which each treatment had two parts and each part consisted of two tasks. The first part was intended to elicit the antecedent of AM and the second to verify its implication. The first part was very simple. In task 1 the subject was asked to choose between two options R (Right) and L (Left). If she chose R she got \$85 and if she chose L she got \$75. In task 2, she again had to choose between two options R and L. But now if she chose R she received \$35 and if she chooses L she received \$25.

In task 3 a subject was asked to choose between the "dominant" lottery R and a the "dominated" lottery L as shown in the Figure 1, panel (a). AM implies that if the subject chose the higher amount in both tasks 1 and 2 then the same subject should prefer the dominant lottery R, with prizes \$85 and \$35, to the dominated lottery L with prizes \$75 and \$25[7].

Figure 1: Task 3

Fixed Columns

	Left	Right
B-ball Num. > 20	75	85
B-ball Num. ≤ 20	25	35

(a)

Switched Columns

	Left	Right
B-ball Num. > 20	85	75
B-ball Num. ≤ 20	25	35

(b)

Note: "B-ball Num." refers the number on the ball chosen from the bingo cage.

Interesting situations occur when the subject takes the lower amount in one of the tasks 1 or 2. Then, Fixed Columns (FC) version of task 3, as shown

[7] Our payoffs are very similar to those used by Shafir and Tversky (1992b) and Croson (1999) who have both found a large amount of disjunct behavior in the prisoner's dilemma game with almost identical payoffs. The only difference is that in our case the lower payoffs of L and R are 25 and 35 instead of 30 and 35.

in the Figure 1 panel (a), is a test of AM *only* if we are willing to believe that the subject had made a mistake in tasks 1 or 2. But if say the lower amount were truly preferred to the higher amount in task 1, then the appropriate test of AM is the Switched Columns (SC) formulation of task 3 as shown in panel (b). Similar SC formulation is needed for a choice of the lower amount in task 2. We ran one of our treatments (the Uncertainty treatment) under both conditions FC and SC[8]. The other treatment (Ambiguity) was run only under SC condition.

The lotteries R and L in task 3 were resolved with the same randomizations device - a bingo cage. The balls in the bingo cage were uniquely labeled with numbers between 1-40. The higher prize of the chosen lottery was paid out whenever the number on the ball drawn from the bingo cage exceeded 20. Otherwise, the subject earned the lower prize. In the "Uncertainty" treatment we gave the subjects all the details about the bingo cage. Subjects were told that there were exactly 40 uniquely labeled balls in the bingo cage. Then, they were all invited come to the front of the room[9] where all the balls from the bingo-cage were lined up and ordered in the ascending order so they could easily inspect that all we told them was true. In the second "Ambiguity" treatment subjects were told that the balls were uniquely labeled from 1-40 but we did not reveal anything about how many balls were in the bingo cage. The bingo-cage contained 35 balls. Subjects did see the bingo cage placed in the front of the room but were not invited to come and inspect the contents.

The final task of the experiment was the prisoner's dilemma game with

[8] Only one treatment was run under both conditions because in the experiment only a few subjects had chosen the lower amount in tasks 1 or 2. It would have been very costly to run both treatments under both conditions. The Uncertainty treatment was deemed more appropriate because there the behavior of subjects who had chosen the higher amount in both tasks 1 and 2 was very convincing: they committed virtually zero violations in task 3. Therefore, for example under the assumption that subject's choice of the lower amount in one of the tasks 1 or 2 was due to a mistake and mistakes are not correlated across tasks, we would have had a clear prediction of no violation of AM under FC condition even for this group of subjects. We refer the reader to the results section for further discussion.

[9] This was done row-by-row.

payoffs we've been using all along. Subjects were told that in this task (and only in this task) they are matched with one randomly chosen participant. The frame of the PD is presented in Figure 2

Figure 2: Task 4

	Left	Right
The person who you are matched with chose: Left	75 , 75	25 , 85
The person who you are matched with chose: Right	25 , 85	35 , 35

To minimize the chance of distortions due to possible peer and experimenter effects[10] we minimized social distance by adopting a double-blind protocol. Subjects were separated from each other by blinders that fully surrounded each of them and provided complete privacy. In the experiment each subject was identified by a number that was inscribed on a card randomly

[10] Tasks 1 and 2 are so simple and transparent that they may seem unnecessary. After all, who would ever take less money if more is available? We can think of at least two reasons why this could happen. For instance, subjects may try to avoid the shame from appearing greedy in front of their peers or the experimenter (who might become their future professor). There is an established literature on the importance of double-blind protocol in preventing these peer-effects in experiments (Hoffman *et al.* 1996, Eckel and Grossman 1996).

We have had our own share of experience. In our early pilots that were run pen-and-paper in a nonanonymous classroom setting we found as much as 50% of subjects taking the lower amount in tasks 1 and 2. This stands in stark contrast with the data we obtained in the actual experiment in which we used (i) double-blind procedure and (ii) we stated clearly in the instructions that the experiment was funded by an external grant in order to mitigate the possibility that subjects think they are taking money out of our own pocket.

The second explanation for taking the lower amount could be that some subjects are simply more prone to making mistakes than others. Their behavior might be qualitatively different form those who do not make mistakes. In either case, whether the lower amount is taken due to preferences or mistakes, our initial two tasks 1 and 2 are able to detect such person and allow us to analyze these types separately from the rest.

drawn from a hat. This number was entered by the subject on the opening screen of the software. At the end of the experiment the experimenter put all payments in the respective envelopes with the corresponding numbers written on the top of them. One of the subjects was again randomly selected to hand out the envelopes to everyone else in the room[11,12].

To get at our questions we used a within subject design in which tasks come in sequence. Since under SC, payoffs in task three were contingent on task 1 and 2 choices, tasks were unfolded to the subject one at a time. A computer which observed choices in tasks 1 and 2, constructed and presented the subject with task three.

Sequencing of tasks could cause order effects. We control for order effects by randomizing the order of tasks. To preserve the natural structure of the AM implication we only randomized the order of the tasks 1 and 2 and the order of tasks 3 and 4. Furthermore, we were worried that responding to tasks may become automatic if the same column with the higher amount(s) is always associated with the same button, e.g., Right. For this reason we also randomized the assignment of columns to buttons for each task and each subject.

The experiment was run at ITAM in the computer laboratory. The software was written in Visual Basic 6.0. Together 162 subjects participated in the experiment. The Ambiguity and Uncertainty treatments consisted of 3 and 6 sessions respectively with 12 - 20 students per session. The students were recruited form the 1st year introductory courses offered at ITAM, i.e., they had only minimal exposure to economics. The experiment was run in

[11] In exchange for the envelope she collected the card with the number that matched the envelope. The cards were then handed back to the experimenter.

[12] One of the major difficulties with double blind procedure is with having subjects sign the payment receipts. At that point a name and face is clearly related to the amount (and decisions) made in the experiment. We by-passed this problem by having each subject sign a payment form with the average amount earned by a subject in the experiment. This procedure was explained to subjects verbally.

Spanish[13,14]. Our assistant who is a native Spanish speaker has read the instructions aloud for the whole class. This was followed by a round of privately answering subjects' individual questions. The opening screen of the software contained a page of comprehension questions that had to be answered correctly by everyone before the experiment could begin. The experiment lasted for about 45 minutes. Subjects were paid 50 Pesos as a show fee and half of their total point earnings in the experiment. The average payment was 155 Pesos.

3.2 Hypotheses

In our experiment we observe choices and not preferences nor the subjects' construct of the state space. Therefore, before we state our hypotheses it is necessary to define an observational equivalent of the AM for our experiment. This we call the monotonicity principle:

Monotonicity Principle: Choice satisfies the principle of monotonicity if the dominant lottery is chosen in task three.

Recall that in the Uncertainty treatment we carefully explained to subjects all details of the randomization device in task 3. If this was understood by them, then they must have considered only two kinds of states of the world. Let's refer to them as the high state (h), in which the number on the ball drawn from the bingo cage is higher than 20, and the low state (l), in which the number is lower than 20. Then, the state space can then represented by $S = \{h, l\}$. Let c denote choice in task t and let a denote the alternative which was not chosen. Task 1 can be viewed as a choice conditional on h such that the utility of the chosen amount is preferred to the alternative, i.e., $u(c_1, h) \succ_h u(a_1, h)$. Similarly for task 2 we have, $u(c_2, l) \succ_l u(a_2, l)$. Since

[13] The instructions and the software were initially written in English, then translated to Spanish by our assistant, and consequently translated back to English by our second assistant to ensure the accuracy of the translation.

[14] The English version of the instructions can be found in the Appendix.

in task 3 the dominant lottery gives prizes $u(c_1, h)$ on h and $u(c_2, l)$ on l, by conditional preference it is preferred on both h and l. Then, by AM, it must be preferred to the dominated lottery. This implies our first hypothesis.

H1: If AM holds under uncertainty, then we will observe zero (negligible number of) dominated lottery choices in the Uncertainty treatment.

In the Ambiguity treatment the randomization process is obscure and objective probabilities cannot be computed. If a subject does not fully understand all the details of how lotteries are resolved then she may resort to an alternative (perhaps simpler) heuristics. *For example*, she may start filling the missing pieces based on her own experiences that are readily available to her. This could lead her to consider as relevant some states of the world that do not physically exist. If that happens, however, then the decision problem as perceived by the subject could be represented in the Savagian language as follows: in addition to states $\{h, l\}$ there may be an additional state m (middle say) that conditions the outcome on the choice of the lottery, i.e.,

	L	R
h	75	85
m	75	35
l	25	35

Notice that *if* this is an accurate representation of the perceived decision problem then AM is vacuously true in our setting[15]. To minimize the possibility of this type of mis-perception we decided to use a physical randomization device - the bingo cage - which was pre-loaded with balls and was

[15] An alternative formulation of the problem would provide a test of AM even under this state-space specification. If the payoffs were reshuffled as follows

	L	R
h	75	85
l	25	35

then the lottery R gives the higher amount in all states than L. A more complicated strategic version of this decision problem has been run as a treatment by Croson (1999). She found no violations AM.

on display in the front of the room for the duration of the experiment. The experimenters made sure they did not touch the bingo cage. Based on the assumption that subjects did not mis-percieve the physical structure of the experiment we obtain the following hypothesis.

H2: If AM holds under both uncertainty and ambiguity, then we will observe zero (negligible number of) dominated lottery choices in both Uncertainty and Ambiguity treatments.

H2 can be viewed as a joint hypothesis of AM *and* the absence of non-relevant states form the decision problem. This view will prove useful later when we give interpretation of our results. Our next question of interest is how does a violation of AM affect the play in the prisoner's dilemma. To choose a strategy in a game the player has to form a belief about the opponent's strategy. This necessarily puts the player in the situation of ambiguity. We speculate that the same factors that are responsible for the violation of AM in the decision theoretic setting will also contribute to the choice of the dominated strategy (cooperation) in the PD game.

H3: Subjects who violated AM will cooperate in the prisoner's dilemma at higher rate than subjects who did not violate AM.

4 Results

As the first step we check the consistency of behavior in tasks (1, 2, and 4) that were unaffected by treatment variations.

The Table 1 shows that there are indeed no differences. In addition, there is nothing irregular about the behavior in three tasks 1, 2 and 4. In tasks 1 and 2 most of the subjects revealed preferences for money and took the higher amount. Somewhat surprisingly, however, a minority (about 15-20%) took the lower amount at least once. We will return to this subgroup in the later

Table 1: Choice frequencies in tasks 1, 2 and 4

	Treatments	
	Ambiguity	Uncertainty
Tasks 1 and 2: took the higher amount	85%	82.4%
Task 4: chose the dominated strategy	45%	47.1%
No. of observations	60	102

Note: Task 3 is left out intentionally and is analyzed in detail in Table 2.

section. Task 4 was the PD game. The observed cooperation rates are 45-47% and this is consistent with the previous findings in the literature[16]. Next we turn to our main result - the evidence on the violation of monotonicity.

4.1 Violation of the Monotonicity Axiom

Task 3 presented the subjects with a choice between two lotteries. The dominant lottery, in which the prizes were the chosen amounts in the initial two tasks, and the dominated lottery, with prizes equal to the amounts left unchosen.

Table 2: Dominated lottery choices in Task 3

	Treatments			
	Ambiguity	Uncertainty		
		Pooled	SC	FC
Higher amount in both task 1 and 2	17.7%	2.4%	4.4%	0%
	(9/51)	(2/82)	(2/45)	(0/39)
Lower amount in task 1 or task 2	66.7%	-	72.7%	28.6%
	(6/9)	-	(8/11)	(2/7)
Combined	25%	-	17.9%	4.4%
	(15/60)	-	(10/56)	(2/46)

Note: The ratios in parenthesis give the actual number of observations.

The top row presents number of violations for the sub-sample (82-85%) of subjects who have chosen the higher amount in both initial two tasks. This

[16] Fisher's exact test shows no significant differences between frequencies for Tasks 1 and 2 (p-value = 0.687) and also for Task 4 (p-value = 0.429).

gives the cleanest test of AM because for these subjects we are confident that the antecedent of AM is satisfied. Furthermore, for this subgroup there was no difference between SC and FC conditions. All four tasks were exactly the same. Therefore, we present a test based on the pooled data. The incidence of dominated lottery choices between Ambiguity was as high as 17.7% and Uncertainty only 2.4%. The difference is significant on 1% level with the p-value of one-sided (two-sided) Fisher's exact test[17] equal to 0.006 (0.008)[18]. Based on this evidence we accept H1 and reject H2.

Result 1: AM holds in the most basic decision-theoretic setting. Under uncertainty only 2.4% of subjects violate AM. Under ambiguity a significant 17.7% of subjects violate AM.

The subjects who chose the lower amount in at least one of the tasks 1 or 2 account for about 15-20% of the data. This makes them less important but nonetheless a quite interesting group. Their dominated lottery choices are shown in the second row of Table 2[19]. For these subjects we have much less confidence that the antecedent of AM is satisfied because the choice of the lower amount in task 1 or 2 seems somewhat strange. It could be that this choice is preference driven but it could also be that it is a product of a mistake.

We cannot and would not want to rule out either of the explanations. Under the assumption that all choices in our experiment are preference driven the appropriate test of AM is the formulation of task 3 with switched columns, i.e., condition SC. On the other hand, if the choice of the lower amount occurred due to a mistake, then the appropriate test of AM is task 3 with the fixed columns, i.e., condition FC.

The data in Table 2 show that under preference assumption it would be

[17] The p-values in the reminder of this paper are based on this test.

[18] The difference between Ambiguity and individual uncertainty conditions is also significant. One-sided (two-sided) p-values for the Ambiguity vs. SC condition are 0.064 (0.108) and for the Ambiguity vs. FC condition are 0.009 (0.011).

[19] Additional tables allowing a deeper look in the data can be found in the Appendix.

that majority of subjects, 72.7%, violate AM. Under the mistakes assumption the proportion of would-be violators drops down to 28.8% but it is still far from negligible.[20] From our experiment we cannot determine whether choices are preference- or mistake-driven. But irrespective of which one it is the data indicate that the number of AM violators is likely to be higher for this group then for the previous group[21].

4.2 Play in the Prisoner's Dilemma

In the previous sections we have shown that a substantial proportion of subjects violate AM. This brings up a question about the implications of this finding for the dominance play in games. Recall from our previous discussion that both Shafir and Tversky (1992, 1993) and Croson (1999) have found a large proportion of what appeared to be AM violations (in the order of 30%) in the prisoner's dilemma game. They called this the disjunction effect. However, PD game arguably involves more complexity than our nonstrategic setting in task 3. Cooperation can occur for other reasons that are unrelated to the monotonicity of choice. For instance, in the PD game players split the payoffs between the two of them which can bring in social preferences to play a role in their decisions[22]; secondly, in the PD game players face strategic uncertainty which requires that they form beliefs about the others' preferences and strategy choices. This is certainly a great leap from simple bingo-cage-type uncertainty with 50/50 chances. However, here we are only interested in a very simple question: is the violation of AM one of the drivers of cooperation in the PD game. Our experiment allows us to screen each subject for her type, either she is an AM violator or not, and then relate her type to her behavior in the PD game.

[20] The test of the hypotheses that the proportions AM violators in the subsample of those who always took the higher amount (2/82) and those who did not under FC condition (2/7) is rejected on the 5% level (p-value is 0.045)

[21] The power of this conclusion is limited by the small sample-size that it's based on.

[22] For example due to distribution-based (e.g., see Bolton and Ockenfels, 2000 or Fehr and Schmidt, 1999) or belief-based (Rabin 1993) preferences.

We only look at the data for subjects who chose the higher amount in both tasks 1 and 2. Only for this subgroup we have sufficient confidence that a choice of dominated lottery in task 3 is a violation of AM. It is natural to suppose that those subjects who violate AM in our simple setting would also tend to violate AM in the more complex strategic setting. Based on this we would expect AM violators to cooperate in the PD game at higher rate than non-violators. But surprisingly we cannot quite conclude this from the data. The proportion of cooperators amongst those who took the higher amount in tasks 1 and 2 and did not violate AM is 42.8% (18/42). Amongst those who did violate AM the proportion is higher, 55.6% (5/9), but the difference is not significant (the p-value is 0.45).

Result 2: Violation of AM does not imply cooperative behavior in the prisoner's dilemma game.

Thus, we can reject the H3 hypothesis. This suggests that violation of AM does not have an important impact on the cooperation rates in the PD game. As a result, there may be a fundamental disconnection between violation of AM and what is known in the literature as the disjunction effect.

4.3 Discussion

Savage's theory is prescriptive. It guides a decision maker in making consistent decisions. A crucial ingredient in this process is the formulation of the state space. The state space so constructed has to do with only those phenomena that are beyond the decision maker's perceived control. As such, the decision maker should not believe that her decisions affect the occurrence or nonoccurrence of states. Acts have to be independent of beliefs over states. Dependence, if any, should be built into the model.

Take our task three for example. Readers may agree that the "relevant" decision problem is as depicted below:

	L	R
h	75	85
l	25	35

This may indeed be so, in the sense that a decision maker can be easily persuaded that this is indeed the "right" state space to consider. In fact, in our Uncertainty treatment most of our subjects seem to have such a construct without any persuasion. However, in real life matters may be complex and decision makers may not agree on the right way to formulate the state space. For example, in the Ambiguity treatment, decision makers may believe that the experimenter tampers with the number of balls *after* she has made her decision In such a case, she may add a hypothetical state m as depicted below. If so, then her choice of L over R would not violate AM at all.

	L	R
h	75	85
m	75	35
l	25	35

To reduce the chance of this happening, in our experiments, we took adequate measures. i.e., the bingo cage not touched by the experimenters throughout the experiment. If we did indeed succeed in ruling out such perceptions, then what could have gone wrong?

Our hunch is along the lines of Gilboa and Schmeidler (1995). Constructing a state space is not always easy. The decision maker may not have enough time or appropriate advice, to formulate the "correct" state space. In our Ambiguity treatment, simple as it is, the subject may start wondering about the actual number of balls in the bingo cage. Yes, the number of balls should not matter. But the point is that the subject should be able to figure it out. If not, then how should she decide?

A way out could be that the subject ignores the state space. Instead, she draws on her past experience. For example, she could determine that choosing the dominant lottery is akin to her being greedy. Historically, our subject had been unlucky whenever she was greedy. So she concludes that if she were to choose the dominant lottery she would get \$35 with a very high probability and if she were to choose the dominated lottery she would get \$75 with a very high probability. As such she chooses the dominated lottery. Savage would have rightly concluded that her acts and beliefs are not independent. We agree. In our simple experiments we could perhaps teach subjects how to "correctly" formulate the state space; or repeating the experiment might do the trick. But our basic point is that in real life people are not trained and even if they were there is no guarantee they could be specify the state space correctly.

Ahn (2008) has more persuasive arguments along these lines. So much so, that figuring out the right state space may be a fruitless (very costly) exercise. Ahn shows how to avoid the complexities of the state space by directly assigning probabilities to lotteries. In this framework Ahn obtains a representation but notice, since there is no state space, AM is vacuous.

5 Conclusions

Our objective was to design a simple and direct test of the Axiom of Monotonicity. This axiom is fundamental for most theories of choice under uncertainty. Yet, to this date we do not have a reliable test of AM. In a nutshell, the axiom says that if the decision maker would have made the same choice in all contingencies after the uncertainty is resolved she would have made the same choice even before the uncertainty is resolved. This statement is so simple and intuitive that it would be difficult to doubt its empirical validity. However, an intriguing evidence from the literature on the disjunction effect points to a possible violation of AM in real decisions, which suggests that a careful test of AM may be needed. We designed such a test and presented the results in this paper.

Our test was placed in the decision-theoretic setting and examined behavior both under simple uncertainty and under ambiguity. Results could be summarized as follows: (i) AM in not violated in the most basic environment under simple uncertainty but it is violated in a significant proportion under ambiguity; (ii) a 15-20% of our subjects took lower amount of money when higher was available and for this subgroup AM is also likely to be violated in a significant proportion; and finally, (iii) AM violators do tend to cooperate more in the strategic setting of the prisoner's dilemma but this difference is not significant.

The evidence indicates that one should be careful when applying monotonicity in the decision problems without objective probabilities. Our results favor approaches to theoretical modeling that do not assume AM. Amongst these are the case-based approach by Gilboa and Schmeidler and the state-free approach of Ahn. Lastly, our experiments also illustrate the divide between decision theory and game theory. Strategic uncertainty and possible social (or other regarding) preferences are likely the dominant determinant of the behavior in the game like prisoner's dilemma.

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Appendix

A Instructions

Below is the English version of the instructions. The instructions below are those used in the Uncertainty treatment. In the Ambiguity treatment the changes were that the text in [] was added and the text in {} was deleted.

Instructions

Welcome to the experiment. From this moment on no talking is allowed. If you have a question after we finish reading the instructions, please raise your hand and the experimenter will approach you and answer your question in privacy.

The experiment consists of 4 tasks that will be presented to you in sequence (one after another). In each task you will be asked to make a single decision.

Earnings

The amount you earn in this experiment will be paid to you in cash at the end of the experiment. The funding for this experiment was provided by an external grant from Asociation Mexicana de Cultura.

You will be paid according to the following rule:

50 pesos for coming on time to the experiment + $\frac{1}{2}$ * (the points that you earn in each of the four tasks of the experiment).

Privacy

In this experiment you are completely anonymous. The experimental procedure that will be described to you in detail insures that NO ONE including the experimenters will be able to know which decision was made by you.

Tasks and Decisions

You will be seated at the computer terminal which is shielded by blinders to insure your complete privacy. In front of you there is a folded card with a number which will identify you throughout the experiment. You will use this number to make your decisions and also to redeem your payment.

After we finish reading these instructions and answer any questions that you may have, you will be asked to follow the instructions on the computer screen. The software will guide you through the tasks of the experiment.

When we start the experiment you will see the following screen:

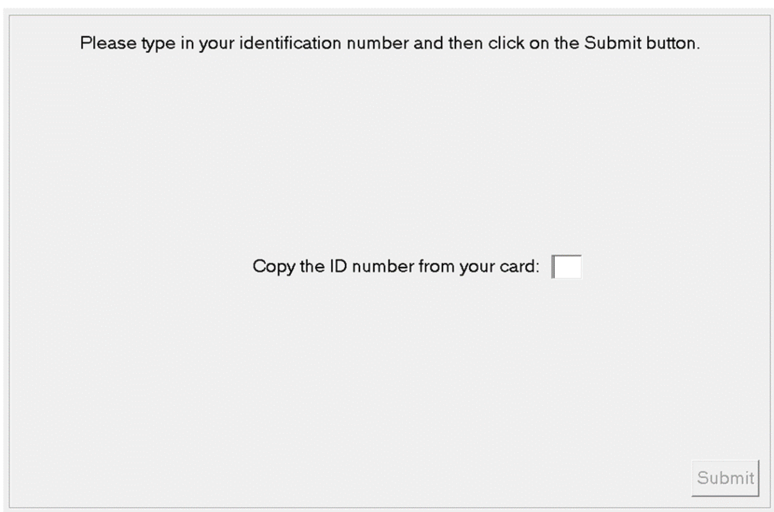

Figure 1

Please enter your identification number which is written on the card in front of you. Make sure that you copy the number correctly. If you make a mistake we will not be able to pay you your earnings.

Next you will be asked to complete a series of comprehension questions. These questions ensure that you have properly understood the instructions. You will not be allowed to proceed with the experiment unless you have answered all questions correctly. Once you have answered the instructions the Task 1 of the experiment begins.

TASK 1: The task is very simple. On the screen (Figure 2) you see two boxes each containing a single number.

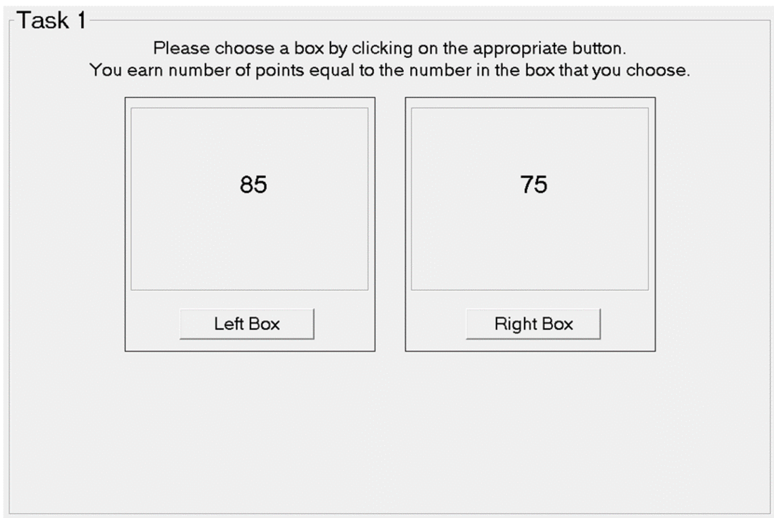

Figure 2

This screen is just an example. In the experiment the numbers in boxes may be switched.

All you have to do is to choose a box (left or right) by clicking on the appropriate button labeled either "Left" or "Right." The number inside of the box that you choose represents the number of points that you earn in this task.

TASK 2: The instructions for task 2 are exactly the same as for task 1. The only difference between tasks 1 and 2 is the numbers in the two boxes.

TASK 3:

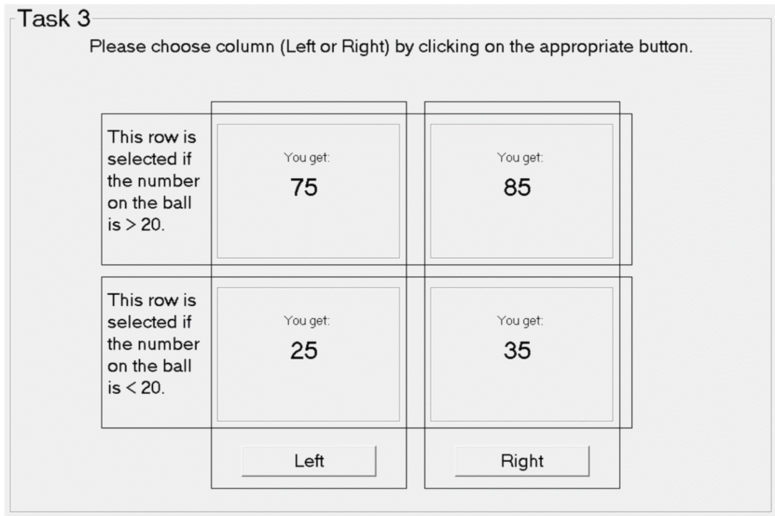

Figure 3

This screen is just an example. In the experiment the numbers in boxes may be switched.

In this task (Figure 3) you see 4 boxes. The boxes are grouped horizontally into two rows and also vertically into two columns.

You are asked to choose a column (left or right) by clicking on the appropriate button labeled either "Left" or "Right."

The number of points you earn is equal to the number in box which is

(i) inside of the column that you have selected and also

(ii) inside of the row which will be decided randomly at the end of the experiment by a draw of a single ball from a bingo cage. This is done in the following way:

The bingo cage in front of the room contains [forty] balls that are labeled with numbers between 1 and 40. No two balls have the same number. {The number of balls in the bingo cage is decided by the experimenter.} After everyone has completed the experiment one of the participants will be randomly selected to spin the bingo cage and draw a single ball. If the number on the ball is 20 then the top row is chosen. If the number on the ball is different from 20 then the bottom row is chosen.

TASK 4:

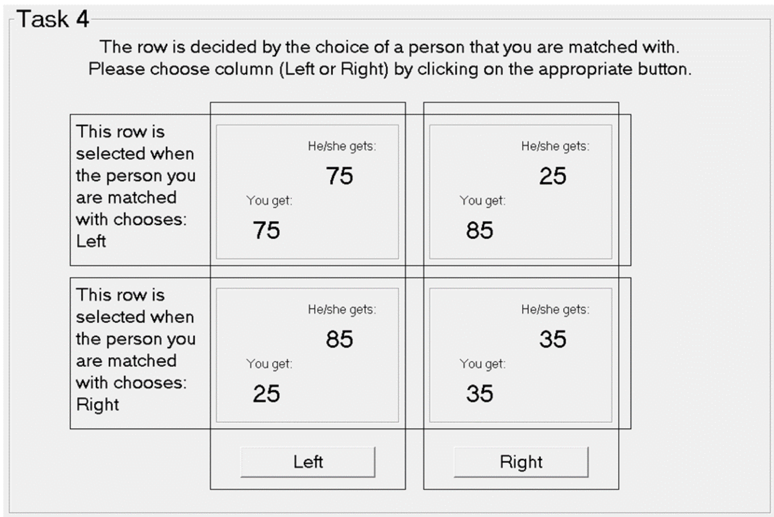

Figure 4

This task is similar to task 3. The difference is that now you are randomly matched with one other person in this room. In Figure 4 you see 4 boxes. The boxes are grouped horizontally into two rows and also vertically into two columns. Each box contains two numbers. The number labeled "You earn:" represents the number of points that you earn when that box is selected. Similarly, the number labeled "He/she earns:" represents the number of points that the person you are matched with earns when that box is selected.

Which box is selected depends on your decision as well as on the decision of the other person that you are matched with. Both you and the other person simultaneously choose a column (left or right) by clicking on the appropriate button labeled either "Left" or "Right." The box which is selected for payment lies

(i) inside of the column that you have chosen and also

(ii) it is inside of the row which depends on what the other person has done: if he/she chose left, then the top row is selected; and if he/she chose right, the bottom row is selected.

The order of tasks

In the experiment you will first complete tasks 1 and 2 but the order in which they appear is decided randomly. This means that you may encounter task 2 as first and task 1 as second. Then you complete tasks 3 and 4. Again their order is decided randomly and you may complete task 4 before you complete task 3. When everyone is finished with all four tasks you will be asked to fill out a short questionnaire. After that one randomly chosen participant will draw a ball from the bingo cage to determine which row is played in the Task 3.

Payment

When everyone is finished with the experiment the experimenter will put earnings of each student into a separate envelope and write the student's identification number on the top of the envelope. Then, one of the students in the room will be randomly selected to distribute the envelopes to everyone else in the room. To receive your earnings you will be asked to exchange the card with your identification number for the envelope which contains your earnings and has the same number written on the top of it. When you get your envelope you may leave the room.

B Additional tables

Table 3: Dominated lottery choices in Task 3

	Took lower amount in tasks 1 or 2			
	Order		Payoffs	
	First task encountered	Second task	Task 1 75 vs. 85	Task2 25 vs. 35
Ambiguity	75%	0%	60%	66%
	(6/8)	(0/1)	(3/5)	(4/6)
Uncertainty SC	66.7%	80%	75%	66.7%
	(4/6)	(4/5)	(6/8)	(2/3)
Uncertainty FC	50%	0%	0%	40%
	(2/4)	(0/3)	(0/2)	(2/5)
Total	66.7%	44.4%	60%	57.1%
	(12/18)	(4/9)	(9/15)	(8/14)

Note: The ratios in parenthesis give the actual number of observations.

Table 4: Cooperation in PD by groups

	Treatments			
	Ambiguity	Uncertainty		
		Pooled	SC	FC
Task 3: Satisfied AM				
T1&2: Both high	18/42	35/82	20/43	15/39
	42.9%	42.7%	46.5%	38.5%
T1&2: One low	2/3	7/8	2/3	5/5
	66.7%	87.5%	66.7%	50%
Task 3: Violated AM				
T1&2: Both high	5/9	0/2	0/2	0/0
	55.6%	0%	0%	-
T1&2: One low	2/6	6/10	4/8	2/2
	33.3%	60%	50%	100%
Total	45%	47.1%	46.4%	47.8%
	27/60	48/102	26/56	22/46