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Abstract. This paper investigates household decisions when individual utility depends on a consumption reference level. The desire to "keep up with the Joneses" represents one such example. The prior literature shows that, in a Ramsey model, consumption externalities have *no* impact on steady state behavior, once labor supply is exogenous. In contrast, this paper argues that — once there is (exogenous) technological change — consumption externalities *always* affect steady state behavior, even if labor supply is exogenous. The nature of the effects depends on the consumption externality's impact on a household's elasticity of marginal utility of consumption.

Keywords and Phrases: Consumption externality, keeping up with the Joneses, Ramsey model, intertemporal elasticity of substitution.

JEL Classification Numbers: D91, E21, O41

1 Introduction

This paper considers the impact of consumption externalities on a decentralized steady state equilibrium of a standard Ramsey model *with inelastic labor supply*. The consumption externality is introduced in that a representative household derives utility not only from own consumption but also from a consumption reference level, which here is given by the *current* average consumption level of society. An example widely discussed in the literature is the desire to *keep up with the Joneses*. But the framework considered encompasses any positive or negative consumption externality that is based on current average consumption.

The prior literature shows that, in a standard Ramsey model, with neoclassical production, consumption externalities have no impact on the steady state equilibrium, once labor supply is exogenous.[1]

This paper demonstrates that in the presence of exogenous technological change, a consumption externality always affects the steady state equilibrium, even if labor supply is inelastic. The consumption externality affects the elasticity of marginal utility of consumption. Once there is technical change, the elasticity of marginal utility enters the Euler equation. As a consequence, the elasticity of marginal utility becomes a channel through which a consumption elasticity affects the steady state equilibrium — even in the absence of elastic labor supply and a consumption-labor tradeoff.

[1] For example, Brekke and Howarth (2002, p.142) argue that "we have established that augmenting a standard neoclassical growth model to incorporate a concern for relative consumption has no impacts on long-run economic behavior." Fisher and Hof (2000, p.249) show that the result that "relative consumption does not affect the long-run steady state...is robust with respect to the specification of the instantaneous utility function." Liu and Turnovsky (2005, p.1106) state that "[w]ith exogenous labor supply, consumption externalities, which impact through the labor-consumption tradeoff, have no channel to affect steady state output." Rauscher (1997, p.38) argues that "conspicuous consumption does not affect the long-run steady state."

2 The Model

Firms. There is a large number of identical, fully competitive, profit-maximizing firms, producing a homogeneous product, Y, according to the production function:

$$Y(t) = F(K(t), E(t)\, L(t))\,, \tag{1}$$

where $K(t)$ is capital input, $L(t)$ is labor input, and $E(t) = e^{\gamma\, t}$ is the level of technology, which grows at a constant rate $\gamma \geq 0$. Function $F(.)$ exhibits the usual properties of a neoclassical production function. Specifically, it exhibits constant returns to scale with respect to $(K, E\, L)$, and both inputs have positive and strictly declining marginal products. Define $\hat{y} \equiv Y/(E\, L)$, and $\hat{k} \equiv K/(E\, L)$. We can then express the production function in intensive form:

$$\hat{y}(t) = f(\hat{k}(t))\,, \tag{2}$$

with $f(0) = 0$, $lim_{\hat{k}\to\infty}\, f'(\hat{k}) = 0$, and $lim_{\hat{k}\to 0}\, f'(\hat{k}) \to \infty$.

A competitive firm takes the interest, r, and wages rates, w, as given. It maximizes profits by setting:

$$f'(\hat{k}) = r + \delta\,, \quad [f(\hat{k}) - \hat{k}\, f'(\hat{k})]\, e^{\gamma\, t} = w(t)\,, \tag{3}$$

where δ is the rate of depreciation of capital.

Households. Population grows at a constant rate n: $L(t) = e^{n\, t}$. A representative household derives utility not only from own consumption, c, but also from a consumption reference level: *current* average consumption of society, $\bar{c}$. Let instantaneous utility be given by: $u(c(t), \bar{c}(t))$, which is strictly concave in c, with $u_c(.) > 0$.[2] A consumption externality is said to be negative (positive) if $u_{\bar{c}}(.) < 0$ (if $u_{\bar{c}}(.) > 0$). In case the marginal utility of consumption increases in the reference level, $u_{c\bar{c}}(.) > 0$, the consumption externality is referred to as a keeping up with the Joneses (KUJ) externality (Dupor and Liu, 2003). We shall impose the following restrictions, in order to preclude a consumption externality to dominate the direct effect of individual consumption on utility: $u_c(.) + u_{\bar{c}}(.) > 0$, and $u_{cc}(.) + u_{c\bar{c}}(.) < 0$.

[2] Subscripts refer to partial derivatives.

Taking $\bar{c}(t)_{t=0}^{\infty}$ as given, a household chooses $c(t)_{t=0}^{\infty}$ such as to maximize its present value of instantaneous utility, discounted at the household's pure rate of time preference ρ subject to its flow budget constraint and a transversality condition:

$$U = \int_0^{\infty} u(c(t), \bar{c}(t))\, L(t)\, e^{-\rho\, t}\, dt\,. \tag{4}$$

To ensure convergence of the integral, $\rho > n + g_c^*\, [u_c(.) + u_{\bar{c}}(.)]/[u(.)/c]$, where g_c^* is the steady state rate of growth of per capita consumption, and the right hand term represents a ratio of marginal to average utility. Let a represent a household's assets. The budget constraint and transversality condition are:

$$\dot{a}(t) = r(t)\, a(t) + w(t) - c(t) - n\, a(t)\,, \quad \lim_{t\to\infty} a(t)\, e^{-\int_0^{\infty} [r(s)-n]\, ds} = 0\,. \tag{5}$$

The transversality condition implies the No-Ponzi game condition. The Hamiltonian becomes:

$$H(c(t), a(t), \mu(t), t) = u(c(t), \bar{c}(t))\, L(t)\, e^{-\rho\, t} + \mu(t)\, [r(t)\, a(t) + w(t) - c(t) - n\, a(t)]\,, \tag{6}$$

where $\mu(t)$ represents the costate variable. Every individual household considers $\bar{c}$ as given. Ex post, however, we consider a symmetric equilibrium with $c(t) = \bar{c}(t)$. Define the effective elasticity of marginal utility, $\hat{\theta}$ by:

$$\hat{\theta}(c(t)) = -\frac{[u_{cc}(.) + u_{c\bar{c}}(.)]c(t)}{u_c(.)}|_{\bar{c}(t)=c(t)} > 0\,. \tag{7}$$

Pontryagin's principle implies the Euler equation:

$$r(\hat{k}(t)) = \rho + \hat{\theta}(c(t))\, \frac{\dot{c}(t)}{c(t)}\,. \tag{8}$$

Equilibrium. Equilibrium requires $c(t) = \bar{c}(t)$, and $a(t) = k(t)$. Considering (3) and (5):

$$\dot{\hat{k}}(t) = f(\hat{k}(t)) - \hat{c}(t) - (\gamma + \delta + n)\, \hat{k}(t)\,. \tag{9}$$

3 Steady State Effects of Consumption Externalities

Let g_x denote the growth rate of variable x. In a steady state, $\hat{k}$ and $\hat{y}$ are constant. That is, in a steady state, $g^* = g_Y = g_K = g_C = g_c + n$. Considering (1), $g^* = \gamma + n$, and $g_c = \gamma$.

Proposition 1 *Suppose $\gamma > 0$, and the consumption externality affects $\hat{\theta}(c)$. Then the consumption externality has an impact on average consumption and capital levels in the Ramsey model — even if labor supply is exogenous.*

Proof. The steady state version of Euler equation (8) is:

$$f'(\hat{k}) - \delta = \rho + \hat{\theta}(c)\,\gamma\,. \tag{10}$$

If $\gamma = 0$, the steady state capital level is determined by the Keynes-Ramsey rule — according to which: $f'(\hat{k}) = \rho + \delta$ — independently of the consumption externality. Consumption is determined by the market clearing condition (9), also independently of the consumption externality. Once, however, steady state consumption growth is different from zero, the consumption externality affects the steady state capital level via the elasticity of marginal utility, $\hat{\theta}(c)$. ||

Once $\gamma > 0$, the consumption externality may affect the Euler equation via the elasticity of marginal utility, $\hat{\theta}(c)$. To illustrate one case, suppose the consumption externality lowers $\hat{\theta}(c)$. The right hand side of (10) can be interpreted as the benefit of consuming today as opposed to postponing consumption. If $\gamma > 0$, tomorrow's consumption level is higher than today's. That is, marginal utility of consumption tomorrow is lower than the present one. The two benefits of consuming a marginal unit today rather than tomorrow then consist of consuming earlier in time (time preference) and enjoying a higher marginal utility. If a consumption externality lowers $\hat{\theta}(c)$ — that is, the decline in marginal utility with increasing consumption — the benefit from consuming today rather than tomorrow declines. The Euler equation then requires a household to shift consumption from the present to the future. This is done by a rise in savings, which lowers the rate of interest to the point at which (10) is satisfied.

The mechanism illustrated is consistent with a *keeping up with the Joneses externality.* Such an externality raises the marginal utility of consumption and (in important frameworks) lowers the elasticity of marginal utility, $\hat{\theta}(c)$. As a consequence, if

$\gamma > 0$, households postpone consumption as the marginal utility declines less strongly in the presence of a keeping up with the Joneses externality. The market clearing condition (9), together with strict concavity of $f(\hat{k})$, implies that the steady state level of consumption increases, while the propensity to consume out of accumulated wealth, c/k decreases due to the consumption externality.

This result is noteworthy, as the prior literature generally argues that in the framework of a Ramsey model *without technological progress* and with exogenous labor supply, consumption externalities have no effect on the steady state equilibrium. Proposition 1 adds an important qualifier to this finding. In the *presence of technological change* consumption externalities generally exert an impact on the steady state equilibrium via the elasticity of marginal utility of consumption.

Example: A Constant Elasticity Utility Function. Let the instantaneous utility function be given by:

$$u(c(t), \bar{c}(t)) = \frac{[c(t)\,\bar{c}(t)^{-\eta}]^{1-\theta} - 1}{1-\theta}\,, \quad \theta > 0\,, 0 \leq \eta < 1\,, \tag{11}$$

where η is called the "reference parameter," which measures the importance of the consumption reference level. The reference parameter introduces a KUJ consumption externality. Parameter θ governs the intertemporal elasticity of substitution, which, in a steady state, is given by $\hat{\theta}^{-1} = [1 - (1-\theta)(1-\eta)]^{-1}$. Parameter $\hat{\theta}$ represents the (absolute value of the) effective elasticity of marginal utility of consumption. For (4) to converge, we need: $\rho > n + \gamma(1-\eta)(1-\theta)$.

Proposition 2 *Let instantaneous utility be given by* (11)*, and* $\theta \neq 1$*. If* $\gamma > 0$*, the consumption externality has an impact on average consumption and capital levels in the Ramsey model. In particular:* $\partial\, k/\partial\, \eta \gtreqless 0 \Leftrightarrow \theta \gtreqless 1$.

Proof. The Euler equation, in steady state, becomes: $f'(\hat{k}) - \delta = \rho + \hat{\theta}\,\gamma$. Moreover, $\partial\,\hat{\theta}/\partial\,\eta = 1 - \theta$ implies: $\hat{\theta}_\eta \lesseqgtr 0 \Leftrightarrow \theta \gtreqless 1$. The result of Proposition 2, then, follows from strict concavity of $f(\hat{k})$. $||$

If $\theta > 1$, a rise in the reference parameter lowers the effective elasticity of marginal utility. The Euler equation then requires households to shift consumption to the future and raise savings. As a consequence, steady state capital and consumption levels increase. The opposite occurs if $\theta < 1$, in which case steady state consumption and capital levels decline and the steady state propensity to consume out of wealth increases due to a rise in η.

Corollary 1 *If $\theta = 1$, the consumption externality has no impact on average consumption and capital levels in the Ramsey model.*

If $\theta = 1$ then $\hat{\theta} = 1$ and the consumption externality does not affect the elasticity of marginal utility, *regardless of the presence or absence of technological change.*

4 Discussion and Conclusion

The prior literature argues that in a standard Ramsey model with neoclassical production, a consumption externality affects a decentralized steady state equilibrium only through the labor-consumption tradeoff. If, however, labor supply is exogenous, there is no channel for a consumption externality to affect the steady state equilibrium.

This paper identifies a further channel through which a consumption externality affects the steady state equilibrium in a standard Ramsey model: the elasticity of marginal utility. Once there is technological change, the elasticity of marginal utility enters the Euler equation. If the consumption externality affects the elasticity, it always affects the steady state consumption and capital levels — even if labor supply is exogenous.

It must be emphasized, though, that this result refers to consumption externalities, for which the consumption reference level is given by *current* average consumption. A parallel result was previously shown for a neoclassical growth model in which the consumption reference stock is a weighted average of *current and past* consumption levels (Alvarez-Cuadrado *et al.*, 2004).

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