A benefit of uniform currency

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Abstract

The role of distinct currencies is studied using a random-matching model with randomized trades. The equilibrium concept is the pairwise core in meetings. We show that there exist equilibria in which home and foreign currency play distinct roles and in which the quantities of trade and output are less than the optimal quantities. The benefit of a uniform currency is the elimination of such inferior equilibria. Specifically, any equilibrium in which home and foreign currency play distinct roles is dominated in terms of ex ante welfare by the best one-currency equilibrium – for some parameters weakly, for some strongly.

JEL classifications: E42, F33.

1 Introduction

There is a widespread belief that gains are achieved when a country or a group of countries operates under a uniform currency—as the EMU now does. An objective of the US National Banking System, established in 1863, was the creation of a uniform currency for the entire country (see Friedman and Schwartz [7]). And underlying the notion of optimum currency areas (see Mundell [15]) is the belief that a uniform currency for the entire world would be best were there no nominal rigidities. However, nominal rigidities aside, neither Mundell nor anyone else has presented a model of the gains. In this paper we do so.

Our model, a pairwise matching model, is a simple depiction of the long-held notion that trading opportunities arise in pairwise meetings in which there are absence-of-double-coincidence difficulties. The benefit of a uniform currency in our model is that it eliminates some inferior equilibria. They are ones in which the currencies play distinct roles and in which, as a consequence, the quantities of trade and output are less than the optimal quantities. Among them are equilibria in which observed prices, calculated from the trades that occur, are higher in terms of foreign currency than in terms of home currency. This last feature is consistent with the distinct roles of different currencies that we often see, and, in particular, with the advice that appears in many travel guides: if you pay in foreign currency, then you get a bad deal.

We are not the first to apply matching models to multiple currencies and countries. Moreover, some previous versions display a closely related multiplicity of equilibria. In the two-country, two-money matching models of Matsuyama, Kiyotaki, and Matsui [13], Trejos and Wright [16], and Zhou [18], there exists an equilibrium in which people do not distinguish between home and foreign money. And if the potential gains from international trade are sufficiently small, then there exists another inferior equilibrium in which they are distinguished. Even aside from the qualification about potential gains from international trade, a qualification we do not need, there is at least one concern about those results: the existence of the equilibrium in which currencies are distinguished seems to depend on extraneous assumptions that rule out small trades.

In [13], [16], and [18], money is indivisible, individual holdings of money are bounded at one unit, and trades are deterministic. In [13] and [16], a country is defined by a pairwise meeting pattern: meetings between people from the same country occur more frequently than meetings between people from different countries. There the equilibrium in which currencies are distinguished has no foreign trade and has no one accepting foreign money. A deterministic deviation from this outcome would have to have a producer accept foreign money in exchange for production. In [13], goods are also indivisible so that the producer would experience as much disutility as he would to obtain a unit of home money. But if he accepts the foreign money, then, given the strategies of others, he has to wait to meet a foreigner in order to pass it on. Therefore, if such meetings are rare enough, then there would be no deviation from such non acceptance, even cooperatively by the pair in the meeting. In [16] goods are divisible so that the producer would be willing to produce a small amount to get the unit of foreign money. However, the consumer would not surrender the (entire) unit of his home money for too small an amount of production. So again, there is no defection. In [18], a country is defined by a distribution of taste shocks: people in a country are more likely to receive shocks which make them prefer only home goods than they are to receive shocks which make them prefer only foreign goods. There, too, there is a no-trade aspect to equilibria in which the currencies are distinguished: producers with a current preference for home goods do not accept the foreign currency, a non-acceptance which again seems to depend on the impossibility of small trades.

Therefore, the inferior equilibria in [13], [16], and [18] rest on shaky ground.[1] And if the inferior equilibria do not exist, then those models do not

[1] The model in Kocherlakota and Krueger [11] shares features with [13], [16], and [18], but has a very different message. They provide a model in which distinct monies are

display a benefit of uniform currency. In this paper, we work with the model in [13], except that we assume divisible goods as in [16]. And, although we maintain the assumption of indivisible money and a unit upper bound on individual holdings, we permit randomization as in Berentsen, Molico, and Wright [4] so that goods can be traded for a probability of receiving money. Such randomization in meetings permits small trades to occur—a small trade being one in which a small amount of production trades for a low probability of receiving the indivisible unit of money. In that respect and in others, our model is formulated so that the main results stand a good chance of holding in versions with divisible money.

We define an outcome to be an equilibrium if the trades in meetings satisfy two conditions: the trades are individually rational and pairwise efficient given the future values of the two monies. In other words, an outcome is an equilibrium if it is in the pairwise core in each meeting. This concept of equilibrium is natural in a model in which trade occurs in momentary meetings between two strangers against the background of a world populated by a large number of strangers. It does, however, give rise to a large set of equilibria—in part because conditional on the future values of monies there are many pairwise-core outcomes, many ways of dividing the gains from trade in meetings.[2] A subset of them are ones in which the two monies are not distinguished. We call those uniform-currency equilibria.

We show that a best equilibrium is a uniform-currency equilibrium. We also show that there are inferior equilibria in which the currencies play distinct roles. The second result is established by showing that there can be

optimal in that they permit people to credibly signal private information about preferences concerning the source, by country, of the goods to be consumed. As in [13], [16], and [18], Kocherlakota and Krueger assume indivisible money, a unit upper bound, and deterministic trades. They wonder, as do we, whether the signalling function of distinct monies would survive if money were divisible or if randomization were allowed.

[2]The same kind of equilibrium concept, described somewhat differently, is used by Engineer and Shi [6] to show that there can be a role for money even when there are no absence-of-double-coincidence difficulties.

discrimination against foreign currency in the sense that the gains from trade are divided in a way that is more favorable to consumers who have the producer's home currency than to consumers who have the producer's foreign currency.

Finally, the existence of inferior equilibria in which currencies are distinguished has nothing to do with distinctions between the countries or their policies; the mere presence of different currencies is enough to permit such inferior equilibria to exist. In the model, the countries are identical, the money supplies are identical and fixed, and there are no policies. Indeed, the inferior equilibria in which currencies are distinguished have a fixed exchange rate. In these equilibria, there are trades of one currency for the other at each date and the trades are always one for one. Thus, the model gives rise to a distinction between uniform currency and fixed exchange rates, a distinction that, as noted by Alvarez [2] and Kehoe [9], is missing in many discussions of uniform currency.

2 A symmetric two-country environment

Time is discrete and the horizon is infinite. There are two identical countries. There are $N \geqslant 3$ perishable goods at each date and a $[0,1]$ continuum of each of N types of people in each country. A type n person consumes only good n and is at most able to produce good $n+1$ (modulo N). Each person maximizes expected discounted utility with discount parameter $\beta \in (0,1)$. The period utility function is $u(x)-y$, where x is consumption of the relevant good and y is production of the relevant good. The function u, defined on R_+, is bounded, strictly concave, and increasing, and satisfies $u(0)=0$ and $u'(0)=\infty$. We let $g(y) \equiv u(y)-y$ and let $y^* \equiv argmax_y g(y)$.

At each date, each person meets someone from his country with probability θ and meets someone from the other country with probability $1-\theta$. Conditional on the country of residence of the meeting partner, the meeting partner is a random draw from the population.

There are two distinct monies in fixed supply. The amount of each is m per specialization type and per country, where $m \in (0, 1)$. In each country, the fraction m of each specialization type is endowed initially with *one* unit of one of the monies. Moreover, at any date, those who begin a period with money are unable to produce.[3] Although people can freely dispose of money, such disposal does not permit them to produce.

We assume that a person's specialization type, nationality, and holding of money are observable. We also assume that people cannot commit to future actions and that each person's history, except as revealed by money holdings, is private.

The assumption that people with money cannot produce, which was also made in [13], [16], [18], and [11], is critical for our conclusion that a best equilibrium is a uniform currency equilibrium. With indivisible money and a unit bound on individual holdings, if people with money can produce, then having distinct monies can be helpful (see Aiyagari, Wallace, and Wright [1] and Cavalcanti [5]). Whether or not randomization is allowed, there is an equilibrium in which the distinction between the monies is ignored and in which, therefore, a producer with money does not *want* to trade. However, there are also equilibria in which one money is more valuable than the other and in which a producer with the less valuable money is *willing* to offer it along with some output to obtain the more valuable money. Because there is more trade in the second set of equilibria, welfare can be higher with distinct monies.

This beneficial effect of distinct monies, which is like the benefit of being able to make change, seems to arise entirely from the assumed indivisibility of money and the bound on individual holdings. In particular, a plausible surmise is that it would disappear if money were divisible. Therefore, if we want to obtain results that are robust to more general individual holdings

[3]One consequence is that the economy's total productive capacity is tied to the amount of money. That being so, we treat the amount of money as a given, not as something to be chosen by the society.

and to divisible money, then we should exclude this benefit of distinct monies. The assumption that people with money are unable to produce accomplishes that.

The modeling of countries by meeting patterns also merits comment. Although very different from the way countries are defined in international trade theory, that specification, introduced in [13], seems well-suited for the study of currency substitution. For obvious reasons, border areas between countries are the first places to look for currency substitution. As a simple representation of border areas, consider the following one-dimensional spatial model. There are K identical "cities" arrayed as K equally spaced points on a line segment. Residents of each city meet each other more frequently than they meet residents from adjoining cities and people from non adjoining cities never meet. If there are two monies, then an obvious question is whether there are outcomes in which the K cities split into two contiguous sets with those to one side of an endogenous border specialized in the use of one money and those to the other side specialized in the use of the other money. The answer to that question seems to hinge entirely on what can happen in the two border cities. The specification in [13], which we are adopting, is the special case in which $K = 2$ and in which, therefore, there is only one possible border. Obviously, that is the first case to study.

3 Symmetric equilibrium

We restrict attention to allocations which are symmetric across countries and across specialization types. Each person starts a period in one of 3 situations, which we call states: holds no money, state 0; holds a unit of foreign money, state 1; holds a unit of home money, state 2. (Thus, if the two people in a meeting are from different countries and if both are in either state 1 or in state 2, then they are holding distinct monies; if one is in state 1 and the other is in state 2, then they are holding the same money.)

An allocation describes time paths of the distribution of residents of each country over states and actions in meetings. Under the assumed symmetries, we let $p_t \equiv (p_{0t}, p_{1t}, p_{2t})$, a probability distribution over $\{0,1,2\}$, be the beginning of date t distribution over states in each country, where p_{it} is the fraction of every specialization type in state i at date t and where p_0 is the initial condition. There are meetings in which production can occur, single-coincidence meetings in which the potential producer does not have money and in which the potential consumer has money. We call such meetings production meetings. The only non production meetings that matter are those between people from different countries who hold different monies. For non production meetings, we let s_{jt} be the probability that the monies are exchanged at date t when both persons start in state j, $j = 1, 2$, and we let $s_t = (s_{1t}, s_{2t})$.[4] For production meetings, we have to distinguish between those involving two people from the same country and those involving people from different countries. For the former, we let $y_{jt} \in R_+$ and $\tau_{jt} \in [0,1]$ be output and the probability that money is transferred at date t when the consumer is in state j, $j = 1, 2$.[5] For the latter, we let y'_{jt} and τ'_{jt} denote the output and the probability that money is transferred at date t when the consumer is in state j, $j = 1, 2$. As shorthand, we let $y_t = (y_{1t}, y_{2t})$ and similarly for τ_t, y'_t, and τ'_t and let $A_t = (p_t, s_t, y_t, \tau_t, y'_t, \tau'_t)$. An *allocation* is a sequence $\{A_t\}_{t=0}^{\infty}$.

The law of motion for p_t can be expressed in terms of the transition matrix implied by p_t, s_t, τ_t, and τ'_t. That is,

$$p_{t+1} = p_t T_t, \tag{1}$$

[4] Given the unit bound on money holdings, this description is sufficient.

[5] Although this specification seems restrictive, it is not. Let λ denote a probability measure over $R_+ \times \{0,1\}$, where the first set is production and the second is "exchange" of states, 0 denotes no exchange of states and 1 denotes an exchange of states. Because the payoffs are additively separable in output and the state, only marginal distributions appear in those payoffs. That and the strict convexity of the preferred set for the consumer imply that equilibrium allocations do not have random output. In addition, only the marginal distributions for state transitions appear in the transition law for p_t.

where

$$NT_t = \begin{bmatrix} N(1-T_{12}-T_{13}) & p_{1t}\theta\tau_{1t} + p_{2t}(1\text{-}\theta)\tau'_{2t} & p_{2t}\theta\tau_{2t} + p_{1t}(1\text{-}\theta)\tau'_{1t} \\ p_{0t}[\theta\tau_{1t} + (1\text{-}\theta)\tau'_{1t}] & N(1-T_{21}-T_{23}) & Np_{1t}(1\text{-}\theta)s_{1t} \\ p_{0t}[\theta\tau_{2t} + (1\text{-}\theta)\tau'_{2t}] & Np_{2t}(1\text{-}\theta)s_{2t} & N(1-T_{31}-T_{32}) \end{bmatrix},$$

and where T_{ij} denotes the probability of a transition from state $i-1$ to state $j-1$. Although it is convenient to define this transition matrix, the cross-country symmetry implies that for all t, $p_{0t} = 1-m$ and $p_{1t}+p_{2t} = m$. Therefore, (1) can be expressed entirely in terms of a transition for p_{1t}.

In order to define equilibrium allocations, it is convenient to first define expected discounted utilities. We let v_{it} denote the expected discounted utility at the start of a date, prior to meetings, of someone in state i and let $v_t \equiv (v_{0t}, v_{1t}, v_{2t})'$. Then, the sequence $\{v_t\}$ satisfies

$$v_t = R_t + \beta T_t v_{t+1}, \tag{2}$$

where

$$R_t = \frac{1}{N}\begin{bmatrix} -\sum_{j=1}^{2} p_{jt}[\theta y_{jt} + (1-\theta)y'_{jt}] \\ p_{0t}[\theta u(y_{1t}) + (1-\theta)u(y'_{1t}) \\ p_{0t}[\theta u(y_{2t}) + (1-\theta)u(y'_{2t}) \end{bmatrix}.$$

For future reference, we note that given an allocation, there is exactly one bounded sequence $\{v_t\}$ that satisfies (2).[6]

Now we define equilibrium allocations.

Definition 1 *Given an initial condition p_0, an allocation $\{A_t\}_{t=0}^{\infty}$ is an equilibrium if (1) holds and if there exists a bounded sequence $\{v_t\}$ satisfying (2) such that the trade components of A_t are individually rational and pairwise efficient (are in the pairwise core) given v_{t+1}.*

We next define what we mean by a uniform-currency equilibrium. In a uniform-currency equilibrium, people do not distinguish between home and foreign money, between states 1 and 2.

[6]Consider the space of bounded sequences, $\{v_t\}$, with the sup norm. Let the mapping $f = (f_0, f_1, ..., f_t, ...)$ from this space into itself be defined by $f_t(\{v_t\}) = R_t + \beta T_t v_{t+1}$. Then, f satisfies Blackwell's sufficient conditions for contraction.

Definition 2 *A uniform-currency equilibrium is an equilibrium that satisfies* $y_{1t} = y_{2t}$*, and* $\tau_{1t} = \tau_{2t}$ *and* $y'_{1t} = y'_{2t}$*, and* $\tau'_{1t} = \tau'_{2t}$*.*

It follows from (2) and the definition of the transition matrix T_t that $v_{1t} = v_{2t}$ for a uniform currency equilibrium. It also follows that the v_t sequence implied by a uniform-currency equilibrium does not depend on those for s_t and p_t. In other words, a uniform-currency equilibrium is a one-money equilibrium which makes trades of the two monies and the distribution between the two monies in each country irrelevant.

Our welfare criterion is ex ante expected utility prior to the assignment of money holdings to individuals; namely, the inner product $p_0 v_0 \equiv w$. It follows from (2) that

$$p_t v_t = p_t R_t + \beta p_t T_t v_{t+1} = p_t R_t + \beta p_{t+1} v_{t+1}.$$

Therefore,

$$w = \sum_{t=0}^{\infty} \beta^t p_t R_t = \sum_{t=0}^{\infty} \beta^t \{ \frac{p_{0t}}{N} \sum_{j=1}^{2} p_{jt} [\theta g(y_{jt}) + (1-\theta) g(y'_{jt})] \} \tag{3}$$

where, recall, $g(x) \equiv u(x) - x$.

4 Best equilibria

We describe best equilibria in each of two mutually exclusive and exhaustive regions of the parameter space, regions which can be thought of as the low β region and the high β region. For low β, we show that best equilibria are uniform-currency equilibria. For high β, we show that there exist uniform-currency equilibria which are best equilibria.

In each case, we propose an equilibrium. Let $\gamma \equiv \frac{\beta(1-m)}{N(1-\beta)}$ and let $\hat{x}(\gamma)$ denote the unique positive solution for x to $x = \gamma g(x)$.[7] Existence and uniqueness of $\hat{x}(\gamma)$ follow from the properties of g. Another consequence of those

[7] It is well-known (see, for example, [4]) that $\hat{x}(\gamma)$ is the largest constant output that satisfies individual rationality for producers for a one-country, one-money version of our model.

properties, used below, is that if $x \le \gamma g(x)$, then $x \le \hat{x}(\gamma)$. We show that any best equilibrium has output equal to $\min\{\hat{x}(\gamma), y^*\}$ in every production meeting. Moreover, for low β—more precisely, for $\hat{x}(\gamma) \le y^*$—this best outcome for output is attained only by uniform-currency equilibria. For high β ($\hat{x}(\gamma) > y^*$), it is attained by many equilibria, among which are uniform-currency equilibria. The low β case might be regarded as especially relevant because in that case (when it holds with strict inequality) paying interest on money would be desirable if it were feasible.

Proposition 1 *Let $\gamma \equiv \frac{\beta(1-m)}{N(1-\beta)}$ and let $\hat{x}(\gamma)$ denote the unique positive solution for x to $x = \gamma g(x)$. If $\hat{x}(\gamma) \le y^*$, then best equilibria satisfy $y_{jt} = y'_{jt} = \hat{x}(\gamma)$ and $\tau_{jt} = \tau'_{jt} = 1$, and, therefore, are uniform-currency equilibria. If $\hat{x}(\gamma) > y^*$, then there exist uniform-currency equilibria which are best equilibria.*

The proof is given in section 6. The main part of the proof involves showing for the case $\hat{x}(\gamma) \le y^*$ that there is no better equilibrium.

The result for the case $\hat{x}(\gamma) > y^*$ cannot be strengthened. If $\hat{x}(\gamma) > y^*$, then there are non uniform-currency equilibria that are best equilibria. For example, for θ close to unity, an equilibrium with output equal to y^* in every production meeting, with $\tau_{1t} = \tau'_{2t} = 1$ and $\tau_{2t} = \tau'_{1t} = 1 - \varepsilon$ for some positive and small ε is a best equilibrium for some initial condition and implies $v_{2t} > v_{1t} > v_{0t}$. This kind of example also implies that if we distinguish people by initial asset holding, then it is not true that any non uniform-currency equilibrium is Pareto dominated by some uniform-currency equilibrium.[8]

[8]The individual differences in initial asset holdings in this model are dictated by the restrictive assumptions about individual money holdings: the indivisibility of money and the unit upper bound. In a version without such restrictive assumptions, individuals could be assumed to have identical initial asset holdings.

5 Inferior equilibria

For the case $\hat{x}(\gamma) \leq y^*$ we now show that there exist equilibria in which the monies play distinct roles and, in particular, are such that observed prices, as implied by the trades that occur, are higher in foreign currency than in home currency. This demonstration serves two purposes. First, it shows that proposition 1 is not vacuous in the sense that there do exist non uniform-currency equilibria. Second, and more important, the existence of such equilibria is our explanation for the distinct roles of different monies that we often see.[9]

An obvious way to get such a differential price equilibrium is to have the gains from trade in meetings divided up in different ways depending on whether or not the consumer can offer the producer the producer's home money. This is our way of modeling the idea of getting a relatively bad deal with foreign money. With one money, it is known that maximizing the value of money corresponds to having the consumer make take-it-or-leave-it offers and that a valued money steady state does not exist if the producer makes take-it-or-leave-it offers. The former gives all the gains from trade in meetings to the consumer, while the latter gives them all to the producer. In the proof of the next proposition, if a potential consumer can offer state 2 to the producer, then the consumer makes a take-it-or-leave-it offer; while if a potential consumer can offer state 1 to the producer, then the producer makes a take-it-or-leave-it offer.

To set the stage for the proof, we describe the date t solutions to these special bargaining problems for any v_{t+1} that satisfies the inequalities, $v_{0t+1} < v_{1t+1} \leq v_{2t+1} \leq \frac{y^*}{\beta}$, the last of which is implied by $\hat{x}(\gamma) \leq y^*$. To simplify the notation here, we suppress time subscripts.

Lemma 1 *Let $\Delta_i \equiv v_i - v_0$ for $i = 1, 2$. If the consumer can offer the*

[9] There are many models which use government policies that penalize holdings of foreign money or favor holdings of home money to get distinct roles for home and foreign money. See, for example, Li and Wright [12] and Waller and Soller Curtis [17].

producer the producer's home money, if the consumer makes a take-it-or-leave-it offer, and if $0 < \Delta_1 \leq \Delta_2 \leq \frac{y^*}{\beta}$*, then* $y_2 = y_1' = \beta\Delta_2$ *and* $\tau_2 = \tau_1' = 1$.

Proof. Here the producer's individual rationality (IR) constraint must hold at equality. Substituting from that constraint into the consumer's objective, we get $u(y) - y\frac{\Delta_i}{\Delta_2}$, where i is the consumer's state. Let $x = \arg\max[u(y) - y\frac{\Delta_i}{\Delta_2}]$. It follows from $\Delta_i \leq \Delta_2$ that $x \geq y^* \geq \beta\Delta_2$. Therefore x violates IR for the producer, which implies that $y = \beta\Delta_2$ and $\tau = 1$ is the only candidate solution. Because this candidate satisfies consumer IR, it is the solution. ■

Lemma 2 *Let* $\Delta_i \equiv v_i - v_0$ *for* $i = 1, 2$*. If the consumer can offer the producer the producer's foreign money, if the producer makes a take-it-or-leave-it offer, and if* $0 < \Delta_1 \leq \Delta_2 \leq \frac{y^*}{\beta}$*, then* $y_1 = u^{-1}(\beta\Delta_1)$*,* $\tau_1 = 1$*,* $y_2' = \min\{\eta(\frac{\Delta_2}{\Delta_1}), u^{-1}(\beta\Delta_2)\}$*, and* $\tau_2' = \frac{u(y_2')}{\beta\Delta_2} > 0$*, where* $\eta(\frac{\Delta_2}{\Delta_1}) \equiv \arg\max_y[\frac{\Delta_1}{\Delta_2}u(y) - y]$.

Proof. Here the consumer's IR constraint must hold at equality. Let i be the consumer's state. If $\Delta_i = \Delta_1$ and if, for the moment, we ignore the constraint $\tau \leq 1$, then the problem is to maximize $g(y)$. But because the maximizer of g is y^*, and $u(y^*) > y^* \geq \beta\Delta_1$, output at y^* violates the consumer's IR constraint at equality. Therefore, the unique candidate solution is $y = u^{-1}(\beta\Delta_1)$ and $\tau = 1$. This is the solution because it satisfies producer IR . Now suppose $\Delta_i > \Delta_1$, which implies that $\Delta_i = \Delta_2$. Substituting from the consumer's IR constraint into the objective, the objective becomes $\frac{\Delta_1}{\Delta_2}u(y) - y$. It follows that the only candidate solution is $y = \min\{\eta(\frac{\Delta_2}{\Delta_1}), u^{-1}(\beta\Delta_2)\}$ with τ determined by the consumer's IR constraint at equality. Because producer IR is satisfied at this candidate, it is the solution. ■

The trades described in lemmas 1 and 2 are functions of $\Delta \equiv (\Delta_1, \Delta_2)$ and are continuous in Δ. Those properties are used in the proof of the next proposition.

Proposition 2 *If* $\theta > \frac{1}{2}$ *and* $\hat{x}(\gamma) \leq y^*$*, then for any* p_0 *there exists an equilibrium with* $v_{0t} < v_{1t} < v_{2t}$*,* $s_t \equiv (s_{1t}, s_{2t}) = (1, 0)$*, and* $p_{1,t+1} \in (0, m)$.

The proof appears in section 6. It applies the sequence of truncated economies approach used in Balasko and Shell [3] and Brouwer's fixed point theorem for each truncated economy.

Corollary 1 *In a proposition* 2 *equilibrium consistent with the bargaining schemes of lemmas* 1 *and* 2*, prices at each date in terms of foreign currency exceed prices in terms of home currency.*

Proof. According to the model, observed prices in home currency are $\frac{\tau_{2t}}{y_{2t}}$ and $\frac{\tau'_{1t}}{y'_{1t}}$, while observed prices in foreign currency are $\frac{\tau_{1t}}{y_{1t}}$ and $\frac{\tau'_{2t}}{y'_{2t}}$. By lemma 1, prices in home currency satisfy $\frac{\tau_{1t}}{y_{1t}} = \frac{\tau'_{2t}}{y'_{2t}} = \frac{1}{\beta\Delta_{2t+1}}$. By lemma 2 and proposition 2, prices in terms of foreign currency satisfy: $\frac{\tau_{1t}}{y_{1t}} = \frac{1}{u^{-1}(\beta\Delta_{1t+1})} > \frac{1}{(\beta\Delta_{1t+1})} > \frac{1}{\beta\Delta_{2t+1}}$ and $\frac{\tau'_{2t}}{y'_{2t}} = \frac{u(y'_{2t})/\beta\Delta_{2t+1}}{y'_{2t}} > \frac{1}{\beta\Delta_{2t+1}}$. ■

A proposition 2 equilibrium has some foreign trade. In fact, some foreign trade occurs whenever an allocation gives rise to $v_{1t} > v_{0t}$, a higher value of holding foreign currency than of producing. When that inequality holds, any pairwise core outcome has trade in a meeting between a producer and a consumer who can offer the producer state 1, the producer's foreign money. Moreover, because $v_{1t} > 0$ is a consequence of $v_{2t} > 0$ (by way of meetings with foreigners), a necessary condition for valuable home money and no foreign trade is $v_{0t} \geq v_{1t} > 0$. Although there exist such equilibria for some initial conditions and some parameters, we do not think they should be taken seriously.[10] In a model with divisible money in which anyone can always produce, the analogue of the inequality $v_{0t} \geq v_{1t}$ cannot hold. (In our model, even $v_{0t} > v_{1t}$ is possible because disposing of money does not give a person the capability to produce.)

[10] To construct such allocations, set $p_{1t} = y'_{2t} = \tau'_{2t} = y_{1t} = \tau_{1t} = 0$ and choose scalars $y_{2t} \equiv y_2$ and $\tau_{2t} \equiv \tau_2$ so that $v_{2t} \equiv v_2 > v_{0t} \equiv v_0 > 0$. There are many such (y_2, τ_2). (Any stationary one-money, one-country monetary allocation for a model in which people meet no one with probability $1-\theta$ will do except those consistent with take-it-or-leave-it offers by consumers.) For such allocations, the only meetings that contribute to making $v_{1t} \equiv v_1$ positive are meetings with foreign producers. It follows that the implied $v_1 \to 0$ as $\theta \to 1$. Therefore, for θ near enough to unity, such allocations exist.

6 Proofs of the propositions

This section contains the proofs of propositions 1 and 2.

Proposition 1. Let $\gamma = \frac{\beta(1-m)}{N(1-\beta)}$ and let $\hat{x}(\gamma)$ denote the unique positive solution for x to $x = \gamma g(x)$. If $\hat{x}(\gamma) \leq y^*$, then best equilibria satisfy $y_{jt} = y'_{jt} = \hat{x}(\gamma)$ and $\tau_{jt} = \tau'_{jt} = 1$, and, therefore, are uniform-currency equilibria. If $\hat{x}(\gamma) > y^*$, then there exist uniform-currency equilibria which are best equilibria.

Proof. Assume that $\hat{x}(\gamma) \leq y^*$.

First we show that there exists $\{p_{t+1}, s_t\}$ such that it and $y_{jt} = y'_{jt} = \hat{x}(\gamma)$ and $\tau_{jt} = \tau'_{jt} = 1$ is an equilibrium. There exist many such $\{p_{t+1}, s_t\}$. We can let $\{s_t\}$ be arbitrary and then let $\{p_{t+1}\}$ be determined by (1). For $\{v_t\}$, we propose $v_{0t} = 0$ and $v_{1t} = v_{2t} = \frac{\hat{x}(\gamma)}{\beta}$. Direct substitution shows that these satisfy (2). The final step is to show that $y_{jt} = y'_{jt} = \hat{x}(\gamma)$, $\tau_{jt} = \tau'_{jt} = 1$, and any s_t is in the pairwise core when $v_{0t} = 0$ and $v_{1t} = v_{2t} = \frac{\hat{x}(\gamma)}{\beta}$. With $v_{1t+1} = v_{2t+1}$, any s_t is in the pairwise core. As regards $y_{jt} = y'_{jt} = \hat{x}(\gamma)$, $\tau_{jt} = \tau'_{jt} = 1$, it is sufficient to show that these solve the following problem: choose scalars $y \geq 0$ and $\tau \in [0, 1]$ to maximize $u(y) + \tau \cdot 0 +$ $(1-\tau)\hat{x}(\gamma) -$ $\hat{x}(\gamma) = u(y) - \tau\hat{x}(\gamma)$ subject to $-y + (1-\tau) \cdot 0 + \tau\hat{x}(\gamma) = -y + \tau\hat{x}(\gamma) \geq 0$. The solution is $y = \min\{\hat{x}(\gamma), y^*\}$ with the constraint at equality. The assumption, $\hat{x}(\gamma) \leq y^*$, implies that the solution is $y = \hat{x}(\gamma)$ and $\tau = 1$.

Let w^* denote ex ante welfare implied by $y_{jt} = y'_{jt} = \hat{x}(\gamma)$ according to (3). Now we turn to the main part of the proof and show that any equilibrium which gives welfare no smaller than w^* also has $y_{jt} = y'_{jt} = \hat{x}(\gamma)$ and $\tau_{jt} = \tau'_{jt} = 1$. Let A denote an equilibrium which gives welfare no smaller than w^*. If A does not satisfy $y_{jt} = y'_{jt} = \hat{x}(\gamma)$ and $\tau_{jt} = \tau'_{jt} = 1$, then A must differ either in some y component or some τ component at some date. If the former, then some y component at some date must exceed $\hat{x}(\gamma)$ (Otherwise, $w(A) < w^*$ because g is increasing on $[0, y^*]$). By producer IR, it follows that $\Delta_{jt}(A) > \frac{\hat{x}(\gamma)}{\beta}$ for some t and j. (Recall that $\Delta_{jt} = v_{jt} - v_{0t} = \frac{\hat{x}(\gamma)}{\beta}$ and $\tau_{jt} = \tau'_{jt} = 1$ for the candidate uniform-currency equilibrium.) If not the former—and, therefore, only the latter—then, again by producer IR, $\Delta_{jt}(A) > \frac{\hat{x}(\gamma)}{\beta}$ for some t and j. We show that the condition $\Delta_{jt}(A) > \frac{\hat{x}(\gamma)}{\beta}$ gives rise to a contradiction. Let $\Delta_{Mt}(A) = \max\{\Delta_{1t}(A), \Delta_{2t}(A)\}$. We first show that $\Delta_{jt}(A) > \frac{\hat{x}(\gamma)}{\beta}$ implies that $\{\Delta_{Mt+k}(A)\}_{k=0}^{\infty}$ is strictly increasing.

Claim: If $\Delta_{Mt}(A) > \frac{\hat{x}(\gamma)}{\beta}$, then $\Delta_{Mt+1}(A) > \Delta_{Mt}(A)$.

To simplify the expressions, we omit the argument A in what follows. It is to be understood, that the allocation and the sequence $\{v_t\}$ pertain to A. By (2), for $j = 1, 2$,

$$\begin{aligned} \Delta_{jt} \quad = \quad & \beta\Delta_{jt+1} + \frac{\theta}{N}\{p_{0t}[u(y_{jt}) - \tau_{jt}\beta\Delta_{jt+1}] + \sum_{i=1}^{2} p_{it}(y_{it} - \tau_{it}\beta\Delta_{it+1})\} + \\ & \frac{(1-\theta)}{N}\{p_{0t}[u(y'_{jt}) - \tau'_{jt}\beta\Delta_{jt+1}] + \sum_{i=1}^{2} p_{it}(y'_{it} - \tau'_{it}\beta\Delta_{i't+1})\} + \\ & (1-\theta)s_{jt}p_{jt}\beta(\Delta_{j't+1} - \Delta_{jt+1}) \end{aligned} \tag{4}$$

This expresses $v_{jt} - v_{0t}$ as the discounted value of itself plus the gains from trade of starting in state j minus the gains from starting in state 0. By producer IR and the definition of Δ_{Mt+1}, it follows that

$$\Delta_{jt} \leq \beta\Delta_{Mt+1} + \frac{p_{0t}}{N}\{\theta[u(y_{jt}) - \tau_{jt}\beta\Delta_{jt+1}] + (1-\theta)[u(y'_{jt}) - \tau'_{jt}\beta\Delta_{jt+1}]\}.$$

Using producer IR again and the definition of g, we have

$$\Delta_{jt} \leq \beta\Delta_{Mt+1} + \frac{p_{0t}}{N}[\theta g(\tau_{jt}\beta\Delta_{jt+1}) + (1-\theta)g(\tau'_{jt}\beta\Delta_{jt+1})].$$

Then, by concavity of g,

$$\Delta_{jt} \leq \beta\Delta_{Mt+1} + \frac{p_{0t}}{N}g(\tau\beta\Delta_{jt+1}), \tag{5}$$

where $\tau \equiv \theta\tau_{jt} + (1-\theta)\tau'_{jt}$. Because (5) holds for $j = 1, 2$, it follows that for some j,

$$\Delta_{Mt} \leq \beta\Delta_{Mt+1} + \frac{p_{0t}}{N}g(\tau\beta\Delta_{jt+1}). \tag{6}$$

Now suppose by contradiction that $\Delta_{Mt+1} \leq \Delta_{Mt}$. Then for some j, we have

$$\Delta_{Mt} \leq \beta\Delta_{Mt} + \frac{p_{0t}}{N} g(\tau\beta\Delta_{jt+1})$$

or

$$\Delta_{Mt} \leq \frac{p_{0t}}{(1-\beta)N} g(\tau\beta\Delta_{jt+1}) \tag{7}$$

or multiplying by β and using $p_{0t} = 1 - m$,

$$\beta\Delta_{Mt} \leq \gamma g(\tau\beta\Delta_{jt+1}). \tag{8}$$

Now we deal separately with $\beta\Delta_{Mt} \leq y^*$ and $\beta\Delta_{Mt} > y^*$. If $\beta\Delta_{Mt} \leq y^*$, then $\tau\beta\Delta_{jt+1} \leq \beta\Delta_{Mt+1} \leq \beta\Delta_{Mt}. \leq y^*$, which implies that $g(\tau\beta\Delta_{jt+1}) \leq g(\beta\Delta_{Mt})$. This and (8) imply $\beta\Delta_{Mt} \leq \gamma g(\beta\Delta_{Mt})$. But by the definition of $\hat{x}(\gamma)$, this inequality contradicts the hypothesis of the claim. If $\beta\Delta_{Mt} > y^*$, then (8) implies $y^* < \gamma g(y^*)$. This inequality contradicts the definition of $\hat{x}(\gamma)$ and the assumption $\hat{x}(\gamma) \leq y^*$. Hence, the claim is established.

Because $\{\Delta_{Mt+k}(A)\}_{k=0}^{\infty}$ is strictly increasing and bounded, it has a limit, L, where $\beta L > \hat{x}(\gamma)$. As demonstrated above, (6) is a consequence of the assumption about allocation A. The limit conclusion and (6) imply that

$$L - \varepsilon \leq \beta L + \frac{p_{0t}}{N} g(\tau\beta\Delta_{jt+1}) \tag{9}$$

or

$$\beta L - \frac{\beta\varepsilon}{1-\beta} \leq \gamma g(\tau\beta\Delta_{jt+1}) \tag{10}$$

for any arbitrarily small $\varepsilon > 0$ and $\tau\beta\Delta_{jt+1} < \beta L$. But (10) produces the same kind of contradiction obtained at the end of the proof of the claim. If $\beta L \leq y^*$, then (10) implies $\beta L - \frac{\beta\varepsilon}{1-\beta} \leq \gamma g(\beta L)$, which contradicts $\beta L > \hat{x}(\gamma)$. If $\beta L > y^*$, then (10) implies $y^* - \frac{\beta\varepsilon}{1-\beta} < \gamma g(y^*)$, which contradicts $\hat{x}(\gamma) \leq y^*$. This completes the case, $\hat{x}(\gamma) \leq y^*$.

Now suppose $\hat{x}(\gamma) > y^*$.

In this case, there are many uniform currency best equilibria. All satisfy $y_{jt} = y'_{jt} = y^*$. To find all possible uniform probabilities of transferring money, we begin by using the stationary version of (4). If $y_{jt} = y'_{jt} = y^*$ and $\tau_{jt} = \tau'_{jt} = \tau > 0$, then the stationary version of (4) implies

$$\tau\beta\Delta[1 + \frac{N(1-\beta)}{\beta\tau}] = (1-m)u(y^*) + my^*. \tag{11}$$

If we set $\tau\beta\Delta = y^*$, as implied by binding producer IR, then (11) implies $\tau = \frac{y^*}{\gamma g(y^*)} < 1$, where the inequality follows from $\hat{x}(\gamma) > y^*$. Now consider $y_{jt} = y'_{jt} = y^*$ and $\tau_{jt} = \tau'_{jt} = \tau^* \in [\frac{y^*}{\gamma g(y^*)}.1]$. As above, let $\{s_t\}$ be arbitrary and let $\{p_{t+1}\}$ be determined by (1). We have only to show that any such allocation is an equilibrium. (Because y^* is the unconstrained maximum of g, there is no better allocation.)

Let Δ^* be the solution for Δ from (11) when $\tau = \tau^*$. We have only to show that (y^*, τ^*) is in the pairwise core when $\Delta = \Delta^*$. That is, it is enough to show that $(y, \tau) = (y^*, \tau^*)$ satisfies consumer IR and is the solution to the following problem for some $k \geq 0$: choose (y, τ) to maximize $u(y) - \tau\beta\Delta^*$ subject to $-y + \tau\beta\Delta^* \geq k$. It is evident that $(y, \tau) = (y^*, \tau^*)$ is the solution if $k = -y^* + \tau^*\beta\Delta^*$. Thus, we need to confirm that $-y^* + \tau^*\beta\Delta^* \geq 0$. By construction, $-y^* + \tau^*\beta\Delta^* = 0$ when $\tau^* = \frac{y^*}{\gamma g(y^*)}$. By (11), $\tau\beta\Delta$ is increasing in τ. Therefore, $-y^* + \tau^*\beta\Delta^* \geq 0$ for all $\tau^* \in [\frac{y^*}{\gamma g(y^*)}, 1]$. Finally, satisfaction of consumer IR $(u(y^*) \geq \tau^*\beta\Delta^*)$ follows from (11) and $u(y^*) > y^*$. ■

Proposition 2. If $\theta > \frac{1}{2}$ and $\hat{x}(\gamma) \leq y^*$, then for any p_0 there exists an equilibrium with $v_{0t} < v_{1t} < v_{2t}$, $s_t \equiv (s_{1t}, s_{2t}) = (1, 0)$, and $p_{1,t+1} \in (0, m)$.

Proof. We use the truncated economy approach in [3]. The first step involves applying Brouwer's fixed point theorem to establish existence for a T-period economy conditional on a given terminal condition for $\Delta = (\Delta_1, \Delta_2)$.

We begin by defining a (one-period) mapping, H. In effect, the mapping is from $\Delta_{t+1} = (\Delta_{1t+1}, \Delta_{2t+1})$ and p_t to Δ_t and p_{t+1} via (1) and (2) using the trades implied by lemmas 1 and 2 and $s_t \equiv (s_{1t}, s_{2t}) = (1, 0)$. To simplify the notation, we drop time subscripts when it will not cause confusion.

For $j = 1, 2$, let $h_j(\Delta, p_1)$ denote the right-hand side of (4) and let $h_3(\Delta, p_1)$ be the second component of $p_t T_t$ (see (1)) when $p_t = (1 - m,$

p_1, $m - p_1$), and when $(y_t, y'_t, \tau_t, \tau'_t)$ is given by the conclusions of lemmas 1 and 2 and $s_t = (1, 0)$. Then, let $H(\Delta, p_1) \equiv (h_1(\Delta, p_1), h_2(\Delta, p_1), h_3(\Delta, p_1))$.

Our next task is to choose a suitable domain for H. We begin by constructing lower bounds for h_1 and h_2 for Δ satisfying $0 < \Delta_1 \leq \Delta_2 \leq \frac{y^*}{\beta}$. By definition,

$$\begin{aligned} h_1(\Delta, p_1) &= \beta\Delta_1 + \frac{\theta p_1}{N}(y_1 - \tau_1\beta\Delta_1) + \frac{(1-\theta)p_0}{N}[u(y'_1) - \tau'_1\beta\Delta_1] \\ &\quad + \frac{(1-\theta)}{N}\{p_2(y'_2 - \tau'_2\beta\Delta_1) + p_1 N\beta(\Delta_2 - \Delta_1)\}. \end{aligned} \tag{12}$$

Therefore, using $\Delta_2 \geq \Delta_1 \geq 0$ and gathering all the terms in Δ_1, we have

$$h_1(\Delta, p_1) \geq \beta\Delta_1[1 - \frac{1}{N}] + \frac{(1-\theta)p_0}{N}[u(y'_1)] \geq \frac{(1-\theta)p_0}{N}[u(y'_1)]. \tag{13}$$

Moreover, by lemma 1, $y'_1 = \beta\Delta_2$. Therefore,

$$h_1(\Delta, p_1) \geq \frac{(1-\theta)(1-m)}{N}[u(\beta\Delta_2)]. \tag{14}$$

Thus, a positive lower bound on Δ_2 implies a positive lower bound on $h_1(\Delta, p_1)$.

Now we turn to $h_2(\Delta, p_1)$. By definition,

$$\begin{aligned} h_2(\Delta, p_1) &= \beta\Delta_2 + \frac{\theta}{N}p_0[u(y_2) - \tau_2\beta\Delta_2] + \frac{\theta}{N}p_1(y_1 - \tau_1\beta\Delta_1) \\ &\quad + \frac{(1-\theta)}{N}p_2(y'_2 - \tau'_2\beta\Delta_1). \end{aligned} \tag{15}$$

Therefore, using $\Delta_2 \geq \Delta_1 \geq 0$ and gathering all the terms in Δ_2, we have

$$h_2(\Delta, p_1) \geq \beta\Delta_2(1 - \frac{1}{N}) + \frac{\theta p_0}{N}u(y_2) \geq \frac{\theta p_0}{N}u(y_2). \tag{16}$$

Because $y_2 = \beta\Delta_2$ by lemma 1,

$$h_2(\Delta, p_1) \geq \frac{\theta(1-m)}{N}u(\beta\Delta_2). \tag{17}$$

Now let $\hat{b}$ be the unique positive solution to $b = \frac{\theta(1-m)}{N} u(\beta b)$ and let $e \in (0, \hat{b})$. It follows that if $\Delta_2 \geq e$, then $h_2(\Delta, p_1) \geq e$.

We use this lower bound and that in (14) to construct the domain for H. Let $(\varepsilon_1, \varepsilon_2)$ satisfy the following three conditions: $\varepsilon_2 \in (0, \hat{b})$, $\varepsilon_1 = \frac{(1-\theta)(1-m)}{N} u(\beta\varepsilon_2)$, and $max(\varepsilon_1, \varepsilon_2) < \frac{\hat{x}(\gamma)}{\beta}$. (Because $u(0) = 0$, these conditions can always be met.) Let

$$S \equiv \{\Delta \in R^2 : \Delta_i \in [\varepsilon_i, \frac{\hat{x}(\gamma)}{\beta}] \text{ and } \Delta_1 \leq \Delta_2\} \text{ and } D \equiv S \times [0, m]. \tag{18}$$

Our proposed domain for H is D. Notice that D is nonempty, compact, and convex.

The next task is to show that $H(D) \subset D$. For any $z \in D$, it is immediate that $h_3(z) \in [0, m]$. Also, $z \in D$ implies that the restrictions on Δ assumed in lemmas 1 and 2 hold. Therefore, by construction, the lower bounds in S are preserved by the mapping H.

It remains to show that if $z \in D$, then $h_1(z) \leq h_2(z) \leq \frac{\hat{x}(\gamma)}{\beta}$. We start with the first inequality. From (12) and (15),

$$\begin{aligned} h_2(z) - h_1(z) &= \beta(\Delta_2 - \Delta_1)[1 - (1-\theta)p_1] + \\ & \frac{\theta p_0}{N}[u(y_2) - \tau_2\beta\Delta_2] - \frac{(1-\theta)p_0}{N}[u(y_1') - \tau_1'\beta\Delta_1]. \end{aligned}$$

By lemma 2, $(y_2, \tau_2) = (y_1', \tau_1') = (\beta\Delta_2, 1)$. Therefore,

$$\begin{aligned} h_2(z) - h_1(z) &= \beta(\Delta_2 - \Delta_1)[1 - (1-\theta)p_1] + \\ & \frac{\theta p_0}{N}[g(\beta\Delta_2)] - \frac{(1-\theta)p_0}{N}[g(\beta\Delta_2) - \beta(\Delta_2 - \Delta_1)] \\ &= \beta(\Delta_2 - \Delta_1)\lambda + \frac{p_0 g(\beta\Delta_2)}{N}(2\theta - 1) > 0, \end{aligned} \tag{19}$$

where $\lambda \equiv [1 - (1-\theta)p_0(1 + \frac{1}{N})] > 0$ because $\theta > \frac{1}{2}$. Therefore, $\theta > \frac{1}{2}$ implies the strict inequality in (19). We now show that $z \in D$ implies $h_2(z) \leq \frac{\hat{x}(\gamma)}{\beta}$.

Only the first two terms on the right side of (15) can be positive. Therefore,

$$\begin{aligned} h_2(z) &\leq \beta\Delta_2 + \frac{\theta(1-m)}{N}[g(\beta\Delta_2)] = \beta\Delta_2 + \frac{1-\beta}{\beta}\gamma[g(\beta\Delta_2)] \\ &\leq \hat{x}(\gamma) + \frac{1-\beta}{\beta}\gamma g(\hat{x}(\gamma)) = \hat{x}(\gamma) + \frac{1-\beta}{\beta}\hat{x}(\gamma) = \frac{\hat{x}(\gamma)}{\beta} \end{aligned}$$

where the last inequality uses the hypothesis $\hat{x}(\gamma) \leq y^*$ to conclude that $\beta\Delta_2 \leq \hat{x}(\gamma)$ implies $g(\beta\Delta_2) \leq g(\hat{x}(\gamma)$.

Now we turn to the truncated economy. Let $T \geq 1$. Fix $(\Delta_T, p_{10}) \in D$ and let $z^T \equiv (z_0, z_1, ..., z_{T-1})$, where $z_t = (\Delta_{1t}, \Delta_{2t}, p_{1,t+1}) \in D$. Notice that $z^T \in D^T$. Let $H^T(\cdot; \Delta_T, p_{10}) \equiv (H_0, H_1, ..., H_{T-1}) : D^T \to D^T$ be defined by $H_t(z^T; \Delta_T, p_{10}) \equiv H(\Delta_{t+1}, p_{1t})$. The conclusion that H^T maps D^T into itself follows from the fact, that H maps D into itself. Because H is a continuous function on D, it follows that $H^T : D^T \to D^T$ is a continuous function on a non-empty, compact, and convex domain. Therefore, by Brouwer's fixed point theorem, H^T has a fixed point. Any such fixed point satisfies the equilibrium conditions for dates $t = 0, 1, ..., T-1$ given the terminal condition Δ_T.

Now let

$$F^T(p_{10}) = \{z^T \in D^T : H^T(z^T; \Delta_T, p_{10}) = z^T \text{ for some } \Delta_T \in S\}. \qquad (20)$$

That is, F^T is the set of fixed points of H^T—as the terminal condition, Δ_T, ranges over all of S. By definition, $F^T(p_{10}) \subset D^T$. Now let $\Omega_0 \equiv \times_0^\infty D$, $\Omega_1 \equiv F^1 \times (\times_1^\infty D), ..., \Omega_T \equiv F^T \times (\times_T^\infty D), ...$ It follows that $\Omega_0 \supset \Omega_1 \supset ... \supset \Omega_T \supset ...$, so that the sequence $\{\Omega_t\}_{t=0}^\infty$ is a sequence of non-empty, compact, nested sets. It follows from Tychonoff's Theorem that $\Omega \equiv \cap_{t=0}^\infty \Omega_t$ is not empty. Let $\tilde{z}$ denote an element of Ω. We can associate with $\tilde{z}$ the sequence of trades implied by lemmas 1 and 2. Finally, we can associate with those trades a sequence $\{v_t\}_{t=0}^\infty$ implied by (2); namely,

$$v_t = R_t + \sum_{i=t+1}^{\infty} \beta^{i-t}(\Pi_{j=t}^{i-1} T_j) R_i. \qquad (21)$$

Because T_j is a transition matrix and R_i as determined by $\tilde{z}$ is bounded, v_t exists. Finally, any such $\{v_t\}_{t=0}^{\infty}$ is consistent with $\tilde{z}$. Hence, there is an equilibrium associated with $\tilde{z}$. Call it $\{\tilde{A}_t\}_{t=0}^{\infty}$. The last step is to verify the strict inequalities not guaranteed by the domain D.

Suppose, by way of contradiction, that $\{\tilde{A}_t\}_{t=0}^{\infty}$ implies $v_{1t} = v_{2t}$. By the definition of D, $v_{1t+1} \leq v_{2t+1}$. But since v_{t+1} and v_t have to be consistent with the mapping H, this violates the inequality in (19). Next, consider the claim that $\tilde{p}_{t+1}$ has full support. If $\tilde{p}_t$ has full support, then so does $\tilde{p}_{t+1}$ because not everyone trades. If $\tilde{p}_t$ does not have full support, then $\tilde{p}_{t+1}$ does because, according to lemmas 1 and 2, there is a positive inflow into the other state through trade with foreigners. In fact, this shows that the $\tilde{p}_t$ sequence cannot converge to a non full-support distribution. ■

7 Concluding remarks

According to our model, the benefit of a uniform currency is that it avoids a class of equilibria in which the different currencies play distinct roles, a class which is superfluous but not innocuous. The class is superfluous in the sense that the optimum is always among the equilibria in which the monies do not play distinct roles. The class is not innocuous because, as proposition 2 shows, it contains inferior equilibria.[11] Although we demonstrated this for a very particular model, a divisible goods version of [13] with randomized trade, the results seem to depend on only two ingredients of the model. One is that trade occurs in single-coincidence meetings in which the gains-from-trade associated with trading goods for money are such that the pairwise core contains many elements. The other is that the sole requirement for equilibrium is that trades be in that core. (In fact, small groups could replace

[11] The existence of inferior equilibira distinguishes our multiplicity from that in the exchange-rate indeterminacy literature (see Kareken and Wallace [8] , Manuelli and Peck [14], and King, Wallace, and Weber [10]). Unless the models in that literature are augmented by assumptions which rule out risk-sharing markets, the multiplicity is innocuous; multi-currency outcomes are no worse than one-currency outcomes.

pairs without substantially changing the results.[12])

Those two ingredients give rise to a multiplicity of equilibria that many readers will find troublesome. In the model, in the low discount factor case, any best equilibrium has unique trades: each unit of money trades for a particular of amount of the good. Such trades are *consistent* with take-it-or-leave-it offers by consumers, but are not implied by those offers. (Even with such offers, no trade of any sort is a possible equilibrium, and in richer settings there are other possible equilibria.) Because the best trades are *inconsistent* with any other way of dividing the gains from trade in meetings, inferior equilibria are easy to construct. One way is to make the division depend on which money is offered, proposition 2 being one extreme instance. Of course, there are also inferior uniform-currency equilibria—for example, an equilibrium with no trade at all or equilibria in which trades depend on the nationalities of consumers and producers.

Despite the implied multiplicity, the two main ingredients should not be quickly dismissed. The notion that trade occurs in pairs has been part of discussions of money for a very long time and not because anyone thought that it led to a simple and tractable model. And if trade occurs in pairs against the background of a large economy, then the pairwise core is an appealing concept of equilibrium. In a world in which those two ingredients are approximately valid, it could easily happen that countries end up with distinct roles for home and foreign monies. That is the kind of world in which the benefit of a uniform currency set out above applies.

However, the result that the best equilibrium is a uniform-currency equilibrium may not hold if countries differ–for example, in preferences. If countries differ, then having different distributions of money within each country may be desirable because the distributions affect individual-rationality constraints. And the set of equilibrium distributions is likely to be larger with distinct currencies than with a uniform currency. Therefore, differences

[12]But centralized markets would change the results because the core would contain only one allocation.

among countries could overturn the result that the best equilibrium is a uniform-currency equilibrium.

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