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In this paper the results of six different estimation methods applied to a linear aggregated model of the Italian economy are at first displayed. Afterwards, the inherent dynamic characteristics and the simulation properties of the six sets of estimates are analyzed. In no case the obtained results show a clear cut prevalence of one estimation method on the others, at least as far as the used indicators are concerned.

KEYWORDS: Econometric Models, Estimation Methods, Simulation.

1. Introduction

It is well known that each estimation method is, in general, characterized by some desirable statistical properties. It should be therefore possible to catalogue the different estimation methods as well as, perhaps, to determine the best estimator, i.e. the one which, among the estimators of the same class, possesses the greatest number of these properties. Unfortunately, many of these properties are asymptotic, that is they are valid for large samples; consequently the advantages coming from the use of estimation methods "better" (in the sense previously explained) than others are strongly reduced and even compromised because, very often, the available samples for econometric applications are extremely small. Furthermore, the small sample properties of these estimators are far from being known, at least a-priori (see Kmenta(1971)), so that a choice among them will not be univocal and resort will be made to a-priori information (in our case related to economic theory) about the expected value of the estimates. Nevertheless, very often (see Seaks(1974) and Klein(1969)), alternative estimates of the same model, besides being statistically not rejectable, are considered reasonable and comparable also from an economic point of view. One could therefore decide to use indifferently any of the alternative estimates, for example in experiments of economic policy. However, this is not the case because, as remarked by Klein(1969), small differences in the point estimation can generate significant system-wide effects, so that, in such situations, further instruments should be used for investigating the validity of alternative estimates.
Starting from this last consideration, in this paper the inherent dynamic characteristics and the simulation properties of a model estimated by means of six different methods will be analyzed. The six methods will be hereafter referred to with the following

symbology: OLS (Ordinary Least Squares), CORC (Cochrane-Orcutt method), 2SLS (Two Stage Least Squares), LISE (Limited Information Single Equation), 3SLS (Three Stage Least Squares), FIML (Full Information Maximun Likelihood). The inherent dynamic properties of the model will be examined in terms of its characteristic roots. The simulation properties will be analyzed in terms of some non-parametric measures, so called because, at this stage of our knowledge, they cannot be subjected to classic statistical tests, being, as noted by Dhrymes et alii(1972,pag.314), measures "merely descriptive and geared to specialized model uses". Among these non-parametric measures, the most widely used (see Howrey(1972), Dhrymes et alii(1972)) for the validation of econometric models have been chosen; particularly, in order to study the behaviour of each endogenous variable, single-variable measures like Root Mean Squared Error (RMSE) (see Klein(1974)), Inequality Coefficient (U) of Theil(1966), and Mean Absolute Percentage Error (MAPE) (see Klein(1974)) have been used.

2. The model

The model analyzed in this paper is a linear aggregated model of the Italian economy developed by some researchers (Sitzia and Tivegna(1975)) of the Study Group of the Bank of Italy. Its theoretical assumptions have been influenced by previous studies of Klein(1950): its structure is therefore similar to that of the Klein-I model. In Table 1 the structural form of the model is displayed; it consists of five behavioural (stochastic) equations and two national accounting identities, so that there are seven jointly determined variables (endogenous). The whole specification involves eight predetermined variables (exogenous and lagged endogenous) as well.

Table 1. The model

$$
\begin{aligned}
&1 - \underline{CPN} &&= a_{10} + a_{11}(\underline{WIT}+WG+X2) + a_{12}(\underline{PIT}+PAF) + a_{13}(PIT+PAF)_{-1} + u_1 \\
&2 - \underline{ILIT} &&= a_{20} + a_{21}PIT_{-1} + a_{22}\underline{KOCC} + a_{23}ILIT_{-1} + u_2 \\
&3 - \underline{M} &&= a_{30} + a_{31}(\underline{CPN}+\underline{ILIT}) + a_{32}T + u_3 \\
&4 - \underline{WIT} &&= a_{40} + a_{41}(\underline{WIT}+\underline{PIT}) + a_{42}\underline{KOCC} + a_{43}DUS70 + a_{44}WIT_{-1} + u_4 \\
&5 - \underline{KOCC} &&= a_{50} + a_{51}(\underline{ILIT}+ILIT_{-1}+ILIT_{-2}) + a_{52}(ILIT_{-1}+2ILIT_{-2}) + u_5 \\
&6 - \underline{RNLCF} &&= \underline{CPN} + \underline{ILIT} + WG + X1 - \underline{M} - TI \\
&7 - \underline{PIT} &&= \underline{RNLCF} - \underline{WIT} - WG - PAF - X2
\end{aligned}
$$

The endogenous variables are underlined. The indexes -1 and -2 refer to endogenous or exogenous variables lagged 1 or 2 years. a_{ij} are the structural coefficients, u_i are the disturbance terms of the stochastic equations. In Table 2 a list of all the variables included in the model is presented.

Table 2. List of variables

<u>CPN</u>	Total private consumption.
<u>ILIT</u>	Gross private investment in the industrial and tertiary sectors.
<u>M</u>	Imports of merchandise.
<u>WIT</u>	Wage bill in the industrial and tertiary sectors.
<u>KOCC</u>	Index of capacity utilization in the manufacturing sector.
<u>RNLCF</u>	Gross national product at factor prices.
<u>PIT</u>	Residual incomes in the industrial and tertiary sectors.
WG	Government wage bill.
PAF	Residual incomes in the other private sectors.
X2	Miscellaneous wage earning including agriculture.
X1	Exports and other miscellaneous demand variables (including investments in other sectors).
TI	Indirect taxes.
T	Time (1952=1,...., 1971=20).
DUS70	Dummy variable (1970=0.25, 1971=0.75, other years=0.).
1	Constant term.

3. The results of the alternative methods of estimation

The model presented in the previous section was at first estimated by means of five methods (OLS, 2SLS, LISE, 3SLS, FIML). The sample data are yearly observations from 1952 to 1971. The coefficient estimates and, in parentheses, the associated standard errors, are displayed in Table 3.

Table 3. Estimated coefficients and standard errors

Equation	Coefficient of	OLS	2SLS	LISE	3SLS	FIML
1. CPN	1	2464.0	2327.5	1156.1	2303.5	2299.1
		(765.)	(700.)	(911.)	(729.)	(732.)
	(WIT+WG+X2)	.77127	.75178	.58286	.75013	.73863
		(.110)	(.101)	(.134)	(.105)	(.102)
	(PIT+PAF)	.57348	.58277	.64478	.64665	.49489
		(.137)	(.125)	(.156)	(.126)	(.122)
	$(PIT+PAF)_{-1}$	-.08135	-.06039	.13951	-.12299	.04628
		(.249)	(.228)	(.296)	(.232)	(.223)
2. ILIT	1	-7048.2	-7223.5	-8409.7	-8056.0	-9504.8
		(870.)	(799.)	(992.)	(823.)	(1610.)
	PIT_{-1}	.17195	.17198	.17213	.16262	.15384
		(.025)	(.022)	(.024)	(.022)	(.033)
	KOCC	80.422	82.391	95.712	91.823	107.47
		(9.71)	(8.93)	(11.0)	(9.17)	(17.7)
	$ILIT_{-1}$	.51464	.51344	.50530	.53031	.56241
		(.070)	(.062)	(.067)	(.061)	(.103)
3. M	1	-3322.9	-3303.8	-3232.5	-3701.7	-3918.4
		(464.)	(435.)	(461.)	(421.)	(455.)
	(CPN+ILIT)	.39905	.39735	.39102	.43258	.45363
		(.040)	(.038)	(.040)	(.036)	(.039)
	T	-152.58	-150.79	-144.15	-187.72	-211.76
		(43.1)	(40.4)	(42.9)	(38.7)	(41.3)

4. WIT	1	-5052.2	-4812.5	-4313.8	-6007.1	-5925.3
		(1742.)	(1572.)	(1727.)	(1562.)	(3084.)
	(WIT+PIT)	.24466	.23815	.22763	.15532	.09977
		(.076)	(.068)	(.073)	(.068)	(.109)
	KOCC	51.937	49.383	44.009	64.509	65.538
		(20.5)	(18.5)	(20.3)	(18.3)	(35.3)
	DUS70	1010.6	980.67	925.93	938.26	1145.0
		(295.)	(258.)	(266.)	(265.)	(425.)
	WIT_{-1}	.60755	.62007	.64055	.76830	.86020
		(.138)	(.123)	(.134)	(.124)	(.202)
5. KOCC	1	88.815	88.825	88.955	89.461	90.299
		(1.61)	(1.49)	(1.50)	(1.59)	(1.74)
	$(ILIT+ILIT_{-1}+ILIT_{-2})$	.00401	.00397	.00354	.00461	.00351
		(.0003)	(.0008)	(.0008)	(.0007)	(.0008)
	$(ILIT_{-1}+2ILIT_{-2})$	-.00392	-.00389	-.00346	-.00462	-.00358
		(.0009)	(.0008)	(.0008)	(.0007)	(.0008)

It can be inferred from the preceding table that all the coefficients are statistically significant, except the coefficient of $(PIT+PAF)_{-1}$ in the consumption equation. Moreover, for OLS, 2SLS and 3SLS the sign of this coefficient is not positive as expected; furthermore, the D.W. statistic for this equation ranges from 0.916 (FIML) to 1.023 (OLS), thus indicating the presence of a first order positive autocorrelation, which can be removed only by means of some "ad hoc" estimation procedures. In this study the iterative procedure by Cochrane-Orcutt(1949) has been applied to the OLS estimate of the consumption equation; an autoregression coefficient $\hat{\rho}$ = 0.631(.177) and the following equation coefficients have been obtained:

$$\hat{CPN} - \hat{\rho}CPN_{-1} = \underset{(902.)}{892.9}(1-\hat{\rho}) + \underset{(.103)}{.5936}((WIT+WG+X2)-\hat{\rho}(WIT+WG+X2)_{-1}) +$$
$$+ \underset{(.125)}{.4980}((PIT+PAF)-\hat{\rho}(PIT+PAF)_{-1}) + \underset{(.220)}{.2946}((PIT+PAF)_{-1}-\hat{\rho}(PIT+PAF)_{-2})$$

The D.W. statistic so obtained is 1.978, which means absence of first order autocorrelation.
Before entering into details about the simulation results, we want to display the characteristic roots of the different estimated versions of the model (Table 4). The results referred to CORC have been obtained utilizing OLS estimates for all the equations except the consumption equation.

Table 4. Characteristic roots

OLS	.701(cos.905 ± i sin.905)	.663		-.064
CORC	.752(cos.983 ± i sin.983)	.649	.351(cos1.47 ± i sin1.47)	
2SLS	.726(cos.899 ± i sin.899)	.663		-.048
LISE	.747(cos.873 ± i sin.873)	.577		.135
3SLS	.908(cos.968 ± i sin.968)	.807		-.096
FIML	.827(cos.910 ± i sin.910)	.876		.032

Except CORC, whose introduction, from a dynamic point of view, involves transformations in the structure of the model, so that it is perhaps not completely comparable with the other methods, the analysis of Table 4 indicates that among the four characteristic roots two are always real. The modulus of each root is always (also for CORC) less than 1 (stability condition, Goldberger(1970)); in no case a clear prevalence of monotonic or oscillatory behaviour can be observed, because the modulus of the greatest real root is comparable with the modulus of the conjugate complex pair. The period of the latter is in any case about 7 years (in the case of CORC a further 4 years component is present).
We must therefore conclude that, as far as the inherent dynamic properties are concerned, no significant differences can be observed.

4. Simulations with the six sets of estimates

A summary analysis of the previous results does not point out any significant difference among the estimation methods, so that we try to get some further information from the results of simulations performed on the six estimated versions of the model. To maintain the sample correspondence between estimation and simulation phases, the displayed results refer to the period 1952-1971 only.
Due to the presence of lagged endogenous variables in the model, both one step and dynamic simulations (referred to as total and final method, respectively, in Goldberger(1970)) have been performed.
The simulation properties have been analyzed, as usual, in terms of goodness of fit between computed and observed values. It must be remarked that this comparison is statistically appropriate only in the case of one step simulation (Klein(1969)). In dynamic simulation, in fact, the problem arises of asymmetric treatment given to the lagged endogenous variables that are predetermined and fixed in the estimation phase, but derived from previous solutions in the simulation phase. This surely causes problems of autocorrelation and eteroschedasticity in the reduced form disturbances, as pointed out by Howrey and Kelejian(1969), and other problems not yet well investigated (Seaks(1974)).
The first indicator we have analyzed is the Root Mean Squared Error, which, for each endogenous variable, can be defined as in Klein(1974):

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(C_t - O_t)^2}$$

where O_t is the observed value in period t, C_t is the simulated value in the same period and T is the sample period length. RMSE is a measure of the dispersion of computed around observed values and, in the case of one step simulation, its square is equal to the variance of the corresponding variable in the reduced form.
In Tables 5 and 6 the numerical values of RMSE, respectively for one step and dynamic simulation, are displayed.

Table 5. RMSE (One step simulation)

	CPN	ILIT	M	WIT	KOCC	PIT	RNLCF
OLS	437.7	198.6	225.5	229.3	2.451	267.9	343.6
CORC	343.1	198.6	252.1	228.7	2.451	202.4	276.2
2SLS	433.7	199.0	222.1	227.5	2.450	265.1	339.1
LISE	445.1	200.1	205.6	216.6	2.411	279.0	326.4
3SLS	437.5	214.9	220.0	241.0	2.688	298.3	325.1
FIML	415.6	198.9	225.3	267.1	2.451	296.9	321.5

Table 6. RMSE (Dynamic simulation)

	CPN	ILIT	M	WIT	KOCC	PIT	RNLCF
OLS	409.9	328.8	270.5	242.0	2.580	282.1	361.5
CORC	471.0	321.7	265.2	253.9	2.595	283.8	380.6
2SLS	409.9	325.1	266.9	240.4	2.574	281.1	358.2
LISE	448.4	302.8	242.4	229.8	2.552	277.5	349.1
3SLS	427.0	302.4	264.3	227.1	2.625	300.0	345.9
FIML	413.3	318.5	263.0	291.9	2.601	331.5	357.7

In the case of one step simulation there seems to be a slight prevalence of the results obtained by CORC, probably because of the gain in efficiency in the estimation of the structural equation of consumption (see Goldberger(1962)). In the case of dynamic simulation, however, the CORC's performances are generally worse than those of the other methods, almost certainly for the presence of a larger number of lagged endogenous variables. A further analysis of Table 5 indicates that the lowest values, in all the cases where CORC does not prevail, are relative to LISE and that, looking at the relative positions, the second best performance is furnished by 2SLS. This tendency is confirmed by Table 6 where the RMSE of LISE are practically the lowest for all the variables, CPN excepted. With regard to RMSE, LISE can be therefore considered the best method.
The relative superiority of LISE is confirmed by Table 7 where, for the dynamic case, the mean values (simulated and actual) of each variable are displayed. The simulation values referred to LISE are always the closest to the actuals, with the exception of PIT, for which, however, the difference is the second best.

Table 7. Mean values (Dynamic simulation)

	CPN	ILIT	M	WIT	KOCC	PIT	RNLCF
OLS	19036.	3249.	3968.	9512.	90.82	8528.	26226.
CORC	18909.	3237.	3912.	9459.	90.83	8498.	26142.
2SLS	19037.	3249.	3968.	9514.	90.82	8527.	26226.
LISE	19048.	3252.	3974.	9526.	90.84	8523.	26235.
3SLS	19036.	3252.	3969.	9508.	90.80	8535.	26228.
FIML	19036.	3239.	3963.	9522.	90.81	8514.	26221.
ACTUAL	19044.	3251.	3972.	9532.	90.84	8515.	26232.

Let us analyze now Tables 8 and 9 holding the numerical values (respectively for one step and dynamic simulation) of Theil's inequality coefficient, which is defined as the square root of:

$$U^2 = \sum_{t=1}^{T}(c_t - o_t)^2 \Big/ \sum_{t=1}^{T} o_t^2$$

where o_t and c_t are the annual percentage changes of O_t and C_t defined as in the RMSE formula.

Table 8. Theil's U (One step simulation)

	CPN	ILIT	M	WIT	KOCC	PIT	RNLCF
OLS	.5347	.6440	.6980	.4085	.9222	.6105	.3362
CORC	.4820	.6440	.7621	.3816	.9222	.5913	.3004
2SLS	.5251	.6455	.6922	.4061	.9220	.6152	.3338
LISE	.4634	.6477	.6490	.3898	.9136	.6443	.3169
3SLS	.5406	.7340	.6951	.4135	.9853	.6726	.3337
FIML	.4768	.6970	.7071	.4235	.9214	.6610	.3027

Table 9. Theil's U (Dynamic simulation)

	CPN	ILIT	M	WIT	KOCC	PIT	RNLCF
OLS	.4952	.7672	.7392	.3614	.9977	.6525	.3113
CORC	.4011	.7738	.7245	.3678	1.008	.5956	.2785
2SLS	.4877	.7637	.7365	.3583	.9967	.6501	.3083
LISE	.4397	.7464	.7144	.3487	.9906	.6217	.2884
3SLS	.5157	.8037	.7860	.3366	1.030	.7001	.2971
FIML	.4306	.8046	.7767	.3559	1.012	.6701	.2746

The conclusions that can be drawn are not completely analogous to those of RMSE. In fact, even if in the one step simulation CORC maintains a pre-eminent position, it is not the worst in the dynamic simulation, but directly follows LISE and preceeds 2SLS. On the other hand, LISE prevails again in the dynamic case and gains positions in the one step case. It is interesting to analyze for the variable CPN the asymmetric behaviour of RMSE and U for LISE in one step simulation (CORC in dynamic). This asymmetry can be easily explained by reconsidering the different information given by the two indicators: the simulation path for LISE (CORC) moves away from the actual more than for the other methods (highest RMSE), but the outline of the observed values is followed more correctly (lowest U).
The last indicator which has been analyzed is MAPE, defined as:

$$\text{MAPE} = \frac{1}{T} \sum_{t=1}^{T} \frac{|C_t - O_t|}{O_t}$$

Table 10. MAPE (One step simulation)

	CPN	ILIT	M	WIT	KOCC	PIT	RNLCF
OLS	1.963	4.803	4.897	1.600	1.955	2.610	1.157
CORC	1.500	4.803	5.533	1.528	1.955	2.171	0.957
2SLS	1.956	4.797	4.848	1.589	1.955	2.635	1.153
LISE	2.109	4.816	4.825	1.552	1.947	2.973	1.142
3SLS	2.010	5.546	4.491	1.618	2.388	3.120	1.108
FIML	1.831	5.229	4.779	1.692	2.053	3.197	1.027

Table 11. MAPE (Dynamic simulation)

	CPN	ILIT	M	WIT	KOCC	PIT	RNLCF
OLS	1.847	7.433	5.680	1.776	2.244	2.549	1.075
CORC	2.394	7.397	7.110	2.244	2.270	2.842	1.351
2SLS	1.858	7.385	5.626	1.773	2.238	2.544	1.080
LISE	2.165	7.115	5.383	1.923	2.207	2.631	1.148
3SLS	1.914	7.999	5.975	1.674	2.364	2.947	0.998
FIML	1.800	8.093	6.022	2.347	2.379	3.552	1.002

The preceding Tables 10 and 11 hold the numerical values of MAPE. In one step simulation the results seem to confirm what already said about RMSE, even if, for LISE, there appears a tendency to have a forecast error relatively greater in correspondence with lower actual values. This is confirmed in the dynamic case where, on average, LISE has a behaviour similar to OLS and 2SLS, so that we could even speak of a slight prevalence of 2SLS.

5. Conclusions

A joint analysis of all the results shows that the different methods of estimation result in anything but the same sequence. Moreover, except particular cases (i.e. CORC), the various measures are (for each variable) sufficiently concentrated. We could therefore say that, since the obtained results are functions of random variables (the estimates), for the model considered no estimation method prevails in absolute on the others. On the other hand, the partial synthesis we have performed by analyzing the results relative to the various measures, does not appear to be completely supported by the theoretical expectations. First of all, the division between inconsistent (OLS, CORC) and consistent methods (all the others) is not observed, even if a certain tendency can be observed passing from the dynamic to the one step simulation. In the latter, the OLS performance is always poor, even if not in such a way as to be dominated by any consistent method. Moreover, among the consistent methods, a systematic prevalence of the single equation (LISE, 2SLS) on the system methods can be observed (analogous results are obtained by Seaks(1974)). In this connection, it is interesting to point out that in the dynamic simulation it seems to appear a gain in consistency, contrarily to all the expectations; in fact, it is well known (see Wonnacott and Wonnacott(1970), Klein(1969)) that, in system methods, the specification errors are passed on the whole system.
We should therefore conclude that perhaps more correct is the scheme, partially originated by some Monte Carlo studies, proposed by Johnston(1963) and Christ(1966), who suggest to use different methods according to the purposes attributed to the econometric models (estimation of the structural coefficients, estimation of the reduced form, accuracy of the forecasts). The use of OLS is, for example, suggested for the estimation of structural relationships, but in forecast (especially with overidentified models) it is convenient to use simultaneous methods for the estimation of the structural form and to solve the estimated structure to have the reduced form. This is particularly confirmed by our results (2SLS and LISE better than OLS) but is partially denied by a recent Monte Carlo work (Mikhail(1975)) in which the independence of the results from the above mentioned purposes is proved. On the other hand, if it is true that, from the forecast point of view, 2SLS and LISE are equivalent (as in our case), it is true as well that the derived reduced form presents very great divergences between the two methods (see Corsi(1976)), putting into evidence the economist's dilemma in the choice of alternative policies. We should therefore conclude that it is quite right what pointed out in Wonnacott and Wonnacott(1970,pag.400): "The selection of the estimator and evaluation of results is still based largely on judgement".

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