Modelling and forecasting the volatility of the portuguese stock index PSI-20

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MODELING AND FORECASTING THE VOLATILITY OF THE PORTUGUESE STOCK INDEX PSI-20

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Abstract

The volatility clustering often seen in financial data has increased the interest of researchers in applying good models to measure and forecast stock returns. This paper aims to model the volatility for daily and weekly returns of the Portuguese Stock Index PSI-20. By using simple GARCH, GARCH-M, Exponential GARCH (EGARCH) and Threshold ARCH (TARCH) models, we find support that there are significant asymmetric shocks to volatility in the daily stock returns, but not in the weekly stock returns. We also find that some weekly returns time series properties are substantially different from properties of daily returns, and the persistence in conditional volatility is different for some of the sub-periods referred. Finally, we compare the forecasting performance of the various volatility models in the sample periods before and after the terrorist attack on September 11, 2001.

Keywords: EGARCH, forecasting, GARCH, GARCH-M, leverage effect, PSI-20 index, TARCH, volatility.

1. INTRODUCTION

Most of the traditional time series econometric tools are concerned with modelling the conditional mean of a random variable. However, many of interesting economic theories are designed to work with the conditional variance, or volatility, of a process. The volatility clustering often seen in financial markets has increased the interest of

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researchers in applying good models to measure and forecast stock return volatilities. Some important empirical applications of the Autoregressive Conditional Heteroskedasticity (ARCH) model, introduced by Engle (1982) and generalized by Bollerslev (1986) in GARCH model and its various extensions are to forecast volatility in stock return series, to measure the risk of asset management and security pricing, to analyse foreign exchange rate movements and the relationships between long and short term interest rates. See the surveys by Bollerslev, Chou and Kroner (1992), Bollerslev, Engle and Nelson (1994) and Kroner and Ng (1998) for discussion.

Volatility is a measure of the intensity of unpredictable changes in asset returns and it is commonly time varying dependent as recognized by Baillie (1997) among others, so we can think of volatility as a random variable that follows a stochastic process. The task of any volatility model is to describe the historical pattern of volatility and possibly use this to forecast future volatility. An important characteristic of financial stock markets is that the periods of high volatility tend to be more persistent than periods of lower volatilities. Another stylized effect in financial data is that the stock return series exhibit non-normality and excess of kurtosis.

In this paper we estimate common GARCH, GARCH-M, TARCH and EGARCH models for the return rate (or the growth rate) of the daily and weekly PSI-20 Index of the Lisbon and Oporto Stock Exchange (BVLP). Our tasks are (1) to measure the persistence on volatility of the daily and weekly stock return, (2) to analyse the statistical properties of daily and weekly PSI-20 returns, (3) to allow for asymmetric effects on conditional volatility following the methods suggested by Nelson (1991), Glosten, Jagannathan and Runkle (1993) and Zakoian (1994), and (4) to evaluate the one-step ahead and multi-step forecasts of the conditional mean and variance of the daily and weekly PSI-20 Index in different sub-periods.

The paper is organised as follows. Section 2 provides a detailed account of previous research. Section 3 starts with a brief discussion on the models used in the empirical work. Section 4 contains the data description and the empirical results for the estimated models of the PSI-20 stock returns, including forecasting. Last section concludes.

2. LITERATURE REVIEW

Over the past two decades, there have been many applications of ARCH and GARCH models to stock indices returns. For example, see French, Schwert and Stambaugh (1987) for daily returns of the S&P stock index, Chou (1988) for the weekly NYSE value-weighted returns, Akgiray (1989) for index returns, Lamoureux and Lastrapes (1990) for daily returns of US stocks, Schwert (1990) for future returns in US, Attanasio (1991) for monthly returns on the S&P500 index, and Engle and Mustafa (1992) for individual stock returns. For empirical applications of GARCH models using Portuguese stock return series, see for instance Costa and Leitão (2001) and Martins, Couto and Costa (2002).

More recently, asymmetric volatility models have been proposed to incorporate the *leverage effect*[1] (Glosten, Jagannathan and Runkle, 1993 and Zakoian, 1994). The empirical results in the literature on this topic are somewhat different. Glosten, Jagannathan and Runkle (1993) find empirical evidence that positive and negative shocks to returns have vastly different effects on volatility and the conditional variance effects of the monthly stock returns is not highly persistent, while Nelson (1991) finds high persistence in the volatility of daily stock returns. Engle and Ng (1993) find a bigger coefficient in the conditional variance for negative returns than for positive returns, but both shocks led to variance increase. In contrast, Glosten, Jagannathan and Runkle (1993) find that positive and negative returns have vastly different impacts on conditional volatility, so that positive residuals have a negative impact in the conditional variance.

Franses and van Dijk (1996) compare the volatility forecasting performance of the GARCH, Quadratic-GARCH (Sentana, 1995) and TARCH models to the random walk model for weekly Dutch, German, Italian Spanish and Swedish stock index returns. They concluded that the random walk model performs better when the crash of 1987 is included in the estimation sample, while the Quadratic-GARCH performs well upon its exclusion.

McMillan, Speight and Apgwilym (2000) analyses the performance of a variety of volatility models, including GARCH, TARCH, EGARCH and Component-GARCH (Engle and Lee, 1993) models to forecast the volatility of the daily, weekly and monthly UK FTA and FTSE 100 stock indices. They have found that GARCH and moving average models provided the most consistent forecasting performance for all frequencies.

Siourounis (2002) estimated GARCH type models for daily returns of the Athens Stock Exchange Market, an Emerging Capital Market. His findings are that negative shocks have an asymmetric impact on the daily return series and political instability increases capital markets volatility over time. Ratner (1996) and Dockery and Vergari (1996) have also investigated the behaviour of smaller Emerging Capital Markets in developed countries.

Blair, Poon and Taylor (2002) compared the volatility of the S&P 100 index and all its constituent stocks by estimating simple ARCH and TARCH models. They concluded

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[1] "negative correlation between current returns and future volatility" (Bollerslev, Chou and Kroner,

that the majority of stocks have a greater volatility response to negative returns than to positive returns and the asymmetry is higher for the index than for most stocks.

Ng and McAleer (2004) used simple GARCH(1,1) and TARCH(1,1) models for testing, estimation and forecasting the volatility of daily returns in S&P 500 Composite Index and the Nikkei 225 Index. Their empirical results indicate that the forecasting performance of both models depends on the data set used. The TARCH(1,1) model seems to perform better with S&P 500 data, whereas the GARCH(1,1) model is better in some cases with Nikkei 225.

3. BRIEF DISCUSSION ON THE METHODOLOGY

Engle (1982) introduced the ARCH(q) model assuming that the conditional variance depends on past volatility measured as a linear function of past squared values of the process ε_t, i.e., $\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2$, where $\varepsilon_t = u_t \sigma_t$ and u_t is an independent and identically distributed sequence with zero mean and unit variance. The application of the linear ARCH model has problems concerning the need of a long lag length q and the non-negativity conditions imposed in parameters. An alternative and potentially more parsimony parameter structure was the Generalized ARCH, or GARCH(p,q) model proposed by Bollerslev (1986),

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 = \omega + \beta(L)\sigma_t^2 + \alpha(L)\varepsilon_t^2 , \tag{1}$$

where $\beta(L)\sigma_t^2$ is the GARCH term of order p and $\alpha(L)\varepsilon_t^2$ is the ARCH term of order q. The necessary conditions for the model (1) to be variance and covariance stationary are: $\omega > 0$; $\alpha_i \geq 0, i = 1,..., q$; $\beta_i \geq 0, i = 1,..., p$; and $\sum \alpha_i + \sum \beta_i < 1$. Last summation

1992). See Black (1976) and Christie (1982) for further discussion quantifies the shock persistence[2] to volatility. In most applications, the simple GARCH(1,1) model has been found to provide a good representation of a wide variety of volatility processes as discussed in Bollerslev, Chou and Kroner (1992).

In many empirical applications using high frequency financial data one often observes extreme persistence in the conditional variance, so that in the common GARCH(1,1) model the sum of parameters is close to one; i.e., $\alpha + \beta \approx 1$. This presence of an approximate unit root in the conditional variance has led Engle and Bollerslev (1986) to propose the Integrated GARCH, or IGARCH model. The simple IGARCH(1,1) model with $\alpha + \beta = 1$ can be defined as

$$\sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + \alpha \varepsilon_{t-1}^2 . \quad (2)$$

In this model, the minimum mean square error forecast for the conditional variance s steps ahead can be expressed as $E\left(\sigma_t^2\right) = (s-1)\omega + \sigma_{t+1}^2$. Consequently, the uncondicional variance for the IGARCH(1,1) does not exist and the general IGARCH(p,q) is not defined. Nelson (1990) has shown that the IGARCH model is strictly stationary, but not stationary in covariance.

Further extension of the GARCH model includes the GARCH-in-Mean or GARCH-M specification (Engle, Lilien and Robins, 1987) that incorporates the conditional variance in the mean equation. If one assumes that the return series r_t follows an m order autoregressive process, then the GARCH-M is expressed in the form:

$$r_t = \phi_0 + \sum_{i=1}^{m} \phi_i r_{t-i} + \lambda \sigma_t^2 + \varepsilon_t , \quad (3)$$

where σ_t^2 is defined as in equation (1) and the parameter λ may be interpreted as a measure of the risk-return trade-off. For details of GARCH-M specifications and interpretations, see Merton (1980).

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[2] A higher persistence indicates that periods of high (slow) volatility in the process will last longer.

In financial stock markets it is often observed that positive and negative shocks have different effects on the volatility, in the sense that negative shocks are followed by higher volatilities than positive shocks of the same magnitude (Engle and Ng, 1993). To deal with this phenomenon, Glosten, Jagannathan and Runkle (1993) and Zakoian (1994) introduced independently the Threshold ARCH, or TARCH model[3], which allows for asymmetric shocks to volatility. The conditional variance for the simple TARCH(1,1) model is defined by

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 d_{t-1}, \tag{4}$$

where $d_t = 1$ if ε_t is negative, and 0 otherwise. In this model, volatility tends to rise with the *bad news* ($\varepsilon_{t-1} < 0$) and to fall with the *good news* ($\varepsilon_{t-1} > 0$). Good news has an impact of α while bad news has an impact of $\alpha + \gamma$. This model is concerned with the leverage effect sometimes observed in stock returns. If $\gamma > 0$ then the leverage effect exists. If $\gamma \neq 0$, the shock is asymmetric, and if $\gamma = 0$, the shock is symmetric. The persistence of shocks to volatility is given by $\alpha + \beta + \gamma/2$.

An alternative for asymmetric volatilities is the Exponential GARCH, or EGARCH model, introduced by Nelson (1991). The EGARCH(p,q) model is defined by

$$\log\sigma_t^2 = \omega + + \sum_{i=1}^{p}\beta_i \log\sigma_{t-i}^2 + \sum_{i=1}^{q}\alpha_i\left[\phi z_{t-i} + \gamma\left(\left|z_{t-i}\right| - E\left|z_{t-i}\right|\right)\right]. \tag{5}$$

In this model is not necessary to assume non-negativity restrictions for the parameters α_i and β_i and thus, the representation in (5) is basically like an unrestricted ARMA(p,q) model for $\log\sigma_t^2$. The conditional variance of the simple EGARCH(1,1) model in EViews[4] specification is a little different from the Nelson model,

[3] This model is also called the GJR (Glosten, Jagannathan and Runkle, 1993) model.

[4] Statistical program used in our empirical work.

$$\log \sigma_t^2 = \omega + \beta \log \sigma_{t-1}^2 + \alpha \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} . \qquad (6)$$

In the estimation of this asymmetric model, it is assumed a normal distribution for the term error, while Nelson (1991) assumes a generalized error distribution for the errors. The exponential leverage effect is presented if $\gamma < 0$, and the shock is asymmetric when $\gamma \neq 0$. The shock persistence in the EGARCH(p,q) model is measured by $\sum \beta_i$.

4. EMPIRICAL RESULTS

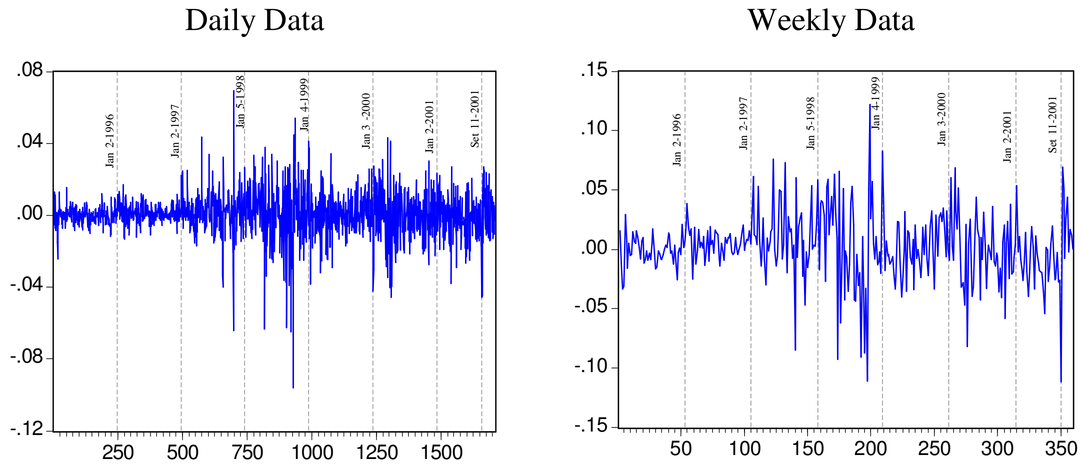
4.1 Data and Descriptive Statistics

The data used in this study cover are the daily and weekly PSI-20 index[5] and its return $r_t = \log(I_t / I_{t-1})$ where I_t is the level of the index in the end of the day/week *t*, and cover the period from January 2, 1995 to November 23, 2001 for a total of 1708 and 359 observations, respectively. In order to compare the statistical properties of daily and weekly returns and to evaluate the performance of the various volatility models over different forecast horizons, the daily data were re-sampled in a weekly returns frequency.

The return series are graphed in Figure 1. The graphs clearly show volatility clustering, especially in the last quarter of 1997 and in some periods after the historical highs reached in 1998 with the public privatisations, that seems to be associated with the instability in international markets, such as Asian crisis in 1997 and financial crises in Russia and Latin America in the second half of 1998.

FIGURE 1

Daily and Weekly PSI-20 Stock Return (r_d and r_w)



During the period 1997-2001 the Portuguese stock market becomes highly sensitive to fluctuations in international markets due to the integration in the euro area markets. Moreover, the reduced size of the Portuguese financial market suggests that the behaviour of national stock returns is closer to the behaviours of stock returns in European and American markets. The sub-sample period January 2, 2001 to November 23, 2001 was characterized by a climate of economic and political instability in Europe and United States due to the high value of the dollar against the euro, the Israel-Palestinian conflict, and the terrorist attacks on September 11 and the subsequent climate of uncertainty, with negative impacts on the financial markets, including the Portuguese stock market.

Table 1 presents the summary statistics for the daily and weekly PSI-20 stock return series during the periods January 2, 1995 to November 23, 2001 (sample period) and January 2, 2001 to November 23, 2001 (sub-sample), respectively.

[5] The PSI-20 Index is a price index calculated based on 20 share issues obtained from the universe of Portuguese companies listed to trade on the Main Market, and was designed to became the underlying element of futures and options contracts. For details, see BVLP(2001)

TABLE 1

Summary Statistics for PSI-20 Stock Return

	Jan 2, 1995 to Nov 23, 2001		**Jan 2, 2001 to Nov 23, 2001**	
	Daily Data	Weekly Data	Daily Data	Weekly Data
Mean	0.000	0.002	–0.001	–0.006
Median	0.000	0.001	–0.002	–0.004
Maximum	0.069	0.122	0.027	0.069
Minimum	–0.096	–0.112	–0.046	–0.112
Standard Deviation	0.012	0.028	0.012	0.030
Stand.Dev./Mean	29.789	15.500	–9.760	–5.237
Skewness	–0.598	–0.182	–0.359	–0.386
Kurtosis	9.790	5.673	4.071	5.923
Jarque-Bera	3380.688	108.590	15.521	17.516
P-value	(0.000)	(0.000)	(0.000)	(0.000)
Observations	1707	358	224	46

The daily and weekly stock returns in the sample range are both leptokurtic, however daily return series (1707 observations) has more excess of kurtosis than weekly return series (358 observations). In the sub-sample period from January 2, 2001 to November 23, 2001, the kurtosis coefficients of the daily and weekly PSI-20 return series (4.071 and 5.923) are also different but closer than in the sample range.

The standard deviations of the weekly returns (0.028 and 0.030, respectively in the sample range and in the sub-sample) are much larger than in the daily returns (0.012 in both periods), but in contrast, the coefficients of variation (standard deviation/mean) in weekly returns are smaller, in absolute value, than in daily returns. The Jarque-Bera test clearly rejects the normal distribution in all the series and the negative skewness coefficients for the return series show the distributions have long left tails.

4.2 Estimation Results

Table 2 shows the estimates for typical and parsimonious GARCH(1,1), TARCH(1,1), EGARCH(1,1), EGARCH(2,1) and EGARCH(1,1)-M models[6] for daily PSI-20 stock return. The variance equation contains a seasonal dummy variable for Monday (MON) day to capture the weekend non-trading effect[7], and the mean equation has autoregressive terms of order 1 and 3 and includes also a dummy variable for the day after the weekend.

In the GARCH(1,1) all estimated coefficients (except the weekend effect in the variance equation) are significant at conventional levels and have the appropriate signs, however, the sum of the ARCH and GARCH estimates ($\hat{\alpha}_1 + \hat{\beta}_1 = 1.014$) suggests the conditional variance to be non-stationary in covariance. These results are not surprisingly, since many empirical applications of the class of ARCH models to stock returns have found highly significant ARCH effects[8].

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[6] The use of GARCH type models with low orders for the lengths *p* and *q* seems sufficient to model the conditional variance even over vary large sample periods as discussed by Bollerslev, Chou and Kroner (1992) among others.

[7] As discussed in French and Roll (1986) and Nelson (1991) the variance of stock returns tend to be higher on days following the weekend.

[8] For example, Costa and Leitão (2001) estimated a GARCH(1,1) model for daily returns of the BVL-30 Index from January 4, 1993 to August 31, 1999 (1650 observations) and the persistence estimate was 1.017. They also reported the persistence of shocks for the subperiods from January 4, 1993 to December 31, 1996 (989 observations) and January 2, 1997 to August 31, 1999 (661 observations), the persistence estimates were close to one (0.972 and 0.905, respectively). Siourounis (2002) estimated a GARCH(1,1) model for daily returns of the Athens Stock Exchange from January 1, 1988 to October 30, 1998 (2692 observations), the persistence estimate was 1.014.

TABLE 2

Models for Volatility of the Daily PSI-20 Stock Return in the Period from January 2, 1995 to November 23, 2001 (1708 observations)

$$r_t = \phi_1 r_{t-1} + \phi_3 r_{t-3} + \lambda\sigma_t^2 + \delta MON_t + \varepsilon_t$$

GARCH(1,1): $\sigma_t^2 = \omega + \beta_1\sigma_{t-1}^2 + \alpha_1\varepsilon_{t-1}^2 + \pi MON_t$

TARCH(1,1): $\sigma_t^2 = \omega + \beta_1\sigma_{t-1}^2 + \alpha_1\varepsilon_{t-1}^2 + \gamma_1\varepsilon_{t-1}^2 d_{t-1} + \pi MON_t$

EGARCH(1,1): $\log\sigma_t^2 = \omega + \beta_1 \log\sigma_{t-1}^2 + \alpha_1\left|\varepsilon_{t-1}/\sigma_{t-1}\right| + \gamma_1(\varepsilon_{t-1}/\sigma_{t-1}) + \pi MON_t$

EGARCH(2,1): $\log\sigma_t^2 = \omega + \beta_1 \log\sigma_{t-1}^2 + \beta_2 \log\sigma_{t-2}^2 + \alpha_1\left|\varepsilon_{t-1}/\sigma_{t-1}\right| + \gamma_1(\varepsilon_{t-1}/\sigma_{t-1}) + \pi MON_t$

	GARCH(1,1)	GARCH-M(1,1)	TARCH(1,1)	EGARCH(1,1)	EGARCH(2,1)
ϕ_1	0.214	0.211	0.224	0.219	0.208
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ϕ_3	0.059	0.058	0.064	0.061	0.051
	(0.023)	(0.025)	(0.012)	(0.017)	(0.037)
δ	0.001	0.001	0.001	0.000	0.000
	(0.038)	(0.061)	(0.063)	(0.198)	(0.163)
λ	—	3.732	—	—	—
	—	(0.075)	—	—	—
ω	0.000	0.000	0.000	−0.512	−0.567
	(0.088)	(0.000)	(0.146)	(0.000)	(0.000)
β_1	0.847	0.843	0.857	0.974	0.655
	(0.000)	(0.000)	(0.000)	(0.000)	(0.005)
β_2	—	—	—	—	0.318
	—	—	—	—	(0.167)
α_1	0.167	0.171	0.116	0.319	0.388
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
γ_1	—	—	0.075	−0.057	−0.060
	—	—	(0.116)	(0.068)	(0.005)
π	−0.000	−0.000	0.000	0.112	0.072
	(0.923)	(0.543)	(0.610)	(0.478)	(0.645)
Persistence	1.014	1.014	1.011	0.974	0.973
Log-likelihood	5559.282	5560.460	5564.253	5576.936	5581.996
AIC	−6.517	−6.517	−6.521	−6.536	−6.541
BIC	−6.494	−6.491	−6.496	−6.511	−6.512
Skewness	0.087	0.114	0.175	0.126	0.118
Kurtosis	4.795	4.864	4.992	4.674	4.601
$Q(10)$	9.655	12.301	9.797	10.346	12.420
	(0.290)	(0.138)	(0.280)	(0.242)	(0.133)
$Q^2(10)$	15.645	14.506	12.072	14.387	12.325
	(0.048)	(0.069)	(0.148)	(0.072)	(0.137)
LM(10)	14.319	13.311	11.246	13.474	15.564
	(0.159)	(0.207)	(0.339)	(0.198)	(0.315)

Notes: Numbers in parentheses are the probability values; AIC is the Akaike information criterion; BIC is the Schwarz criterion; $Q(10)$ is the Ljung-Box statistic for up to tenth order autocorrelation in the residuals; $Q^2(10)$ is the Ljung-Box for up to tenth order autocorrelation in the square normalized residuals; and LM(10) is a Lagrange multiplier test for ARCH effects up to order 10 in the residuals (Engle, 1982). The standard error estimates were obtained using the methods proposed by Bollerslev and Wooldridge (1992).

In the TARCH(1,1) model, the *good news* has an impact on conditional volatility of 0.116 while the *bad news* has an impact of 0.191. The leverage effect in the EGARCH(2,1) is significantly positive, while in the EGARCH(1,1) is statistically positive only at the 10% level (p-value = 0.068) indicating that the conditional variance is higher in the presence of negative innovations. The residuals diagnostic checking indicates that there are any ARCH effects left up to order 10 in the standardized residuals of the variance equations[9]. The estimated coefficient of the conditional variance in the GARCH-M model yields evidence of a statistically significance effect at the 10% level (p-value = 0.075) of volatility on PSI20 stock returns.

Table 3 presents the estimation results for the weekly stock returns. In the simple GARCH(1,1) the sum of the ARCH and GARCH terms also exceeds one (1.016) and the leverage coefficient γ_1 is not statistically different from zero in all asymmetric models. We also included the weekly index in levels in the variance equation of the GARCH(1,1) following the suggestions by Kupiec (1990) and Gallant, Rossi and Tauchen (1992) but the leverage effect in the weekly PSI-20 index was not statistically significant (p-value = 0.323). In the GARCH-M(1,1) model, the estimated parameter λ in the mean equation is insignificant at both 5% and 10% levels indicating that the return series does not depend on the conditional variance.

The diagnostic tests show that the models for daily returns perform better than the models for weekly returns in terms of the mean equation but not in terms of the variance equation. Shocks persistence to volatility in TARCH(1,1), EGARCH(1,1) and EGARCH(2,1) models for the period from January 1995 to November 2001 are found to be 1.011, 0.974 and 0.983 for daily data and 1.017, 0.975 and 0.971 for weekly data.

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[9] The tenth order of lag seems to be sufficient for detecting serial correlation in the errors. If we choose too large a lag, the test may be has low power as discussed by Harvey (1993).

TABLE 3

Models for Volatility of the Weekly PSI-20 Stock Return in the Period from January 2, 1995 to November 19, 2001 (359 observations)

$$r_t = \phi_1 r_{t-1} + \phi_3 r_{t-3} + \lambda\sigma_t^2 + \varepsilon_t$$

GARCH(1,1)a: $\sigma_t^2 = \omega + \beta_1\sigma_{t-1}^2 + \alpha_1\varepsilon_{t-1}^2$

TARCH(1,1): $\sigma_t^2 = \omega + \beta_1\sigma_{t-1}^2 + \alpha_1\varepsilon_{t-1}^2 + \gamma_1\varepsilon_{t-1}^2 d_{t-1}$

EGARCH(1,1): $\log\sigma_t^2 = \omega + \beta_1 \log\sigma_{t-1}^2 + \alpha_1|\varepsilon_{t-1}/\sigma_{t-1}| + \gamma_1(\varepsilon_{t-1}/\sigma_{t-1})$

EGARCH(2,1): $\log\sigma_t^2 = \omega + \beta_1 \log\sigma_{t-1}^2 + \beta_2 \log\sigma_{t-2}^2 + \alpha_1|\varepsilon_{t-1}/\sigma_{t-1}| + \gamma_1(\varepsilon_{t-1}/\sigma_{t-1})$

GARCH(1,1)b: $\sigma_t^2 = \omega + \beta_1\sigma_{t-1}^2 + \alpha_1\varepsilon_{t-1}^2 + \varphi PSI20_t$

	GARCH(1,1)a	GARCH-M(1,1)	TARCH(1,1)	EGARCH(1,1)	EGARCH(2,1)	GARCH(1,1)b
ϕ_1	0.154	0.145	0.155	0.147	0.168	0.192
	(0.007)	(0.009)	(0.008)	(0.015)	(0.005)	(0.002)
ϕ_3	0.111	0.109	0.111	0.131	0.137	0.154
	(0.081)	(0.094)	(0.091)	(0.043)	(0.036)	(0.004)
λ	—	2.813	—	—	—	—
	—	(0.189)	—	—	—	—
ω	0.000	0.000	0.000	−0.405	−0.512	−0.000
	(0.303)	(0.266)	(0.296)	(0.068)	(0.038)	(0.026)
β_1	0.859	0.843	0.859	0.975	0.460	0.601
	(0.000)	(0.000)	(0.000)	(0.000)	(0.106)	(0.003)
β_2	—	—	—	—	0.511	—
	—	—	—	—	(0.078)	—
α_1	0.157	0.176	0.153	0.299	0.404	0.118
	(0.001)	(0.001)	(0.021)	(0.001)	(0.000)	(0.117)
γ_1	—	—	0.010	−0.002	0.013	—
	—	—	(0.942)	(0.976)	(0.906)	—
φ	—	—	—	—	—	0.000
	—	—	—	—	—	(0.323)
Persistence	1.016	1.019	1.017	0.975	0.971	0.719
Log-likelihood	809.708	809.629	809.734	810.508	811.673	818.093
AIC	−4.534	−4.527	−4.528	−4.532	−4.533	−4.575
BIC	−4.479	−4.462	−4.463	−4.467	−4.457	−4.510
Skewness	−0.184	−0.132	−0.167	−0.078	−0.183	−0.174
Kurtosis	5.425	5.337	5.396	5.113	5.240	5.152
$Q(10)$	15.416	17.395	15.352	13.469	12.754	9.400
	(0.052)	(0.026)	(0.053)	(0.097)	(0.121)	(0.310)
$Q^2(10)$	4.673	4.169	4.645	6.293	4.541	6.853
	(0.792)	(0.842)	(0.795)	(0.614)	(0.805)	(0.553)
LM(10)	4.545	4.159	4.486	5.839	4.397	6.852
	(0.919)	(0.940)	(0.923)	(0.829)	(0.928)	(0.739)

Note: As Table 2.

Table 4 shows the estimates for daily PSI-20 stock return models in the sub-period from 2001:1:2 to 2001:11:23 (225 observations) in which there was a sharp drop in the Portuguese stock market, both in terms of stock prices and in terms of volume of transactions, as was the case in most international stock markets.

The conditional variance of the GARCH(1,1) is not highly persistent since the sum of the ARCH and GARCH coefficients ($\hat{\alpha}_1 + \hat{\beta}_1 = 0.778$) is much lower when compared with the GARCH models estimated in Tables 2 and 3. The GARCH-M parameter estimate ($\hat{\lambda} = 0.011$) provides no evidence to support a contemporaneous relationship between expected returns and volatility in this sub-sample period. The weekend non-trading effect is also not statistically significant in this period.

From Tables 2 and 4, one can see that reducing the sample size period for estimation from January 1995−November 2001 to January 2001−November 2001, the persistence estimates for the asymmetric TARCH(1,1), EGARCH(1,1) and EGARCH(2,1) models decreases from 1.017 to 0.767, from 0.975 to 0.825 and from 0.971 to 0.830, respectively.

The leverage effect is statistically different from zero in the TARCH model at the 1% level and in EGARCH models at the 2.5% level for the sub-sample period, clearly indicating the existence of an asymmetric shock on the volatility of the daily PSI-20 index return. For instance, in the TARCH(1,1) model positive news has an impact of 0.007 on volatility while negative news has an impact of 0.410, indicating much more asymmetry than the same model specification in Table 2.

Usually what happens in the Portuguese market and in many other developed countries is that small investors get panic from these negative shocks and sell their stocks in order to avoid higher losses. Consequently, an increase in volatility is observed.

TABLE 4

Models for Volatility of the Daily PSI-20 Stock Return in the Period from January 2, 2001 to November 23, 2001 (1708 observations)

$$r_t = \phi_1 r_{t-1} + \phi_3 r_{t-3} + \lambda\sigma_t^2 + \delta MON_t + \varepsilon_t$$

GARCH(1,1): $\sigma_t^2 = \omega + \beta_1\sigma_{t-1}^2 + \alpha_1\varepsilon_{t-1}^2 + \pi MON_t$

TARCH(1,1): $\sigma_t^2 = \omega + \beta_1\sigma_{t-1}^2 + \alpha_1\varepsilon_{t-1}^2 + \gamma_1\varepsilon_{t-1}^2 d_{t-1} + \pi MON_t$

EGARCH(1,1): $\log\sigma_t^2 = \omega + \beta_1 \log\sigma_{t-1}^2 + \alpha_1\left|\varepsilon_{t-1}/\sigma_{t-1}\right| + \gamma_1(\varepsilon_{t-1}/\sigma_{t-1}) + \pi MON_t$

EGARCH(2,1): $\log\sigma_t^2 = \omega + \beta_1 \log\sigma_{t-1}^2 + \beta_2 \log\sigma_{t-2}^2 + \alpha_1\left|\varepsilon_{t-1}/\sigma_{t-1}\right| + \gamma_1(\varepsilon_{t-1}/\sigma_{t-1}) + \pi MON_t$

	GARCH(1,1)	GARCH-M(1,1)	TARCH(1,1)	EGARCH(1,1)	EGARCH(2,1)
ϕ_1	0.143	0.143	0.159	0.163	0.160
	(0.046)	(0.046)	(0.026)	(0.021)	(0.023)
ϕ_3	0.145	0.144	0.108	0.107	0.108
	(0.048)	(0.055)	(0.113)	(0.123)	(0.130)
δ	−0.001	−0.000	−0.001	−0.001	−0.001
	(0.716)	(0.740)	(0.613)	(0.604)	(0.589)
λ	—	0.011	—	—	—
	—	(0.999)	—	—	—
ω	0.000	0.000	0.000	−1.782	−1.732
	(0.010)	(0.009)	(0.007)	(0.043)	(0.061)
β_1	0.552	0.567	0.558	0.825	0.740
	(0.000)	(0.000)	(0.000)	(0.000)	(0.012)
β_2	—	—	—	—	0.090
	—	—	—	—	(0.734)
α_1	0.206	0.202	0.007	0.300	0.308
	(0.006)	(0.007)	(0.922)	(0.008)	(0.011)
γ_1	—	—	0.403	−0.198	−0.204
	—	—	(0.008)	(0.023)	(0.019)
π	−0.000	−0.000	−0.000	−0.279	−0.317
	(0.309)	(0.343)	(0.610)	(0.308)	(0.253)
Persistence	0.758	0.769	0.767	0.825	0.830
Log-likelihood	689.480	689.480	693.454	694.115	694.143
AIC	−6.066	−6.057	−6.093	−6.099	−6.090
BIC	−5.960	−5.936	−5.971	−5.977	−5.954
Skewness	−0.240	−0.245	−0.115	−0.121	−0.131
Kurtosis	3.360	3.353	3.003	2.959	2.973
$Q(10)$	7.052	6.990	8.769	9.085	8.973
	(0.531)	(0.538)	(0.362)	(0.335)	(0.345)
$Q^2(10)$	11.137	11.000	16.064	15.598	15.034
	(0.194)	(0.202)	(0.041)	(0.049)	(0.058)
LM(10)	11.585	11.372	18.447	17.130	16.538
	(0.314)	(0.329)	(0.048)	(0.072)	(0.085)

Note: As Table 2.

4.3 Forecasting Results

Table 5 presents the forecast error statistics Root Mean Square Prediction Error (RMSPE), Mean Absolute Prediction Error (MAPE) and Mean Absolute Percentage Prediction Error (MAPPE) for each model, obtained by sequences of both 100 one-day ahead and 20 one-week ahead forecasts (static forecast) and by sequences of both daily and weekly multi-step forecasts (dynamic forecast) of the PSI-20 index in levels for the periods July 4, 2001 to November 23, 2001 (last 100 daily observations) and July 7, 2001 to November 19, 2001 (last 20 weekly observations). In Table 5, we also present the MAPPE measure for the volatility models estimated in Tables 2, 3 and 4 for the sub-periods before and after the terrorist attacks in September 11, 2001.

From Table 5, one can see that forecast error statistics by using the static forecasting procedure are more or less the same on both daily data sets (1708 and 255 observations). As expected, the MAPE for the weekly stock index models are much larger than for the daily stock index models. Comparing the two periods before and after the terrorist attack, we conclude that the one-step forecasts are less accurate in the period of the volatility increasing as a result of the bad news than in the period before.

The forecast results by using the dynamic forecasting procedure are somewhat different. The volatility models of the daily PSI-20 Index for the period from January 2, 2001 to November 23, 2001 (225 observations) provide better forecasts than the models estimated for all the sample period (1708 observations). The EGARCH(2,1) model perform better for daily data, while the GARCH(1,1)b model provide better weekly forecasts. Another interesting result is that, despite the poor forecasting performance of the GARCH(1,1)-M model in almost all the periods and data frequencies, it provide the best multi-step forecast on the MAPE statistic for both daily and weekly PSI-20 data sets (1708 and 359 observations) in the sub-period after September 11, 2001.

Table 5

Comparison of Forecast Performance Measures

Sample Period	Sample Forecast	Model	RMSPE	MAPE	MAPPE	MAPPE Before	MAPPE After
I. Static Forecst							
Jan 2, 1995 to	Jul 4, 2001 to	GARCH(1,1)	91.202	71.560	0.955	0.813	1.091
Nov 23, 2001	Nov 23, 2001	GARCH-M(1,1)	91.216	71.559	0.955	0.812	1.092
(1708 daily	(last 100 daily	TARCH(1,1)	**91.154**	71.490	0.954	0.812	**1.090**
observations)	observations)	EGARCH(1,1)	91.213	**71.466**	**0.953**	**0.810**	1.091
		EGARCH(2,1)	91.323	71.555	0.955	0.811	1.093
Jan 2, 1995 to	Jul 9, 2001 to	GARCH(1,1)a	254.862	177.405	2.435	1.630	3.240
Nov 19, 2001	Nov 19, 2001	GARCH-M(1,1)	254.867	177.456	2.435	1.634	3.237
(359 weekly	(last 20 weekly	TARCH(1,1)	254.864	177.398	2.435	1.629	3.240
observations)	observations)	EGARCH(1,1)	254.294	174.720	2.400	1.619	3.180
		EGARCH(2,1)	254.290	174.323	2.395	1.608	3.182
		GARCH(1,1)b	**254.216**	**172.700**	**2.374**	**1.589**	**3.159**
Jan 2, 1995 to	Jul 4, 2001 to	GARCH(1,1)	91.140	72.135	0.961	0.816	1.101
Nov 23, 2001	Nov 23, 2001	GARCH-M(1,1)	**91.131**	72.121	0.961	0.896	1.101
(225 daily	(last 100 daily	TARCH(1,1)	91.237	71.733	0.957	0.811	**1.096**
observations)	observations)	EGARCH(1,1)	91.226	**71.707**	**0.956**	**0.810**	**1.096**
		EGARCH(2,1)	91.243	71.724	**0.956**	**0.810**	**1.096**
II. Dynamic Forecast							
Jan 2, 1995 to	Jul 4, 2001 to	GARCH(1,1)	918.333	813.870	10.957	8.075	9.549
Nov 23, 2001	Nov 23, 2001	GARCH-M(1,1)	1514.56	1371.29	18.251	10.548	**4.920**
(1708 daily	(last 100 daily	TARCH(1,1)	917.469	813.207	10.948	8.094	9.669
observations)	observations)	EGARCH(1,1)	898.524	793.777	10.686	7.959	9.709
		EGARCH(2,1)	**896.088**	**790.419**	**10.649**	**7.909**	9.560
Jan 2, 1995 to	Jul 9, 2001 to	GARCH(1,1)a	691.170	553.695	7.575	5.233	6.826
Nov 19, 2001	Nov 19, 2001	GARCH-M(1,1)	870.873	748.096	10.135	6.434	**6.055**
(359 weekly	(last 20 weekly	TARCH(1,1)	690.870	553.339	7.570	5.229	6.834
observations)	observations)	EGARCH(1,1)	680.233	540.468	7.401	5.074	6.931
		EGARCH(2,1)	674.775	533.962	7.315	4.998	7.039
		GARCH(1,1)b	**661.699**	**518.259**	**7.108**	**4.816**	7.237
Jan 2, 1995 to	Jul 4, 2001 to	GARCH(1,1)	830.448	720.259	9.726	7.548	10.369
Nov 23, 2001	Nov 23, 2001	GARCH-M(1,1)	832.915	722.971	9.761	7.564	10.352
(225 daily	(last 100 daily	TARCH(1,1)	811.883	699.584	9.454	7.375	**10.223**
observations)	observations)	EGARCH(1,1)	811.229	698.879	9.445	7.375	10.245
		EGARCH(2,1)	**808.632**	**695.970**	**9.407**	**7.355**	10.251

Notes: Minimum forecast errors are indicated in bold. Sub-sample periods before and after the terrorist attacks are July 4, 2001 to September 10, 2001 and September 12, 2001 to November 23, 2001, respectively.

Figures 2 and 3 plot the graphs of both the static and dynamic forecasts of the conditional variance for the GARCH(1,1), GARCH(1,1)-M, TARCH(1,1) and EGARCH(2,1) models presented in Tables 2 (daily returns) and 3 (weekly returns).

By using the one-step ahead forecasting procedure for the last 100 daily and 20 weekly observations in sample, one can see that that the volatility shocks were highly persistent days after the terrorist attacks[10]. In the end of the sub-sample period, the conditional variance seems converge to a relative stability state.

Since the persistence estimates of GARCH(1,1), GARCH(1,1)-M and TARCH(1,1) models exceeds one and for the EGARCH(2,1) model is very close to one, the in-sample dynamic forecasts of the PSI-20 conditional variance indicate a gradually increased in the volatility of stock returns across the period from July to November 2001. The exception is the forecasting of the volatility for the EGARCH(2,1) model on daily returns, where the conditional variance follows a downward trend over the sample forecast period.

5. CONCLUDING REMARKS

In this paper, we found that some statistical properties of the daily PSI-20 return series are different from the weekly return series. The conditional volatility of the stock returns is more persistent in daily data then in weekly data, confirming empirical results obtained by Nelson (1991) and Glosten, Jagannathan and Runkle (1993).

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[10] The estimation of the GARCH(1,1)-AR(1) model for the period from September 12, 2001 to November 23, 2001 provide a persistence estimate of 0.940.

FIGURE 2

Static Forecast of the Conditional Variance

Volatility Models of Daily and Weekly PSI-20 Stock Returns

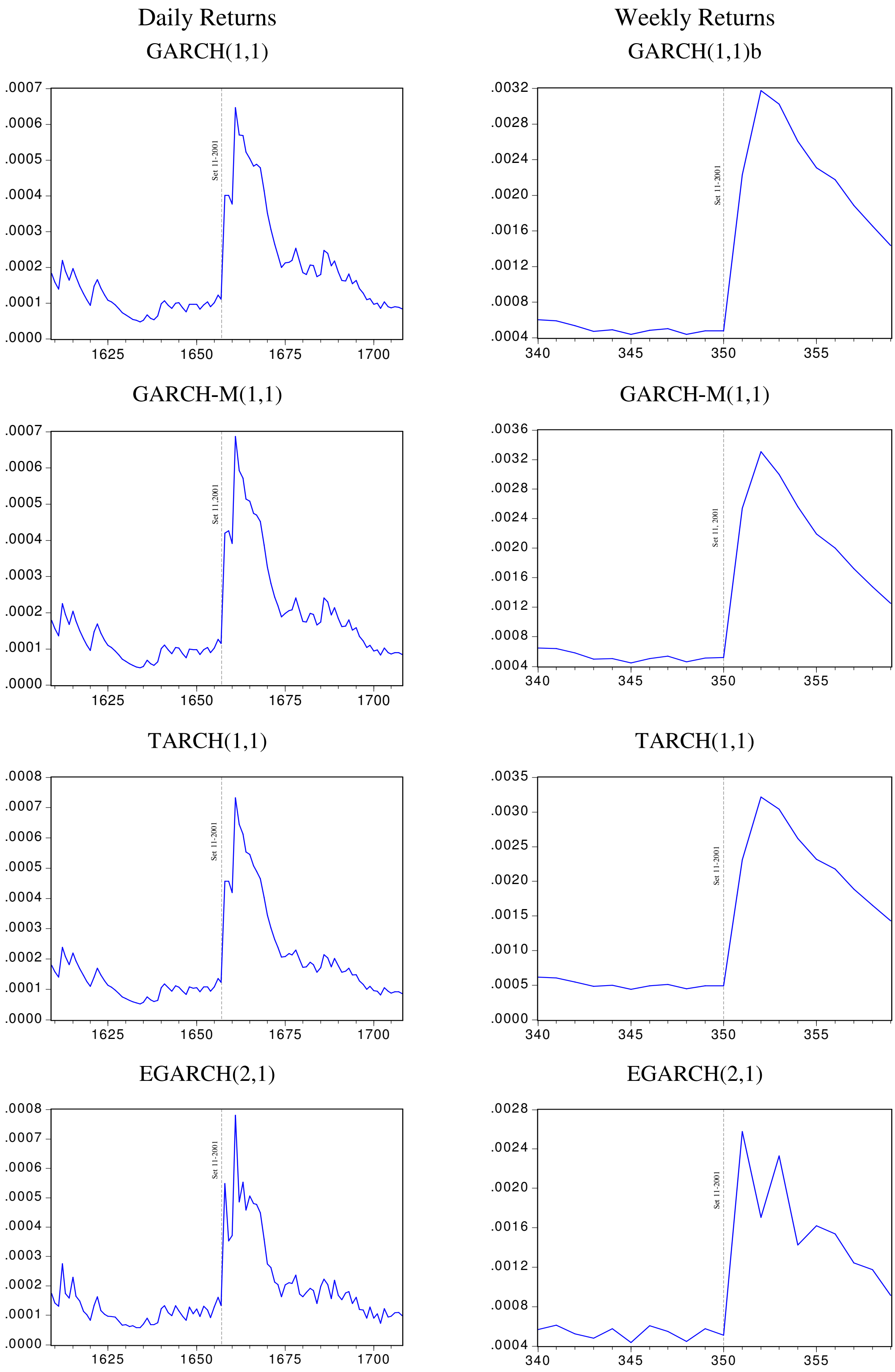


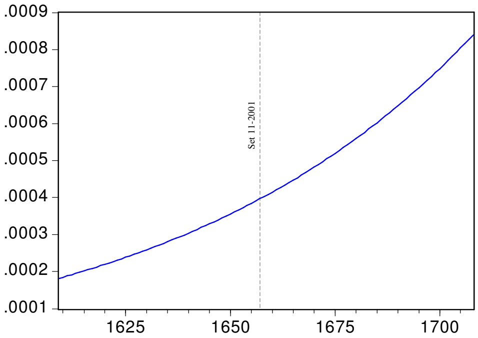
FIGURE 3

Dynamic Forecast of the Conditional Variance

Volatility Models of Daily and Weekly PSI-20 Stock Returns

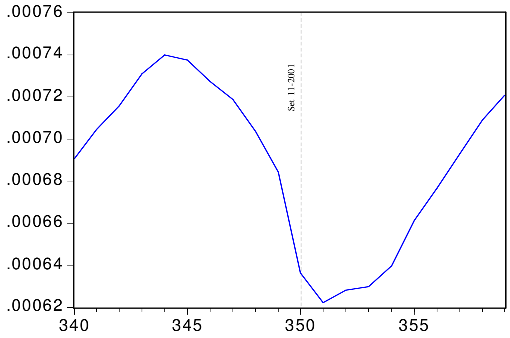
Daily Returns

GARCH(1,1)

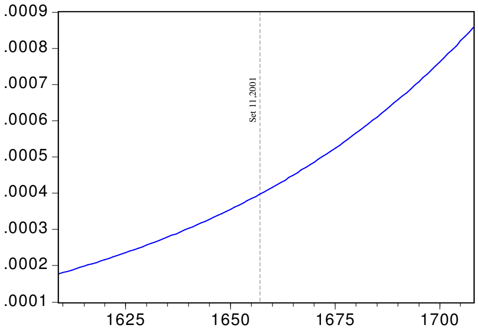


Weekly Returns

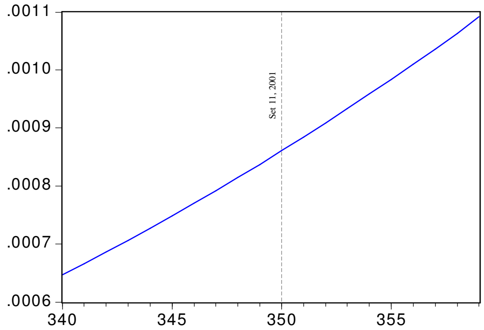
GARCH(1,1)b



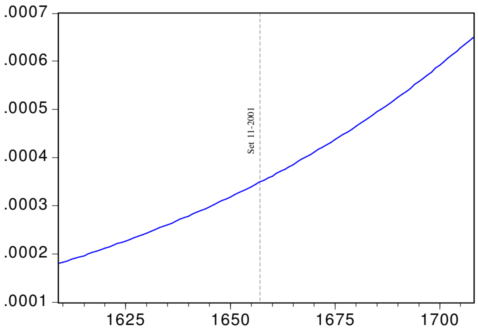
GARCH-M(1,1)



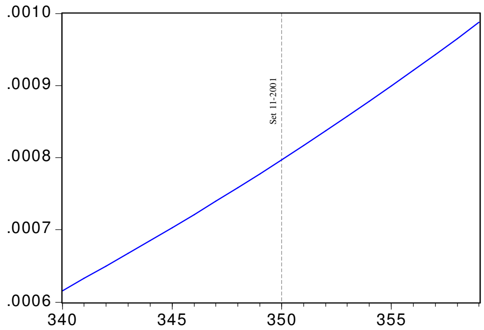
GARCH-M(1,1)



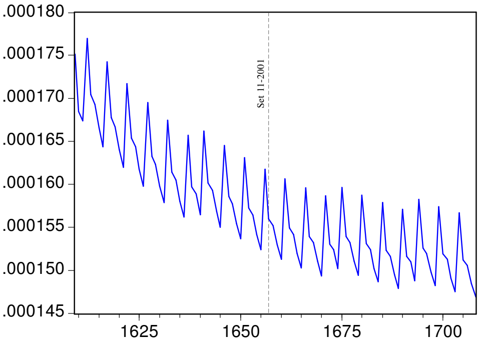
TARCH(1,1)



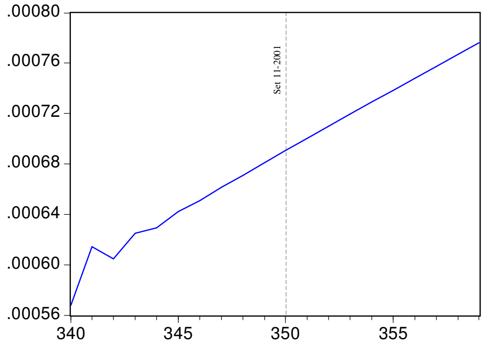
TARCH(1,1)



EGARCH(2,1)



EGARCH(2,1)



We also found that the leverage effect of daily stock returns in the sub-period from January 2, 2001 to November 23, 2001, in which the PSI-20 index declined by 24,42 per cent, is significantly different from zero, indicating that the Portuguese stock market becomes more nervous when negative shocks take place. Those results are consistent with the asymmetric shocks to volatility discussed in Engle and Ng (1993), Zakonian (1994), and Nelson (1991) among others in the sense that the *bad news* has a greater impact on conditional volatility than the *good news*. By contrast, the conditional volatility of the weekly stock returns has no asymmetric effect. Our findings indicate also that there is no evidence of higher movements in the volatility of the PSI20 stock return on days following the weekend.

The simple GARCH, GARCH-M, TARCH and EGARCH models performed better daily and weekly forecasts in the period before September 11, 2001, than in the subsequent period, in which the Portuguese financial market was characterised by a high volatility, as a result of the strong disturbance in US financial markets. For multi-step forecasting the EGARCH models are found to provide better daily forecasts, while the GARCH model with the levels of the PSI-20 index included in the variance equation provide superior weekly forecasts. We came also to the important conclusion that reducing the sample period for estimation improves the accuracy of predicting future observations of the PSI-20 index and stock returns.

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