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The Spatial Diffusion of Social Conformity: The Case of Voting Participation

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Abstract

Social interaction combined with social conformity spreads attitudes and behaviors through a society. This paper examines such a process geographically for compliance with the norm that good citizens should vote. The diffusion of conformist behavior affects the local degree of conformity with the norm and produces highly specific and predictable patterns of behavior across a country. These are demonstrated with qualitative and quantitative spatial analyses of voter turnout in the United States and Russia.

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The Spatial Diffusion of Social Conformity: The Case of Voting Participation

Political research has increasingly demonstrated that interaction between people can spread political attitudes, norms, and behavior through a local population (Kenny, 1992; Mutz, 1992 and 2002; Huckfeldt and Sprague, 1995; McClurg, 2003). The underlying cause for much of this behavioral effect is social conformity, which can mean either conformity with an ethical norm or with others' behavior (Cialdini, 1993; Coleman, 2007a). People may change their behavior because of overt social pressure or simply to fit in with others. As Cialdini reports, people are increasingly likely to conform with others as the proportion of other people doing something increases. Even the thought that relatively more people are doing something is enough to prompt conformist behavior in many individuals. This is a self-limiting process, however, as not everyone can be brought into conformity.

Studies on social conformity also point to the importance of spatial effects. The willingness of people to comply with social norms, such as voting, recycling, obeying laws, or giving to charity, can vary significantly from place to place (Coleman, 2007a). And the degree of conformity with a norm can change when people in one area are influenced by the behavior of people in other locations. In a natural social context, the influence of conformity on an individual is related to the distance from other people as well as to the relative number of people who may express a position or behavior. The joint influence of a group increases with a power function of the number (usually an exponent of about 0.5), but decreases approximately with the square of the distance to the individual (Nowak and Vallacher, 1998: 225).

Voting, especially in a national election, is a good case to study the diffusion of compliance or conformity with an important social norm. Considerable research backs up the fact that people vote mainly because of the widely held norm that good citizens should vote (Blais, 2000), and social pressure or information about others' voting behavior can increase voting participation (Knack, 1992; Gerber, Green, and Larimer, 2008; Gerber and Rogers, 2009). Much of this research has been at the individual level, but conformity operates at individual, group, and societal levels (Cialdini, 1993), so one would expect to see a spatial effect on political behavior at higher levels of aggregation, such as neighborhoods, counties, or states.

The impact of social conformity also can extend across different social behaviors or norms, strengthening its community-wide effect. This happens when conformity with one norm or behavior spills over to bring people into conformity with other norms (Cialdini, Reno, and Kallgren, 1990.) People collectively tend to behave with a consistent degree of conformity in different situations, such as voting, abstaining from committing crimes, giving to charity, and answering the Census. Knack and Kropf (1998) show this at the county level and Coleman (2002, 2007a) at the state and county levels. Coleman (2002, 2004, 2007a, 2010) also shows that conformity with the voting norm in a state or region can spill over to affect voting for political parties. So as this analysis shows the diffusion of voting participation, one can imagine a corresponding diffusion of behavior on related social norms.

A growing number of studies demonstrate spatial effects in political behavior over larger areal units. One example is when voters change their voting choice to align with the local party majority in a constituency, as research on British voters shows (MacAllister et al., 2001). Tam Cho and Rudolph (2008) analyze political activities of individuals in and around large American cities. They conclude that the spatial pattern of behavior around cities is consistent with a diffusion model and cannot be reduced to socio-demographic differences in the population. Other spatial analyses showing broad regional or community effects, all with aggregated data, concern voter turnout in Italy (Shin, 2001; Shin and Agnew, 2007), the Nazi vote in Germany in 1930 (O'Loughlin, Flint, and Anselin, 1994), realignment in the New Deal (Darmofal, 2008), and voting in Buenos Aires, Argentina (Calvo and Escolar, 2003). One also sees spatial effects at larger geographic scales in the diffusion or contagion of homicide rates (Cohen and Tita, 1999; Messner, et al, 1999); in collective violence such as riots (Myers, 2000); and in the negative association of lynching rates across Southern counties (Tolnay, Deane, and Beck, 1996). Although such evidence points toward a social diffusion process, this has not been demonstrated conclusively. The methods of spatial analysis used in most of these studies were developed for exploratory data analysis and do not lead directly to a test for the presence of social diffusion.

The measure of voting participation here is voter turnout in national elections. Analysis is applied at the state level to presidential elections in the United States and at a regional level in a recent Russian parliamentary election. Because Russia and the United States have such different political histories and political cultures, yet have large geographical extension, a test of diffusion in both countries offers a good starting point for establishing the generality of the theory.

Methods²

This analysis uses the geographical software GeoDa 0.9.5 developed primarily by Luc Anselin, who pioneered many of the methods used in spatial analysis. The software has good capabilities for geographical analysis, including spatial autocorrelation and regression, but must be supplemented with a statistical program for more complex data manipulation and other statistical analysis. GeoDa is available at no charge via the Internet from Arizona State University.³ Getting the right data in the right format is a further complication. GeoDa follows the ArcView standard for geometric area data files developed by ESRI, Inc. To construct a map and analyze the corresponding data, a set of at least three different files are required: a shape file (*.shp) that describes the geometry of each unit, an index file (*.shx), and a data file in dBase (*.dbf) format. It is burdensome to construct these files, but fortunately many such files already exist and are available online without charge.⁴ One can modify the data file to include data for analysis, but one cannot easily change the map layout. All these files must be coordinated by a unique identifier for each case and have the same number of cases. Missing data is not allowed.

The elemental principles of spatial analysis are that distance matters and that being closer means a having a stronger effect, which is in accord with research on social conformity. The definition of distance is open to discussion, however. If spatial dependency is present, one expects to see an association or autocorrelation between neighboring areas on the same behavioral dimension. But one might also observe the same correlation in the absence of any spatial effect, perhaps because each area had been simultaneously affected by a remote influence, or because of random chance events or historical circumstances. So an analysis must first determine whether an observed spatial autocorrelation is not random and also is a function of distance. Because spatial dependency weakens with increasing distance from a location, the analysis must focus on areas or regions around a location where one might reasonably find a strong autocorrelation. For each areal unit one identifies its nearest

² For a general reference on spatial analysis, see Haining (1990 or 2003).

³ <http://geodacenter.asu.edu>

⁴ Map files (shape files) for administrative units of most countries are available from www.maplibrary.org. See also the Centers for Disease Control, www.cdc.gov/epiinfo/maps.htm.

neighboring units where one would expect to see the strongest spatial autocorrelation. The selection of neighbors is somewhat arbitrary, however, which is one of several research issues that make spatial analysis much more complex than classical statistical analysis. In this analysis the neighbors are the units that share a common border with the unit of interest.

With geographically based data at hand, and neighborhoods identified, one can move on to investigate spatial autocorrelation. A spatial autocorrelation may refer to an attribute of an entire country, or it may refer to regions within a country. This analysis reports countrywide estimates of spatial autocorrelation, as well as identifying nonrandom regional clusters. The spatial autocorrelation for a state is the correlation between its turnout and its spatial lag. Under the definition of neighbors used here, spatial lag is the average turnout in the bordering states.

As in classical statistical analysis one can graduate from correlation to regression analysis (Ward and Gleditsch, 2008). Here again, many problems arise, chief among them being the likely fact that the units of analysis are not independent of one another as to the values of the variables. This situation is typically ignored in cross-sectional analysis, although it can result in faulty inferences owing to Type I error. But it is at the heart of spatial regression analysis. Indeed if it were not a problem, one could dispense with spatial analysis. A further complication is that the regression model itself may have a spatial dependence owing to local clustering. Examples of spatial regression can be found in Tam Cho and Rudolph (2008), Brunson, Fotheringham, and Charlton (1998), and Beck, Gleditsch, and Beardsley (2006). This analysis uses OLS and spatial regression models, but the concern here is more to identify whether specific types of diffusion models fit the data than to estimate coefficients for the purpose of explaining turnout. In that sense the analysis is as much qualitative as quantitative. The emphasis on theoretical model identification over regression estimates reflects that view that in much social research an over-reliance on regression estimates in specific cases has hindered development of a predictive social science (Coleman, 2007b; Taagepera, 2008).

Models

It may come as a surprise to most political scientists that there is a large body of research on diffusion models of voting, because this

research has been done by physicists. This line of research draws on models from physics which are explored using computer simulations. Here I try to present the essentials of the method and main results; for an exhaustive review see Castellano, Fortunato, and Loreto (2009). This research tries to model a very simple abstraction of individual behavior in an artificial social context. Imagine that people in a population are represented as points on a lattice, and that people are assigned a value of, say, one or zero depending on whether they will vote or not. Now one can add various complexities to the model by making an individual's hypothetical voting decision dependent on the decisions of his neighbors on the lattice. This is where the model of social conformity enters. In a simple model one might introduce a rule that each person or agent makes his behavior agree with the next neighbor on the lattice. One can start with a random distribution of voters and nonvoters, and then run a computer simulation to see what will happen under the rule. At successive computer iterations, the status of each agent is modified sequentially according to the rule on social influence. This type of model can become very complex depending on the degree of influence among neighboring agents and their rules of behavior; probabilistic behavior can be added for increased realism.⁵

This is called an Ising model after its discoverer who proposed it to explain ferromagnetism. A magnet consists of a very large number of atomic mini-magnets, each spinning in one of two directions. The overall behavior depends on statistical properties of the ensemble and factors such as dimension and temperature. Physicists have applied such models to a variety of social phenomena, including voting, political party choice, the spread of opinions, language dynamics, hierarchy emergence, and crowd behavior. (See, for example, Fosco, Laruelle, and Sanchez (2009); Dodds and Watts, 2008; and Sznajd-Weron and Sznajd, 2001).

These physics-based models (as with other agent-based computer models) face several great challenges: the need for realistic micro-level models of behavior, the problem of inferring macroscopic phenomena from the microscopic dynamics, and the compatibility of results with empirical evidence (Castellano, Fortunato and Loreto, 2009). In their critique, they write, "Very little attention has been paid to a stringent quantitative validation of models and theoretical results"

⁵ For a more accessible analysis of social conformity by social scientists using agent-based models, see Nowak and Vallacher (1998). Similar models are used in epidemiology and biology but the field is too extensive to review here. Although these models refer to spatial dimensions, they usually have little connection to an actual geographical system.

(p. 3). Even if macroscopic behavior seems to mimic reality, it has not been proved that it is unique to the micro-level model. In the simplest voting models, the result of a computer simulation is that every agent ends up voting or not voting, which is not realistic. But clusters of agents with different behaviors can persist for long periods. Much attention in these analyses is on the path of change over time in aggregate behavior measures, cluster patterns across the lattice, and their degree of stability. These findings do not concern us here, however, because the focus of this analysis is on the final outcome of change over time.

The Ising model is an early prototype of cellular automata models, which originated with von Neuman and others in the 1940s. In the Ising model the agent is in only one two possible states, voting or not voting. But one can extend the model to continuous cellular automata where the agent can have a value over a continuous range, usually $[0,1]$. This type of model is better suited to an areal spatial analysis where one must consider an aggregate, continuous quantity such as voting turnout. Instead of individual agents on a lattice, the model here uses agents that represent voter turnout in a small areal unit.

The model assumes that one can represent a country by a large number of small geographic areas much like a chess board; each areal unit is identified by a point on the lattice, say at its geographical center. And assume that voter turnout u is known for each small area. Let each area be identified by its x_i and y_j location on the (x,y) geographical coordinates of the lattice with i counting lattice points from left to right and j from top to bottom. A small unit at (x_i, y_j) has four neighbors (x_i, y_{j+1}) , (x_{i+1}, y_j) , (x_i, y_{j-1}) , (x_{i-1}, y_j) . Consider next how an individual in the center unit is influenced by turnout in the neighboring units. A rule is needed, as in other cellular models, to describe how each unit will change at each iteration. By the Nowak and Vallacher (1998) model and Cialdini's (1993) research, influence is proportional to the relative frequency of people in neighboring units who are expected to vote. The neighboring units are equidistant from the center, so distance is not a factor. What might be the net result on voter turnout in the center unit? Suppose that two of the neighboring units have turnout 50% and two have 70%. One would expect people in the center who are closer to the 50% neighbors to shift their voting behavior in that direction, while voters closer to the 70% areas would tend that way. So a commonsense prediction would be that turnout in the center would tend toward the average, 60%. So for the moment consider as a working hypothesis that turnout in the center unit will be

approximately the average of turnout in the neighboring units. The analysis subsequently will try to validate this hypothesis.

More formally, let us express the idea that because of the influence of social conformity each unit becomes more like its neighbors, with the turnout at (x_i, y_j) tending toward the average of the turnouts in the four neighbors. The units might have any turnout values initially. One can extrapolate what will happen in this arrangement by a mental or computer simulation similar to the procedure used in the physics models. At each iteration one successively replaces the turnout value at each point by the average turnout of its four neighbors. That is, at each turn for every point let

$$u(x_i, y_j) = \frac{1}{4} u(x_i, y_{j+1}) + \frac{1}{4} u(x_{i+1}, y_j) + \frac{1}{4} u(x_i, y_{j-1}) + \frac{1}{4} u(x_{i-1}, y_j)$$

If one does this simulation the result is that after some large number of iterations all units end up with the same turnout value. But this would be an unrealistic outcome. With one additional hypothesis, however, this becomes an interesting and realistic model, namely, that turnout values in the units on the geographic boundary of the country (or lattice) do not change, or at least change very little in relation to change in the interior. This seems reasonable because each boundary unit interacts with two neighbors that are also boundary units but with only one interior unit; change in the interior will propagate slowly to the boundary. The analysis subsequently will check how realistic this hypothesis is.

What can one say about the result of this model after a simulation of many iterations? As it turns out, it is not necessary to simulate this on a computer to know the general form of the result. No matter what the initial turnout values are, or the boundary values, this model leads to a distribution of turnout values across the country or lattice that is unique and depends only on the values on the boundary. If the simulation continues until no further change occurs—the steady state—the distribution of turnout values fits a mathematical function $u(x, y)$ known as a harmonic or potential function (Garabedian, 1964: 458ff). It is this type of function that interests us, not the actual turnout values. Such a function is a solution of the Laplace equation (1), namely that the sum of the continuous partial derivatives of a differentiable function equals zero,

$$u_{xx} + u_{yy} = 0 \quad (1)$$

This is a famous equation of mathematics and physics. To solve it for a given area one must know the values on the boundary. If the boundary values are held constant, finding a solution to the values across the interior is known as the Dirichlet problem.⁶ This was a very difficult problem for mathematicians of the 1800s to solve analytically, and more recently it was discovered that one can also solve the problem numerically by a computer simulation of the type just described (Garabedian, 1964: 485ff).⁷ This problem arises in physics when one tries to explain the effect of gravitation, electrostatic charge, or the diffusion of heat, across a distance on a surface or sphere. The analogy of heat diffusion fits best here as, for example, the daily weather map that shows contours of temperature change across the country.

A harmonic function has unique properties (Kellogg, 1953): (1) The product of a harmonic function multiplied by a constant is harmonic, as is the sum or difference of two such functions. (2) It is invariant under translation of the axes. (3) The function over an area is completely determined by the values on the boundary; the solution is unique. (4) A harmonic function over a closed, bounded area takes on its maximum and minimum values only on the boundary of the area (if it is not a constant). (5) If a function is harmonic over an area, the value at the center of any circle within the area equals the arithmetic average value of the function around the circle. This implies that averages around concentric circles are equal. The converse is also true. If the averages around all circles equal the values at their centers, the function is harmonic. Harmonic functions have many other, more complex properties as well.

Examples of harmonic functions are:

- (1) A plane surface $Ax + By + Cz + D = 0$ for constants A, B, C, D
- (2) In polar coordinates, $f(r) = c/r$ or c/r^2
- (3) $f(x,y) = \ln(x^2 + y^2)$
- (4) $f(x,y) = e^x \sin(y)$
- (5) constant functions

Because a harmonic function is the unique solution to the diffusion problem represented by the lattice model of social conformity, one can

⁶Dirichlet's interest in proving the stability of the solar system led to the study of harmonic functions.

⁷The Laplace equation is solved by approximation with its Taylor expansion to a difference equation that, by rearranging terms, is exactly the equation used in this model. The boundary must be fairly smooth. For Monte Carlo solutions to the Laplace equation see Haji-Sheikh and Sparrow (1966). In 1944 Kakutani had showed that a numerical solution is also possible with random walks.

use the properties of harmonic functions as approximate tests for the validity of the model. Here three properties of harmonic functions are tested: (1) that the geographical distribution of turnout is a harmonic function; (2) that turnout averages around concentric circles are equal; and (3) that the maximum and minimum turnouts are in border areas. These hypotheses would be satisfied trivially if the distribution of turnout were random or constant, so these situations must be ruled out as well. A broad class of alternatives to the harmonic function can be tested with quadratic equations, such as $u(x,y) = a x^2 + b x + c$ or $u(x,y) = a x^2 + b x y + c y^2 + d$ when $a + b + c \neq 0$. If the geographic distribution fits these models, it is not harmonic. The analysis is limited, however, to testing these hypotheses with areal data, which lacks precision as to location. So the hypotheses must be adapted to fit this type of data.

Analysis

United States. The analysis begins with an exploratory examination of the spatial distribution of voter turnout in three presidential elections in the 48 contiguous states, and the harmonic distribution hypothesis is tested on these three elections. The other two hypotheses about harmonic functions are tested in all elections from 1920 to 2000. The first research question is to determine if spatial dependency is present and, if so, that it is not the result of random chance. Three elections—somewhat arbitrarily chosen—are scrutinized as to regional spatial autocorrelations and national patterns; these are 1920, 1968, and 1992. The elections in 1920 and 1968 come after important expansions of eligibility for voting participation. The 1920 election was the first with women's suffrage; 1968 followed the Voting Rights Act of 1965. The 1992 data was chosen for convenience because it was already in the map database. As it happens, the general distribution of turnout across the states does not change much, so it is not necessary to look in detail at spatial autocorrelation in every election.

As stated previously, for this analysis the local area or region around each state is defined as the set of states that have a boundary in common with it; this is called rook contiguity by analogy with chess. This is a gross approximation of the lattice model discussed earlier but is sufficient to begin testing the model. In the US this identification of neighbors leads to different numbers for the states.⁸ The most

⁸ Because the boundary values completely determine the solution to the Laplace equation, it does not matter what the exact geometric arrangement of states is or how many share borders. This arrangement can affect the rate of convergence toward the steady-state solution, however.

common number of neighbors is four, and forty states have between three and six states sharing a border.

The rule for change in the lattice model, which leads uniquely to the harmonic function hypotheses, is to set each unit's turnout equal to the average of its neighbors at each iteration. So the analysis first checks on how well this applies to states. The result is in Table 1, which shows the OLS regression of turnout in each state against its spatial lag, which is the average turnout in the contiguous states. If the state turnout approximately equals the average, the coefficient should be very close to 1. Indeed for all elections the coefficients are close to 1, especially for 1992, and less than one standard error from 1; one cannot reject the statistical hypothesis that the coefficient equals 1. The constant terms are not statistically significant. So the model is on firm ground as to the working hypothesis of the lattice model.

Table 1. OLS regression of turnout against spatial lag (average turnout in contiguous states).

Election	Constant (error)	Coefficient (error)	p	R square	Mean	Std. Dev.
1920	4.0 (7.0)	0.91 (0.13)	<.0001	0.50	50	18
1968	7.6 (8.2)	0.88 (0.13)	<.0001	0.50	62	7.9
1992	1.8 (8.1)	0.97 (0.14)	<.0001	0.51	57	7.2

For each election the analysis shows both a state-level map of the distribution of turnout and a map of regions that have significant spatial autocorrelations; see Figures 1, 3, and 5. As the first map in each figure shows, the lowest turnout values typically are in the South and higher values are in the North. The second map in each figure identifies local clusters of nonrandom spatial autocorrelation that are statistically significant ($p < .05$) by a permutation test (repeated 999 times) using the LISA (Local Indicators or Spatial Association) method⁹ (Anselin, 1994); in 1920, for example, these clusters were the South and a group of states from Illinois to Pennsylvania. (Darker states are significant at lower p values.)

⁹ For each unit the neighbors are replaced by other units randomly selected and the local statistic calculated; this is repeated many times to approximate a statistical distribution for the null hypothesis. For more on permutation tests see Efron and Tibshirani (1998).

Spatial autocorrelation for the entire country is assessed with Moran's I . This is a measure of spatial autocorrelation with range $[-1,1]$.¹⁰ As with Pearson's correlation, Moran's I can be positive or negative, and zero implies no correlation. It is based on the aggregate of autocorrelations in the neighborhoods of all states. When states with above average turnout are neighbors of states that also have above average turnout, the I value increases; the same holds when below average turnout states border other low turnout states. As seen in Figure 2, for example, the I value for 1920 is 0.55 ($p < .0001$), indicating significant spatial autocorrelation across the country. The significance levels of the Moran's I estimates are determined by a permutation test (repeated 9,999 times). In 1968 higher turnout values are in the North but extend broadly across the country, and there is a high turnout cluster from Montana to Wisconsin (Figs. 3 and 4). Again, the South stands out as a local cluster of low turnout. Moran's I is 0.57 ($p = .0001$). The spatial dependency patterns for 1992 are in Figure 5. Results show a familiar north-south gradient with regional clustering of high turnout states in the North, as in 1968, and low turnout states in the southeast. Moran's I is 0.53 ($p = .0001$).¹¹

Because Moran's I refers to the entire country, it can cause one to overlook heterogeneity at the regional level. To overcome this weakness in interpretation one can examine the Moran scatterplot (Anselin, 1993). It is like a regression of a spatially lagged turnout against turnout with the slope of the regression line equal to I , as represented by the solid line. The figure also shows the null hypothesis (the dashed line along the x-axis) and the 95% confidence bands—the paired dashed lines—of the *null* hypothesis; when the solid Moran regression line is outside of those the result is significant. The origin is at the mean turnout value; units are standard deviations. Points in the upper right quadrant are states with higher than average turnout that border other states with higher turnout; in the lower left quadrant are states with lower than average turnout that border other states with low turnout. States in the other two quadrants are not like their neighbors; that is, one finds a high turnout state next to a low turnout state. While it is normal to find some of these cases, a large number would suggest areas with little or no spatial autocorrelation or, perhaps, negative autocorrelation.

¹⁰ $I = (N/\sum_i \sum_j w_{ij}) \cdot \sum \sum w_{ij} (X_i - X_{avg}) (X_j - X_{avg}) / \sum (X_i - X_{avg})^2$ for weighted neighbor pair w_{ij} . The w matrix has a weight for each connected pair and equals zero if not connected.

¹¹ Results are similar if the neighbors chosen for spatial analysis are the four nearest states, regardless of whether they share a border or not.

Figure 1. US, 1920, turnout quantiles by state and statistically significant regional clusters as determined by a permutation test.



Figure 2. US, 1920, Moran scatterplot.

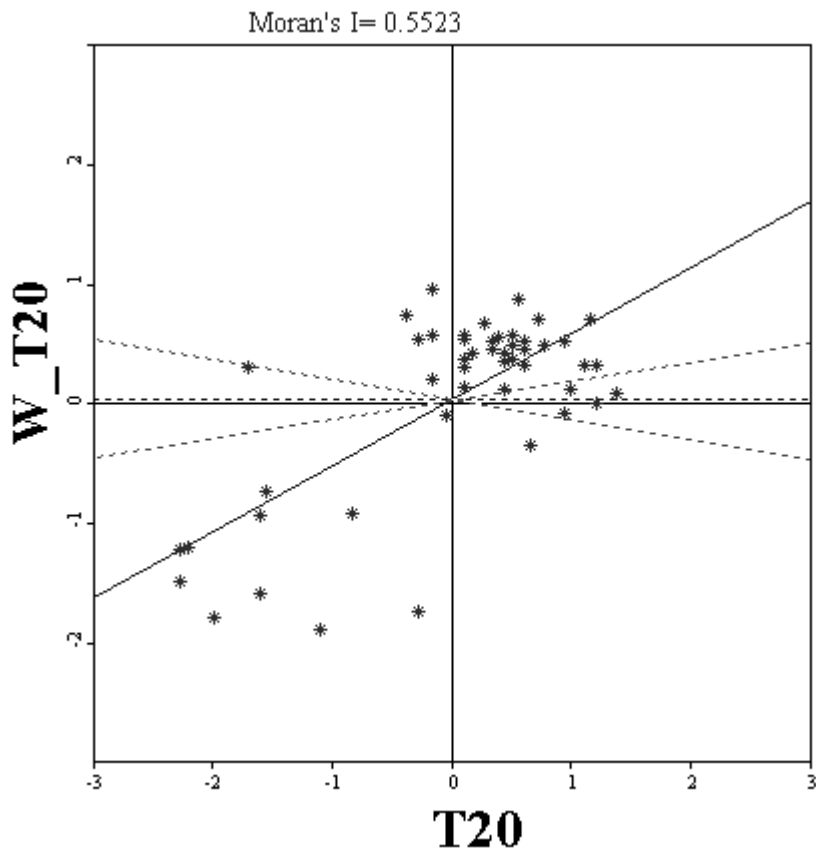


Figure 3. US, 1968, turnout quantiles by state and statistically significant regional clusters as determined by a permutation test.



Figure 4. US, 1968, Moran scatterplot.

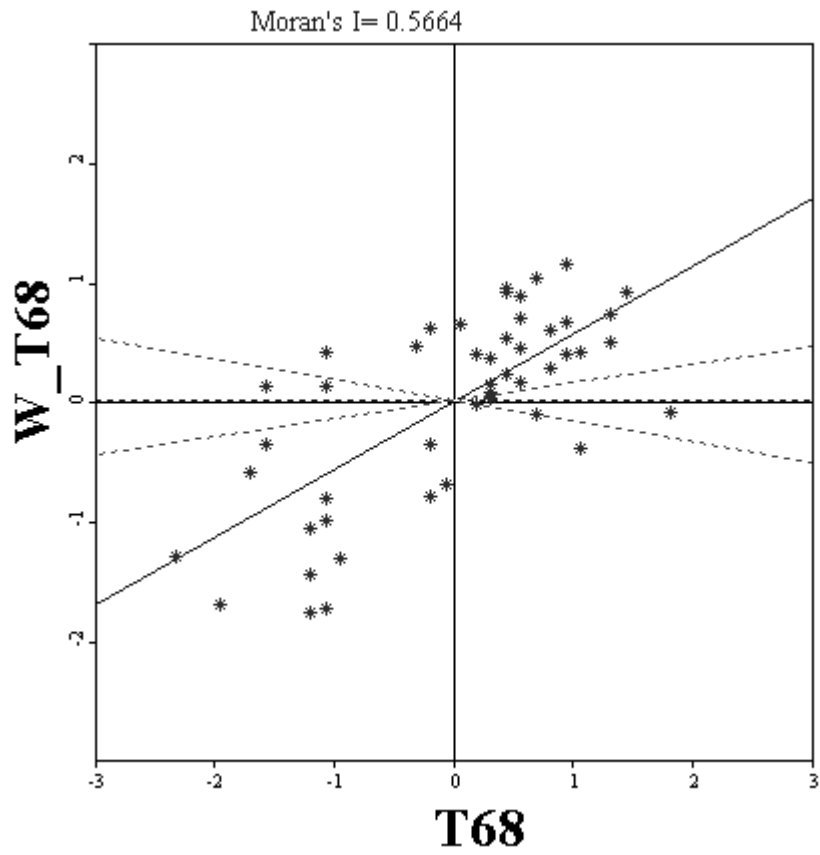


Figure 5. US, 1992, turnout quantiles by state and statistically significant regional clusters as determined by a permutation test.

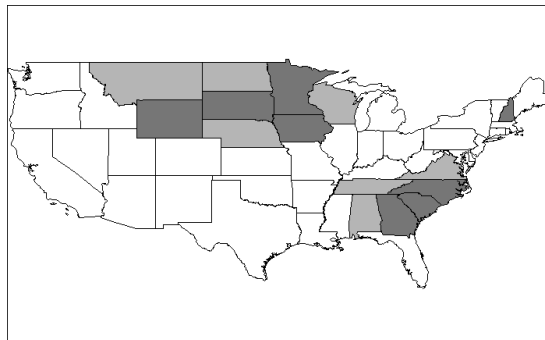
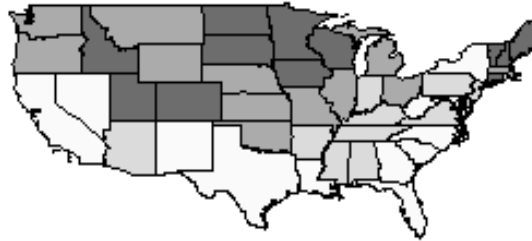
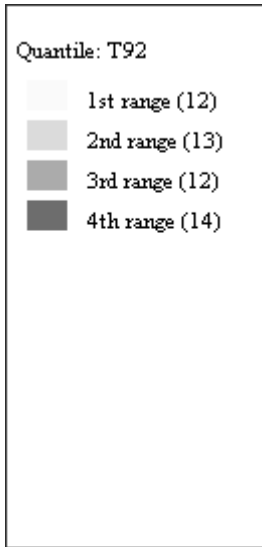
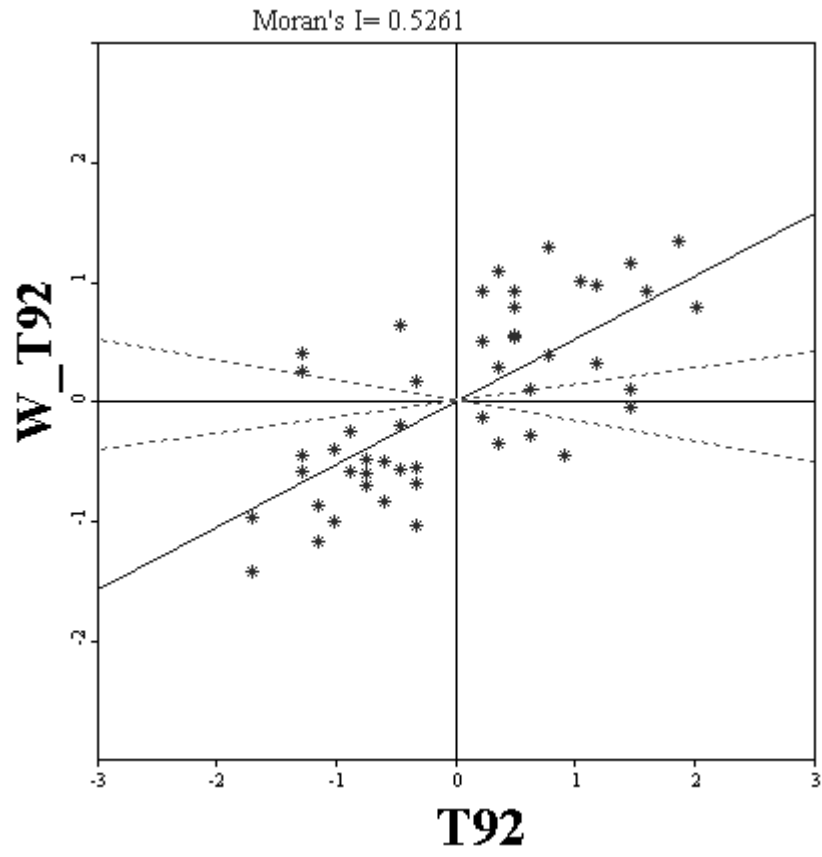


Figure 6. US 1992, Moran scatterplot.



The spatial autocorrelation analysis shows that turnout has a distinct and nonrandom distribution across American states. The strong north-south gradient in the turnout data suggests modeling the state turnout distribution as a function of latitude. The map shapefile contains information on the longitude and latitude of the polygon used to map each state. For each state GeoDa can compute a centroid, which is the latitude-longitude location of the geometric center of gravity of the state. This location is used in the analysis. Table 2 shows the results of linear regression of turnout against latitude at the state centroid. Longitude is not statistically significant.

Table 2. Regression model: turnout % = constant + b * latitude

Year Turnout	Constant (error)	Coefficient b (error)	Significance	R squared
1920	-39.9 (17.6)	2.28 (0.44)	<.00001	0.36
1968	13.9 (6.5)	1.23 (0.16)	<.00001	0.55
1992	13.2 (6.0)	1.12 (0.15)	<.00001	0.55

Note: Latitude is at the centroid.

One can see from Table 2 that the relationship with latitude strengthened after 1920 but with a gradient that was less steep; 1968 and 1992 are more alike in that regard. Checking for curvature with a quadratic model, one finds a better model (with errors) for 1920,

$$\text{turnout} = -457 (117) + 24.1 (6.1) \text{latitude} - 0.281 (0.088) \text{latitude}^2$$

For this estimate, R square = 0.51, and the fitted quadratic surface has a maximum at about latitude 43 degrees (the latitude of Madison, Wisconsin). As one can see on Figure 1, turnout drops a bit in the north-central states.

As an additional check on the regression models, they were re-estimated with a spatially lagged turnout term. As seen in Table 3, the lagged term has marginal statistical significance in 1992. So, for 1992, latitude almost completely suffices to encompass spatially autoregressive turnout effects. The same analysis for 1968 shows that the spatial term is not significant, but it is significant in 1920.¹²

¹² On interpretation of spatial regression models, see also Ward and Gleditsch (2008).

Table 3. Regression model for 1992 with spatially lagged turnout.

Variable	Coefficient	Std. Error	p
Constant	6.25	6.7	.35
Lagged Turnout	0.30	0.16	.06
Latitude	0.85	0.20	<.0001

Likelihood ratio test for spatial dependence = 3.3, $p = 0.7$, R square = .59.

OLS model w/o lagged term, Lagrange Multiplier for lag = 3.9, $p = .05$

OLS model w/o lagged term, Lagrange Multiplier for error = 4.3, $p = .04$

As another check for regression problems that might be caused by omitted variables, a regression model with SES and demographic variables was tested for 1992; no problem was found. (See the Appendix.) The goal here is not to explain turnout, however, but to test whether the distribution fits the predicted harmonic equation model.

The regression analysis shows that a plane dependent only on latitude fits the turnout data well in 1968 and 1992, but not so well in 1920 when the distribution is curved. Recall that a plane is a harmonic function, so 1968 and 1992 satisfy the diffusion hypothesis but not 1920. The plane is a good fit to the data in 1968 and 1992, though unexplained variability remains. The finding that turnout varies linearly with latitude also supports the working hypothesis of the lattice model that turnout in the center unit is approximately the average of values in neighboring units. Of course, precision is limited by use of state-level data.

Although the regression analysis leads to a harmonic function in 1968 and 1992, it is not necessarily the case that the estimated function is the solution for the given boundary values. If it is not an approximate solution, one can anticipate continued change in turnout across the country until a steady state is attained. Because the steady-state solution is completely determined by the boundary values, one can compare the previous regression to one based only on values in boundary states. Classification of boundary states is a bit subjective for a few states, but here 30 states are identified as boundary states and 18 as interior states.¹³ Results are in Table 4. Comparing Tables 2 and 4, one finds that the coefficients for latitude trend toward equality in 1992, but with higher R square in the boundary regression. So the

¹³ Boundary states are: WA, OR, CA, AZ, NM, TX, LA, MS, AL, FL, GA, SC, NC, VA, MD, DE, NJ, NY, CT, RI, MA, VT, ME, OH, MI, WI, MN, IL, ND, MT. Interior states: ID, NV, UT, CO, WY, SD, OK, AK, IA, IN, KY, WV, TN, NH, PA, NE, KS, MO.

distribution of turnout has approached that of a steady state over this period. Theoretically one could try to solve the equation numerically for the given boundary values, but this might not lead to the exact mathematical form of the solution and a numerical result would still be an approximate solution because state-level data lacks geographic precision.

Table 4. Regression model: turnout % = constant + b * latitude, for boundary states only (N = 30).

Year Turnout	Constant (error)	Coefficient (error)	Significance	R squared
1920	-46.7 (18.5)	2.35 (0.47)	<.00001	0.47
1968	14.4 (6.3)	1.18 (0.16)	<.00001	0.66
1992	13.2 (6.4)	1.10 (0.16)	<.00001	0.62

Note: latitude is for the centroid. Longitude and quadratic terms are not statistically significant.

The second hypothesis test for harmonic functions is that the average value around circles equals the value at the center. Instead of trying to draw a circle on the US map, however, the analysis divides the states into two groups: 30 on the boundary or border, and 18 in the interior. The harmonic property suggests that to an approximation the average value of turnout in the boundary states should equal the average in the interior states. This is tested with a t-test for every election.

The trend from 1920 to 2000 is strongly toward equality of means as seen in Figures 7. Of the 21 elections in the analysis, the boundary and interior means are equal (the null hypothesis is not rejected) in 13, at a significance level of $p = .05$. (T-tests were adjusted for unequal variance but not corrected for multiple tests.) Elections with statistical rejection of equal means run from 1920 to 1936 and 1952 to 1960. But in the seven elections from 1976 on, the difference between mean boundary and interior turnouts is consistently less than 2 percentage points and is less than 1 point in four elections.

As seen in Figure 8, which plots the trend in the difference in means, there is a remarkably consistent convergence of the difference to zero. The trend is strongly linear (linear regression, R square = 0.94), and the difference between boundary and interior averages decreases at a rate of about 0.2 percentage points per year or 0.8 points per

election.¹⁴ The strong linearity of the change, meaning a constant rate of change, would not be the expected result. Typically in models like this one expects that the rate of change would depend on the difference—larger differences would lead to faster change—so that the rate of convergence would be exponential.¹⁵

The third hypothesis test of a harmonic function is that the maximum and minimum are on the boundary. Over almost all the elections the minimum has been on the boundary, namely in a southern state. The maximum has been less often on the boundary, but from 1976 has been in Minnesota or Maine. Utah or Idaho had the top values in elections from 1944 to 1968. From 1976 on, the minimum was in South Carolina five times, and once each in Nevada and Arizona. So six of the seven elections from 1976 to 2000 satisfy the hypothesis. The chance of either the maximum or minimum being on the boundary in a given election is about 0.62 if all combinations are equally likely; for both to be on the boundary about 0.38. By the binomial distribution the probability of six of the seven elections having the predicted result by chance is $p = 0.013$. So the analysis confirms the hypothesis for the group of elections from 1976, which agrees with the other results that the country has gradually converged toward a harmonic distribution from 1920 to 2000.

A final test is whether the boundary values are stable, which was hypothesized when developing the lattice model. Analysis of linear trends from 1920 to 2000 shows that average turnout of interior states is decreasing ($p = .01$), but there is no trend for boundary states ($p = .08$). The turnout in boundary states remained in a narrow range with the average turnout for boundary states 54.7% and 95% CI [52.5-56.8]. This is consistent with other findings the pointed toward a harmonic distribution.

Analysis shows that the geographic distribution of turnout across the states has increasingly approximated a harmonic function, namely a plane, with the results closest to prediction from about 1980 on. Over half the variation in state turnout rates can be accounted for by the latitudes of the states. Regional spatial autocorrelations have remained significant over this time, but one can conclude that the US as a whole now shows a much broader pattern of diffusion over its entirety. It cannot be determined, however, if the locally significant spatial association in the South is a result of diffusion or the shared historical

¹⁴ Linear model: Difference = $364 - 0.18 \text{ Year}$. Std error 0.009; 95% CI = [-0.20, -0.16].

¹⁵ In the analogy of a thermodynamic system, the expected result would follow Newton's Law of Cooling.

circumstances of the states. Variation in turnout has decreased greatly, the standard deviation of turnout falling from 18 in 1920, to 7.9 in 1968, and to 7.2 in 1992 (Table 1). As one can see in the decreasing difference between boundary and interior states, regional differences have moderated. Moreover, the steady convergence of interior and boundary mean turnout for at least 80 years suggests a process toward social homogeneity that is little affected by short-term political or economic changes. In essence the US has undergone a slow averaging or smoothing of turnout across its territory, as assumed in the lattice model of social diffusion and caused by social conformity.

Figure 7. US elections, 1920-2000, average interior and boundary turnout with LOWESS smoothing.

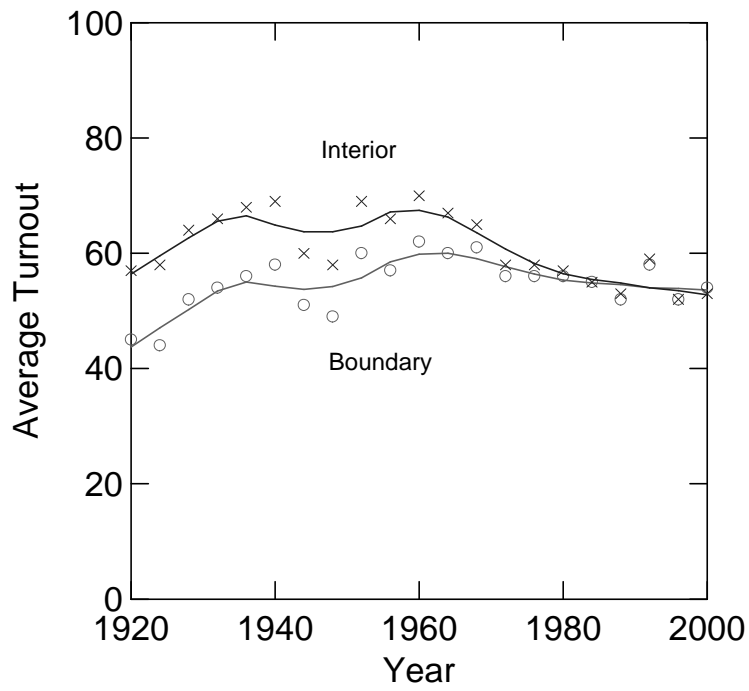
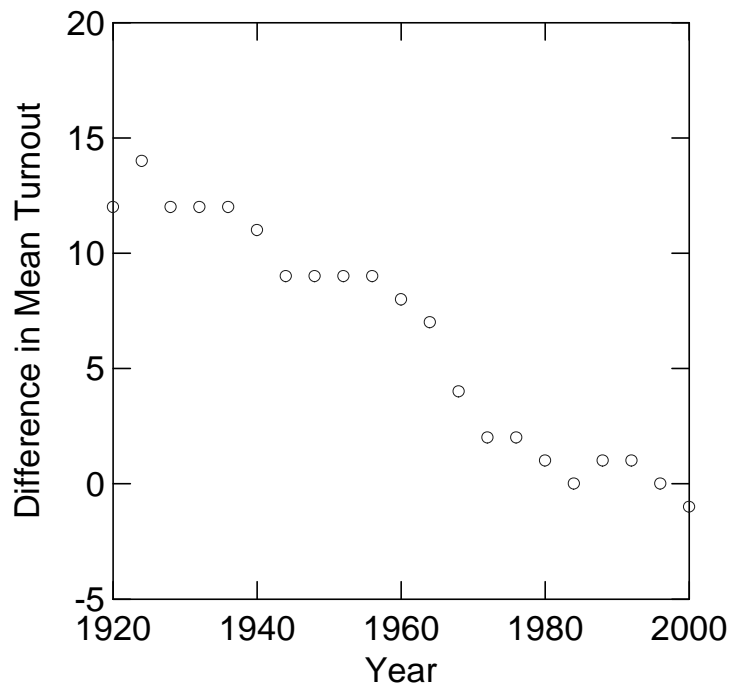


Figure 8. US elections, 1920-2000, difference between average interior state turnout and average boundary state turnout.



Russia. Previous research has showed a moderate to strong influence of social conformity in Russian national elections (Coleman, 2004, 2007a, 2010; Borodin, 2005). So one would expect evidence of spatial autocorrelation in voting. Apparently no specific research on this has been published, but several econometric studies demonstrate significant spatial effects at the regional level (Kholodilin, Oshchepkov, and Silivertovs, 2009; Ledyeva, 2007; Buccellato, 2007). For a general review of geographical voting in Russia see Clem (2006).

The analysis concerns the Russian parliamentary (Duma) election of 2007 for 85 federal subjects, which are a mixture of different kinds of governmental units. The spatial analysis shown on the maps and the spatial autocorrelations cover only the 55 units of European Russia, however, because of availability of the geographic shape file for this area.¹⁶ Other analyses include all units. A minor issue that can be safely ignored is that the boundaries of some units have changed occasionally, so that the map displayed here may not align exactly with election data for all the units. The federal subjects in 2007 are not the same election reporting units as in earlier elections, however. Election data is available in Russian from the Central Election Commission of the Russian Federation.¹⁷

The map analysis follows the path of US elections. The rook method is used to define neighbors, and forty units have between three and six bordering units, most commonly five. Figure 9 shows the level of turnout across European Russia and several regional clusters therein. Figure 10 shows the Moran scatterplot. The measure of spatial autocorrelation Moran's I is weaker than in the US at 0.19 ($p = .02$). The highest turnout was in Chechnya and lowest in St. Petersburg--both on the border (this applies to all of Russia though both are in European Russia). Chechnya is in a high turnout cluster in the Caucasus region, where turnout was so high that it raised questions about ballot stuffing. There are clusters of low turnout areas in the north. To check on the sensitivity of the results to the definition of neighbors, Moran's I was recalculated using the four nearest neighbors method. This gives Moran's I equal to 0.32 ($p = 0.0004$), a somewhat stronger indication of spatial autocorrelation.

¹⁶ The Federal cities of Moscow and St. Petersburg are not included as shapes in the map file and in the corresponding analysis, but the cities are included in the other statistical analysis for the entire country..

¹⁷ <http://www.cikrf.ru>. For English language data, see website of Electoral Geography 2.0, <http://www.electoralgeography.com/new/en/>.

The turnout in European Russia can also be fitted to a regression model as a function of latitude and longitude at the centroids of the units. The model (with errors) is

$$\text{turnout \%} = 99 (15) + 0.44 (0.20) \text{ longitude} - 0.97 (0.23) \text{ latitude}$$

For this model R square equals 0.30, $p < .00001$; squared terms were not statistically significant. Again the estimated surface is a plane and a harmonic function. For boundary units of European Russia only (N = 27), the model is

$$\text{turnout \%} = 125 (16) - 1.12 (0.30) \text{ latitude}$$

R square equals 0.36, $p < .0001$. Longitude is not statistically significant. The coefficients for latitude in these models nearly equal latitude coefficients in the corresponding US models for 1968 and 1992 with the direction reversed. But the spatial effect is weaker in Russia, and the country is farther from a steady-state distribution.

Additional checks on the OLS regression model were done to determine if the model captured all spatially lagged effects. Unlike the US case, SES variables were not available, so a spatial regression model was used. Results are in Table 5.

Table 5. Regression model with spatially lagged turnout.

Variable	Coefficient	Std. Error	p
Constant	104	18.1	<.00001
Lagged Turnout	-0.070	0.14	.63
Longitude	0.50	0.22	.03
Latitude	-1.02	0.25	<.0001

Diagnostic test for spatial dependence, Likelihood Ratio Test = 0.10, $p = 0.74$
 OLS, w/o lagged term, Lagrange Multiplier test for lag effect = 0.07, $p = .80$.
 OLS, w/o lagged term, Lagrange Multiplier test for error effect = 0.06, $p = 0.81$.

The OLS results with rook neighbors confirm the significance of latitude and longitude while rejecting any additional significance of spatially lagged turnout or the possibility of spatially clustered omitted variables and spatial dependence between errors.¹⁸ If the spatially lagged turnout term is included in the regression, it is not statistically significant ($p = .63$); and the spatial lag dependence likelihood ratio =

¹⁸ On interpretation of spatial regression models, see also Ward and Gleditsch (2008).

0.10, $p = 0.74$. So latitude and longitude are sufficient to determine the lagged spatial effect of turnout.

The last statistical test is for the property of harmonic functions that the averages around concentric circles equal the average at the center. To approximate this, all 85 units are sorted into three groups: boundary units, interior units adjacent to the boundary units, and central units another step removed from the boundary. Descriptive statistics for these groups are in Table 6. Differences between the means are not statistically significant¹⁹ (ANOVA, $F = 0.23$, $p = .79$).

Table 5. Russia, 2007, turnout in concentric regions.

Region	N	Mean	Median	SD	Maximum	Minimum
Boundary	45	66.7	63.2	13.1	99.5	51.6
Interior	26	65.4	63.2	9.5	89.7	52.2
Central	14	64.5	61.5	11.1	94.5	54.8

¹⁹ Because the means are so nearly equal, the statistical test lacks power to detect a difference and should not be given much weight in this context.

Figure 9. European Russia, 2007, turnout quantiles by Federal subject and statistically significant regional clusters as determined by a permutation test.

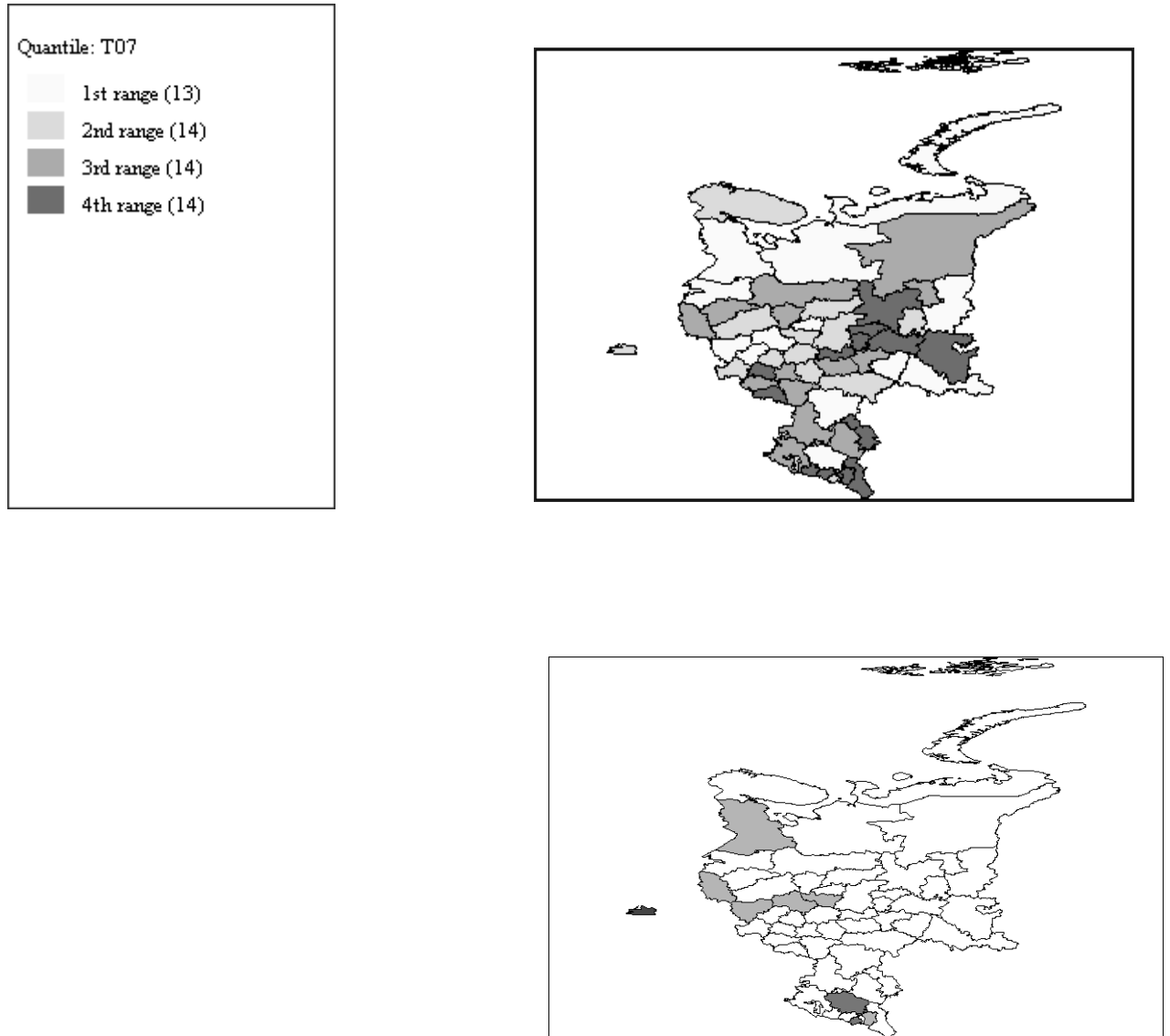
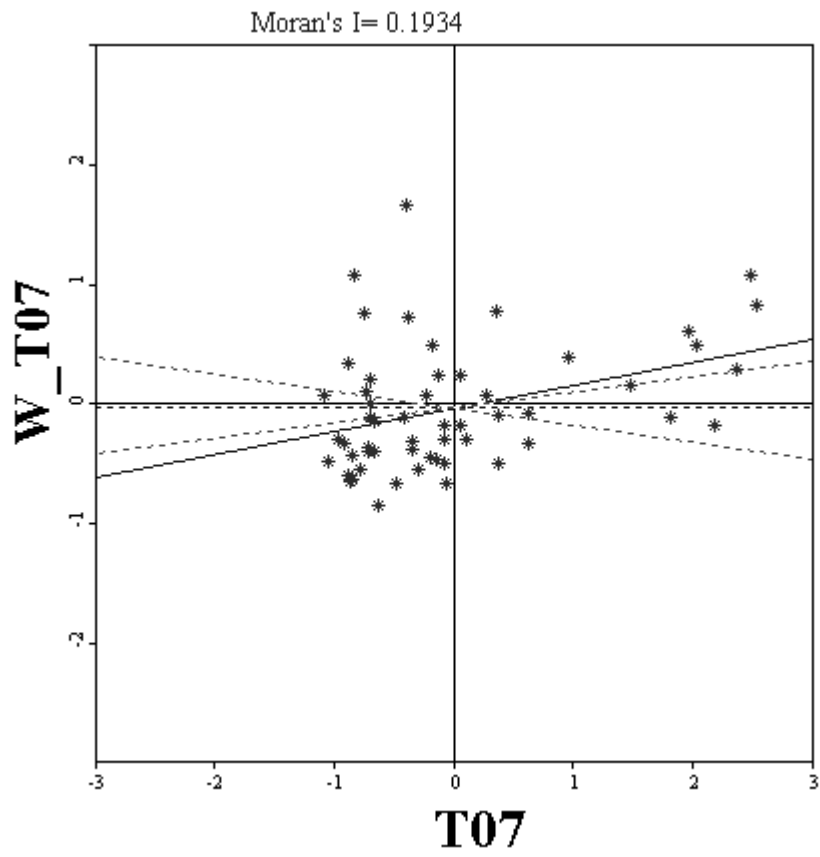


Figure 10. European Russia 2007, Moran Scatterplot.



Discussion

The degree of social conformity with an important norm, such as voting, can vary across both time and geography. As people in one area influence those in the next, and so on, the degree of conformity can change across a landscape, with a general trend toward a smooth transition in behavior from one area to the next. Because conformity is a universal human characteristic one can expect to see this process at work in every society, and a general model of diffusion should be the goal of research. The methods of spatial analysis were developed primarily for exploratory data analysis, however, and they do not help much in developing and testing general theories about spatial dependency. The analysis here adds another layer of explanation to what is offered by spatial analysis—a layer more aligned with theory construction and testing. The methods can be extended to other social norms beside voting.

The goal here was not to explain voter turnout, but to examine how the diffusion of conformity has affected the degree of compliance with the norm for voting. Nevertheless, one can readily see from the results that studies of voting behavior should include spatial lags or geographical location, which has not been common practice. Location matters as to compliance with social norms.

Two research findings stand out. First is the fact that from 1968 on the US looks similar to Russia in the spatial diffusion of voting participation. In both cases statistical tests show good evidence of spatial autocorrelation, and a harmonic function—a plane—fits the spatial data well; the rate of change across latitude is almost the same in each country though the slope of the north-south gradient is reversed. This speaks strongly to the generality of the model. The second important finding is the very slow, exceptionally steady rate of change in voting participation over time in the US, as average turnout in interior states converged toward that of the boundary states, and the country as a whole began to show the characteristics of diffusion. The diffusion model did not fit the US in 1920 but it did by 1968. Clearly, the degree of compliance with the social norm of voting does not change easily. There are situations when conformist change can diffuse rapidly through a society; fashions, fads, and crime waves are examples. But they look more like epidemics in their rapid and transient spread, which would suggest a different type of mathematical model than the Laplace equation. (Epidemics, for example, can show spatial wave patterns, which cannot result from a Laplace model.) But in the US elections one sees a diffusion process in voting participation that has

taken several generations and 80 years or more to reach its current, nearly harmonic distribution close to a steady state. This also means that the turnout distribution is not going to change much from now on. Local bumps might get smoothed out, but the north-south gradient will remain mostly as it is for the foreseeable future.

Appendix

Expanded regression models for USA state turnout in 1992 including latitude, and SES and demographic variables.

Variable	Coefficient	Std. Error	p
Constant	-21.4	11.4	.07
Latitude	0.82	0.14	<.00001
Verbal SAT	0.083	0.021	.0003
Marriage Rate	- 0.095	0.042	.03
Population 1990	- 0.00026	0.00012	.04
Income	0.00043	0.00018	.02
Pop. Density	- 0.0074	0.0036	.05

N = 48, R square = 0.75, F = 21. Average Verbal SAT test scores are an indicator of educational achievement among students applying for college. Latitude is at the centroid. Latitude 95% CI = [0.54, 1.10]

Above model with latitude and verbal SAT only.

Variable	Coefficient	Std. Error	p
Constant	-20.9	8.9	.02
Latitude	1.01	0.13	<.00001
Verbal SAT	0.085	0.02	.00004

R square = 0.69, F = 50.

Model for boundary states (for variables with p < .05).

Variable	Coefficient	Std. Error	p
Constant	-15.1	12.0	.22
Latitude	1.00	0.15	<.00001
Verbal SAT	0.073	0.027	.01

N = 30, R square = 0.70, F = 32. 95% CI for Latitude = [0.69, 1.32].

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