Stochastic simulation: a package for Monte Carlo experiments on econometric models

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STOCHASTIC SIMULATION: A PACKAGE FOR MONTE CARLO EXPERIMENTS ON ECONOMETRIC MODELS

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The stochastic simulation of an econometric model is an application of Monte Carlo methods.

Let us formalize an econometric model in its "structural form" as a system of n equations [5, page 269]:

$$f_i(y_{1,t},\ldots,y_{n,t},\ y_{1,t-1},\ldots,y_{n,t-p};\ x_{1,t},\ldots,x_{m,t}) = e_{it}$$

$$\begin{cases} i = 1,2,\ldots,n \\ t = 1,2,\ldots,T \end{cases}$$

where $y_{i,t}$ are the jointly dependent endogenous variables at time t, $y_{i,t-k}$ are the lagged endogenous variables, $x_{i,t}$ are the exogenous variables and $e_{i,t}$ are the structural disturbances at time t.

During the estimation phase, the coefficients of the structural functions f_i are derived on the basis of the historical values (time series) of the involved variables and of the hypotheses about the distribution of the disturbance terms.

Deterministic simulation is performed setting $e_{i,t}=0$ [2, page 132], so that, for each endogenous variable and for every period, one value is generally obtained as solution. Stochastic simulation, on the contrary, takes into account the disturbance terms, solving the model after adding a vector of pseudo-random numbers drawn from a prespecified multivariate distribution. "The joint distribution from which these stochastic elements are drawn should, naturally, reflect the true model structure as fully as possible" [11]. For this reason the suggested algorithms are related to the residuals obtained in the estimation phase, in order to generate pseudo-random numbers with the same statistical properties of the structural disturbances $e_{i,t}$. Replicated solutions, as usual in Monte Carlo experiments, generate a distribution of outcomes for each endogenous variable, allowing to draw some statistical inferences.

A package to perform stochastic simulation of linear and nonlinear econometric models has been implemented at the Scientific Center of IBM Italy in Pisa.

Algorithms

The generation of the vector of pseudo-structural disturbances to be added in the simulation phase, as above mentioned, requires in this package three main steps:

1) Generation of pseudo-random numbers with continuous uniform distribution in the open interval (0,1). The power residue method is used in the program for this purpose, using as a modulus the prime number $2^{31}-1$ and its primitive root 7^5 as a multiplier, with some minor modifications from the algorithm described in [6]. The numbers generated by this algorithm are then shuffled using a simple 128 cells algorithm, addressed by the lowest-order 7 bits of each number [7].

2) Generation of independent pseudo-random numbers with univariate normal distribution, zero mean and unit variance. Two alternative methods can be used in this phase; they are the Box-Müller (or polar) method [1] and the inverse algorithm [3, page 83]. Both the methods that use the uniform numbers as input are theoretically exact, not involving asymptotic properties, even if they involve, of course, numerical approximations. The bias arising in the joint use of the power residue method and the Box-Müller transformation [10] should be avoided by the above-mentioned intermediate phase of shuffling.

3) Generation of pseudo-random numbers with multivariate normal distribution, zero mean and assigned variance-covariance matrix. Also in this stage two alternative methods are available. The first is based on the triangularization of the covariance matrix (among equations) as it is computed in the estimation phase; it was proposed for application to the stochastic simulation of econometric models by Nagar [9]. The second, proposed by McCarthy [8], is directly based on the use of the estimated residuals. The last method is of more general applicability, being Nagar's algorithm applicable only when the number of stochastic (behavioral) equations is smaller than the sample period length (number of residuals in each equation).

The numerical solution of the system of equations representing the econometric model is performed by means of the Gauss-Seidel iterative procedure. It has been preferred to other methods because of the simplicity of use and the applicability to linear and nonlinear models without differences. In default of other specifications, the initial values for the iterative procedure are chosen equal to the historical values of the endogenous variables at the same year.

Input Requirements

First of all it must be underlined that the package runs on an IBM System/370 model 168, under the operating system VM-370/CMS [4], and its use is strictly based on the facilities of this system.

The package is written in FORTRAN IV and ASSEMBLER 370 languages. It consists of approximately one thousand statements, in addition to the statements necessary to formalize the model.

The required storage for the program is nearly 60 kilobytes. A large work matrix is then required to hold intermediate and final results of the computation; its dimensions depend on the model's size (number of equations, number of exogenous variables, sample period length, etc.).

To perform stochastic simulation of a model, two files must be prepared on the work minidisk of the user's virtual machine; one, containing the model's equations, is a FORTRAN program; the other contains the time series values, the estimated residuals and some indications, such as sample period and simulation period length and number of replicated solutions to be performed.

The choice among the alternative procedures to generate the pseudo-random disturbances must be specified in the starting command at the interactive terminal [4].

Displayed Results

For every simulation period a printout of all the endogenous variables is displayed. It presents, for each variable, the actual (observed) value, the deterministic solution, the estimated mean and standard deviation, the minimum and maximum of the stochastic solutions among the replications.

The same information is presented on outputs per variable, together with the first relative differences of the actual, deterministic and mean stochastic values. Some other statistics and empirical indicators of goodness-of-fit are then computed. They are:

1) The mean over the simulation period of the actual, deterministic and mean stochastic values.

2) The Root Mean Square Error (RMSE) of the deterministic and mean stochastic solutions [5], [11].

3) The Mean Absolute Percentage Error (MAPE) of the deterministic and mean stochastic solutions [5].

4) Theil's inequality coefficient (U) of the deterministic and mean stochastic solutions [12].

5) The coefficients and standard errors of the regression (with intercept) of the observed values on the deterministic or mean stochastic solutions [12].

6) The coefficients and standard errors of the regression (without intercept) of the first relative differences of the observed values over those of the deterministic or mean stochastic solutions [12].

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