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Relative and Absolute Preference for Quality*

CONSTANTINE ANGYRIDIS[†] DEBAPRIYA SEN[‡]

June 6, 2010

Abstract

This paper seeks to explain two related phenomena: (i) it is often the case that when the new variety of a product is launched, some consumers do not purchase the latest variety and (ii) the quality of the latest variety of a product is often not significantly superior compared to the existing variety. We consider a simple model of monopoly with two types of consumers: “regular” (type R) who cares only about the *absolute* quality of the product and “fastidious” (type F) who cares about the *relative* quality vis-a-vis the existing variety. We show that it is never optimal for the monopolist to exclusively serve type F . Moreover, we identify situations where although it is optimal for the monopolist to upgrade the quality of the product, this upgrade is not sufficient to meet the standards of type F . As a result, only type R buys the upgraded variety while type F chooses not to buy it.

Keywords: Relative Preference, Absolute Preference, Singular Menu, Separating Menu

JEL Classification: D42, D82, D86, L15

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1 Introduction

It is often the case that when a new variety of a product is launched, it does not receive a uniformly positive response from consumers. Some consumers do not buy the new variety as they do not consider it to be of significantly superior quality compared to the existing variety. There are two well known examples that validate these observations.

The first example involves the launching of “New Coke” by Coca Cola Co. on April 23, 1985. Despite a prolonged and aggressive advertising campaign, the public’s reaction to the new product was overwhelmingly negative. Soon people were stockpiling cases of the old Coke. According to Hartley (2006, p.34): “...by mid-May 1985 calls were coming in at the rate of 5,000 a day, in addition to a barrage of angry letters.” The Newsweek magazine reported in June 1985 that a case of the old Coke was sold in the black market for as much as \$30. These events prompted Coca Cola to withdraw New Coke from shelves on July 11. Furthermore, company officials apologized to the public and brought back the old Coke in the market. Until this day, the launching of New Coke is widely considered to be one of the biggest marketing fiascos in business history.¹

The launching of the Operating System (OS) Windows Vista by Microsoft in early 2007 is another example. Many users did not consider Vista to be of significantly higher quality compared to the existing system Windows XP. The launching of Vista prompted the media outlet InfoWorld to set up the “Save XP” petition that was signed by more than 75,000 people by February 2008. A survey of 961 IT professionals conducted in late November 2007 found 90% of participants had major concerns about the new OS. The most important issue was the lack of stability of Vista relative to predecessor XP. According to InfoWorld,²

“...many businesses and consumers were not excited about dispensing their time and money upgrading to a new OS that they believe does not offer enough considerable advantages over XP, and are not keen on dealing with the incompatibility issues upgrading invariably causes. Their argument is simple—‘if it ain’t broke, don’t fix it’.”

These examples bring out an interesting aspect of consumer behavior: when the new variety of a product is launched, at least some consumers judge its quality not in *absolute* terms, but rather in *relative* terms by comparing it with the existing variety. Accordingly, these consumers may not purchase the latest variety if the net quality improvement is not deemed to be sufficiently significant. This paper proposes a simple theoretical framework that models such consumer preference and seeks to provide an explanation of the cases described above.

Specifically, we consider a model of a product market that is served by a monopolist. The monopolist can produce an existing variety of low quality at zero cost. In addition, it can also produce a higher quality product by incurring additional costs. Each buyer in the market buys either one or zero unit of the good. The buyers are classified into two types according to the nature of their preference: type *R* (regular) buyers care about the *absolute* quality of the product and type *F* (fastidious) buyers care about its *relative* quality vis-a-vis the existing variety. The monopolist can offer either a single variety (singular menu) or a

¹Michael E. Ross, April 22, 2005. “New Coke and Other Marketing Fiascos,” *MSNBC.com*
<http://www.msnbc.msn.com/id/7209828/>

²Andrew Hendry, February 6, 2008. “Microsoft responds to Save XP petition,” *Computerworld*
[http://www.computerworld.com.au/article/203968/microsoft_responds_save_xp_petition/?](http://www.computerworld.com.au/article/203968/microsoft_responds_save_xp_petition/)

pair of varieties (separating menu) consisting of both the existing variety as well as a new variety of higher quality.

The nature of the preference of type F buyers plays a crucial role in our model. Indeed, this is the major point of departure from the existing literature that generally considers buyers who only care about the absolute quality of a product. When buyers care only about absolute quality (and there are two types), there is an exogenously fixed classification of buyers according to their willingness to pay for the product: “low type” and “high type”. In that case, regardless of the level of quality, high type buyers have a uniformly higher willingness to pay compared to low type. In contrast, there is no exogenously fixed low or high type in our model and whether a buyer is low or high type depends on the quality of the product. Type F is the high type if the new variety is of significantly superior quality compared to the existing variety, while type R is the high type otherwise.

Type R buyers have a standard absolute preference for quality in our model. Accordingly, for type R we obtain the well known result that it is left with positive surplus in any optimal separating menu when it has a higher willingness to pay (see, e.g., Salanié (2005)). This is not the case with type F . A type F buyer, having a relative preference for quality, never buys the existing variety. As a result, when a separating menu (i.e. a menu consisting of both the existing variety as well as a new variety of higher quality) is offered, the incentive compatibility constraint of type F is never binding. This enables the monopolist to charge the maximum possible price for the high-quality variety from type F to extract its entire surplus. Consequently, type F is always left with zero surplus in any separating menu. Then it follows that it cannot be optimal for the monopolist to exclusively serve type F as by choosing a separating menu that serves both types, the monopolist can obtain additional profits from type R . Finally, we identify situations where it is optimal for the monopolist to offer a new variety exclusively for type R . The quality of this new variety, though higher than the existing level, is not significantly superior and as a result type F buyers do not buy it. This outcome closely resembles the examples of New Coke and Vista, where the new variety, though presumably superior, was not purchased by many consumers. To summarize our main results:

1. In any optimal separating menu, a type F buyer is always left with zero surplus, while a type R buyer may be left with positive surplus.
2. It is never optimal for the monopolist to exclusively serve type F buyers.
3. There are situations where it is optimal for the monopolist to offer a new variety of higher quality exclusively for type R buyers. Thus, although the monopolist offers a higher quality product, the quality improvement is not sufficiently significant to meet the standards of type F buyers and they do not buy the new product.

This paper relates to a large literature that considers different aspects of quality provision by monopoly sellers in markets where buyers have different willingness to pay for quality. The literature can be traced back to Spence (1975), who argues that market failure may arise due to the ineffectiveness of prices to convey reliable information on the quality of the product. Mussa and Rosen (1978) show that when a monopolist endogenously chooses quality to serve heterogenous consumers, it is optimal to choose separating menus. It is generally argued in this literature that the monopolist has the incentive to distort the quality choice

of some consumers to extract higher surplus from others (e.g., White (1977), Mussa and Rosen (1978), Maskin and Riley (1984)). Besanko et al. (1987) find in this regard that setting minimum quality standards may have ambiguous welfare implications as it may lead to exclusion of some consumers. Some recent papers include Kim and Kim (1996) who show that in the presence of spill-over effects in costs, higher willingness to pay may not necessarily lead to the provision of higher quality and it may be optimal for the monopolist to choose a pooling menu (i.e. only one price-quality pair). Acharyya (1998) argues that a separating menu may not be optimal without restrictions on cost functions and the extent of market coverage and moreover, there may not be any distortion on quality. Our result that it may be optimal for the monopolist to offer a pooling menu exclusively for type R is consistent with both Kim and Kim (1996) and Acharyya (1998). However, in contrast to these and other papers of the literature, the main driving force behind our results is the relative nature preference of type F buyers.

The monopoly model of this paper is arguably a reasonable approximation with respect to the examples of Coca Cola and Microsoft given before. For instance, Microsoft has continually enjoyed the lion's share of the Operating System market over the past several years. According to the IT company NetApplications, Microsoft has maintained a market share of more than 90%.³ In the case of Coca Cola, its share in the Carbonated Soft Drinks market may not be as dominant, but it is also quite significant. According to the magazine Beverage-Digest, the share of Coca Cola Co. in recent years is around 42% while its closest rival PepsiCo has a share of around 30%.⁴ From a theoretical viewpoint, the monopoly model enables us to see the effect of relative preference in particular clarity. It is an important benchmark which can be extended to incorporate other relevant factors such as the effect of competition in the product market.

The paper is organized as follows. We present the model and derive the optimal price-quality schedules in Section 2. Section 3 endogenizes the choice of quality of the new variety. We conclude in Section 4. Most proofs are relegated to the Appendix.

2 The Model

Consider a good that is produced by a monopolist M . There is an existing variety of the good that has quality $L > 0$, which can be produced at zero cost. By incurring cost $c(H) > 0$, M can produce an additional variety of superior quality $H > L$. For this section, the level of superior quality H is considered to be exogenously given and we denote $c(H) \equiv c$. This assumption is relaxed in Section 3.

Any consumer in this market buys either one unit of the good or nothing. The market size is normalized to be 1, so each consumer can be viewed as a point in the interval $[0, 1]$. A consumer can be either of the following two types: F (fastidious) and R (regular). Let $\lambda \in (0, 1)$ denote the fraction of type F consumers. The type presents the nature of preference for quality of a consumer. A type R consumer cares about the *absolute* quality of the good while a type F consumer cares about its *relative* quality vis-a-vis the existing quality L . For $X \in \{L, H\}$ and $t \in \{R, F\}$, let $v_t(X)$ denote the valuation of a consumer of type t for the

³Top Operating System Share Trend, *Net Applications*
<http://www.netmarketshare.com/os-market-share.aspx?qprid=9>

⁴Beverage-Digest, March 24, 2010, Vol. 56, No.7. <http://www.beverage-digest.com>

good of quality X . We assume that $v_R(X) = X$ and there is a constant $\theta > 1$ such that $v_F(X) = \theta(X - L)$. That is,

$$v_R(L) = L, v_R(H) = H, v_F(L) = 0 \text{ and } v_F(H) = \theta(H - L) \quad (1)$$

The specification in (1) captures the preferences of two types of consumers in a simple way. The valuation of type R is standard. Since type F cares about the relative quality, its valuation depends on the net quality improvement $X - L$ rather than the absolute quality X . The parameter $\theta > 1$ captures the fact that type F is willing to pay more on the basis of the net quality improvement.⁵ To make this point precise, denote

$$\tau(\theta, L) := \theta L / (\theta - 1) \quad (2)$$

Observe that

$$v_F(H) = \theta(H - L) \begin{matrix} \geq \\ \leq \end{matrix} v_R(H) = H \Leftrightarrow H \begin{matrix} \geq \\ \leq \end{matrix} \tau(\theta, L) \quad (3)$$

This shows that the willingness of type F to pay for quality H is higher than the willingness of type R only when H is sufficiently high. Thus, if $H \geq \tau(\theta, L)$, then type F is the “high type” and type R the “low type” while if $H < \tau(\theta, L)$, the converse is true.

The product menu offered by M can be classified into the following categories, where $p_X \geq 0$ stands for the price of quality X :

- (i) It is *varied* if it has two price-quality pairs $\langle (L, p_L), (H, p_H) \rangle$. A varied menu is called *separating* if it is optimal for type R to buy a variety (X, p_X) and it is optimal for type F to buy a different variety (Y, p_Y) .
- (ii) It is *singular* if it has only one price-quality pair (X, p_X) where $X \in \{L, H\}$.

Without loss of generality, we shall only consider singular and separating menus.

Consumers have separable utilities. A consumer who does not buy anything obtains zero utility. A type t consumer who buys variety X at price p_X obtains utility $v_t(X) - p_X$.

2.1 Separating menus

When a separating menu $\langle (L, p_L), (H, p_H) \rangle$ is offered, a consumer of type t has three choices: (i) buy (L, p_L) to obtain $v_t(L) - p_L$ or (ii) buy (H, p_H) to obtain $v_t(H) - p_H$ or (iii) buy nothing and obtain zero. Let $X, Y \in \{L, H\}$ and $X \neq Y$. A type t consumer buys (X, p_X) if and only if both of the following hold:

$$\text{IR}_t \text{ (Individual rationality of type } t\text{): } v_t(X) - p_X \geq 0 \quad (4)$$

$$\text{IC}_t \text{ (Incentive compatibility of type } t\text{): } v_t(X) - p_X \geq v_t(Y) - p_Y \quad (5)$$

Since $v_F(X) = \theta(X - L)$, it follows by (4) that for any positive p_L , $v_F(L) = 0 < p_L$, so type F never buys variety L . Therefore, in order to determine the optimal menus for M in the

⁵The valuations in (1) can have the following alternative interpretation. A type F consumer already purchased the variety of existing quality L in a previous period and as a result has a “reservation utility” θL . If type F switches to the variety of quality H , it will obtain θH , so it compares θH with θL . In contrast, a type R consumer is a first-time buyer of the product and has zero reservation utility, so type R compares H or L with 0.

class of all separating menus, it is sufficient to consider menus where type R buys (L, p_L) and type F buys (H, p_H) . As the fraction of type F is λ , the cost of variety H is c and the cost of L is zero, the profit of M from such a separating menu is

$$\Pi(p_L, p_H) = \lambda(p_H - c) + (1 - \lambda)p_L \quad (6)$$

By (1), (4) and (5), M 's problem is choosing $p_L, p_H \geq 0$ to maximize $\Pi(p_L, p_H)$ subject to

$$\begin{aligned} \text{IR}_R \text{ (Individual rationality of type } R\text{): } & p_L \leq L \\ \text{IC}_R \text{ (Incentive compatibility of type } R\text{): } & H - p_H \leq L - p_L \\ \text{IR}_F \text{ (Individual rationality of type } F\text{): } & p_H \leq \theta(H - L) \end{aligned}$$

Constraint IR_R implies that buying (L, p_L) is better for type R than buying nothing; similarly IR_F says that for type F , buying (H, p_H) is better than buying nothing. Constraint IC_R says that for type R , buying (L, p_L) is better than buying (H, p_H) . There is no incentive compatibility constraint for type F since it never buys variety L .

Proposition 1 (Optimal separating menus) *M has a unique optimal separating menu $\langle (L, p_L), (H, p_H) \rangle$ that has the following properties.*

- (I) IR_F always binds, i.e., $p_H = \theta(H - L)$, so type F is always left with zero surplus.
- (II) (Full Surplus Extraction (FSE)) If $H \geq \tau(\theta, L)$, IR_R binds and type R is also left with zero surplus. The optimal menu has $p_H = \theta(H - L)$ and $p_L = L$. M extracts full surplus from both types to obtain $\lambda[\theta(H - L) - c] + (1 - \lambda)L$.
- (III) (Partial Surplus Extraction (PSE)) If $H < \tau(\theta, L)$, IR_R does not bind and type R is left with positive surplus. The optimal menu has $p_H = \theta(H - L)$ and $p_L = L - (H - p_H) = (\theta - 1)(H - L) < L$. M extracts full surplus from type F and partial surplus from type R to obtain $\lambda[\theta(H - L) - c] + (1 - \lambda)(\theta - 1)(H - L)$.
- (IV) IC_R binds if $H \leq \tau(\theta, L)$ and it does not bind if $H > \tau(\theta, L)$.

Proof See the Appendix. ■

An interesting conclusion is that type F is *always* left with zero surplus under the optimal separating menu. One standard result in contract theory (see, e.g., Salanié, 2005) is that under the optimal separating contract, the type that has a lower willingness to pay (“low type”) is left with zero surplus. In our model, a specific type does not have a uniformly lower willingness to pay for variety H . The low type is F if $H < \tau(\theta, L)$, while it is R if $H \geq \tau(\theta, L)$. Accordingly, type F obtains zero surplus if $H < \tau(\theta, L)$ and type R obtains zero if $H \geq \tau(\theta, L)$, which is along the lines of the existing literature. The question is why is type F left with zero surplus even when $H \geq \tau(\theta, L)$ (i.e. when it has a higher willingness to pay)? The reason is that as type F never buys variety L , M has the opportunity to raise p_H to the highest level possible without facing the risk that F will switch to variety L . This is not possible in a standard model with absolute preferences. In such a case, if variety H is too expensive, the incentive compatibility constraint of the high type will be violated which results in the high type always being left with positive surplus. In our model, this is indeed the case for type R . This type has absolute preferences and is left with positive surplus when it has a higher willingness to pay (i.e. when $H < \tau(\theta, L)$).

2.2 Singular menus

When a singular menu (X, p_X) is offered, a consumer of type t has two choices: (i) to buy the menu to obtain utility $v_t(X) - p_X$ or (ii) to not buy at all and obtain zero. Therefore a type t consumer buys if and only if

$$\text{IR}_t \text{ (Individual rationality of type } t\text{): } v_t(X) - p_X \geq 0 \quad (7)$$

By (1) and (7), the optimal choices of consumers for singular menus are

- (i) (L, p_L) : Type F never buys and type R buys iff $p_L \leq L$.
- (ii) (H, p_H) : Type F buys iff $p_H \leq \theta(H - L)$ and type R buys iff $p_H \leq H$.

Note by (2) that

$$\theta(H - L) \underset{\leq}{\overset{\geq}{\cong}} H \Leftrightarrow H \underset{\leq}{\overset{\geq}{\cong}} \tau(\theta, L) \quad (8)$$

Therefore, for a menu (H, p_H) , we have

- (a) If $H \geq \tau(\theta, L)$, both types buy the menu when $p_H \leq H$, only type F buys when $H < p_H \leq \theta(H - L)$ and no one buys when $p_H > \theta(H - L)$.
- (b) If $H < \tau(\theta, L)$, both types buy the menu when $p_H \leq \theta(H - L)$, only type R buys when $\theta(H - L) < p_H \leq H$ and no one buys when $p_H > H$.

Proposition 2 (Optimal Singular Menus)

- (I) Consider the set $S_L = \{(L, p_L) | p_L \geq 0\}$ of all singular menus where M offers only variety L . The optimal menu over this set has $p_L = L$ where type R is left with zero surplus, type F does not buy and M obtains $(1 - \lambda)L$.
- (II) Consider the set $S_H = \{(H, p_H) | p_H \geq 0\}$ of all singular menus where M offers only variety H . The optimal menu over this set has the following properties.
 - (a) For $H \geq \tau(\theta, L)$:
 - (i) If $c \geq \theta(H - L)$, M does not obtain positive profit from any menu in S_H .
 - (ii) If $H \leq c < \theta(H - L)$, M sets $p_H = \theta(H - L)$. Type R does not buy, type F buys and is left with zero surplus and M obtains $\lambda[\theta(H - L) - c] > 0$.
 - (iii) If $c < H$, $\exists \bar{\lambda}(H) \equiv (H - c) / [\theta(H - L) - c]$ such that
 - If $\lambda > \bar{\lambda}$, M sets $p_H = \theta(H - L)$; type R does not buy, type F buys and is left with zero surplus and M obtains $\lambda[\theta(H - L) - c] > 0$.
 - If $\lambda \leq \bar{\lambda}$, M sets $p_H = H$; both types buy, type R is left with zero surplus, type F with positive surplus and M obtains $H - c > 0$.
 - (b) For $H < \tau(\theta, L)$:
 - (i) If $c \geq H$, M does not obtain positive profit from any menu in S_H .
 - (ii) If $\theta(H - L) \leq c < H$, M sets $p_H = H$. Type F does not buy, type R buys and is left with zero surplus and M obtains $(1 - \lambda)(H - c) > 0$.

- (iii) If $c < \theta(H - L)$, $\exists \lambda^*(H) \equiv [H - \theta(H - L)] / (H - c)$ such that
- If $\lambda > \lambda^*$, M sets $p_H = \theta(H - L)$; both types buy, type F is left with zero surplus, type R with positive surplus and M obtains $\theta(H - L) - c > 0$.
 - If $\lambda \leq \lambda^*$, M sets $p_H = H$; type F does not buy, type R buys and is left with zero surplus and M obtains $(1 - \lambda)(H - c) > 0$.

Proof See the Appendix. ■

2.3 Optimal menus

Having identified optimal menus for M in two different classes, separating (Prop 1) and singular (Prop 2), now we are in a position to characterize globally optimal menus for M . Towards this end, we begin with Lemma 1, which is also of some independent interest.

Lemma 1 *Offering a singular menu exclusively for type F can never be a globally optimal choice for M .*

Proof We know that type F will never buy a singular menu that has variety L , so consider a singular menu (H, p_H) that is offered exclusively for type F . If this menu is optimal for M , then by Prop 2(II), we must have $H \geq \tau(\theta, L)$ and $p_H = \theta(H - L)$ that gives M the profit $\Pi(\theta(H - L)) = \lambda[\theta(H - L) - c]$. By Prop 1(II), if $H \geq \tau(\theta, L)$, then M obtains $\lambda[\theta(H - L) - c] + (1 - \lambda)L > \Pi(\theta(H - L))$ from its optimal separating menu, so the singular menu $(H, \theta(H - L))$ cannot be globally optimal. ■

The lemma above shows that it is never optimal for M to exclusively serve type F (the type that cares about relative quality). To see the intuition, first note that when variety H is not of significantly superior quality ($H < \tau(\theta, L)$), type F is willing to pay less than type R . In this case, type R is the high type and it is clear that it must be served in any singular menu that exclusively serves one type. In contrast, if variety H is of significantly superior quality ($H \geq \tau(\theta, L)$), then it may be optimal for M to offer an exclusive menu for type F and extract the entire surplus from this type. However, we have seen in Proposition 1 that due to the relative preference of type F , the monopolist always extracts the entire surplus from F in its optimal separating menu. Consequently, it is better for the monopolist to choose this separating menu as it enables M to obtain additional profits from type R .

In view of Proposition 2 and Lemma 1, in our search for optimal menus, it is sufficient to compare optimal separating menus from Proposition 1 with:

- (i) Low quality exclusive (LQE) menu for type R : $p_L = L$ and M obtains $(1 - \lambda)L$.
- (ii) High quality exclusive (HQE) menu for type R : $p_H = H$ and M obtains $(1 - \lambda)(H - c)$.
- (iii) High quality inclusive (HQI) menu: both types buy. If $H \geq \tau(\theta, L)$, then $p_H = H$, type R is left with zero surplus, type F with positive surplus and M obtains $H - c$. If $H < \tau(\theta, L)$, then $p_H = \theta(H - L)$, leaving type F with zero surplus, type R with positive surplus and M obtains $\theta(H - L) - c$.

Proposition 3 *The optimal menus of M are given as follows.*

- (I) If $c \geq \theta(H - L)$, it is the LQE menu with $p_L = L$ and M obtains $(1 - \lambda)L$.

(II) If $c < \theta(H - L)$ and $H \geq \tau(\theta, L)$:

(a) For $H - L \leq c < \theta(H - L)$, it is the FSE separating given in Prop 1.

(b) For $c < H - L$, $\exists \hat{\lambda}(H) \in (0, 1)$ such that

(i) if $\lambda > \hat{\lambda}$, it is the FSE separating menu given in Prop 1.

(ii) if $\lambda \leq \hat{\lambda}$, it is the HQI menu with $p_H = H$ and M obtains $H - c$.

(III) If $c < \theta(H - L)$ and $H < \tau(\theta, L)$, $\exists \lambda^*(H) \equiv [H - \theta(H - L)] / (H - c) \in (0, 1)$ such that

(a) For $H - L \leq c < \theta(H - L)$:

(i) if $\lambda > \lambda^*$, it is the PSE separating menu given in Prop 1.

(ii) if $\lambda \leq \lambda^*$, it is the LQE menu with $p_L = L$ and M obtains $(1 - \lambda)L$.

(b) For $c < H - L$:

(i) if $\lambda > \lambda^*$, it is the HQI menu with $p_H = \theta(H - L)$ and M obtains $\theta(H - L) - c$.

(ii) if $\lambda \leq \lambda^*$, it is the HQE menu with $p_H = H$ and M obtains $(1 - \lambda)(H - c)$.

Proof See the Appendix. ■

To see the intuition for the results of Proposition 3, first observe that if $c \geq \theta(H - L)$, it is prohibitively expensive for M to produce variety H , so it chooses only variety L to exclusively serve type R (Prop 3(I)). If $c < \theta(H - L)$, it is potentially worthwhile for M to produce H and its optimal menu depends on the threshold $\tau(\theta, L)$ (Prop 3(II)-(III)). This threshold captures the degree of superiority of H relative to the existing variety L . Figure 1 presents the valuations of two types $v_R(H)$ and $v_F(H)$ and using (3), identifies $\tau(\theta, L)$ as the point of intersection of these two functions.

If $H \geq \tau$, variety H is of significantly superior quality, so that type F buyers have a higher willingness to pay for H . For this case, F is the high type and R the low type. In the optimal separating menu, type R buys L and the monopolist sets a large price for variety H to extract full surplus from type F (FSE menu, Prop 1(II)). Alternatively, M has the option of offering a singular menu consisting of only variety H where both types buy (HQI menu). The price of H here has to be lower than the corresponding price in the FSE menu to induce the low type (type R) to buy. The trade-off between setting a high price of H where only type F buys H (FSE) and a lower price where both types buy H (HQI) is settled by cost c as well as the relative fractions of the two types in the market.

To see this, consider an arbitrary $H \geq \tau$, given by H' in Figure 1. The monopolist can either offer H to type F and L to type R by setting $p_H = AH'$ and $p_L = OL$ (FSE), or it can offer H to both types by setting $p_H = BH'$ (HQI). If it switches from FSE to HQI, it incurs the loss λAB from type F and gains $(1 - \lambda)(BH' - c - OL) = (1 - \lambda)(BE - OL)$ from type R . If c is large ($c \geq H - L$), BE is small, causing M to choose the FSE menu (Prop 3(II)(a)). If $c < H - L$, BE is large and provided that the fraction of type R is also large ($\lambda \leq \hat{\lambda}$), it is optimal for M to choose the HQI menu. Otherwise ($\lambda > \hat{\lambda}$), it is optimal to still choose the FSE menu (Prop 3(II)(b)).

If $H < \tau$, then R is the high type and F the low type. Switching the roles of R and F , the trade-off between separating and singular menus follows along the same lines as in the

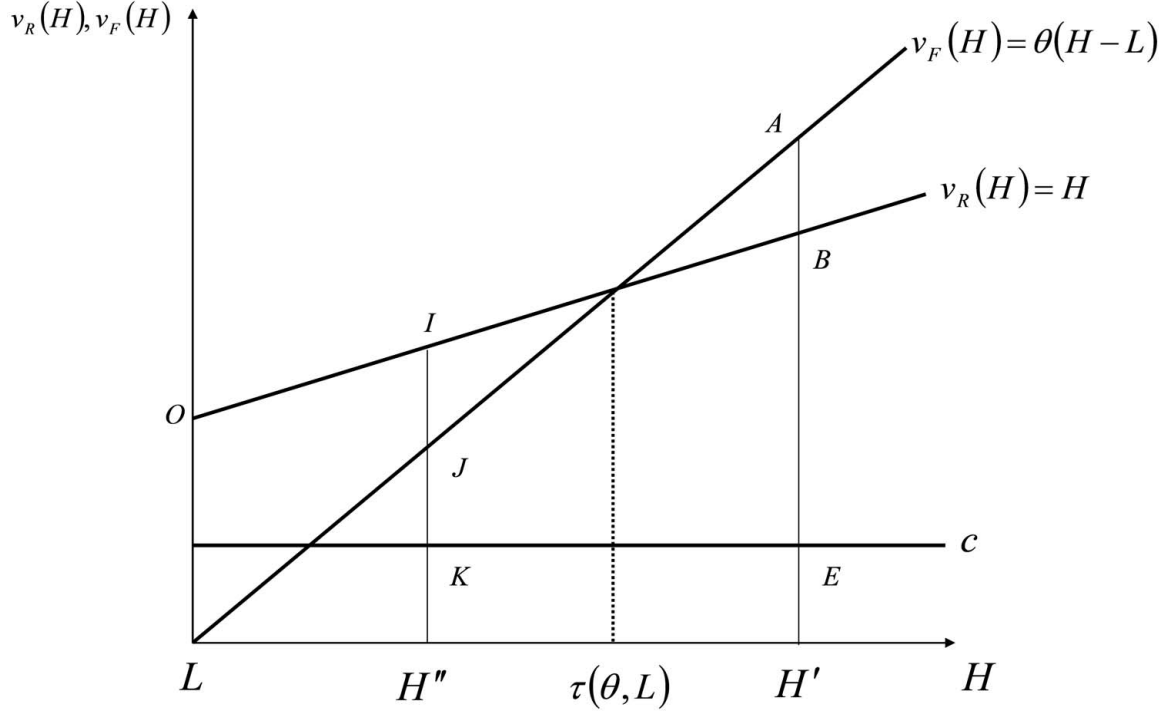


Figure 1: $v_F(H), v_R(H)$ and $\tau(\theta, L)$

previous case. However, there is an important difference. Since the high type of the previous case (type F) had a relative preference, it never bought L , enabling M to extract full surplus from both types in its optimal separating menu (Prop 1(II)). However, for the present case, the high type (type R) has an absolute preference and in the optimal separating menu it has to be left with positive surplus (Prop 1(III)). For this reason, the separating menu loses some of its appeal for $H < \tau$, as evident by comparing parts (II) and (III) of Proposition 3.

To see the intuition for the results of Prop 3(III), consider an arbitrary $H < \tau$ given by H'' in Figure 1. The monopolist can either offer variety H to type F (the low type) by setting $p_H = JH''$, or it can set $p_H = IH'' > JH''$ to serve only R and exclude F .

When the fraction of F is large ($\lambda > \lambda^*$), it is optimal for M not to exclude type F (Prop 3(III)(a)(i),(b)(i)). Regarding type R , M has two options. It can offer only variety H at $p_H = JH''$ so that both types buy H and type R is left with surplus $IH'' - JH'' = IJ$ (HQI menu) or it can offer L at $p_L = OL - IJ$ for type R , making R indifferent between the two varieties, so that R buys L and F buys H (PSE menu). If M switches from HQI to PSE, it incurs the loss $(1 - \lambda)(JH'' - c) = (1 - \lambda)JK$ and gains $(1 - \lambda)(OL - IJ)$ from type R . For higher levels of c , JK is sufficiently small causing M to choose the PSE menu (Prop 3(III)(a)(i)), while for lower levels of c , it is optimal to choose the HQI menu (Prop

3(III)(b)(i)).

When the fraction of F is small ($\lambda \leq \lambda^*$), it is optimal for M to exclude type F and serve type R exclusively (Prop 3(III)(a)(ii),(b)(ii)). In particular, it is not optimal to offer a separating menu for this case. M can either offer variety H at $p_H = IH''$ (HQE) or variety L at $p_L = OL$ (LQE). If M switches from HQE to LQE, it incurs the loss $(1 - \lambda)(IH'' - c) = (1 - \lambda)IK$ and gains $(1 - \lambda)OL$. As before, for higher levels of c , IK is sufficiently small resulting in M choosing the LQE menu (Prop 3(III)(a)(ii)), while for lower levels of c , it is optimal to choose the HQE menu (Prop 3(III)(b)(ii)).

The last part of Proposition 3 ((III)(b)(ii)) is of particular interest as it connects our model with the motivating examples of Windows Vista and New Coke. It identifies situations where the monopolist offers a new variety of superior quality H , but a segment of consumers (type F) does not to buy it. This result is driven by the relative preference of type F consumers. Comparing the quality of the new variety H in relation to the existing level L , they find the quality improvement not sufficiently significant to meet their standards. Accordingly, they are not willing to pay much for variety H , which drives M to offer H exclusively for type R . Observe that offering such an HQE menu is optimal for M only when the fraction of type F is small in the market ($\lambda \leq \lambda^*$). Regarding Microsoft's launching of Vista, it can be argued that although many consumers found Vista to be not of significantly superior quality in relation to the existing system XP, these consumers still possibly constituted only a small segment of the market. Given that, Microsoft might have found it optimal to launch an upgrade that was only moderately superior and excluded this segment in the process. The case of launching of New Coke is different where Coca Cola was eventually compelled to withdraw the new product from the market. From the description of the events following the launching of New Coke, it is evident that most consumers were strongly attached to the existing variety of Coke, suggesting a relative nature of preference for these consumers. Therefore, one plausible explanation of the New Coke fiasco could be the failure on part of Coca Cola to recognize that a very large segment of its consumers had a relative preference.

3 Endogenous H

The level of the superior quality H was considered to be exogenously given in the last section. For any such exogenous H , we characterized optimal menus in Proposition 3. In this section we address the issue of whether our conclusions are robust when the choice of H is endogenous, i.e., when M can choose any $H > L$ rather than an exogenously given level. Instead of characterizing the optimal menus under endogenous H for all possible values of the parameters of the model, for clarity of presentation we shall focus on the primary motivating question of this paper: when M can choose any $H > L$, are there situations where it finds it optimal to choose a level of H where only type R buys the new variety and type F is excluded? We shall identify sufficient conditions on the parameters of the model where this will be case.

We endogenize the choice of H by allowing M to produce a variety of superior quality of any level $H > L$. The production cost $c(H)$ is assumed to be increasing and convex, with $c(L) = 0$. Specifically, we assume that

$$c(H) = \alpha(H - L)^2 \text{ where } \alpha > 0 \quad (9)$$

The parameter α captures how costly it is to produce a high quality good. Denote

$$\tilde{H} \equiv L + 1/\alpha \text{ and } \bar{H} \equiv L + \theta/\alpha \quad (10)$$

Since $\alpha > 0$ and $\theta > 1$, we have $L < \tilde{H} < \bar{H}$. Note from (9) that

$$\begin{aligned} c(H) < H - L \text{ if } H \in (L, \tilde{H}], H - L \leq c(H) < \theta(H - L) \text{ if } H \in [\tilde{H}, \bar{H}) \\ \text{and } c(H) \geq \theta(H - L) \text{ if } H \geq \bar{H} \end{aligned} \quad (11)$$

Using (11) in Prop 3(I), for $H \geq \bar{H}$, the optimal menu is the LQE menu (L, L) where M obtains $(1 - \lambda)L$. Therefore, the profit of M is invariant of H for $H \geq \bar{H}$ and to determine optimal H , it is sufficient to consider $H \in [L, \bar{H}]$. Comparing $\tau(\theta, L) = \theta L/(\theta - 1)$ and \bar{H} ,

$$\tau(\theta, L) \geq \bar{H} \Leftrightarrow \theta \leq \bar{\theta} \text{ where } \bar{\theta}(\alpha, L) := (1 + \sqrt{1 + 4\alpha L})/2 \quad (12)$$

Note that if $\theta < \bar{\theta}$, then $H < \tau(\theta, L)$ for all $H \in [L, \bar{H}]$ (by (11), $c(H) \leq \theta(H - L)$ in this interval). In that case, part (III) of Prop 3 applies, where the optimal menus depend on whether or not $\lambda < \lambda^*(H)$. Using (9), we have

$$\lambda^*(H) = [H - \theta(H - L)]/[H - c(H)] = [H - \theta(H - L)]/[H - \alpha(H - L)^2] \quad (13)$$

Lemma 2 *If $\theta < \bar{\theta}$, then*

- (i) $H < \tau(\theta, L)$, $H > \theta(H - L)$, $H > c(H)$ for all $H \in [L, \bar{H}]$.
- (ii) $\lambda^*(H)$ is well defined and positive for all $H \in [L, \bar{H}]$ and $\lambda^*(L) = \lambda^*(\bar{H}) = 1$.
- (iii) $\exists \underline{H} \in (L, \bar{H})$ such that $\lambda^*(H)$ is decreasing for $H \in [L, \underline{H})$, increasing for $H \in (\underline{H}, \bar{H}]$ and attains its minimum at $H = \underline{H}$.
- (iv) $\exists \tilde{\theta} < \bar{\theta}$ given by $\tilde{\theta}(\alpha, L) := (2\alpha L + 1)/(\alpha L + 1)$ such that $\underline{H} \leq \tilde{H} \Leftrightarrow \theta \leq \tilde{\theta}$.
- (v) If $\theta < \tilde{\theta}$, then the profit of M is either constant or decreasing for $H \geq \tilde{H}$.

Proof See the Appendix. ■

Denote

$$H_E \equiv L + 1/2\alpha \text{ and } H_I \equiv L + \theta/2\alpha \quad (14)$$

Observe that $L < H_E < H_I$.

Proposition 4 *Let $\theta < \bar{\theta}$. There is $\tilde{\lambda}(\theta, \alpha, L) \in (0, 1)$ such that*

- (i) *If $\lambda > \tilde{\lambda}$, the optimal menu for M is the HQI menu with $H = H_I$ and $p_H = \theta(H_I - L)$. Both types buy and M obtains $\theta(H_I - L) - c(H_I) = \theta^2/4\alpha$.*
- (ii) *If $\lambda \leq \tilde{\lambda}$, the optimal menu for M is the HQE menu with $H = H_E$ and $p_H = H_E$. Only type R buys and M obtains $(1 - \lambda)[H_E - c(H_E)] = (1 - \lambda)(L + 1/4\alpha)$.*

Proof See the Appendix. ■

Proposition 4 identifies situations ($\theta < \tilde{\theta}$) such that if the fraction of type F in the market is small ($\lambda \leq \tilde{\lambda}$), then although M finds it optimal to upgrade its product from existing quality L to a superior quality $H_E > L$, this upgrade is not significant to meet the standards of type F . As a result, only type R buys the new variety while type F is excluded. However, if the fraction of type F is large ($\lambda > \tilde{\lambda}$), then it is optimal for M to upgrade its product to a higher level of quality $H_I > H_E$. Such a quality improvement is sufficiently significant so that both types F and R buy the new variety. It can be also noted that when $\lambda \leq \tilde{\lambda}$, the quality of the new variety H_E is independent of θ (the parameter that captures the willingness to pay for type F). However, when $\lambda > \tilde{\lambda}$, the upgraded quality H_I depends on θ and both the quality and the profit of M is monotonically increasing in θ .

To see the economic interpretation of the sufficient condition $\theta < \tilde{\theta}$, observe from Lemma 2 that $\tilde{\theta}$ is increasing in αL . Thus, an implication of the condition $\theta < \tilde{\theta}$ is that either α is large (i.e. it is costly to produce a high quality variety) or L is large (i.e. the quality of the existing variety is quite good). The parameter α simply captures the cost effect. The role of the parameter L is more interesting: with large L , the effect of relative preference becomes more dominant, because when the existing variety is already of reasonably good quality, it becomes more difficult to meet the standards of type F .

4 Concluding remarks

This paper provides a simple theoretical framework to understand two related phenomena: (i) when the new variety of a product is launched, some buyers are reluctant to purchase the latest variety and (ii) the quality of the new variety of a product may not be significantly superior compared to its previous version. We consider a monopoly model with two types of buyers who differ in terms of the nature of their preference: one type cares about the absolute quality of a variety (type R), while the other type cares about its relative quality (type F). We show that type F is always left with zero surplus in any optimal separating menu (Proposition 1) and it is never optimal for the monopolist to exclusively serve type F (Lemma 1). Then we identify situations where it is optimal for the monopolist to offer a high quality variety exclusively for type R (Proposition 3). Furthermore, we show that when the quality choice is endogenous, the quality of the upgraded product is responsive to the preference of type F consumers only when they constitute a substantial segment of the market. Otherwise, it is optimal for the monopolist to only carry out incremental improvements in quality that results in type F consumers not buying the upgraded product (Proposition 4).

The main driving force behind our results is the relative preference of type F consumers. This is the salient point of departure of this paper with the existing literature. One implication of the relative preference is that there is no exogenously fixed classification of buyers according to their willingness to pay for the product, rather it depends on the quality of the product. The monopoly model provides a simple and transparent theoretical framework that illustrates the role of relative preference on firm behavior and quality choice. This framework can be readily extended to incorporate competition in the product market. This extension is left for future research.

Appendix

Proof of Proposition 1 (I): Consider a menu where IR_F does not bind, i.e. $p_H < \theta(H - L)$. If IR_R also does not bind ($p_L < L$), then M can choose a new menu where the prices of both varieties are raised to $p_L + \varepsilon$ and $p_H + \varepsilon$ with $\varepsilon > 0$ sufficiently small. For the new menu IC_R will continue to hold, while both IR_R and IR_F will hold for small values of ε . As $\varepsilon > 0$, M 's profit in the new menu is higher, so the old menu cannot be optimal.

Now consider a menu where IR_F does not bind, but IR_R does ($p_L = L$.) Using $p_L = L$ in IC_R , we have $p_H \geq H$. Observe by (2) that if $H < \tau(\theta, L)$, then $\theta(H - L) < H$, so it is not possible to have both $p_H < \theta(H - L)$ and $p_H \geq H$. Consider $H \geq \tau(\theta, L)$, which implies that $H \leq \theta(H - L)$. Having $p_H \in [H, \theta(H - L))$ cannot be optimal for M as it can raise p_H to improve its profit. This proves that an optimal menu must have $p_H = \theta(H - L)$.

(II)-(IV): Taking $p_H = \theta(H - L)$ in (6), the problem of M reduces to choosing p_L to maximize $(1 - \lambda)p_L$ subject to (a) $p_L \leq L$ (IR_R) and (b) $p_L \leq (\theta - 1)(H - L)$ (IC_R). Clearly, it is optimal for M to choose $p_L = \min\{L, (\theta - 1)(H - L)\}$. Noting that

$$(\theta - 1)(H - L) \lesseqgtr L \Leftrightarrow H \lesseqgtr \tau(\theta, L),$$

it follows that if $H \geq \tau(\theta, L)$, then M chooses $p_L = L \leq (\theta - 1)(H - L)$ that binds IR_R (and does not bind IC_R except if $H = \tau(\theta, L)$). If $H < \tau(\theta, L)$, then M chooses $p_L = (\theta - 1)(H - L) < L$ that binds IC_R , but where IR_R does not bind. This proves (II)-(IV). ■

Proof of Proposition 2 (I): Follows immediately by noting that type F never buys variety L and type R buys (L, p_L) only if $p_L \leq L$.

(II): Observe that when menu (H, p_H) is offered, type R buys only if $p_H \leq H$ while type F buys only if $p_H \leq \theta(H - L)$.

(II)(a) By (2), if $H \geq \tau(\theta, L)$, then $H \leq \theta(H - L)$. If $p_H > \theta(H - L)$, then no type buys and M obtains zero. If $H < p_H \leq \theta(H - L)$, then only type F buys; over this interval, it is optimal for M to set $p_H = \theta(H - L)$ that gives it profit $\Pi(\theta(H - L)) = \lambda[\theta(H - L) - c]$. Finally, if $p_H \leq H$, then both types buy; over this interval, it is optimal for M to set $p_H = H$ where type R is left with zero surplus, type F with positive surplus and M obtains $\Pi(H) = H - c$.

(i) If $c \geq \theta(H - L) \geq H$, both $\Pi(\theta(H - L))$ and $\Pi(H)$ are at most zero which proves (i).

(ii) If $H \leq c < \theta(H - L)$, then $\Pi(H) \leq 0 < \Pi(\theta(H - L))$ which proves (ii).

(iii) If $c < H$, then both $\Pi(H)$ and $\Pi(\theta(H - L))$ are positive and (iii) follows by noting that $\Pi(\theta(H - L)) \gtrless \Pi(H) \Leftrightarrow \lambda \gtrless \bar{\lambda}$.

(II)(b) By (2), if $H < \tau(\theta, L)$, then $H > \theta(H - L)$. If $p_H > H$, then no type buys and M obtains zero. If $\theta(H - L) < p_H \leq H$, then only type R buys; over this interval, it is optimal for M to set $p_H = H$ that gives it profit $\Pi(H) = (1 - \lambda)[H - c]$. If $p_H \leq \theta(H - L)$, then both types buy; over this interval, it is optimal for M to set $p_H = \theta(H - L)$ where type F is left with zero surplus and type R is left with positive surplus. M obtains profit $\Pi(\theta(H - L)) = [\theta(H - L) - c]$.

(i) If $c \geq H > \theta(H - L)$, both $\Pi(\theta(H - L))$ and $\Pi(H)$ are at most zero which proves (i).

(ii) If $\theta(H - L) \leq c < H$, then $\Pi(\theta(H - L)) \leq 0 < \Pi(H)$ which proves (ii).

(iii) If $c < \theta(H - L)$, then both $\Pi(H)$ and $\Pi(\theta(H - L))$ are positive and (iii) follows by noting that $\Pi(H) \geq \Pi(\theta(H - L)) \Leftrightarrow \lambda \geq \tilde{\lambda}$. ■

Proof of Proposition 3 We determine globally optimal menus by comparing the profit of M from (i) the optimal separating menu, (ii) the optimal (singular) menu over $S_H = \{(H, p_H) | p_H \geq 0\}$ and (iii) the optimal (singular) menu over $S_L = \{(L, p_L) | p_L \geq 0\}$. The nature of the menu appears as superscripts in the profit of M . By Prop 2(I), the optimal menu over S_L is the LQE menu with $p_L = L$ where M obtains $\Pi_\lambda^{\text{LQE}} = (1 - \lambda)L > 0$.

(I) Let $c \geq \theta(H - L)$. If $H \geq \tau(\theta, L)$, then by Prop 1(II), the optimal separating menu is the FSE menu and

$$\Pi_\lambda^{\text{FSE}} \leq (1 - \lambda)L = \Pi_\lambda^{\text{LQE}}$$

If $H < \tau(\theta, L)$, then by Prop 1(III), the optimal separating menu is the PSE menu and

$$\Pi_\lambda^{\text{PSE}} \leq (1 - \lambda)(\theta - 1)(H - L) < (1 - \lambda)L = \Pi_\lambda^{\text{LQE}}$$

since for this case, $(\theta - 1)(H - L) < L$ (by (8)). Thus, it is sufficient to compare LQE with the optimal menu over S_H .

If either $H \geq \tau(\theta, L)$ or $H < \tau(\theta, L)$ and $c \geq H$, then no menu in S_H gives positive profit (Prop 2(II)(a)(i),(b)(i)), so LQE is optimal. Finally let $H < \tau(\theta, L)$ and $c < H$. Then by Prop 2(II)(b)(ii), the optimal menu over S_H gives profit

$$(1 - \lambda)(H - c) \leq (1 - \lambda)[H - \theta(H - L)] < (1 - \lambda)L = \Pi_\lambda^{\text{LQE}}$$

since $c \geq \theta(H - L)$ and $\theta > 1$. This proves the optimality of LQE.

(II) Let $c < \theta(H - L)$ and $H \geq \tau(\theta, L)$. Then by Prop 1(II), the optimal separating menu is the FSE menu where M obtains

$$\Pi_\lambda^{\text{FSE}} = \lambda[\theta(H - L) - c] + (1 - \lambda)L > (1 - \lambda)L = \Pi_\lambda^{\text{LQE}}$$

So for this case it is sufficient to compare FSE with the optima menu over S_H .

If either $H \leq c < \theta(H - L)$ or $c < H$ and $\lambda \geq \bar{\lambda}$, then the optimal menu over S_H gives profit $\lambda[\theta(H - L) - c]$ (Prop 2(II)(a)(ii),(iii)), which is less than Π_λ^{FSE} . So let $c < H$ and $\lambda < \bar{\lambda}$. Then the optimal menu over S_H is the HQI menu with $p_H = H$ and $\Pi^{\text{HQI}} = H - c$ (Prop 2(II)(b)(iii)). Let

$$\Delta(\lambda) := \Pi_\lambda^{\text{FSE}} - \Pi^{\text{HQI}} = \lambda[\theta(H - L) - H] - (1 - \lambda)(H - L - c) \quad (15)$$

Since $H \geq \tau(\theta, L)$, by (8), $\theta(H - L) \geq H$. Thus, if $c \geq H - L$, then $\Delta(\lambda) \geq 0$ and the optimal menu is the FSE menu, proving part (II)(a).

To prove (b), let $c < H - L$. Then $\Delta'(\lambda) = [\theta(H - L) - H] + (H - L - c) > 0$. Note from Prop 2(II)(a)(iii) that when $\lambda = \bar{\lambda}(H)$, $\Pi^{\text{HQI}} = H - c = \bar{\lambda}[\theta(H - L) - c]$, so that $\Delta(\bar{\lambda}) = (1 - \bar{\lambda})L > 0$. Since $g(0) = -(H - L - c) < 0$, it follows that $\exists \hat{\lambda}(H) \in (0, \bar{\lambda}(H)) \subset (0, 1)$ such that $\Delta(\lambda) \geq 0 \Leftrightarrow \lambda \geq \hat{\lambda}$. This proves (II)(b).

(III) Let $c < \theta(H - L)$ and $H < \tau(\theta, L)$. Then the optimal separating menu is the PSE menu (Prop 1(III)) where M obtains

$$\Pi_\lambda^{\text{PSE}} = \lambda[\theta(H - L) - c] + (1 - \lambda)[(\theta - 1)(H - L)]$$

In the class of singular menus, the optimal menu is either LQE or HQI with $p_H = \theta(H - L)$ or HQE with $p_H = H$ (Prop 2(II)(b)(iii)). Note that

$$\Pi^{\text{LQE}} = (1 - \lambda)L, \Pi^{\text{HQI}} = \theta(H - L) - c \text{ and } \Pi_\lambda^{\text{HQE}} = (1 - \lambda)(H - c)$$

We observe that

$$\Pi_\lambda^{\text{HQE}} - \Pi_\lambda^{\text{LQE}} = \Pi^{\text{HQI}} - \Pi_\lambda^{\text{PSE}} = (1 - \lambda)(H - L - c) \quad (16)$$

By (16), $\Pi^{\text{HQI}} - \Pi_\lambda^{\text{HQE}} = \Pi_\lambda^{\text{PSE}} - \Pi_\lambda^{\text{LQE}}$. Noting by Prop 2(II)(b)(ii) that $\Pi^{\text{HQI}} \gtrless \Pi_\lambda^{\text{HQE}} \Leftrightarrow \lambda \gtrless \lambda^*$, we conclude

$$\Pi^{\text{HQI}} \gtrless \Pi_\lambda^{\text{HQE}} \Leftrightarrow \Pi_\lambda^{\text{PSE}} \gtrless \Pi_\lambda^{\text{LQE}} \Leftrightarrow \lambda \gtrless \lambda^* \quad (17)$$

To prove (III)(a), let $H - L \leq c < \theta(H - L)$. Then by (16), $\Pi_\lambda^{\text{LQE}} \geq \Pi_\lambda^{\text{HQE}}$ and $\Pi_\lambda^{\text{PSE}} \geq \Pi^{\text{HQI}}$. If $\lambda \geq \lambda^*$, then by (17), $\Pi_\lambda^{\text{HQE}} \leq \Pi^{\text{HQI}} \leq \Pi_\lambda^{\text{PSE}}$ and $\Pi_\lambda^{\text{LQE}} \leq \Pi_\lambda^{\text{PSE}}$. Hence for this case, the optimal menu is PSE, proving part (III)(a)(i). If $\lambda < \lambda^*$, then by (17), $\Pi^{\text{HQI}} < \Pi_\lambda^{\text{HQE}} \leq \Pi_\lambda^{\text{LQE}}$ and $\Pi_\lambda^{\text{PSE}} < \Pi_\lambda^{\text{LQE}}$, so for this case, the optimal menu is LQE, proving part (III)(a)(ii).

For (III)(b), let $c < H - L$. Then by (16), $\Pi_\lambda^{\text{LQE}} < \Pi_\lambda^{\text{HQE}}$ and $\Pi_\lambda^{\text{PSE}} < \Pi^{\text{HQI}}$. If $\lambda \geq \lambda^*$, then by (17), $\Pi^{\text{HQI}} \geq \Pi_\lambda^{\text{HQE}} > \Pi^{\text{LQE}}$. Since $\Pi^{\text{HQI}} > \Pi_\lambda^{\text{PSE}}$, for this case the optimal menu is HQI, proving part (III)(b)(i). If $\lambda < \lambda^*$, then by (17), $\Pi_\lambda^{\text{HQE}} > \Pi^{\text{HQI}} > \Pi_\lambda^{\text{PSE}}$. Since $\Pi_\lambda^{\text{HQE}} > \Pi_\lambda^{\text{LQE}}$, for this case, the optimal menu is HQE, proving part (III)(b)(ii). ■

Proof of Lemma 2 (i) If $\theta < \bar{\theta}$, then by (12), $\bar{H} < \tau(\theta, L)$. So for all $H \in [L, \bar{H}]$, we have $H < \tau(\theta, L)$. Then by (8), $H > \theta(H - L)$. Since $c(H) \leq \theta(H - L)$ for $H \in [L, \bar{H}]$, we have $H > c(H)$.

(ii) The first part is immediate from (i). As $c(L) = 0$ and $c(\bar{H}) = \theta(H - L)$, the last part follows from (13).

(iii) Let $\lambda_1^*(H) \equiv d\lambda^*(H)/dH$. Note that

$$\lambda_1^*(H) \gtrless 0 \Leftrightarrow g(H) \gtrless 0 \text{ where } g(H) := \alpha(H - L)[(1 + \theta)L - (\theta - 1)H] - \theta L \quad (18)$$

As $g'(H) = 2\alpha(\theta - 1)(\tau(\theta, L) - H) > 0$, $g(H)$ is monotonic for all $H \in [L, \bar{H}]$. As $g(L) = -\theta L < 0$ and $g(\bar{H}) = \theta(\theta - 1)[\tau(\theta, L) - \bar{H}] > 0$, $\exists \underline{H} \in (L, \bar{H})$ such that $g(H) \gtrless 0 \Leftrightarrow H \gtrless \underline{H}$ which proves (iii).

(iv) As $g(\tilde{H}) = (1 + \alpha L)(\tilde{\theta} - \theta)/\alpha$, it follows that $\lambda_1^*(\tilde{H}) \gtrless 0 \Leftrightarrow \theta \gtrless \tilde{\theta}$. Then (iv) follows from (iii).

(v) We know that the profit of M is a constant for $H \geq \bar{H}$, so consider $H \in [\tilde{H}, \bar{H}]$. Then by (11), $H - L \leq c(H) \leq \theta(H - L)$ and Prop 3(III)(a) applies.

First suppose $H \in [\tilde{H}, \bar{H}]$ and $\lambda < \lambda^*(H)$. Then by Prop 3(III)(a)(ii), it is best for M to choose the LQE menu that yields profit $(1 - \lambda)L$ which does not depend on H .

Next suppose $H \in [\tilde{H}, \bar{H}]$ and $\lambda \geq \lambda^*(H)$. Since $\theta > \tilde{\theta}$, we have $\tilde{H} > \underline{H}$ (part (iv)), so by part (iii), $\lambda^*(H)$ is increasing for $H \in [\tilde{H}, \bar{H}]$. Hence $\lambda \geq \lambda^*(H) \geq \lambda^*(\tilde{H}) = [\alpha L - (\theta - 1)]/\alpha L$.

By Prop 3(III)(a)(i), for this case it is best for M to choose the PSE menu given in Prop 1 that yields profit

$$\pi(H) = \lambda [\theta(H - L) - \alpha(H - L)^2] + (1 - \lambda)(\theta - 1)(H - L)$$

As $\pi(H)$ is strictly concave and $\pi'(\tilde{H}) = \theta - 1 - \lambda \leq \theta - 1 - \lambda^*(\tilde{H}) = -(2\alpha L + 1)(\hat{\theta} - \theta)/\alpha L < 0$, it follows that $\pi(H)$ is decreasing for all $H \in [\tilde{H}, \bar{H}]$. ■

Proof of Proposition 4 Let $\theta < \tilde{\theta}$. By Lemma 2(v), in search for optimal menus, it is sufficient to consider $H \in [L, \tilde{H}]$, where $c(H) \leq H - L$ (by (11)). Since $\theta < \hat{\theta} < \bar{\theta}$, we have $H < \tau(\theta, L)$ for all $H \in [L, \tilde{H}]$ (Lemma 2(i)). So we can use the results of (III)(b) of Prop 3, where for any H , the optimal menu depends on whether or not $\lambda > \lambda^*(H)$. Denote

$$A_\lambda := \{H \in [L, \tilde{H}] | \lambda \leq \lambda^*(H)\} \text{ and } \bar{A}_\lambda := \{H \in [L, \tilde{H}] | \lambda \geq \lambda^*(H)\}$$

Case 1 $H \in A_\lambda$: By Prop 3(III)(b), the best menu is the HQE menu where M obtains the profit

$$\Pi_\lambda^{\text{HQE}}(H) = (1 - \lambda)[H - c(H)] = (1 - \lambda)[H - \alpha(H - L)^2] \quad (19)$$

Observe that $\Pi_\lambda^{\text{HQE}}(H)$ is increasing for $H < H_E$, decreasing for $H > H_E$ and its unique unconstrained maximum is attained at H_E where $H_E \equiv L + 1/2\alpha \in (L, \tilde{H})$.

If $\lambda \leq \lambda^*(H_E)$, then $H_E \in A_\lambda$ and it is the optimal choice over $H \in A_\lambda$. If $\lambda > \lambda^*(H_E)$, then $H_E \notin A_\lambda$ and any optimal choice over $H \in A_\lambda$ has $\lambda^*(H) = \lambda$, making such a choice feasible for $H \in \bar{A}_\lambda$.

Case 2 $H \in \bar{A}_\lambda$: By Prop 3, the best menu is the HQI menu that yields the profit

$$\Pi^{\text{HQI}}(H) = \theta(H - L) - c(H) = \theta(H - L) - \alpha(H - L)^2 \quad (20)$$

Let $H_I := L + \theta/2\alpha > L$. Since $\theta < \hat{\theta} < 2$, we have $H_I < \tilde{H} = L + 1/\alpha$. We note that $\Pi^{\text{HQI}}(H)$ is increasing for $H < H_I$, decreasing for $H > H_I$ and its unique unconstrained maximum is attained at H_I .

We observe that $H_I > H_E$ and from the condition $\theta > \hat{\theta}$, it follows that $\lambda^*(H_E) > \lambda^*(H_I)$. For $\lambda \in [0, \lambda^*(H_I)]$, $H_E \in A_\lambda$ and $H_I \notin \bar{A}_\lambda$, so the global optimal choice is H_E (HQE menu with $H = H_E$). For $\lambda \in [\lambda^*(H_E), 1]$, $H_I \in \bar{A}_\lambda$ and $H_E \notin \bar{A}_\lambda$, so the global optimal choice is H_I (HQI menu with $H = H_I$).

Finally consider $\lambda \in [\lambda^*(H_I), \lambda^*(H_E)]$. Then $H_E \in A_\lambda$ and $H_I \in \bar{A}_\lambda$. Since H_E is the unique maximizer of (19) and H_I the unique maximizer of (20),

$$\Pi_\lambda^{\text{HQE}}(H_E) > \Pi_\lambda^{\text{HQE}}(H_I) \text{ and } \Pi^{\text{HQI}}(H_I) > \Pi^{\text{HQI}}(H_E) \quad (21)$$

Recall by Prop 3(III)(b) when $\lambda = \lambda^*(H)$ for some H , HQE and HQI menus yield the same profit under that H , so that

$$\Pi_{\lambda^*(H_I)}^{\text{HQE}}(H_I) = \Pi^{\text{HQI}}(H_I) \text{ and } \Pi_{\lambda^*(H_E)}^{\text{HQE}}(H_E) = \Pi^{\text{HQI}}(H_E) \quad (22)$$

Denote $\Delta_\lambda := \Pi^{\text{HQI}}(H_I) - \Pi_\lambda^{\text{HQE}}(H_E)$. As both H_E and H_I are independent of λ , from (19) and (20), Δ_λ is increasing in λ . From the first inequalities of (21) and (22), $\Delta_{\lambda^*(H_I)} < 0$ and from the last inequalities, $\Delta_{\lambda^*(H_E)} > 0$. Therefore, $\exists \tilde{\lambda} \in (\lambda^*(H_I), \lambda^*(H_E))$ such that $\Delta_\lambda \gtrless 0 \Leftrightarrow \lambda \gtrless \tilde{\lambda}$. This proves that the HQI menu with $H = H_I$ is optimal for $\lambda > \tilde{\lambda}$ and the HQE menu with $H = H_E$ is optimal for $\lambda \leq \tilde{\lambda}$. The profits of M are immediate. ■

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