Macroeconomic Production Functions for Eastern Europe

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ABSTRACT

This paper was presented in June 1974 at the Symposium "On the Measurement of Factor Productivities" at the castle Reisensburg in Bavaria, and was published in the book of proceedings from this Symposium. It contains results of several years of research about the economies of six East European countries, namely Czechoslovakia, Poland, Hungary, East Germany, Bulgaria and Rumania. Data on output, capital and labor for 15 – 20 branches of industry and 17 – 20 years were collected for each of the mentioned countries. About 40 different models of production function were estimated using time series, cross-section, and combined time series and cross-section regression analysis. Because the time series regressions were run for each industrial branch in every country and cross-section regressions were run for each year across all branches in every country and almost always 40 different versions of production function were estimated, several thousand regressions were run all together. The results showed that in most cases the production functions had usual form with constant returns to scale and close to unit elasticity of substitution between capital and labor. However the most interesting result was relatively high original but steadily declining growth rate of total factor productivity (technical progress). This indicated that within 10 to 20 future years Eastern Europe would be tremendously lagging behind the West.

This paper[1] presents estimates of macroeconomic production functions for Poland, Czechoslovakia, East Germany, Hungary, Rumania and Bulgaria. The estimated production functions were variants of the general type

(1) $$Y = A(t)F(K, L)U(e, t)$$

where
Y, K, L and t are output, capital, labor and time respectively,
U is random disturbance,
F is either Cobb-Douglas or CES production function and the term
$A(t)$ represents impact of the output augmenting technical change.

All the variants with Cobb-Douglas production function were linearized either by logarithmic or by the "rates of growth" transformation. The CES functions were approximated by Kmenta's formula, so that only ordinary least squares method was used for estimation.

One of the purposes of this paper was to compare results obtained by three different approaches, namely (a) the pure cross-section analysis, (b) the pure time-series analysis and (c) the combination of cross-section and time-series analyses. The sensitivity of the estimated parameters to the variations in the form of production function and to the changes in data was also tested.

Although the main aim of our research was to estimate trends in $A(t)$, it turned out to be necessary to pay also a great attention to the reliable estimation of the shape of the function $F(K,L)$, because both estimates are mutually dependent and both parts of the production function are crucial for understanding the dynamic behavior of the economic system.

Four particular questions were examined:

1. What is the rate of technical progress in the Soviet-type economies?
2. Are the factor elasticities in Soviet-type economies different than those in other countries
3. Does the Soviet-type economy cause significant economies or diseconomies of scale?
4. Is the elasticity of substitution in Soviet-type economies near one, very low or very high?

Models of Production Functions

In this section several variants of the production function (1) will be formulated and assigned labels. These labels will be used later when the estimated parameters of individual models are reported. We shall consider five forms of the term A(t), which differ in assumptions about the rate of technical change $\rho(t) = (dA/dt)/A$

1. No technical progress (NTP)

(2.0) $$A(t) = A_0$$

2. Constant rate of technical progress (CRTP)

(2.1) $$A(t)=A_0 \exp(\rho t)$$

3. Trend in the rate of technical progress (TRTP)

(2.2) $$A(t) =A_0 \exp(\rho t + \lambda t^2/2)$$

4. Recessions in the rate of technical progress (RRTP)

(2.3) $$A(t)= A_0 \exp(\rho t + \mu \Sigma s_\tau)$$

where s_τ is a dummy variable for recession years.

5. Recessions and trend in the rates of technical change (RTRTP)

(2.4) $$A(t)= A_0 \exp(\rho t + \mu \Sigma s_\tau + \lambda t^2/2)$$

Further we shall consider four forms of the term F(K, L) which differ in the assumption about both the elasticity of substitution and the returns to scale.

1. Cobb-Douglas production function with constant returns to scale (CDCRS)

(2.6) $$F(K, L) = K^\beta L^{1-\beta}$$

2. Cobb-Douglas function with non-constant returns to scale (CDNRS)

(2.7) $$F(K,L) = K^\beta L^\gamma$$

3. CES function with constant returns to scale (CESCRS)

(2.8) $$F(K, L) = [dK^{-\omega} + (1 - d)L^{-\omega}]^{-1/\omega}$$

4. CES function with non-constant returns to scale

(2.9) $$F(K, L) = [\delta K^{-\omega} + (1 - \delta)L^{-\omega}]^{-\nu/\omega}$$

Let us denote the factor elasticities by η_K, and η_L

$$\eta_K = (\partial Y/\partial K)(K/Y) \qquad \eta_L = (\partial Y/\partial L)(L/Y)$$

In the Cobb-Douglas case $\eta_K = \beta$ and $\eta_L = \gamma$. In the CES case factor elasticity is generally not constant. They depend on the capital-labor ratio

(2.10) $$\eta_K = \nu\{ 1 + [(1 - \delta)/ \delta] (K/L)^{\omega} \}^{-1}$$

(2.11) $$\eta_L = \nu\{ 1 + [\delta /(1 - \delta)] (K/L)^{-\omega} \}^{-1}$$

The CES parameter δ is dependent on units of measurement, and it is clear from (2.10) that it cannot be compared directly with the Cobb-Douglas β, except if the units of measurement of labor and capital are standardized in such a way that for some specific observation the capital-labor ratio is equal to one. It follows from (2.10) and (2.11) that in such a case

(2.12) $$\eta_K = \nu\beta \qquad \text{and} \qquad \eta_L = \nu\gamma$$

The proper choice of units of measurements can make $\nu\delta$ to represent the capital elasticity of output and $\nu(1 - \delta)$ the labor elasticity. This can be true, however, only for one particular observation.

Finally, two alternative ways of randomization will be assumed

1. For estimates from logarithms (LG)

(2.13) $$U(\varepsilon,t) = \exp(\varepsilon)$$

2. For estimates from rates of growth (RG)

(2.14) $$U(\varepsilon,t) = \exp(\varepsilon t)$$

The combination of five forms of A(t) with four forms of F(K, L) and two ways of randomization gives 40 possible models, not all of which were actually estimated. The label for the whole model will be obtained by combining labels for its parts. For example the label CDCRS-CRTP-RG will mean Cobb-Douglas function with constant returns to scale and constant rate of technical progress, which is estimated in the rates of growth transformation.

The regression equations for the Cobb-Douglas functions are quite obvious. The variants with constant returns to scale were estimated by regressing the labor productivity Y/L on the capital labor ratio K/L and alternative time variables.

The variants with non-constant returns to scale (CDNRS) were estimated mostly by regressing output (Y) on quantities of factors (K, L) and only in few cases by regressing labor productivity (Y/L) on capital output ratio (K/L) and labor (L). Both approaches give identical estimates of parameters, but may give different R^2.

Kmenta's linear approximation of the CES production function were estimated in the logarithmic form only. The regression equation for the model CESCRSRTRTP-LG was

(2.18) $$\lg Y/L = \alpha + \delta \lg K/L + \phi(\lg K/L)^2 + \rho t + (1/2)\lambda t^2 + \mu\Sigma_1{}^t s_\tau + \varepsilon$$

and for the model CESNRS-RTRTP-LG

(2.19) $$\lg Y/L = \alpha+\nu\delta\lg K/L+ \phi(\lg K/L)^2 +(\nu -1)\lg L+\rho t+(1/2)\lambda t^2+\mu\Sigma_1{}^t s_\tau+\varepsilon$$

or

(2.20) $$\lg Y/L = \alpha+\nu\delta\lg K + \phi(\lg K/L)^2+(\nu-1)\lg L+\rho t+(1/2)\lambda t^2+\mu\Sigma_\tau{}^t\ s_\tau+ \varepsilon$$

It is known that (2.18) - (2.20) are approximations, which correspond to some other forms of the production function as well. Kmenta's formula[2] gives good approximation of the CES production function only in the close neighborhood of the Cobb-Douglas function. Then it follows that

(2.21) $$\phi = -(1/2)\,\omega\nu\delta(1-\delta)$$

Having estimated δ, v and ϕ from (2.18), (2.19), or (2.20) we can calculate ω according to

(2.22) $$\omega = -2\,\phi/[\,\nu\delta(1-\delta)\,]$$

The elasticity of substitution can be then obtained as $\sigma = 1/(1+\omega)$. The problem, however, is that ω obtained in this way depends on units of measurement, while the true ω of the CES function does not. The effect of the arbitrary choice of units of measurement can be demonstrated quite easily.[3]

There are two ways, how the problem of units of measurement can be at least partially overcome:

I. Assuming $0 < \delta < 1$ it follows, that whatever the units of measurement are $\delta(1 - \delta) \leq .25$. With regard to (2.22) it must therefore hold

$$\omega = -b(\phi/\nu) \qquad b \geq 8 \tag{2.23}$$

This relation can help in estimating the minimum deviation of the elasticity of substitution from 1.

II. The second way rests on the "standardization" of units of measurement by choosing κ_K and κ_L in such a way that $K^*/L^* = 1$ for certain selected observation. This does not really eliminate the problem, but it provides at least certain common ground for comparison of ω's obtained from different regressions.

Three approaches to the estimation of production functions .

The macroeconomic production functions are frequently estimated from the time-series of aggregated data. Unfortunately, this approach has not worked very well for the East European countries, because the number of observations is still small and multicollinearity high. For these reasons it is virtually impossible to separate the contribution of technical change (time trend) from the contribution of capital and labor.

The attempts to estimate production functions from the cross-sectional data gave frequently much better results. However, if the cross-section data are available for sectors or industrial branches, but not for individual firms, different kind of objections may be raised. It can be argued that fitting the single production function to the observations related to different sectors or branches implies the assumption that all the sectors or branches have identical production function. This is, however, not so. It is possible to assume that each sector or branch has its own individual parameters of the production function, and that only the mean values of these parameters are estimated.

Let us take, for example, the model CDNRS-NTP-LO and suppose that its parameters are generally different in each sector:

$$\lg Y = \alpha_i + \beta_i \lg K + \gamma_i \lg L + e_i \qquad (I = 1, 2, \ldots n) \tag{3.1}$$

The index i here indicates sector. Now, suppose that we have only one observation for each sector. Clearly, in such a case it is impossible to estimate all 3n parameters α_i, β_i and γ_i. It is possible, however, to use the sectoral observations to estimate the mean values α, β and γ.

(3.2) $$\alpha = (1/n)\ \Sigma\ \alpha_i \quad \beta = (\Sigma\beta_i\ lgK_i)/\Sigma\ lgK_i \quad \gamma = (\Sigma\gamma_i\ lgL_i)/\Sigma\ lgL_i$$

by applying the least squares method to the regression equation

(3.3) $$lgY_i = \alpha + \beta lgK_i + \gamma lgL_i + u_i$$

where apparently

(3.4) $$u_i = (\alpha_i - \alpha) + (\beta_i - \beta)lgK_i + (\gamma_i - \gamma)lgL_i + \varepsilon_i$$

If $E(\varepsilon_i) = 0$, then it follows from (3.2) and (3.4) that $E(u_i) = 0$. For a moderately large number of sectors the distribution of u_i may very well be close to normal.

The cross-section approach has one obvious drawback. All the observations relate to the same period of time and, therefore, it is impossible to estimate the change of the total factor productivity. However, if combined with time-series analysis, the cross-sectional estimates may help to improve the reliability of the estimated rates of technical progress. Two such combinations are possible:

1. Parameters of F(K, L) can be estimated first from the cross-section data for some selected year and then inserted into the aggregate production function, so that only parameters of the term A(t) need to be estimated from time-series.[4] This may help to bystep multicollinearity and also to moderately increase the number of observations used in regressions. However, a serious question can be raised: are the parameters estimated from the cross-sectional observations likely to be the same as the "true" parameters of the aggregate production function?
It was shown in (3.2) that the cross-sectional estimates may be interpreted as mean values of individual (sectoral) parameters. The parameters of the aggregate (time-series) production function are also a sort of "mean values", but it follows from (3.1) to (3.4) that α, β, γ would coincide with true parameters of the aggregate production function only if the aggregation was done by summing the logarithms of observed variables. Because this is not the usual way of aggregation, the discussed way of combining cross-section and time-series estimates is inconsistent and should be rejected.

2. The second way of combination is to estimate simultaneously all the parameters, by running regressions on the time-series of cross-sectionally disaggregated data. In this case the full matrix of n x T observations - where n is the number of sectors and T is the number of years - must be available. The advantage of such an approach is obvious: The problem of multi-collinearity is eliminated, the number of observations is considerably increased, and all the parameters are estimated from the disaggregated data so that the inconsistency is avoided. Unfortunately, the usual assumptions about the random disturbance are not valid in this case. The residuals from the regressions using this type of combination approach are almost always time wise auto correlated and cross-sectionally heteroscedastic. This "price" is 'however' not very high, as both autocorrelation and heteroscedasticity are less serious obstacles than multicolinearity.

Let us now formulate more exactly the three possible approaches to the estimation of production functions. Suppose we have all necessary data organized into matrices

Y_{11}	$Y_{12,}$	…	Y_{1n}	K_{11}	$K_{12,}$	…	K_{1n}	L_{11}	$L_{12,}$	…	L_{1n}
Y_{21}	Y_{22}	…	Y_{2n}	K_{21}	K_{22}	…	K_{2n}	L_{21}	L_{22}	…	L_{2n}
…	…	…	…	…	…	…	…	…	…	…	…
Y_{n1}	Y_{n2}	…	Y_{nn}	K_{n1}	K_{n2}	…	K_{nn}	L_{n1}	L_{n2}	…	L_{nn}

Each row of these matrices represents one sector and each column represents one year, so that Y_{it}, K_{it} , and L_{it} are output, capital, and labor in the sector i and the year t respectively.

1. The pure cross-section approach will mean to run separate regressions for each year, i.e. for each column of matrices of observations. For example if the model CDNRS-NTP-LG is to be estimated, then T separate regressions

(3.5 a) $$\lg Y_{it} = \alpha_t + \beta_t \lg K_{it} + \gamma_t \lg L_{it} + \varepsilon_{it}$$

one for each t will be run. It is assumed, that for each t the distribution of ε_{it} is approximately normal and

(3.5b) $$E(\varepsilon_{it}) = 0;\ E(\varepsilon^2_{it}) = \sigma^2_t;\ E(\varepsilon_{it}\,\varepsilon_{jt}) = 0 \quad \text{for } i \neq j.$$

2. The pure time-series approach will mean to run separate regressions for each sector, i.e. for each row of matrices of observations. If the model CDNRSCRTP-LG is to be estimated, n separate regressions

(3.6) $$\lg Y_{it} = \alpha_i + \beta_i \lg K_{it} + \gamma_i \lg L_{it} + \rho_i t + \varepsilon_{it}$$

one for each i will be run. It is assumed, that for each i ε_{it} is approximately normally distributed and

(3.6b) $$E(\varepsilon_{it}) = 0;\ E(\varepsilon^2_{it}) = \sigma^2_i;\ E(\varepsilon_{it}\,\varepsilon_{is}) = 0 \quad \text{for } s \neq t.$$

It is apparent that the pure cross-section approach results in time-series of estimated parameters α_t, β_t and γ_t each of them representing a cross-sectional mean for a given year. The pure time-series approach provides in this case sectoral, rather than aggregate production functions. It will result in n separate sets of estimated parameters α_i, β_i, γ_i and ρ_i which are supposed to be constant over the whole observed period.

Having these results we can check whether the time-series of cross-sectionally estimated parameters of the production function support the assumption of constancy in time. Secondly, we should check, whether the crosssectionally estimated parameters are really mean values of sectoral parameters. Only after such a consistency check we should step forward to the combination.

3. The combination of cross-section and time-series analyses will mean to run one regression over all the observed data. Many variants of the combination approach are possible. In the most simple case only one set of parameters α, β, γ and ρ (model CDNRS-CRTP-LG) is estimated from the regression equation

(3.7a) $$\lg Y_{it} = \alpha + \beta \lg K_{it} + \gamma \lg L_{it} + \rho t + \varepsilon_{it}$$

In this case α, β, γ and ρ are uniform for all sectors and all years. It will be assumed that

(3.7b) $$\varepsilon_{it} = \alpha_i + \varepsilon^*_{it} \quad E(\alpha_i) = 0 \quad E(\varepsilon^*_{it}) = 0 \quad E(\varepsilon_{it}^2) = \sigma_{it}^2$$

The random disturbance is assumed to be cross-sectionally heteroscedastic. It is also likely to be time-wise autocorrelated. In this paper we shall attempt to eliminate the sectoral heteroscedasticity by removing the components α_i from residuals.[5] This will be done by applying the ordinary least squares method with sectoral dummy variables.

DATA

Most of the data needed for estimations were obtained directly from the official statistical yearbooks of individual East European countries. The easiest was to get data for industrial sector in the breakdown to 15-20 industrial branches. Much more difficult and not always successful was our attempt to obtain or reconstruct comparable data for non-industrial sectors, i.e. agriculture, transportation, construction and trade. The length of time-series was in most cases from 1948-1950 to 1967-1968, however, for East Germany and Poland only much shorter time-series were available.

The main difficulty which we faced in the process of data preparation were related to capital stock data. For some countries the capital stock had to be partially or fully constructed from the gross investments without knowing the actual replacement ratio, and in the case of Rumania, even without knowing the initial stocks of capital in individual sectors. In the case of Czechoslovakia and Poland the capital stock was also adjusted for the degree of utilization.

The detailed description of data is given in the Appendix, at this place it will be sufficient to give only the following brief characterization:

1. For Y_{it} either gross value of output (GVO) in East European definition or GNP in Western definition was alternatively used.
2. Labor L_{it} was expressed either in the average number of persons employed or in the number of manhours.
3. For K_{it} the average stock of fixed capital, i.e. buildings, constructions, equipment

and machines was used.

The gross value of output (GVO) was used, because for most countries no other data on output were available in the breakdown to industrial branches.

The sole exception was Czechoslovakia for which also Western[6] estimates of GNP were available and used. The frequent criticism of the use of GVO data may very well be correct for aggregate time-series but not necessarily for the disaggregated approach followed in this paper. One advantage of GVO data is that they are truly independent observations. Unfortunately this is not true for some Western estimates of GNP for East European countries, which are "cross-sectionally" constructed from the "right-hand" variables K and L by summing up wages and some "reasonable" profit per capital. It is obvious that construction of "left-hand" variable from the "right hand" variables violates the elementary requirements of regression analysis.

In most cases, the only data available in the breakdown to sectors and branches of industry were those related to either the state or the socialist sectors. The socialist sector includes both state owned and cooperative establishments. It was impossible to get the data on output, labor and capital for the private sector in the needed breakdown. The exclusion of the private sector is not a serious limitation for recent years because its share in all concerned East European countries (except for Polish agriculture) became negligible. It is, however much less satisfactory for the early post-war years.

The obvious disadvantage of the "gross value of output" is, that it makes the estimated parameters less comparable with the similar estimates for Western countries which are based on GNP.

The "gross value of output" is likely to give relatively higher rates of technical change. This is an outcome of the fact that the indicator of the gross value of output has usually grown faster than GNP. The East European definition of output includes only the so called "productive" sectors and activities, which happened to have grown faster than the "non-productive" sphere. In addition to that, the gross value of output includes the intermediate product, which has also frequently grown faster than the net product.

Very serious problems are related to the prices, in which the official data are valued. Many East European economists as well as Western scholars conclude, that the operation of the Soviet type economy leads to the "inflation of constant prices" and to the distortion of relative prices. The empirical evidence is sufficient enough to support this conclusion[7] but unfortunately not sufficient to estimate how much the constant prices were inflated. Both, the "inflation of constant prices", and the distortion of relative prices may result in biased estimates of the parameters of macroeconomic production functions. In the pure time-series analysis, the inflation of constant prices is likely to "inflate" the estimated rates of growth of the total factor productivity. In the pure cross-section analysis, the distortion of relative prices may lead to the biased estimates of capital and labor elasticities. In the combination of cross-section and time-series analyses both effects may appear. The "inflation of constant prices" is hard to eliminate, but the "relative prices" effect may be at least partially eliminated if sectoral dummy variables are introduced into the regression equations.

Another problem is related to the measurement of inputs. Ideally one should use quantities of labor and capital adjusted for the degree of utilisation. Due to the employment planning and to the rigid rules of labor allocation a certain amount of unutilised labor exists in many sector and industrial branches.

Unfortunately no reliable data about the degree of utilisation of labor are available. The best we can do, is to use the manhours worked, where possible, believing that the changes in the average duration of the working day reflect at least partially the variations in the utilization of labor. Capital was adjusted for the degree of utilization with the use of the shift coefficient and productive consumption of the electric power. The detailed description of the method used is given elsewhere.

RESULTS

Earlier, we have described 40 models of production functions, which were estimated by three different approaches for six East European countries and sometimes even with alternative data specifications. This required to run thousands of regressions, only fraction of which is reported in the Appendix.

Pure cross-section analysis

Cross-sectional estimates of the macroeconomic production functions were obtained for Czechoslovakia, Poland, Hungary, Bulgaria, and Rumania. The estimates of capital elasticities of output β for Czechoslovakia, Poland, and Hungary are very similar - around .25 - .3 - and quite close to the theoretically expected values. Almost all of these estimates are significantly different from 0 on the 2 % level of significance. The estimates of β for Bulgaria and Rumania are considerably smaller (.05-.15), and they are not significantly different from 0. It is not clear whether this difference in estimated parameters reflects true differences in capital elasticities of output or simply the fact that our capital data for Bulgaria and Rumania contained relatively much larger errors of measurement than the capital data for the other countries.

The estimated β fluctuated slightly in short run. In the long run we can see slightly increasing trend in Hungary and Bulgaria but no visible trend in Czechoslovakia, Poland and Rumania. Generally, the assumption of constant β seems quite reasonable. It implies that we may represent estimates of β for individual years by their overall arithmetic mean.

Except for Rumania, only small and statistically insignificant deviations from constant returns to scale were found. The assumption of constant returns to scale is apparently quite reasonable for these countries.

Mean Values of Factor elasticities
(obtained from cross-sectional estimates)

	Model CDCRS		Model CDNRS		
Country	β	γ	β	γ	ν
Czechoslovak economy	.258	.742	.227	.640	.867
Czechoslovak industry	.267	.733	.270	.811	1.081
Polish industry	.265	.735	.279	.811	1.090
Hungarian industry	.294	.706	.298	.691	.989
Bulgarian economy	.095	.905	.093	.894	.987
Rumanian economy	.115	.885	.092	.554	.462

The estimated constants α_t = lgA(t) show in all cases the clear upward trend. The cross-section estimates will be, therefore, compatible with the time-series estimates containing technical progress. Assuming (2.1) it is possible to get rough estimates of the rates of technical progress (total factor productivity) by regressing the time-series of the constants α_t obtained from cross-sectional estimates on the time variable t.

Average rates of technical change
(obtained from constants of cross-section regressions)

Model CDCRS			
Country	ρ	stand. error of ρ	R^2
Czechoslovak economy	.0496	.0021	.9862
Czechoslovak industry	.0503	.0019	.989
Polish industry	.0409	.0040	.9555
Hungarian economy	.0403	.0018	.9830
Bulgarian economy	.0787	.0040	.9783
Rumanian industry	.0607	.0027	.9823

Table 2. shows very high rates of technical change in postwar Eastern Europe. With the exception of extremely high rates for Bulgaria these results roughly correspond to the estimates obtained in both pure time-series and combination of time-series and cross-section approaches.

Note: in the following tables stars indicate levels of significance
* is 10% level, ** is 2% level, *** is 1% level

Table 1. Cross-Section Analysis Model CDCRS

Czechoslovak economy variables (Y2, K2, L2)			
Year	α	β	R^2
1950	-1.34***	.27**	.293
1955	-1.02***	.24**	.229
1960	-.74***	.25**	.232
1965	- .62***	.28**	.252

Czechoslovak industry variables (Y2, K2, L2)			
Year	α	β	R^2
1950	-1.25***	.33**	.472
1955	- .97***	.24**	.339
1960	- .66***	.26**	.384
1965	- .52***	.28**	.405

Polish industry variables (Y1, K2, L3)			
Year	α	β	R^2
1950	-	-	-
1955	3.61***	.26*	.242
1960	3.80***	.28**	.294
1965	4.05***	.27**	.314

Hungarian industry variables (Y1, K2, L1)			
Year	α	β	R^2
1950	1.62***	.23*	.202
1955	1.36***	.28*	.259
1960	1.16***	.30**	.350
1965	.99***	.31**	.378

Bulgarian economy variables (Y2, K2, L2)			
Year	α	β	R^2
1950	3.12***	.11	.036
1955	2.86***	.07	.031
1960	2.40***	.13	.118
1965	2.04***	.15*	.180

Rumanian industry variables (Y2, K3, L2)			
Year	α	β	R^2
1950	4.58***	.04	.003
1955	4.73***	.12	.034
1960	5.04***	.11	.039
1965	5.43***	.07	.016

Table 2. Cross-Section Analysis Model CDNRS

Czechoslovak economy variables (Y2, K2, L2)			
Year	β	ν – 1	R^2
1950	.24	-.10	.357
1955	.19	-.15	.349
1960	.23	-.12	.295
1965	.27	-.15	.334

Czechoslovak industry variables (Y2, K2, L2)			
Year	β	ν – 1	R^2
1950	.33	.07	.502
1955	.25	.07	.356
1960	.26	.11	.430
1965	.28	.11	.448

Polish industry variables (Y1, K2, L3)			
Year	β	γ	R^2
1950	-	-	-
1955	.28	.83	.819
1960	.29	,80	.848
1965	.28	.81	.869

Hungarian industry variables (Y1, K2, L1)			
Year	β	γ	R^2
1950	,24	.72	.788
1955	.28	.71	.845
1960	.30	.68	.855
1965	.33	.69	.854

Bulgarian economy variables (Y2, K2, L2)			
Year	β	γ	R^2
1950	.10	.89	.839
1955	.06	.90	.866
1960	.12	.86	.868
1965	.16	.88	.875

The cross-sectional estimates of the Kmenta's linear approximation of the CES production function for the Czechoslovak economy and industry are shown in Table 3.

Table 3. Czechoslovakia Cross-Section CES. Variables (Y2, K2, L2)

Model CESCRS.

Year	α	δ	$\delta*$	ϕ	σ	R^2
1950	-1.23***	.44**	.48	.049	1.65	.503
1955	-.97***	.28*	.29	.020	1.24	.337
1960	-.68***	.29**	.31	.027	1.34	.396
1965	-.55***	.29**	.32	.028	1.36	.416

Model CESNRS

Year	α	$\nu\delta$	$\nu - 1$	$\delta*$	ϕ	σ	R^2
1950	-2.20**	.48**	.10	.50	.068	1.97	.544
1955	-1.71	.29*	.08	.29	.028	1.34	.366
1960	-1.91	.31**	.13	.30	.042	1.54	.456
1965	-1.89	.31**	.14	.31	.048	1.64	.475

The coefficient $\delta*$ was calculated from the estimates of δ by standardizing the units of measurement so that the capital-output ratio in machine building (sector 4) would be equal to 1 in all years. The coefficient $\delta*$ is thus slightly smaller - by less than $\phi/2$ - than the capital elasticity corresponding to the average capital-labor ratio for the whole economy. The comparison of $\delta*$ with estimates of β in the CD approach show that the assumption of the non-unitary elasticity of substitution does not make any difference for the estimated capital elasticity of output. Unlike the CD approach, we see here quite clear trends in $\delta*$: increasing for the whole economy and declining for industry.

The addition of the CES term to the regression equation made almost no difference for the estimated returns to scale. The estimated elasticity of substitution was generally very high, nevertheless not significantly different from 1.

Pure Time-Series Analysis

The pure time-series approach was used to estimate sectoral production functions for Czechoslovakia, Poland and Bulgaria. Very many models were actually run for each country, however, space allows only for summarizing the results in the tables 3 (a,b,c)

Table 3a. Czechoslovakia

Mean values of Coefficients of Sectoral Production Functions

(obtained from time-series analyses)

Model	Data	β	γ	ν	ρ	λ	μ
CDCRS-CRTP-RG	GVO	.222	.778	1	6.634		
CDCRS-CRTP-RG	GNP	.250	.750	1	4.894		
CDCRS-TRTP-RG	GVO	.384	.616	1	11.282	-.429	
CDCRS-TRTP-RG	GNP	.241	.759	1	6.677	.164	
CDCRS-RRTP-RG	GVO	.224	.776	1	8.563		-4.978
CDCRS-RRTP-RG	GNP	.244	.756	1	6.266		-4.157
CDCRS-RTRTP-RG	GVO	.352	.648	1	11.688	-.340	-4.033
CDCRS-RTRTP-RG	GNP	.341	.659	1	6.896	.061	-3.852
CDCRS-RTRP-LG	GVO	.173	.827	1	.149	-.002	- .046
CDCRS-RTRP-LG	GNP	.222	.778	1	.058	- .000	- .039
CDNRS-RTRP-RG	GVO	.198	.650	.848	12.437	- .332	- .4001
CDNRS-RTRP-RG	GNP	.303	.796	1.099	8.134	-.116	-3.563
CDNRS-RTRP-LG	GVO	.759	.707	1.466	.080	- .002	- .062
CDNRS-RTRP-LG	GNP	.532	.771	1.303	.058	- .001	- .026

Table 3b. POLAND

Mean values of Coefficients of Sectoral Production Functions

(obtained from time-series analyses)

Model	Data	β	γ	ν	ρ	λ
CDCRS-CRTP-RG	GVO	.469	.531	1	4.111	
CDCRS-CRTP-LG	GVO	.511	.489	1	.039	
CDCRS-TRTP-RG	GVO	.471	.529	1	6.984	-.375
CDCRS-TRTP-LG	GVO	.459	.541	1	.062	-.004
CDNRS-TRTP-RG	GVO	-.039	.251	.212	9.780	-.199
CDNRS-TRTP-LG	GVO	.061	.232	.293	.124	-.002

Table 3c. BULGARIA

Mean values of Coefficients of Sectoral Production Functions

(obtained from time-series analyses)

Model	Data	β	γ	ν	ρ	λ
CDCRS-CRTP-RG	GV O	.277	.723	1	4.111	
CDCRS-CRTP-LG	GVO	.155	.845	1	.039	
CDCRS-TRTP-RG	GVO	.332	.668	1	10.754	-.376
CDCRS-TRTP-LG	GVO	.323	.677	1	.088	-.002
CDNRS-TRTP-RG	GVO	.256	.625	.881	14.447	-.451
CDNRS-TRTP-LG	GVO	.43	.563	1.000	.110	-.003

Table 4. Czechoslovakia

Time-Series Analysis Model CDCRS-RTRTP-RG

Number of Years 19 (1949–1967) Variables: Y2, K4, L2

	Sector	β	γ	ρ	λ	μ	R^2	D-W
1.	Electric power	.280**	.720	5.656***	.174	5.393***	.6840	1.7501
2.	Fuels	.642***	.358	4.600***	- .019	-4.060**	.7935	.2224
3.	Metallurgy	.236	.764	8.437***	- .208	-3.034	.1495	1.5414
4.	Machine building	- .201	1.201	14.995***	- .312	-5.102**	.5203	1.7168
5.	Chemicals	.090	.910	12.883***	- .186	-3.648**	.3946	2.4951
6.	Building materials	.061	.939	14.941***	- .329	-4.353	.2989	2.5850
7.	Lumber, woodwork	.090	.909	12.764***	- .446***	- 2.506	.5678	1.2094
8.	Paper	.132	.868	5.639***	- .036	-2.748	.1586	1.7215
9.	Glass and ceramics	.350*	.650	8.856***	- .211	-3.738*	.3920	2.0727
10.	Textile industry	.725***	.275	9.819***	-.369**	-3.044*	.6885	2.41 14
11.	Clothing	.207	.793	14.203***	-.435*	8.433***	.5596	1.7885
12.	Leather and shoes	.534***	.466	8.153***	..270*	-2.054	.6634	2.5521
13.	Printing	.099	.901	6.257**	-.005	-1.727	.0549	2.0581
14.	Food processing	.377	.623	11.852***	-.513***	-2.239	.6489	1.7562
15.	Other branches	1.660*	-.660	36.258**	-1.937	-8.215	.2773	2.2847
	Mean	.352	.64.8	11.688	-.340	-4.033		

Table 5. Czechoslovakia
Time-Series Analysis Model CDCRS-RTRTP-RG

Number of years 19 (1949—1967) variables: Y3, K5, L2

	Sector	β	γ	ρ	λ	μ	R^2	D-W
1.	Electric power	.307***	-.693	4.886***	.1232	-4.028***	.6993	2.4108
2.	Fuels	.273	.737	5.203***	.100	-5.768**	.6040	.2.6221
3.	Metallurgy	.392*	.608	6.100**	-.284	-1.369	.2651	2.2286
4.	Machine building	1.360	-.360	1.214	.004	-5.876	.2668	2.2221
5.	Chemicals	.657	.344	5.982	- .050	-4.651	.2504	2.6494
6.	Building materials	.844	.156	11.890	.381	1.460	.1388	2.7330
7.	Lumber,woodwork	.145	.855	6.896	.232	-1.942	.0226	1.9608
8.	Paper	-.117	1.117	1.827	.177	-1.083	.117	1.6310
9.	Glass and ceramics	.518*	.482	5.338*	- .124	-4.992*	.3772	2.9134
10.	Textile industry	.379	.621	6.480***	-.175	-5.742***	.5326	1.2122
11.	Clothing	-.000	1.000	24.490***	-1.143**	-8.014	.4426	1.0647
12.	Leather and shoes	-.048	1.048	1.509	.130	-.947	.0544	1.9912
13.	Printing	.063	.937	5799*	.019	-5.993*	.2395	1.2227
14.	Food processing	.001	.999	8.927***	- .242	-4.986**	.4657	1.7142
	Mean	.341	.659	6.896	.061	-3.852		

The most obvious conclusion from surveying all time-series regressions was that very few of them gave statistically good and economically meaningful results. Either R^2 were very low, or the parameters were not significant, or they did not have meaningful values. Parameters were also extremely sensitive to the form of model and type of data used. The addition of new variable, quite frequently caused a drastic change in the estimates of coefficients at other variables.

It is, however, interesting to note, that the whole set of all time-series estimates gives much more reasonable picture than individual estimates alone. We can draw from them several interesting conclusions.

(1) The mean values of estimated coefficients calculated from sectoral coefficients[8] of each model (Table 3.) show surprising similarity and with only few exceptions are economically meaningful and consistent with results of cross-sectional analysis. The coefficients in Tables 1, 2 and 3 do not disprove the hypothesis that the cross-sectional estimates are mean values of sectoral parameters.

The mutual compatibility of the cross-sectional and the time-series estimates is obvious at least for Czechoslovakia and production function with constant returns to scale.

The time-series estimates of models with CRS for Poland give the mean value of β somewhat larger than cross-sectional estimates but the estimate of the average rate of technical change is almost exactly the same as the rate implied by the cross-sectional estimates. The results of models with NRS for Poland are bad partly because 13 observations do not provide enough degrees of freedom for reliable estimation of 5 parameters.

The mean values of β obtained for Bulgaria were - with two exceptions - between .25 and .35. This would indicate a normal value of capital elasticity rather than low values suggested by the cross-sectional approach. The average rate of technical change is on the other hand somewhat lower - around 6 % instead of almost 8 % implied by the cross-sectional estimates.

(2) The use of GNP instead of GVO as a left hand variable does not give radically different estimates of capital and labor elasticities of output, however, it gives lower - although not much lower - estimates of the rate of technical change. The rates of technical change are respectable even if measured in terms of Western estimates of GNP.

(3) The estimates of the models with TRTP clearly indicate the existence of declining trends in the rate of technical change in all three countries. Not only the mean values of λ as shown in Table 3. but also the individual values of those parameters for almost all sectors are negative.

The magnitude of the declining trend varied, but most frequently it was between .1 and .5 % annually which implies that the rate of technical change in 20 post war years may have slowed down by approximately 2 to 5 %. It should be noted, that the declining trend is much less apparent if the output is measured by GNP.

(4) Models with the "recession parameter" μ estimated for Czechoslovakia, show very clearly the cyclical fluctuations in the rates of technical change. On the basis of preliminary information the years 1953-1954, and 1962-1965 were selected as recession years. The estimates of models with dummies for recession years show that practically in all sectors the rate of technical change was in recession years lower than in the non-recession years by about 4 %.

(5) Values of estimated parameters obtained from rates of growth are approximately the same as those obtained from logarithms of observed data.

In RG models the R^2 s, were much smaller but the Durbin-Watson statistics were better and standard errors of estimated parameters were almost the same as in the LG models. Models with variable rate of technical change give generally better results than those with constant ρ. On the other hand the results were more economically meaningful when the returns to scale were constrained to unity.

Combination of Cross-Section and Time-Series Analyses

The combination approach was applied in two steps:

(1) In the first step the ordinary least squares method was applied to the pooled cross-section and time-series data

(2) In the second step the regressions were repeated with sectoral dummy variables.

Several common features are visible in the estimates from the pooled data[9]

Table 6. The Combination of Cross-Section and Time-Series Analyses

Country (data)	Number of sectors	years / observations	α	β	ρ	R^2	D-W	SER
			Model CDCRS-CRTP-LG					
Czechoslovakia	14	20	-1.9061***	.4676***	.0310***	.8751	.2732	.2113
Y3, K5, L2		280	(.0276)	(.0123)	(.0022)			
Czechoslovakia	15	20	-1.3261***	.2927***	.0547***	.0694	.1399	.3395
Y2, K1, L1		300	(.0500)	(.0200)	(.0035)			
Czechoslovakia	15	20	-1.6658***	.1713***	.0610***	.5784	.1879	.3825
Y2, K5, L2		300	(.0464)	(.0170)	(.0039)			
Czechoslovakia	20	20	-1.4310***	.2643***	.0533***	.4822	.1070	.4715
Yl, K1, L1		400	(.0624)	(.0247)	(.0042)			
Poland	17	9	6.1921***	.2180***	.0382***	.2864	.0120	.3811
Y2, K1, L1		153	(.1667)	(.0323)	(.0120)			
Poland	15	13	3.5827***	.2664***	.0476***	.4050	.2022	.4144
Y2, K2, L3		195	(.1615)	(.0303)	(.0080)			
Poland	15	13	3.0441***	.2395***	.0473***	.3812	.2027	.4072
Yl, K4, L2		195	(.1507)	(.0301)	(.0079)			
Hungary	15	19	-1.5372***	.2963***	.0325***	.4421	.1975	.3968
Y1, K1, L1		285	(.0747)	(.0253)	(.0043)			
Bulgaria	19	21	-3.0867***	.1413***	.0541***	.3678	.2151	.5224
Y2, K2, L2		399	(.1050)	(.0237)	(.0044)			
Rumania	16	20	-4.3776***	.I217***	.0588***	.2208	.1278	.7423
Y2, K3, L2		320	(.1081)	(.0345)	(.0073)			

Table 7. The Combination of Cross-Section and Time-Series Analyses

Country (data)	Sec.	years Obs.	Model CDCRS-TRTP-LG α	β	ρ	λ	R^2	D-W	SER
Czechoslovakia	14	20	-1.9463***	.4675***	.0419***	-.0010	.8758	.2736	.2111
Y3, K5, L2		280	(.0429)	(.0123)	(.0092)	(.0009)			
Czechoslovakia	15	20	-1.8215***	.1711***	.1035***	-.0040***	.5888	.1904	.3784
Y2, K5, L2		300	(.0731)	(.0168)	(.0159)	(.0015)			
Poland	17	9	6.1636***	.2181***	.0535	-.0031	.2868	.0116	.3822
Y2, K1, L1		153	(.1941)	(.0324)	(.0542)	(.0105)			
Poland	15	13	2.9964***	.2391***	.0669**	-.0028	.3823	.2022	.4079
Yl, K4, L2		195	(.1708)	(.0302)	(.0339)	(.0047)			
Poland	15	13	35079***	.2663***	.0777**	-.0043	.4075	.2017	.4146
Yl, K2, L3		195	(.1816)	(.0303)	(.0345)	(.0048)			
Hungary	15	19	-1.5063***	.2954***	.0231	.0009	.4426	.1983	.3973
Yl, K1, L1		285	(.0946)	(.0254)	(.0181)	(.0018)			
Bulgaria	19	21	-3.1516***	.1417***	.0714***	-.0016	.3693	.2115	.5225
Y2, K2, L2		399	(.1240)	(.0237)	(.0181)	(.0016)			
Rumania	16	20	4.3976***	.1215***	.0535*	.0005	.2209	.1279	.7434
Y2, K3, L2		320	(.1546)	(.0345)	(.0303)	(.0028)			

Table 8. The Combination of Cross-Section and Time-Series Analyses

Country (data)	Sec	years obs.	α	β	ρ	μ	R^2	D-W	SER
Model CDCRS-RRTP-LG									
Czechoslovakia Y3, K5, L2	14	20	-1.9749***	.4675***	.0437***	-.0391**	.8773	.2699	.2098
		280	(.0333)	(.0122)	(.0062)	(.0177)			
Czechoslovakia Y2, K5, L2	15	20	-1.7353***	.1713	.0822***	-.0650**	.5846	.1874	.3803
		300	(.0568)	(.0169)	(.0108)	(.0310)			

Table 9. The Combination of Cross-Section and Time-Series Analyses

Model CDCRS-CRTP-RG

Country data	sec.	obs.	years	α	ρ	R^2	D-W	SER
Czechoslovakia	15		19	.3749***	6.0385***	.0500	1.5180	7.7369
Y2, K5, L2		285		(.0972)	(.5925)			
Czechoslovakia	17		17	.2378***	5.6976***	.0770	1.3838	4.6931
Y2, K5, LI		289		(.0486)	(.3744)			
Poland	15		12	.3645***	4.4768***	.1853	1.2934	3.7831
Yl, K4, L2		180		(.0573)	(.3807)			
Hungary	15		18	.0795*	4.3493***	.0114	2.3266	7.8871
Yl, KI, LI		270		(.0452)	(.4965)			
Bulgaria	15		20	-.0189	8.4311***	.0028	1.7393	10.9964
Y2, K2, L2		300		(.0209)	(.6547)	,		

Model CDCRS-TRTP-RG

Country data	sec.	obs.	years	α	ρ	λ	R^2	D-W	SER
Czechoslovakia	15		19	.3821***	10.0912***	- .3716***	.1156	1.5291	7.4779
Y2, K5, L2		285		(.0939)	(1.0546)	(.0812)			
Czechoslovakia	17		17	.2562***	10.0420***	- .4037***	.2416	1.6456	4.2616
Y2, K2, Li		289		(.0442)	(.6479)	(.0512)			
Poland	15		12	.3408***	6.5345***	-.2603***	.2310	1.3436	3.6859
Yl, K4, L2		180		(.0563)	(.7350)				
Hungary	15		18	.0816*	4.6350***	- .0277	.0117	2.3156	7.9005
Yl, KI, LI		270		(.0458)	(1 .0856)	(.0937)			
Bulgaria	15		20	-.0207	11.4374***	-.2616**	.0216	1.6110	10.9103
Y2, K2, L2		300		(.0207)	(1.4151)	(.1094)			
Rumania	16		19	.0932***	6.4822***	-.0311	.0488	1.3032	6.2171
Yl, K3, L2		304		(.0496)	(.8012)	(.0675)			

Table 10. The Combination of Cross-Section and Time-Series Analyses

Model CDCRS-RRTP-RG

Country data	sec.	obs.	years	β	ρ	μ	R^2	D-W	SER
Czechoslovakia	15		19	.3880***	7.5898***	-5.0170***	.1373	1.5084	7.385
Y2, K3, L2		285		(.0928)	(.6357)	(.9387)			
Czechoslovakia	17		17	.2219***	73934***	-4.5735***	.2783	1.6419	4.157
Y2, K2, L1		289		(.0431)	(.3823)	(.5122)			
Czechoslovakia	17		17	.1965***	7.1949***	-4.4442***	.2791	1.6443	4.154
Y2, K3, L1		289		(.0379)	(.4055)	(.5135)			

Model CDCRS-RTRTP-RG

Country data	sec.	obs.	years	β	ρ	λ	μ	R^2	D-W	SER
Czechoslovakia	15		19	.3910***	I0.3048***	-.2.740	-4.1304	.1703	1.5183	7.255
Y2, K3, L2		285		(.0912)	(1.0245)	(.8197)	(.9596)			
Czechoslovakia	17		17	.2385***	10.1314***	-.2878***	-3.60I6***	.3528	1.7859	3.943
Y2, K2, L1		289		(.0410)	(.5997)	(.05 02)	(.5146)			
Czechoslovakia	17		17	.2039***	9.8702***	-.2778***	-35049***	.3489	1.7777	3.955
Y2, K3, L1		289		(.0361)	(.6193)	(.0503)	(.5176)			

Model CDNRS-RRTP-RG

Country data	sec.	obs.	years	β	γ	ρ	μ	R^2	D-W	SER
Czechoslovakia	17		17	.1693***	.8026***	7.8228***	-4.7265***	.5610	1.5776	4.209
Yl, K2, L1		289		(.0526)	(.0594)	(.4660)	(.5262)			
Czechoslovakia	17		17	.1525***	.8074***	7.6673***	-4.6366***	.5630	1.5778	4.199
Yl, K3, L1		289		(.0446)	(.0587)	(.4843)	(.5282)			

Model CDNRS-RTRTP-RG

Country data	sec.	obs.	years	β	γ	ρ	λ	μ	R^2	D-W	SER
Czechoslovakia	17		17	.1918***	.7872***	10.5242***	-2.884	-3.7428	.6055	1.7182	3.997
Yl, K2, L1		289		(.0501)	(.0565)	(.6509)	(.0510)	(.5290)			
Czechoslovakia	17		17	.1611***	.7964***	10.3601***	-2.800	-3.6917	.6051	1.7054	3.999
Yl, K3, L1		289		(.0425)	(.0559)	(.6722)	(.0509)	(.5315)			

(1) The first step estimates (without sectoral dummy variables) may give low R^2s, very low Durbin -Watson statistics[10] but they give usually highly significant estimates of the regression coefficients. The introduction of sectoral dummy variables increased considerably R^2's and usually also diminished standard errors of estimated coefficients, but the serial correlation has not been removed. The check on sectoral sums of squared residuals indicated also presence of heteroscedasticity.

(2)The most striking feature is the relative stability of estimates from the pooled cross-section and time-series data. Particularly it is evident, that the values of estimated coefficients are not sensitive to the form of the technical change term A(t).

(3)The third interesting feature is the fact that almost all the estimated coefficients have economically meaningful values. Let us take for example the estimates of β. Out of 95 separate estimates of β, only two were negative, two larger than 1 and 85 were in the interval from .1 to .5, (57 of them in the narrower range .15 -. 3). Similarly we can see that the average rate of technical change is almost never unrealistically high or low, the occurrence quite frequent in the pure time-series analysis. Out of 45 separate estimates of ρ no one was negative, only two were smaller than 1 %, one was larger than 8 % but 42 of them were in the interval from 3 to 7 %.

The estimates of production functions obtained from the pooled cross-section and time-series data seem to confirm most of the conclusions which were made previously on the basis of pure time-series and cross-section estimates.

The capital elasticity of output in all six East European countries is most likely to be somewhere between .15 and .3. Even though some models gave different estimates of β, there is no unambiguous evidence that the capital elasticity of any East European country would be out of this range.

The estimates of models where the returns to scale parameter ν was not constrained to unity give very ambiguous results. Although the deviation of ν from unity was in many cases statistically significant, it was frequently not very large. Only some models with sectoral dummy variables resulted in more substantial deviations from constant returns to scale. But taking all the estimations together, no clear evidence for either increasing or decreasing returns to scale can be found for any country.

The estimates of the Kmenta's approximation of the CES production function give also ambiguous results. The estimated ϕ for Poland was not significant and in some cases positive and in other negative. The estimates of ϕ for Czechoslovakia were significant but also alternating signs. Estimates for Bulgaria indicate the elasticity of substitution to be significantly larger than one and those for East Germany significantly smaller than one. Therefore our estimates do not confirm the hypothesis that the Soviet-type economic system creates a consistent deviation of the elasticity of substitution from unity.

There are noticeable similarities in the estimated rates of technical change in the East European countries. Almost all the estimates of the average rate of technical change ρ for Czechoslovakia (GVO), Bulgaria and Romania are between 5 and 6 %. East Germany gives somewhat broader range 3 to 7 %. The average rate of technical change for Poland and Hungary was estimated in the range 3 to 5 %. The estimates of ρ from GNP data for Czechoslovakia gave - as expected - a slower rate of technical change, only around 3 %. This difference is caused partly by the fact that GNP does not grow as fast as GVO and partly by the very high capital elasticity β in the GNP estimations. It is also likely that some portion of the discrepancy between GVO and GNP estimates of ρ may have been due to the "inflation of constant prices" as was suggested earlier.

The models with variable rate of technical change gave radically different results for two groups of countries. The first group, which consists of Czechoslovakia (GVO), East Germany, Poland and Bulgaria, exhibits a clear and sizable declining trend of the rates of technical change. With few exceptions the estimates of the parameter λ indicated a .2 to .4 % annual decline of the rate of technical change in the mentioned countries. Such a trend was not found in Hungary and Romania. The estimates of λ on the basis of GNP data are more ambiguous. Sometimes they show the declining trend, but of much smaller size than the estimates from GVO data suggest.

Finally, the estimations of the production functions with the "recession parameter" μ confirm our previous finding, i.e. the sizable and statistically significant fluctuations of the rate of technical change in Czechoslovakia.

CONCLUSIONS

Four questions were formulated at the end of the introduction to this paper. The analysis of the estimations of many variants of production functions for Czechoslovakia, Poland, East Germany, Hungary, Bulgaria and Romania suggests the following answers to those questions;

(1) The average rates of technical change in the postwar period when measured in terms of the official gross value of output were quite high, however, at least in four out of six countries, i.e. in Czechoslovakia, Poland, Germany and Bulgaria the rates of technical change were clearly declining, so that at the end of the sixties they were only moderate. When measured in terms of GNP, the rates of technical change were smaller but still respectable. On the other hand GNP does not show such a deterioration of the economic performance as GVO does.

(2) No overwhelming evidence about any unusual values of capital and labor elasticities was found. Actually, most of the estimates gave standard values of capital elasticity of output around .2-.3 and labor elasticity of output around .6-.8.

(3) Similarly, no clear evidence of either economies or diseconomies of scale was found. The returns to scale parameter was sometimes above, sometimes below but very frequently quite close to 1.

(4) Finally, the same conclusions can be made about the elasticity of substitution which was sometimes estimated larger than 1 sometimes smaller than 1.

The Cobb-Douglas production function with constant returns to scale is probably still the best description of the Soviet-type economies.

APENDIX

Description of Data

Czechoslovakia

Y1 Gross value of output in 1955 prices

Y2 Revised Gross value of output in 1955 prices

Y3 GNP in 1956 prices, calculated by Dr. Alton

K1 Total stock of fixed capital (productive basic funds) in constant prices of 1955

K2 Total stock of fixed capital adjusted for the degree of utilization

K3 Stock of machines and equipment adjusted for the degree of utilization

K4 Revised data on the total stock of fixed capital in 1967 prices, adjusted for degree of utilization

K5 Revised data on the stock of machines and equipment in 1967 prices, adjusted for the degree of utilization

L1Man-hours worked by workers in the "basic industrial activities"

L2 Revised series of the man-hours worked

Poland

Y1 Gross value of output in 1960 prices

Y2 Revised series of gross value of output

K1 Official data on total stock of fixed capital in 1960 prices, for years 1960-1968

K2 Total stock of fixed capital for the years 1956-1968. The data for 1956-1959 reconstructed from investment

K3 Total stock of capital adjusted for the degree of utilization

L1 Total number of employees

L2 Man-hours worked by workers in the industrial activities

L3 Number of employees in the industrial activities

Hungary

Y1 Gross value of output

K1 Stock of fixed capital reconstructed from investment series

K2 Revised series of the stock of fixed capital

L1 Number of workers

Bulgaria

Y1 Gross value of output 1952-1967

Y2 Revised series of gross value of output 1948-1968

K1 Total stock of fixed capital 1952-1967

K2 Revised series of capital stock 1948-1968

L1 Number of employees 1952-1967

L2 Revised series of number of employees 1948-1968

Romania

Y1 Official indices of the Gross value of output

Y2 Reconstructed Gross value of output

K4 Total stock of fixed capital constructed from investment

K2 Revised capital stock series

K3 Second revision of capital stock series

L1 Number of employees

L2 Number of blue collar workers

East Germany

Y1 Gross value of output

K1 Total stock of fixed capital

L1 Number of manhours worked

FOOTNOTES

1. Most of the research for this paper was done during the stay at the University of California at Berkeley and was supported from its Center for Slavic studies. The stimulating environment and facilities of the Russian Research Center of Harvard University and a grant from the Graduate school of Boston University were very helpful in finishing the manuscript. We are particularly grateful to professors Gregory Grossman, Thomas Marschak, Benjamin Ward, Thomas Rothenberg, Paul Zarembka, Jan Kmenta, Arnold Zellner, Zvi Griliches, Ray Jackson, Jon Hughes and Phillip Swan and Mr. E. Gendel who helped us by their comments at various stages of the research.

2. See Kmenta (5) pp. 462 – 463. Kmenta obtained his formula by taking the Taylor's expansion of

$$\lg F(K/L,1) = -(1/\omega)\lg[\delta (K/L)^{-\omega} + 1 - \delta]$$

3.Suppose, that each variable is expressed in two alternative units of measurement, so that we have two sets of observed variables Y, K, L and Y*, K*, L*, related by the relation

(2.23) $$Y = \kappa_Y Y^*,\ \ K = \kappa_K K^*,\ \ L = \kappa_L L^*$$

Substituting (2.23) in (2.18) we get

(2.24) $$\lg(Y^*/L^*) + \lg(\kappa_Y/\kappa_L) = \alpha + \delta[\lg(\kappa_K/\kappa_L) + \lg(K^*/L^*)] + \varphi\{[\lg(\kappa_K/\kappa_L)]^2 + 2\lg(\kappa_K/\kappa_L)\lg(K^*/L^*) + [\lg(K^*/L^*)^2\} + \rho t + (1/2)\lambda t^2 + \mu\,\Sigma^{t}_{\tau=1} s_\tau + \varepsilon$$

And after simple rearrangements we find

(2.25a) $$\alpha^* = \alpha - \lg\kappa_Y + \delta\lg\kappa_K + (1-\delta)\lg\kappa_L + \varphi[\lg(\kappa_K/\kappa_L)]^2$$

(2.25b) $$\delta^* = \delta[1 + 2\varphi\lg(\kappa_K/\kappa_L)]$$

(2.25c) $$\varphi^* = \varphi;\ \ \rho^* = \rho;\ \ \lambda^* = \lambda;\ \ \mu^* = \mu.$$

It is apparent from (2.25a,b) that any change in units of measurement of output Y does influence the constant α only. Changes of units of measurements of capital K and labor L do influence both the constant α and the distribution parameter δ. The estimates of parameters φ, ρ, λ, and μ are apparently independent of units of measurement. The estimates of parameter ω and therefore also of elasticity of substitution σ, are obtained from estimates of δ and φ. They are, therefore, also dependent on units of measurement.

4.This procedure was used for example by Tlusty and Strnad (13), (14).

5.This type of models were investigated for example by Balestra and Nerlove (1), Mundlak (8), Wallace and Hussain (15), and Kmenta (4). To the large extent we follow here the Kmenta's suggestion of estimating the cross-sectionally heteroscedastic and time-wise autoregressive models.

6. We are grateful to Dr. Thad Alton and his colleagues who provided us with some yet unpublished estimates of GNP of industrial output for Czechoslovakia and some East European countries.

7. See, for example, Novotny (9).

8. Coefficients in the Table 3. Are calculated as simple arithmetic means of sectoral coefficients. The more correct procedure would be to use the weighted arithmetic means, as can be seen from (3.2). However, the results would not be very different. The simple arithmetic mean remains to be correct for calculating the mean values of the technical progress parameters r,.l and m.

9. See Tables 6 – 13.

10. The Durbin-Watson statistics were evaluated in the time-wise direction with "gaps" between sectors., i.e. using formula

$$D = [\Sigma_i^N \Sigma_t^T (e_{it} - e_{it-1})^2] / \Sigma_i^N \Sigma_t^T e_{it}^2$$

This gives the same result as taking the weighted arithmetic mean of sectoral sums of squared residuals as weights.

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