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Uninsurable Risk and Financial Market Puzzles

*By Parantap Basu, Andrei Semenov, and Kenji Wada**

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Abstract

Following Kocherlakota and Pistaferri (2009), we consider two forms of incomplete risk sharing in economies with consumer heterogeneity: (a) where agents are unable to insure their consumption against idiosyncratic skill shocks and (b) where idiosyncratic shocks to skills can be partially insured by striking long term insurance contract with truth revelation constraint. When considering the equity premium, currency premium, risk-free rate, and consumption-real exchange rate puzzles in an integrated framework, we find empirical evidence that although the pricing kernel associated with (a) outperforms the complete risk-sharing stochastic discount factor and the pricing kernel associated with (b), it is still unable to jointly resolve these asset-pricing anomalies.

JEL Classification: E32, G11, G12.

Keywords: Currency Premium, Equity Premium, Exchange Rate.

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1 Introduction

There are four well known puzzles in the macro-finance literature. Two of these puzzles are home based. Mehra and Prescott (1985) and Hansen and Jagannathan (1991) among others observe that the covariance of aggregate per capita consumption growth with the excess return on the market portfolio over a risk-free asset is very low, so that the representative-agent consumption CAPM can explain the observed market premium only if the typical investor is extremely risk averse. This is known as the *equity premium puzzle*. In addition, Weil (1989) observes that given the lack of variability of aggregate consumption growth, the representative agent must have a negative rate of time preference for the model to be able to match the observed mean risk-free rate. This puzzle is known as the *risk free rate puzzle*.

The other two puzzles appear on the international front. Economic theory predicts that the log real exchange rate growth between any two countries equals the difference in the logs of the foreign and domestic stochastic discount factors (Brandt *et al.*, 2006). With a representative agent for each country, the log real exchange rate growth must be perfectly correlated with the difference in the log growth rates of marginal utilities of aggregate per capita consumption of respective countries. This implies that, under the standard assumption of power utility, the log real exchange rate and log relative consumption should be perfectly correlated. In practice however it is observed that the correlation between relative consumption and the real exchange rate is close to zero or even negative. The real exchange rates are more volatile and persistent than the log relative consumption. This is the *consumption-real exchange rate puzzle* documented by Kollmann (1992) and Backus and Smith (1993). The final anomaly with the representative-agent model is that it is unable to reconcile the highly volatile excess return on currency with the smooth aggregate consumption growth rate unless the agent is assumed to bear an implausibly high level of risk aversion. This is the *currency premium puzzle*.¹

A number of recent papers departed from the complete market paradigm with full risk sharing to understand these puzzles.² In a recent influential paper, Kocherlakota and Pistaferri (hereafter KP) (2007) address the consumption-real exchange rate puzzle in a setting where markets are complete with respect to country-specific shocks (individuals can fully insure their consumption against cross-country shocks), but domestic markets are incomplete (individuals cannot completely insure themselves against idiosyncratic skill shocks). They consider two forms of partial insurance against idiosyncratic skill shocks. The first they call domestically incomplete

¹Lustig and Verdelhan (2007) show that the representative-agent consumption CAPM can explain the cross-sectional variation in currency premia only if the representative agent's coefficient of risk aversion is around 100. They find that the estimate of the risk aversion coefficient does not change when the representative agent's Euler equations for the currency portfolios are estimated jointly with the Euler equations for US domestic bond portfolios (sorted by maturity) and stock portfolios (sorted by size and book-to-market ratio).

²Bewley (1982), Mehra and Prescott (1985), Mankiw (1986), Constantinides and Duffie (1996), and Brav *et al.* (2002), among others, argue that consumers' heterogeneity induced by market incompleteness may be relevant for understanding these asset pricing anomalies.

markets (DI). Under this formulation, individuals are unable to insure their consumption against idiosyncratic skill shocks. The second form of partial insurance they label as Private Information Pareto Optimal (PIPO). Here, the agents are able to sign insurance contracts, which allow them to insure themselves against idiosyncratic shocks, subject to the incentive constraint that agents reveal truth about their private skill shocks to the financial intermediary. For each form of partial insurance, KP (2007) derive a restriction relating the growth rate of the real exchange rate to the difference in the growth rates of the moments of the cross-sectional distributions of consumption in two countries. Using household-level consumption data from the US and the UK, they show that the PIPO model fits the data with the relative risk aversion coefficient of around 5, while the complete risk-sharing (CRS) model and the DI model both perform poorly. In another companion paper, KP (2009) demonstrate the superior performance of the PIPO model to explain the observed mean equity premium in the US with a value of the relative risk aversion coefficient between 5 and 6.³

Although both papers of KP make a major methodological contribution to model consumer heterogeneity in the presence of uninsurable risk, there are two problems with their approach. The first problem is about the robustness of this approach to attack various puzzles. Ideally, one expects that the same economic fundamentals should be responsible for understanding home and international financial puzzles described earlier. KP focus on only one puzzle in isolation and the proposed pricing kernels do not work properly if two or more asset pricing anomalies are addressed together in an integrated framework. For example, the stochastic discount factors proposed by KP (2009) fail to explain the observed mean risk-free rate. Likewise, both these pricing kernels yield implausibly low estimates of the subjective time discount factor.⁴

The second problem is about the robustness of KP's estimation results. One of the problems with the pricing kernels proposed in KP (2007, 2009) is that the testing results are very sensitive to the sample design. Kollmann (2009) documents that the KP (2007) results are highly sensitive to the presence of outliers. He also finds that minor specification changes may overturn the KP (2007) findings. Kollmann reestimates KP models and find that the real exchange rate anomaly continues to persist.

These two problems are related. If KP results are so sensitive to sample design, the issue arises whether the superior performance of the PIPO pricing kernel will survive when more than one asset pricing anomalies are addressed together in an integrated framework. In this present paper, we precisely address this issue. While deriving our stochastic discount factors, we address

³Constantinides and Duffie (1996), Brav *et al.* (2002), Semenov (2004), Basu and Wada (2006), and Balduzzi and Yao (2007) also argue that the model with heterogeneous consumers can help explain the excess return on the market portfolio over the risk-free rate with a plausible value of the relative risk aversion coefficient.

⁴The same problem arises in Brav *et al.* (2002). For example, the stochastic discount factors proposed in Brav *et al.* (2002) fail to jointly explain the observed mean equity premium and risk-free rate. At economically plausible estimates of the risk aversion coefficient required to explain the equity premium puzzle, both these pricing kernels yield implausibly low estimates of the subjective time discount factor.

four asset pricing puzzles (the equity premium, risk-free rate, consumption-real exchange rate, and currency premium puzzles) simultaneously using an integrated framework. Thus we propose to get a unique set of estimates for the preference based fundamentals, namely risk aversion and time preference rate which could reconcile all four puzzles simultaneously. This requires careful modelling of international currency trades by motivating the transaction demand for currency in terms of a cash-in-advance constraint. In this respect, our approach is theoretically more elegant than KP (2007, 2009).

Second, to check the robustness of the results of KP as pointed out by Kollmann (2009), we revise the data for the US and the UK. Our quarterly dataset ranges up to 2004 while KP dataset is limited only up to 1999. We compare the results of our dataset with KP.

Finally, following the tradition of Constantinides and Duffie (1996), Sarkissian (2003), Basu and Wada (2006) and Semenov (2008) we use the assumption of lognormality of the cross section consumption process to work out closed form expressions for the pricing kernels for alternative market environments.⁵ These pricing kernels are empirically easily estimable using the Generalized Method of Moments (GMM) and are less vulnerable to the sampling and measurement error problems mentioned in KP.

We perform a number of experiments to understand all four puzzles in an integrated framework. We take a first look at the asset pricing anomalies using a commonly used calibration approach. The key result is that a given pricing kernel is unable to explain all four puzzles. This necessitates the use of GMM to estimate the deep parameters using the moments conditions. We find that when Euler equations for the equity premium, currency premium and risk free rate and the Backus-Smith real exchange rate equation are estimated jointly, all three models, CRS, DI and PIPO pass the diagnostics and provide economically plausible estimates of deep parameters. In other words, we are unable to reject any of the three models. Finally, we put these three models to further scrutiny by testing their forecasting abilities. We find that the DI model slightly outperforms the other two in forecasting the real exchange rates. However, the forecasting ability is still far from satisfactory for all three models under scrutiny.

The rest of the paper is organized as follows. In Section 2, we describe the DI and PIPO environments and derive the associated stochastic discount factors. Section 3 addresses the empirical implementation of the models derived in Section 2. In Section 4, we report the empirical

⁵The assumption of lognormality of the cross sectional consumption process is empirically well grounded. Battistin *et al.* (2007, 2009) show that, within demographically homogeneous groups, the cross-sectional consumption distribution in both the US Consumer Expenditure Survey (CEX) and the UK British Family Expenditure Survey (FES) is approximately log normal. They argue that this is due to the fact that the logic of Gibrat's law applies to consumption. Brzozowski *et al.* (2009) provide empirical evidence that the cross-sectional distribution of consumption within cohort groups in Canada may be very well approximated by a log-normal. Blundell and Lewbel (1999) also provide strong empirical evidence of log-normality of the cross-sectional distribution of consumption in a variety of data sets. Attanasio *et al.* (2004) assume log-normality of the cross-sectional distribution of household consumption when studying the evolution of inequality in consumption in the US both within cohorts and for the all population.

estimation and testing results for these models, as well as investigate how these results may be affected in the presence of the measurement and sampling errors in individual consumption. Section 5 concludes.

2 The DI and PIPO Environments

Following KP (2009), we relax the assumption of market completeness and assume that although international markets are complete (individuals can fully insure their consumption against country-specific (aggregate) shocks), domestic markets are incomplete (individuals can only partially insure their consumption against individual-specific (idiosyncratic) skill shocks). To take into account this partial insurance against shocks, as in KP (2009), we consider two market structures: (a) where agents are unable to insure their consumption against idiosyncratic shocks and (b) where idiosyncratic shocks can be partially insured by striking long term insurance contract with truth revelation constraint (the PIPO form of partial insurance against idiosyncratic shocks).

In this section, we first describe each of these two environments and then derive the associated stochastic discount factors.

2.1 The DI Environment

2.1.1 The Problem

Our DI environment is similar to that in KP (2009) and Golosov and Tsyvinski (2006) except that, within our approach, we have explicit stock, bond, and currency trading. We assume that there are two generic countries in the world: a home country and a foreign country. We further assume that the economy is populated by infinitely many agents with *ex ante* identical preferences. The agents are not country specific in nature and only differ in private history of skill shocks.

At any date t ($t = 0, 1, 2, \dots, T$), an agent experiences an idiosyncratic skill shock θ_t , which is drawn from a finite set Θ . In addition, all agents are exposed to the same aggregate shock z_t that is drawn from an uncountable set Z . The date t private skill shock history $\boldsymbol{\theta}^t = (\theta_1, \theta_2, \dots, \theta_t)$ and public shock history $\mathbf{z}^t = (z_1, z_2, \dots, z_t)$ are the t th components of Θ^T and Z^T , respectively, with respective probabilities $\pi(\boldsymbol{\theta}^t)$ and $\psi(\mathbf{z}^t)$. We assume that the idiosyncratic skill shock and the aggregate shock are drawn independently, so that by observing the aggregate shock one cannot infer anything about the idiosyncratic skill shock. According to the law of large numbers, at any date t there are exactly $\pi(\boldsymbol{\theta}^t)$ agents with the private history $\boldsymbol{\theta}^t$.

Suppose that the home country produces two goods, tradable (y_t^{TR}) and non-tradable (y_t^{NT}), with the following technologies:

$$y_t^i(\mathbf{z}^t, \boldsymbol{\theta}^t) = \phi_t^i(\mathbf{z}^t, \boldsymbol{\theta}^t) l_t^i, \quad i = TR, NT, \quad (1)$$

where l_t^i is the labour used in sector i and ϕ_t^i is the sector i marginal product of labour. Note that the labour productivities depend on the history of the public and idiosyncratic skill shocks, \mathbf{z}^t and $\boldsymbol{\theta}^t$. Both $\boldsymbol{\theta}^t$ and l_t^i are private information to agents.

The aggregate outputs of traded and non-traded goods for the home country are

$$Y_t^i(\mathbf{z}^t) = \sum_{\boldsymbol{\theta}^t} y_t^i(\mathbf{z}^t, \boldsymbol{\theta}^t) \pi(\boldsymbol{\theta}^t), \quad i = TR, NT. \quad (2)$$

Assume that there are the following assets: (a) two home stocks, which are claims to the nominal proceeds from traded and non-traded sectors, (b) a one-period nominal bond that pays a nominal interest rate of r_t , and (c) the home country currency, which is traded in the international spot and forward markets. We further assume that only the spot and forward contracts on currency are traded abroad, while stocks and bonds are not internationally traded. The currency plays a twofold role: (a) as a means of exchange (specified as a cash-in-advance constraint) and (b) as a store of value (the same currency can be invested in the international spot and forward markets). Home and foreign goods are both non-storable.⁶

Financial markets open before the goods market. At the start of the day, agents trade in stocks, bonds, and currency. Once the financial transactions are completed, a household takes the left over cash to transact in goods. Each household has two distinct entities: a shopper and a producer. As a producer, the household produces traded and non-traded goods, while as a shopper it purchases the same goods. Since in the market place there are infinitely many shoppers and producers and a shopper meets a producer randomly, a cash-in-advance constraint is necessitated.

A home country agent faces the following optimization problem:

$$\text{Max } E_t \sum_{j=t}^T \beta^{j-t} \left[\frac{\{u(c_j^{TR}, c_j^{NT})\}^{1-\gamma}}{1-\gamma} - v(l_j^{TR}, l_j^{NT}) \right] \quad (3)$$

s.t.

$$m_j^c + m_j^s + m_j^f + \sum_{i=TR,NT} Q_j^i \xi_j^i + b_j \leq \sum_{i=TR,NT} (D_j^i \xi_{j-1}^i + Q_j^i \xi_{j-1}^i) + \frac{S_j m_{j-1}^s}{S_{j-1}} + \frac{F_{j-1} m_{j-1}^f}{S_{j-1}} + (1 + r_{j-1}) b_{j-1} \quad (4)$$

and

$$\sum_{i=TR,NT} P_j^i c_j^i \leq m_j^c. \quad (5)$$

⁶In order to keep the equity premium puzzle a purely domestic financial puzzle, we assume that the stocks and the bond are non-traded assets. This is an extreme form of “home bias” documented by Tesar and Werner (1995), e.g. As a result, we rule out the possibility of earning the risk-free interest on the currency held from one period to another. This is a simplifying assumption to rule out complex term structure issues of returns on currency based on trading in foreign bonds.

Here, c_j^i is the date j consumption of sector i goods, P_j^i is the date j nominal price of sector i goods, Q_j^i is the date j sector i nominal price of new equity purchases, $D_j^i = P_j^i Y_j^i$ is the date j dividends from sector i ,⁷ ξ_j^i is the date j share of sector i , r_j is the date j nominal risk-free rate of interest, m_j^s is the home money invested in the spot market at date j , m_j^f is the home money committed to the forward market at date j ,⁸ m_j^c is the currency used for purchase of goods at date j , b_j is the risk-free bondholding at date j , F_j stands for the date j forward exchange rate, S_j represents the date j spot exchange rate. All prices are denominated in home money. The instantaneous utility function $u(c_j^{TR}, c_j^{NT})$ is assumed to be linearly homogenous as in Backus and Smith (1993) and function $v(l_j^{TR}, l_j^{NT})$ is monotonically increasing in its arguments with usual regularity conditions as in KP (2009). $E_t[\cdot]$ is an expectations operator. Expectation is computed with respect to the probability measures of \mathbf{z}^{t+1} and $\boldsymbol{\theta}^{t+1}$. Finally, β is the subjective time discount factor and $\gamma > 0$ is the relative risk aversion coefficient.

Non-storability of goods is a crucial assumption, which explains why foreign prices do not enter the flow budget constraint (4). If goods are storable, then home agents can convert foreign currency into foreign goods and enjoy capital gains when foreign prices rise in the following period.

Since within the DI framework agents are unable to insure themselves against idiosyncratic skill shocks, all date t prices, interest rate, and exchange rates are functions of public history of shocks \mathbf{z}^t only. The crucial assumption here is that stocks and bonds do not hedge the idiosyncratic skill shocks. In this respect, the markets are domestically incomplete.

2.1.2 First-Order Conditions

The Lagrangian for the above optimization problem is

$$L = E_t \left[\sum_{j=t}^T \beta^{j-t} \frac{u(c_j^{TR}, c_j^{NT})^{1-\gamma}}{1-\gamma} - v(l_j^{TR}, l_j^{NT}) \right] + E_t \left[\sum_{j=t}^T \mu_j \left(m_j^c - \sum_{i=TR,NT} P_j^i c_j^i \right) \right] + E_t \sum_{j=t}^T \lambda_j \left(\sum_{i=TR,NT} (D_j^i \xi_{j-1} + Q_j^i \xi_{j-1}^i - Q_j^i \xi_j^i) + \frac{S_j m_{j-1}^s}{S_{j-1}} + \frac{F_{j-1} m_{j-1}^f}{S_{j-1}} \right) + (1 + r_{j-1}) b_{j-1} - b_j - m_j^s - m_j^f - m_j^c \right). \quad (6)$$

The corresponding first-order conditions are:

$$c_t^i : u_t^{-\gamma} u_{c_t^i} = \mu_t P_t^i, \quad i = TR, NT, \quad (7)$$

$$l_t^i : v_{l_t^i} = \mu_t \phi_t^i P_t^i, \quad i = TR, NT, \quad (8)$$

⁷Note that there is no labour market. Agents supply their own labour and thus the dividends are simply the proceeds from the sale of outputs in the goods market.

⁸Although money is committed to the forward market at date j , no money literally changes hands until date $j + 1$ by the very definition of a forward contract. The home investor is free to take a long ($m_j > 0$) or short ($m_j < 0$) position at the contracted forward rate F_j that is binding at date $j + 1$.

$$\xi_t^i : -\lambda_t Q_t^i + E_t [\lambda_{t+1}(Q_{t+1}^i + D_{t+1}^i)] = 0, \quad i = TR, NT, \quad (9)$$

$$b_t : -\lambda_t + E_t [(1 + r_t)\lambda_{t+1}], \quad (10)$$

$$m_t^s : -\lambda_t + E_t \left[\lambda_{t+1} \frac{S_{t+1}}{S_t} \right] = 0, \quad (11)$$

$$m_t^f : -\lambda_t + E_t \left[\lambda_{t+1} \frac{F_t}{S_t} \right] = 0, \quad (12)$$

$$m_t^c : -\lambda_t + \mu_t = 0. \quad (13)$$

Here, the subscripts of u and v state for the partial derivatives of u and v with respect to the relevant argument. The first-order condition (13) means that the agents allocate money for transaction purpose so as to equate the marginal benefit of transaction to the marginal opportunity cost of the foregone earnings from currency trading.

Based on the first-order conditions (7) and (8), it is straightforward to verify the following static efficiency condition for the labour supply decision:

$$\frac{u_{c_t^{TR}} \phi^{TR}}{u_{c_t^{NT}} \phi^{NT}} = \frac{v_t^{TR}}{v_t^{NT}}, \quad (14)$$

which shows the equivalence between the ratio of marginal disutilities of labour and the corresponding marginal utilities from consumption in each sector.

2.1.3 Monetary Policy and Initial Distributions of Assets

Monetary policy within this framework represents an initial cross-country distribution of money stocks, namely home money, M_0 , and foreign money, M_0^* , to fix the date 0 spot rate such that

$$M_0 = S_0 M_0^*.^9 \quad (15)$$

In other words, central banks in both home and foreign countries coordinate monetary policies in such a way that the initial spot rate S_0 is pinned down. After this, the central banks let the nominal exchange rate float according to currency trading among countries.

The initial distributions of stocks and bonds are such that

$$\sum_{\theta^0} \xi_0^i(\theta^0, z^0) \pi(\theta^0) = 1, \quad (16)$$

$$\sum_{\theta^0} b_0(\theta^0, z^0) \pi(\theta^0) = 0, \quad (17)$$

and

$$\sum_{\theta^0} m_0(\theta^0, z^0) \pi(\theta^0) = M_0. \quad (18)$$

⁹Hereafter, the asterisk denotes the foreign country.

2.1.4 Composite Good and Price

Following KP (2009), it is convenient to reduce the two good setting to a composite good problem. Exploiting the linear homogeneity of the instantaneous utility function and the duality property, we can write:

$$\bar{P}_t c_t = \sum_{i=TR,NT} P_t^i c_t^i, \quad (19)$$

where \bar{P}_t is the minimum expenditure required to attain one unit of utility.

That is,

$$\bar{P}_t = \underset{c_t^{TR}, c_t^{NT}}{Min} \sum_{i=TR,NT} P_t^i c_t^i \quad (20)$$

s.t.

$$u(c_t^{TR}, c_t^{NT}) = 1, \quad (21)$$

which means that instantaneous utility $u(c_t^{TR}, c_t^{NT})$ is nothing, but the real consumption expenditure or a composite consumption good that we label c_t hereafter.

Based on this composite consumption and exploiting the linear homogeneity of $u(c_t^{TR}, c_t^{NT})$, equations (7) and (13) can be combined to obtain

$$c_t : \beta^t c_t^{-\gamma} - \lambda_t \bar{P}_t = 0. \quad (22)$$

2.1.5 Equilibrium

In equilibrium, the following market-clearing conditions must hold. Given the assumption that stocks and bonds are not internationally traded, the stock and bond markets must domestically clear meaning

$$\sum_{\theta^t} \xi_t^i(\theta^t, \mathbf{z}^t) \pi(\theta^t) = 1 \text{ for each } i \quad (23)$$

$$\sum_{\theta^t} b_t(\theta^t, \mathbf{z}^t) \pi(\theta^t) = 0. \quad (24)$$

The other market-clearing conditions are the traded and non-traded market clearing conditions

$$\sum_{\theta^t} (c_t^{TR}(\mathbf{z}^t, \theta^t) + c_t^{*TR}(\mathbf{z}^t, \theta^t)) \pi(\theta^t) = \sum_{\theta^t} (y_t^{TR}(\mathbf{z}^t, \theta^t) + y_t^{*TR}(\mathbf{z}^t, \theta^t)) \pi(\theta^t) \quad (25)$$

$$\sum_{\theta^t} c_t^{NT}(\mathbf{z}^t, \theta^t) \pi(\theta^t) = \sum_{\theta^t} y_t^{NT}(\mathbf{z}^t, \theta^t) \pi(\theta^t), \quad (26)$$

and the currency market clearing conditions for spot and forward are

$$\sum_{\theta^t} (m_t^s(\theta^t, \mathbf{z}^t) \pi(\theta^t)) = S_t \sum_{\theta^t} m_t^{*s}(\theta^t, \mathbf{z}^t) \pi(\theta^t). \text{ for all } \mathbf{z}^t \quad (27)$$

$$\sum_{\theta^t} (m_t^f(\theta^t, \mathbf{z}^t)) = \sum_{\theta^t} (m_t^{*f}(\theta^t, \mathbf{z}^t)) = 0 \text{ for all } \mathbf{z}^t. \quad (28)$$

The spot rate must be such that the supply of the home currency supplied on the spot exactly equals the corresponding demand. This explains the spot market clearing condition (27). The forward rate at each date is contracted in such a way that agents taking long and short positions in the forward market balance each other for all aggregate history \mathbf{z}^t . This explains the forward market clearing condition (28).

2.1.6 The DI Pricing Kernel

Assume that the world equilibrium, as laid out in the preceding section, exists. Define the real returns on the traded and non-traded stocks as

$$R_{M,t+1}^i = \frac{q_{t+1}^i + d_{t+1}^i}{q_t^i}, \quad i = TR, NT, \quad (29)$$

where $q_t^i(\mathbf{z}^t) = Q_t^i(\mathbf{z}^t)/\bar{P}_t$ is the real i th equity price and $d_{t+1}^i(\mathbf{z}^{t+1}) = D_{t+1}^i(\mathbf{z}^{t+1})/\bar{P}_{t+1}$ is the real dividend from share i .

Based on these two returns, we can define the market portfolio return as

$$R_{M,t+1} = \sum_{i=TR,NT} R_{M,t+1}^i. \quad (30)$$

Define the real risk-free rate as

$$R_{F,t+1} = \frac{(1 + r_{t+1})\bar{P}_t}{\bar{P}_{t+1}}. \quad (31)$$

Based on the first-order conditions (9) and (10), we can thus write the first-order condition for the real market portfolio return as

$$E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\bar{P}_{t+1}}{\bar{P}_t} R_{M,t+1} \right] = 1 \quad (32)$$

and the first-order condition for the real risk-free rate as

$$E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\bar{P}_{t+1}}{\bar{P}_t} R_{F,t+1} \right] = 1. \quad (33)$$

Likewise, the first-order conditions (11) and (12) give the spot and forward rate equations as follows:

$$E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{S_{t+1}}{S_t} \right] = 1 \quad (34)$$

and

$$E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{F_t}{S_t} \right] = 1 \quad (35)$$

or, equivalently,

$$E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\bar{P}_{t+1}}{\bar{P}_t} \frac{S_{t+1}\bar{P}_t}{S_t\bar{P}_{t+1}} \right] = 1 \quad (36)$$

and

$$E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\bar{P}_{t+1}}{\bar{P}_t} \frac{F_t \bar{P}_t}{S_t \bar{P}_{t+1}} \right] = 1. \quad (37)$$

From (32), (33), (36), and (37), it follows that within the DI framework the stochastic discount factor is

$$SDF_{t+1}^{DI} = \frac{\lambda_{t+1}}{\lambda_t} \frac{\bar{P}_{t+1}}{\bar{P}_t}. \quad (38)$$

In the next step, we follow the same principle as in KP (2009) to rewrite the stochastic discount factor (38) in terms of the cross-sectional moments of consumption. Define a generic asset n with the real gross return $R_{n,t}$, which is a function of the aggregate shock history \mathbf{z}^t only. In the present setting, this asset can be an equity, risk-free bond, currency forward, or spot.

Using (22), the Euler equation for such a generic asset can be written as

$$c_t(\boldsymbol{\theta}^t, \mathbf{z}^t)^{-\gamma} = \beta \sum_{\mathbf{z}^{t+1}} R_{n,t+1}(\mathbf{z}^{t+1}) \psi(\mathbf{z}^{t+1} | \mathbf{z}^t) \sum_{\boldsymbol{\theta}^{t+1}} c_{t+1}(\boldsymbol{\theta}^{t+1}, \mathbf{z}^{t+1})^{-\gamma} \pi(\boldsymbol{\theta}^{t+1} | \boldsymbol{\theta}^t). \quad (39)$$

Define

$$E(c_{t+1}^{-\gamma} | \mathbf{z}^{t+1}, \boldsymbol{\theta}^t) = \sum_{\boldsymbol{\theta}^{t+1}} c_{t+1}(\boldsymbol{\theta}^{t+1}, \mathbf{z}^{t+1})^{-\gamma} \pi(\boldsymbol{\theta}^{t+1} | \boldsymbol{\theta}^t) \quad (40)$$

as the $-\gamma$ th non-central cross-sectional moment of composite good consumption conditional on private history $\boldsymbol{\theta}^t$ and public history \mathbf{z}^{t+1} .

Thus, equation (39) can be rewritten as

$$c_t(\boldsymbol{\theta}^t, \mathbf{z}^t)^{-\gamma} = \beta \sum_{\mathbf{z}^{t+1}} R_{n,t+1}(\mathbf{z}^{t+1}) \psi(\mathbf{z}^{t+1} | \mathbf{z}^t) E(c_{t+1}^{-\gamma} | \mathbf{z}^{t+1}, \boldsymbol{\theta}^t). \quad (41)$$

Integrating the both sides of (41) over $\boldsymbol{\theta}^t$ and using the law of iterated expectations, we get

$$E(c_t^{-\gamma} | \mathbf{z}^t) = \beta \sum_{\mathbf{z}^{t+1}} R_{n,t+1}(\mathbf{z}^{t+1}) \psi(\mathbf{z}^{t+1} | \mathbf{z}^t) E(c_{t+1}^{-\gamma} | \mathbf{z}^{t+1}) \quad (42)$$

or, equivalently,

$$E_t \left[\beta \frac{E(c_{t+1}^{-\gamma} | \mathbf{z}^{t+1})}{E(c_t^{-\gamma} | \mathbf{z}^t)} R_{n,t+1} \right] = 1, \quad (43)$$

which immediately shows that

$$SDF_{t+1}^{DI} = \frac{\lambda_{t+1}}{\lambda_t} \frac{\bar{P}_{t+1}}{\bar{P}_t} = \beta \frac{E(c_{t+1}^{-\gamma} | \mathbf{z}^{t+1})}{E(c_t^{-\gamma} | \mathbf{z}^t)} \quad (44)$$

is the stochastic discount factor associated with the DI environment.

This stochastic discount factor differs from the pricing kernels proposed by Constantinides and Duffie (1996), Sarkissian (2003), and Semenov (2008) in that it uses the cross-sectional distribution of consumption in *level*, rather than the cross-sectional distribution of consumption in *growth rates*, as the driving process for the pricing kernel. As we mentioned above, the use of

the cross-sectional distribution of consumption in *growth rates* has a counterfactual implication that the cross-sectional distribution of consumption in *level* is not stationary, implying that the Gini coefficient of consumption distribution goes to infinity.

We are now ready to write the relevant excess return equations, which are namely the equity premium and currency premium equations. Subtracting (33) from (32), we get the following real equity premium equation:

$$E_t [SDF_{t+1}^{DI} (R_{M,t+1} - R_{F,t+1})] = 0. \quad (45)$$

Subtracting (36) from (37), we obtain the real currency premium equation:

$$E_t \left[SDF_{t+1}^{DI} \left(\frac{F_t - S_{t+1}}{S_t} \right) \frac{\bar{P}_t}{\bar{P}_{t+1}} \right] = 0. \quad (46)$$

The real risk-free rate equation (33) is

$$E_t [SDF_{t+1}^{DI} R_{F,t+1}] = 1. \quad (47)$$

Because the log real exchange rate growth between any two countries is equal to the difference in the logs of the foreign and domestic stochastic discount factors (see Brandt *et al.* (2006)), within the DI framework

$$\Delta q_{t+1} = \ln \left(\frac{Q_{t+1}}{Q_t} \right) = \ln (SDF_{t+1}^{DI*}) - \ln (SDF_{t+1}^{DI}). \quad (48)$$

In equations (45)-(48), SDF_{t+1}^{DI} is as defined in (44).

2.2 The PIPO Environment

2.2.1 The Problem

In this alternative setting, agents are able to partially insure against idiosyncratic skill shocks. The model is a dynamic extension of Mirrlees (1971) type private information setting. Trading convention is similar to that in Golosov and Tsyvinski (2006) and KP (2009). All agents are assumed to have *ex ante* identical preferences. There is a continuum of insurance firms, which act on behalf of the households and play the following roles: (a) produce the traded and non-traded goods by hiring workers, (b) sell these goods in national and international markets, (c) trade among themselves in stock, bond, and currency in sequential markets, and, finally, (d) with the resulting profits from this trade insure the households against idiosyncratic skill shocks. Timing of financial and goods markets is the same as in the DI setting. The same cash-in-advance constraint applies to the insurance companies when they trade in goods.

The insurance firms are owned equally by all agents. At date 0, before the realization of aggregate and idiosyncratic shocks, the contract market opens only once. In this market, the competitive insurance firms offer contracts to the households about consumption bundles

of traded and non-traded goods $\{c_t^{TR}, c_t^{NT}\}$, which provide maximum *ex ante* utility to the households. Since the insurance company does not observe the idiosyncratic shock history and labour supply, it stipulates contract about the observed output sequence of traded and non-traded goods $\{y_t^{TR}, y_t^{NT}\}$, such that it is incentive compatible for the agents to reveal the truth about the history of private skill shocks. These contracts are long-term contracts with full commitment on both sides. After the contract market closes, from date 1 onward the insurance firms start trading in goods and financial markets in the same sequential manner as within the DI framework.

A typical insurance company, located in the home country, maximizes the present value of the nominal payoffs to its owners:

$$\underset{\{c_t^{TR}, c_t^{NT}, y_t^{TR}, y_t^{NT}, \xi_t, b_t, m_t^s, m_t^f\}}{\text{Max}} \sum_{t=0}^T \prod_{i=1}^t (1 + \rho_i(\mathbf{z}^i))^{-1} \Pi_t(\mathbf{z}^t) \psi(\mathbf{z}^t) \quad (49)$$

s.t.

$$\begin{aligned} & \Pi_t(\mathbf{z}^t) + m_t^s(\mathbf{z}^t) + m_t^f(\mathbf{z}^t) + m_t^c(\mathbf{z}^t) + \sum_{i=TR, NT} Q_t^i(\mathbf{z}^t) \xi_t^i(\mathbf{z}^t) + b_t(\mathbf{z}^t) \leq \\ & \sum_{i=TR, NT} \xi_{t-1}^i(\mathbf{z}^{t-1}) \sum_{\boldsymbol{\theta}^t} \pi(\boldsymbol{\theta}^t) D_t^i(\boldsymbol{\theta}^t, \mathbf{z}^t) + \sum_{i=TR, NT} \xi_{t-1}^i(\mathbf{z}^{t-1}) Q_t^i(\mathbf{z}^t) + \frac{S_t(\mathbf{z}^t)}{S_{t-1}(\mathbf{z}^{t-1})} m_{t-1}^s(\mathbf{z}^{t-1}) \\ & + \frac{F_{t-1}(\mathbf{z}^{t-1})}{S_{t-1}(\mathbf{z}^{t-1})} m_{t-1}^f(\mathbf{z}^{t-1}) + (1 + r_t(\mathbf{z}^{t-1})) b_{t-1}(\mathbf{z}^{t-1}), \end{aligned} \quad (50)$$

the cash-in-advance constraint

$$\sum_{\boldsymbol{\theta}^t} \sum_{i=TR, NT} \pi(\boldsymbol{\theta}^t) P_t^i(\mathbf{z}^t) c_t^i(\boldsymbol{\theta}^t, \mathbf{z}^t) \leq m_t^c(\mathbf{z}^t), \quad (51)$$

the participation constraint

$$\sum_{t=0}^T \beta^t \sum_{\boldsymbol{\theta}^t, \mathbf{z}^t} \left[\frac{\{u(c_t^{TR}(\boldsymbol{\theta}^t, \mathbf{z}^t), c_t^{NT}(\boldsymbol{\theta}^t, \mathbf{z}^t))\}^{1-\gamma}}{1-\gamma} - v\left(\frac{y_t^{TR}(\boldsymbol{\theta}^t, \mathbf{z}^t)}{\phi_t^{TR}(\boldsymbol{\theta}^t, \mathbf{z}^t)}, \frac{y_t^{NT}(\boldsymbol{\theta}^t, \mathbf{z}^t)}{\phi_t^{NT}(\boldsymbol{\theta}^t, \mathbf{z}^t)}\right) \right] \pi(\boldsymbol{\theta}^t) \psi(\mathbf{z}^t) \geq \underline{u}, \quad (52)$$

and the incentive constraint

$$\begin{aligned} & \sum_{t=0}^T \beta^t \sum_{\boldsymbol{\theta}^t, \mathbf{z}^t} \left[\frac{\{u(c_t^{TR}(\boldsymbol{\theta}^t, \mathbf{z}^t), c_t^{NT}(\boldsymbol{\theta}^t, \mathbf{z}^t))\}^{1-\gamma}}{1-\gamma} - v\left(\frac{y_t^{TR}(\boldsymbol{\theta}^t, \mathbf{z}^t)}{\phi_t^{TR}(\boldsymbol{\theta}^t, \mathbf{z}^t)}, \frac{y_t^{NT}(\boldsymbol{\theta}^t, \mathbf{z}^t)}{\phi_t^{NT}(\boldsymbol{\theta}^t, \mathbf{z}^t)}\right) \right] \pi(\boldsymbol{\theta}^t) \psi(\mathbf{z}^t) \geq \\ & \sum_{t=0}^T \beta^t \sum_{\boldsymbol{\theta}^t, \mathbf{z}^t} \left[\frac{\{u(c_t^{TR}(\boldsymbol{\theta}^t, \mathbf{z}^t), c_t^{NT}(\boldsymbol{\theta}^t, \mathbf{z}^t))\}^{1-\gamma}}{1-\gamma} - v\left(\frac{y_t^{TR}(\boldsymbol{\theta}^t, \mathbf{z}^t)}{\phi_t^{TR}(\boldsymbol{\theta}^t, \mathbf{z}^t)}, \frac{y_t^{NT}(\boldsymbol{\theta}^t, \mathbf{z}^t)}{\phi_t^{NT}(\boldsymbol{\theta}^t, \mathbf{z}^t)}\right) \right] \pi(\boldsymbol{\theta}^t) \psi(\mathbf{z}^t), \end{aligned} \quad (53)$$

where $\Pi_t(\mathbf{z}^t)$ is the date t cash flow of the insurance firm contingent on the shock history \mathbf{z}^t , $\rho_t(\mathbf{z}^t)$ is the \mathbf{z}^t contingent discount rate, and $\boldsymbol{\theta}^t$ is the history of shocks that the household reports to the financial intermediaries. Since the insurance firm does not observe the idiosyncratic shock history, all its relevant choices depend on the aggregate shock history \mathbf{z}^t .

2.2.2 First-Order Conditions

Let $\lambda_t(\mathbf{z}^t)$, $\mu_t(\mathbf{z}^t)$, $\omega_t(\mathbf{z}^t)$, and $\eta_t(\mathbf{z}^t)$ be the Lagrange multipliers associated with (50), (51), (52), and (53), respectively. The first-order conditions for problem (49) through (53) are as follows:

$$\Pi_t(\mathbf{z}^t) : \prod_{i=1}^t (1 + \rho_i(\mathbf{z}^i))^{-1} - \lambda_t(\mathbf{z}^t) = 0, \quad (54)$$

$$c_t^i : \beta^t(\omega_t(\mathbf{z}^t) + \eta_t(\mathbf{z}^t))u_t^{-\gamma}u_{c_t^i} = \mu_t(\mathbf{z}^t)P_t^i, \quad i = TR, NT, \quad (55)$$

$$y_t^i : \beta^t(\omega_t(\mathbf{z}^t) + \eta_t(\mathbf{z}^t))v_{l_t^i} = \lambda_t(\boldsymbol{\theta}^t, \mathbf{z}^t)\xi_{t-1}^i(\mathbf{z}^{t-1})\phi_t^i(\mathbf{z}^t, \boldsymbol{\theta}^t)P_t^i, \quad i = TR, NT, \quad (56)$$

$$\xi_t^i : -Q_t^i(\mathbf{z}^t)\lambda_t(\mathbf{z}^t)\psi(\mathbf{z}^t) + \sum_{\mathbf{z}^{t+1}} (Q_{t+1}(\mathbf{z}^{t+1}) + D_{t+1}(\mathbf{z}^{t+1}))\lambda_{t+1}(\mathbf{z}^{t+1})\psi(\mathbf{z}^{t+1}) = 0, \quad i = TR, NT, \quad (57)$$

$$b_t : -\lambda_t(\mathbf{z}^t)\psi(\mathbf{z}^t) + \sum_{\mathbf{z}^{t+1}} \lambda_{t+1}(\mathbf{z}^{t+1})(1 + r_{t+1}(\mathbf{z}^t))\psi(\mathbf{z}^{t+1}) = 0, \quad (58)$$

$$m_t^s : -\lambda_t(\mathbf{z}^t)\psi(\mathbf{z}^t) + \sum_{\mathbf{z}^{t+1}} \lambda_{t+1}(\mathbf{z}^{t+1})\frac{S_{t+1}(\mathbf{z}^{t+1})}{S_t(\mathbf{z}^t)}\psi(\mathbf{z}^{t+1}), \quad (59)$$

$$m_t^f : -\lambda_t(\mathbf{z}^t)\psi(\mathbf{z}^t) + \sum_{\mathbf{z}^{t+1}} \lambda_{t+1}(\mathbf{z}^{t+1})\frac{F_t(\mathbf{z}^t)}{S_t(\mathbf{z}^t)}\psi(\mathbf{z}^{t+1}), \quad (60)$$

and

$$m_t^c : -\lambda_t(\mathbf{z}^t) + \mu_t(\mathbf{z}^t) = 0. \quad (61)$$

A few clarifications are in order. Based on (54), the Lagrange multiplier λ_t represents the date 0 state claims price of a dollar to be delivered at date t contingent on \mathbf{z}^t . Because of (61), this state claims price is the same as the marginal transaction benefit of a dollar, $\mu_t(\mathbf{z}^t)$. It can be seen that the other first-order conditions are similar to those in the DI setting. Note that the use of (55) and (56) yields the same static efficiency condition as (14).

2.2.3 First-Order Conditions

The monetary policy and the initial distributions of assets are the same as those described by equations (15) through (18) in Section 2.1.2.

2.2.4 Composite Good and Price

As within the DI framework, we can reduce the two good setting to a composite good problem described by equations (20) and (21). This means that the two goods can be reduced to a composite good c_t with an associate composite price \bar{P}_t so that the following equality holds:

$$\bar{P}_t c_t = \sum_{i=TR, NT} P_t^i(\mathbf{z}^t) c_t^i(\boldsymbol{\theta}^t, \mathbf{z}^t). \quad (62)$$

2.2.5 Equilibrium

Following Kocherlakota (2005), we can show that the equilibrium allocation $\{c_t^{TR}, c_t^{NT}, y_t^{TR}, y_t^{NT}\}$ for this decentralized economy solves a constrained social planning problem, where the constraints involve the truth revelation incentive constraint. Because of this optimality, KP (2009) call this allocation Private Information Pareto Optimum. In equilibrium, the market-clearing conditions (23) through (27) hold.

2.2.6 The PIPO Pricing Kernel

From (54), we obtain the following useful relationship between the Lagrange multipliers and the stochastic discount factor:

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{1 + \rho_{t+1}(\mathbf{z}^{t+1})}. \quad (63)$$

Using (61) and (63), and defining the conditional probability $\psi(\mathbf{z}^{t+1}|\mathbf{z}^t) \equiv \psi(\mathbf{z}^{t+1})/\psi(\mathbf{z}^t)$, we get

$$\sum_{\mathbf{z}^{t+1}} \frac{Q_{t+1}^i(\mathbf{z}^{t+1}) + D_{t+1}^i(\mathbf{z}^{t+1})}{Q_t^i(\mathbf{z}^t)} \frac{\psi(\mathbf{z}^{t+1}|\mathbf{z}^t)}{1 + \rho_{t+1}(\mathbf{z}^{t+1})} = 1, \quad i = TR, NT. \quad (64)$$

Likewise, using (58), (59), (60), and (63), we obtain the following equations:

$$\sum_{\mathbf{z}^{t+1}} (1 + r_{t+1}(\mathbf{z}^t)) \frac{\psi(\mathbf{z}^{t+1}|\mathbf{z}^t)}{1 + \rho_{t+1}(\mathbf{z}^{t+1})} = 1, \quad (65)$$

$$\sum_{\mathbf{z}^{t+1}} \frac{S_{t+1}(\mathbf{z}^{t+1})}{S_t(\mathbf{z}^t)} \frac{\psi(\mathbf{z}^{t+1}|\mathbf{z}^t)}{1 + \rho_{t+1}(\mathbf{z}^{t+1})} = 1, \quad (66)$$

and

$$\sum_{\mathbf{z}^{t+1}} \frac{F_t(\mathbf{z}^t)}{S_t(\mathbf{z}^t)} \frac{\psi(\mathbf{z}^{t+1}|\mathbf{z}^t)}{1 + \rho_{t+1}(\mathbf{z}^{t+1})} = 1. \quad (67)$$

To characterize the discount rates $\rho_t(\mathbf{z}^t)$, we follow Kocherlakota (2005) and Golosov *et al.* (2006). Fix the date t history $\boldsymbol{\theta}^t$ and \mathbf{z}^t . Decrease the composite good at date t for this history group by an infinitesimally small amount $\beta\Delta_t$ and increase across the board the date $t + 1$ composite good by Δ_t . This compensating variation leaves the objective function and the incentive and participation constraints unaffected. It only impacts the resource constraints. The insurance company now makes sure to minimize the cost of resources at dates t and $t + 1$ for all possible evolutions of the private and public shocks.

To solve this problem, define

$$\frac{\tilde{c}_t(\boldsymbol{\theta}^t, \mathbf{z}^t)^{1-\gamma}}{1-\gamma} \equiv \frac{c_t(\boldsymbol{\theta}^t, \mathbf{z}^t)^{1-\gamma}}{1-\gamma} - \beta\Delta_t \quad (68)$$

and

$$\frac{\tilde{c}_{t+1}(\boldsymbol{\theta}^{t+1}, \mathbf{z}^{t+1})^{1-\gamma}}{1-\gamma} \equiv \frac{c_{t+1}(\boldsymbol{\theta}^{t+1}, \mathbf{z}^{t+1})^{1-\gamma}}{1-\gamma} + \Delta_t. \quad (69)$$

The insurance company thus chooses Δ_t such that the cost of resources at dates t and $t+1$ evaluated at the respective state claims prices $\lambda_t(\mathbf{z}^t)$ and $\lambda_{t+1}(\mathbf{z}^{t+1})$ is minimized at $\Delta_t = 0$. Using the flow resource constraint (50) and (62), this cost minimization problem can be rewritten as

$$\begin{aligned} \underset{\Delta_t}{\text{Min}} \quad & \lambda_t(\mathbf{z}^t) \bar{P}_t(\mathbf{z}^t) (c_t(\boldsymbol{\theta}^t, \mathbf{z}^t)^{1-\gamma} - \beta(1-\gamma)\Delta_t)^{1/(1-\gamma)} \pi(\boldsymbol{\theta}^t) \\ & + \lambda_{t+1}(\mathbf{z}^{t+1}) \bar{P}_{t+1}(\mathbf{z}^{t+1}) \sum_{\boldsymbol{\theta}^{t+1}} (c_{t+1}(\boldsymbol{\theta}^{t+1}, \mathbf{z}^{t+1})^{1-\gamma} + (1-\gamma)\Delta_t)^{1/(1-\gamma)} \pi(\boldsymbol{\theta}^{t+1}). \end{aligned} \quad (70)$$

The first-order condition with respect to Δ_t evaluated at $\Delta_t = 0$ and the use of (54) and (61) yield the following inverse Euler equation:

$$\beta \bar{P}_t(\mathbf{z}^t) c_t^\gamma(\boldsymbol{\theta}^t, \mathbf{z}^t) \pi(\boldsymbol{\theta}^t) = (1 + \rho_{t+1}(\mathbf{z}^{t+1}))^{-1} \bar{P}_{t+1}(\mathbf{z}^{t+1}) \sum_{\boldsymbol{\theta}^{t+1}} c_{t+1}^\gamma(\boldsymbol{\theta}^{t+1}, \mathbf{z}^{t+1}) \pi(\boldsymbol{\theta}^{t+1}). \quad (71)$$

Next, first integrating the right-hand side of (71) with respect to $\boldsymbol{\theta}^{t+1}$ for a given $\boldsymbol{\theta}^t$ and then integrating the left-hand side of (71) with respect to $\boldsymbol{\theta}^t$, and applying the law of iterated expectations, we get

$$\beta \bar{P}_t(\mathbf{z}^t) E(c_t^\gamma | \mathbf{z}^t) = (1 + \rho_{t+1}(\mathbf{z}^{t+1}))^{-1} \bar{P}_{t+1}(\mathbf{z}^{t+1}) E(c_{t+1}^\gamma | \mathbf{z}^{t+1}) \quad (72)$$

or, equivalently,

$$\frac{1}{1 + \rho_{t+1}(\mathbf{z}^{t+1})} = \beta \frac{E(c_t^\gamma | \mathbf{z}^t) \bar{P}_t(\mathbf{z}^t)}{E(c_{t+1}^\gamma | \mathbf{z}^{t+1}) \bar{P}_{t+1}(\mathbf{z}^{t+1})}. \quad (73)$$

Plugging (73) into (64) through (67), we obtain that within the PIPO framework the stochastic discount factor is

$$SDF_{t+1}^{PIPO} = \frac{1}{1 + \rho_{t+1}(\mathbf{z}^{t+1})} \frac{\bar{P}_{t+1}(\mathbf{z}^{t+1})}{\bar{P}_t(\mathbf{z}^t)} = \beta \frac{E(c_t^\gamma | \mathbf{z}^t)}{E(c_{t+1}^\gamma | \mathbf{z}^{t+1})}. \quad (74)$$

With this pricing kernel, the Euler equations for the equity premium, the risk-free rate, and the currency premium can be rewritten as

$$E_t [SDF_{t+1}^{PIPO} (R_{M,t+1} - R_{F,t+1})] = 0, \quad (75)$$

$$E_t [SDF_{t+1}^{PIPO} R_{F,t+1}] = 1, \quad (76)$$

and

$$E_t \left[SDF_{t+1}^{PIPO} \left(\frac{F_t - S_{t+1}}{S_t} \right) \frac{\bar{P}_t}{\bar{P}_{t+1}} \right] = 0, \quad (77)$$

respectively, with SDF_{t+1}^{PIPO} defined as in (74).

The log real exchange rate growth between two countries equals the difference in the logs of the foreign and domestic pricing kernels and hence within the PIPO framework

$$\Delta q_{t+1} = \ln(SDF_{t+1}^{PIPO*}) - \ln(SDF_{t+1}^{PIPO}). \quad (78)$$

3 Empirical Formulation

3.1 Consumption Process

In a similar spirit as in De Santis (2007), we assume that the log of individual consumption is the sum of the log of aggregate per capita consumption and the uninsurable consumption due to the idiosyncratic uninsurable skill shock:

$$c_{hk,t} = c_{k,t} + \ln(\delta_{hk,t}), \quad (79)$$

where $c_{hk,t} = \ln(C_{hk,t})$ is the log of the date t consumption of the h th investor in country k , $c_{k,t} = \ln(C_{k,t})$ is the log of the aggregate per capita consumption in country k at time t , and $\delta_{hk,t}$ is the idiosyncratic consumption shock.

We assume the following process for $\delta_{hk,t}$:

$$\delta_{hk,t} = \exp\left(u_{hk,t}\sqrt{\text{var}_h(c_{hk,t}) - \frac{\text{var}_h(c_{hk,t})}{2}}\right), \quad (80)$$

where $\text{var}_h(c_{hk,t})$ is the cross-sectional variance of log consumption and $u_{hk,t}$ is a standard normal shock, which is independently and identically distributed across countries, individuals, and time.

The s th non-centred moment of the cross-sectional distribution of consumption is given by

$$E_h(C_{hk,t}^s) = C_{k,t}^s \exp\left(\frac{s^2 - s}{2} \text{var}_h(c_{hk,t})\right). \quad (81)$$

Note that, by construction, the aggregate consumption is the sum of individual consumption, what can be checked by setting $s = 1$. Therefore, this log normal process satisfies the feasibility condition. To check whether this log normal process satisfies the optimality conditions, we follow a reverse engineering approach. If we can find a pricing kernel that supports this allocation of country k consumption and is also independent of the agent's private history, then it must be satisfying the individual optimality conditions.

A problem with using the KP (2007, 2009) DI and PIPO stochastic discount factors is that extreme observations in the right tail of the cross-section distribution of individual consumption may dramatically affect the high-order non-centred cross-sectional consumption moments (especially when the order of the moment, γ , is high (about 5 or 6), as in KP(2007, 2009)) and hence the model estimation results.¹⁰ In section 4.5, we investigate whether the pricing kernels constructed using the assumption of the cross-sectional log normality of individual consumption are sensitive to measurement and sampling errors in the consumption data.

¹⁰See, for example, Kollmann (2009), who shows that the use of the winsorized cross-sectional moments (that are less sensitive to measurement error in individual consumption) overturns the KP (2007) regression results.

3.2 Pricing Kernels

Plugging (81) into (44) and evaluating at $s = -\gamma$, we obtain the following pricing kernel for the DI environment from the home country's perspective:

$$SDF_{t+1}^{DI} = \beta \left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\gamma} \exp \left(\frac{\gamma + \gamma^2}{2} \Delta \text{var}_h (c_{h,t+1}) \right). \quad (82)$$

where \bar{C}_t is the home country aggregate per capita consumption at time t and $\Delta \text{var}_h (c_{h,t+1}) = \text{var}_h (c_{h,t+1}) - \text{var}_h (c_{h,t})$.

Likewise, plugging (81) into (74) and evaluating at $s = \gamma$, we get the stochastic discount factor associated with the PIPO environment from the home country's perspective:

$$SDF_{t+1}^{PIPO} = \beta \left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\gamma} \exp \left(\frac{\gamma - \gamma^2}{2} \Delta \text{var}_h (c_{h,t+1}) \right). \quad (83)$$

These specifications of the DI and PIPO pricing kernels are different from those in KP (2007, 2009) and are new to the literature. As we show in Section 4.5, in contrast with the specifications in KP (2007, 2009), the above stochastic discount factors are robust not only to the observation error in individual consumption, but also to sampling error, what makes the model estimation and testing results more reliable.

Under the assumption of complete risk sharing, within each country the agents are able to equalize state-by-state their IMRS and hence their optimal consumption growth rates. This implies that for both the home and foreign countries the cross-sectional variance of household consumption is constant over time and hence both the DI and PIPO stochastic discount factors for each country reduce to the pricing kernel in the representative-agent framework (the discounted aggregate per capita consumption growth rate in the respective country raised to the power of the negative utility curvature parameter).¹¹ The stochastic discount factor for this environment from the home country's perspective, for example, is:

$$SDF_{t+1}^{CRS} = \beta \left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\gamma}. \quad (84)$$

4 Empirical Investigation

In this section, we assess empirically the potential of the DI and PIPO pricing kernels derived in the previous section to account for the observed fluctuations of the real exchange rate, equity premium, risk-free rate of return, and currency premium and compare the performance of these models with the CRS model. Two countries under investigation are the US (treated as the home country) and the UK (treated as the foreign country).

¹¹When the consumption growth rate is the same for each agent, we can write $C_{h,t+1} = aC_{h,t}$ for each h , where a is a constant. This implies that $\text{var}_h (c_{h,t+1}) = \text{var}_h (\ln(a) + c_{h,t}) = \text{var}_h (c_{h,t})$ and hence $\Delta \text{var}_h (c_{h,t+1}) = 0$. Clearly, the same is true for the foreign country, i.e., $\Delta \text{var}_h (c_{h,t+1}^*) = 0$.

4.1 Data

Consumption. As is conventional in the literature, the consumption measure used in this paper is consumption of nondurables and services. For the US, data on household quarterly consumption of nondurables and services are from the US Consumer Expenditure Survey (CEX), produced by the Bureau of Labor Statistics (BLS). For each household, we calculate quarterly consumption expenditures for all the disaggregated consumption categories offered by the CEX. Then, we deflate obtained values in 2005:Q1 US dollars by the Consumer Price Indexes (CPI's) (not seasonally adjusted, urban consumers) for appropriate consumption categories.¹² Aggregating the household's quarterly consumption across these categories is made according to the National Income and Product Account (NIPA) definition of consumption of nondurables and services.

Following Brav *et al.* (2002), in each quarter we drop households that do not report or report a zero value of consumption of food, consumption of nondurables and services, or total consumption. We also delete from the sample the nonurban households, the households residing in student housing, the households with incomplete income responses, the households that do not have a fifth interview, and the households whose head is under 19 or over 75 years of age.

To calculate the household's quarterly per capita consumption, we divide the quarterly consumption expenditure of each household by the number of people in the household in that quarter.¹³ The within-country consumption variance for each quarter is then calculated as the cross-sectional variance of the log household's quarterly real, per capita consumption. Because of the poor quality of the CEX data before 1982, the sample period is from 1982:Q1 to 2004:Q4.

The US data on quarterly seasonally adjusted US dollar nominal aggregate consumption of nondurables and services are from the US Bureau of Economic Analysis (BEA). The real aggregate consumption of nondurables and services is calculated by dividing the nominal seasonally adjusted aggregate consumption of nondurables and services by the CPI (2005:Q1=1) for nondurables and services (from the BEA). The US aggregate per capita consumption is calculated by dividing the real aggregate consumption of nondurables and services by the US population (from the BEA).

For the UK, we use the UK Family Expenditure Survey (FES), a voluntary survey of a random sample of private households in the UK, conducted by the Office for National Statistics (ONS).¹⁴ The data of approximately 6,500 households are collected throughout the year to cover seasonal variations in expenditures, with either the week or month, in which the fieldwork is

¹²The CPI series are obtained from the BLS.

¹³In Section 3.1, we made the assumption that the individual consumption expenditures are log normally distributed in the cross-section. We use the Jarque-Bera statistic to check this assumption for the US. The evidence is that in the CEX the assumption of the cross-sectional log normality of the household's quarterly per capita consumption is not rejected statistically at the 1% significance level for a large majority of quarters.

¹⁴In April 2001, the FES was replaced by the Expenditure and Food Survey (EFS), which also covered the National Food Survey (NFS).

carried out, being randomly assigned to each individual household. Of the data available in the FES, we use the diary records of daily expenditure, kept for two weeks by each individual aged 16 or over in the household survey.

Using these diary data, the cross-sectional variance of the log household's quarterly real, per capita consumption of nondurables and services for each quarter is computed as follows.¹⁵ First, we calculate the household-wide consumption of nondurables and services by adding the consumption only of nondurables and services (measured in UK pounds) for each individual in the household. The definition of nondurables and services follows that of Attanasio and Weber (1995). Second, given that the household consumption data are for the two week durations only, we multiply them by 6.5, so that the data are converted into quarterly frequency. Third, we divide the quarterly consumption expenditure of each household by the number of people in the household in that quarter to derive quarterly nominal, per capita consumption of nondurables and services. Fourth, we categorize the household consumption data into four quarterly groups, based on the quarter or month the survey was conducted for the household. By dividing the data by the quarterly CPI for all items (not seasonally adjusted)¹⁶ with the basis of 2005:Q1, the quarterly real, per capita consumptions are calculated. Finally, we take the logarithms of the quarterly real, per capita consumptions calculated in the previous step, followed by the calculation of the cross-sectional variance of the log household's quarterly real, per capita consumption for each quarter.

The UK data on seasonally adjusted nominal aggregate consumption of nondurables and services are from the ONS and the UK Data Archive (UKDA). The UK real aggregate consumption of nondurables and services is calculated by dividing the nominal seasonally adjusted aggregate consumption of nondurables and services by the CPI (2005:Q1=1) for nondurables and services (from OECD main economic indicators). We calculate the UK aggregate per capita consumption of nondurables and services as the real aggregate consumption of nondurables and services divided by the UK population (from OECD main economic indicators).

When testing alternative pricing kernels, we use two different sets of household-level data on consumption expenditures from the US and the UK. The first set (we call it the BSW data set) is described above (as we mentioned above, due to the poor quality of the CEX data before 1982, this data set covers the period from 1982:Q1 to 2004:Q4). The second one is the data set used in KP (2007, 2009) (the KP data set, hereafter), which covers the period from 1982:Q1 to 1999:Q4.¹⁷ We refer the reader to KP (2007, 2009) for the description of their data on household-level consumption expenditures. Because the consumption definition and the sample selection

¹⁵Our procedure mimics KP (2009).

¹⁶The CPI is from the OECD main economic indicators.

¹⁷When using the data on household-level consumption expenditures from KP (2007, 2009), we do not consider the data for 1980 and 1981 due to their poor quality in the CEX. As a result, the data set we use in estimation is two years shorter than the data set used by KP (2007, 2009).

procedure in KP (2007, 2009) are different from ours,¹⁸ we believe that estimating the same models using these different data sets on consumption expenditures would provide a good check of whether the estimation and testing results are robust to the used measure of consumption and sample design.

The Spot and Forward Exchange Rates. The nominal spot, S_t , and 3-month forward, F_t , US Dollar to British Pound currency exchange rates are from DATASTREAM (series XUDLUSS and XUDLDS3, respectively). The real spot exchange rate, Q_t , is calculated as

$$Q_t = \frac{S_t CPI_t^*}{CPI_t}, \quad (85)$$

where CPI_t and CPI_t^* are respectively the US and UK CPI's (2005:Q1=1) for consumption of nondurables and services.

Asset Returns. We use three different proxies for the market portfolio return. The first two are the value-weighted, $R_{VW,t}$, and equal-weighted, $R_{EW,t}$, returns (capital gain plus dividends) on all stocks listed on the NYSE, AMEX, and NASDAQ. The data on the nominal quarterly value- and equal-weighted returns on all stocks listed on the NYSE, AMEX, and NASDAQ for the period from 1982:Q1 to 2004:Q4 are obtained from the Center for Research in Security Prices (CRSP) of the University of Chicago. We also view the market portfolio as consisting of five industry portfolio return indices. The nominal quarterly value-weighted returns on five NYSE, AMEX, and NASDAQ industry portfolios ((a) consumer durables, nondurables, wholesale, retail, and some services (laundries, repair shops), $R_{1,t}$, (b) manufacturing, energy, and utilities, $R_{2,t}$, (c) business equipment, telephone and television transmission, $R_{3,t}$, (d) healthcare, medical equipment, and drugs, $R_{4,t}$, and (e) other, $R_{5,t}$) are from Kenneth R. French's web page.

The risk-free rate is the 3-month US Treasury Bill secondary market rate on a per annum basis obtained from the Federal Reserve Bank of St. Louis. In order to convert from the annual rate to the quarterly rate, we raise the 3-month Treasury Bill return on a per annum basis to the power of 1/4.

The real quarterly returns are calculated as the nominal quarterly returns divided by the 3-month inflation rate based on the deflator defined for consumption of nondurables and services. We calculate the equity premium as the difference between the real equity return and the real risk-free rate.

Table I reports the descriptive statistics for the data sets used in estimation.

[Table I]

¹⁸As the measure of consumption, KP (2007, 2009) use non-durable consumption and not consumption of nondurables and services, as in our case. In this paper, we apply the household selection criteria used in Brav *et al.* (2002), which are different from the selection criteria in KP (2007, 2009).

4.2 A First Look at the Financial Market Puzzles

Perhaps the simplest way to assess the plausibility of a candidate stochastic discount factor is to assume joint conditional lognormality and homoskedasticity of the variables in the Euler equations and then to analyze the restrictions on asset returns implied by the pricing kernel.

In the CRS specification, joint conditional lognormality and homoskedasticity imply that the expected equity premium is linear in the variance of the market portfolio return and covariance of the return on the market portfolio with consumption growth:

$$E_t [r_{M,t+1} - r_{f,t+1}] = -\frac{1}{2}var(r_{M,t+1}) + \gamma cov(r_{M,t+1}, \Delta c_{t+1}), \quad (86)$$

where $r_{M,t+1}$ is the log return on the market portfolio, $r_{f,t+1}$ is the logarithm of the risk-free rate of return, and Δc_{t+1} is the logarithm of the aggregate per capita consumption growth rate between t and $t + 1$.

The risk-free rate obeys

$$r_{f,t+1} = -\ln(\beta) - \frac{\gamma^2}{2}var(\Delta c_{t+1}) + \gamma E_t [\Delta c_{t+1}]. \quad (87)$$

Denote $\ln\left(\frac{F_t \bar{P}_t}{S_t \bar{P}_{t+1}}\right)$ as $s_{1,t+1}$ and $\ln\left(\frac{S_{t+1} \bar{P}_t}{S_t \bar{P}_{t+1}}\right)$ as $s_{2,t+1}$. With these notations, we can write the equation for the currency premium as

$$E_t [s_{1,t+1} - s_{2,t+1}] = -\frac{1}{2}(var(s_{1,t+1}) - var(s_{2,t+1})) + \gamma(cov(s_{1,t+1}, \Delta c_{t+1}) - cov(s_{2,t+1}, \Delta c_{t+1})). \quad (88)$$

Under the assumption of complete risk sharing, the equation for the log of the growth rate in the real exchange rate is

$$\Delta q_{t+1} = \gamma(\Delta c_{t+1} - \Delta c_{t+1}^*). \quad (89)$$

Substituting the moments in Table II into equation (86) shows that a relative risk aversion coefficient of 593.68 is required for the CRS model to explain the observed mean excess log return on the market portfolio. As follows from equation (87), this high value of the risk aversion coefficient γ implies a subjective time discount factor of 2.34. These results illustrate the equity premium and risk-free rate puzzles.

[Table II]

One can appreciate the currency premium puzzle by examining equation (88). The moments reported in Table II imply that the expected currency premium predicted by the CRS model is much higher than the observed value when γ is 593.68. To fit the observed mean log currency premium, the representative agent must be highly risk seeking. This is the currency premium puzzle.

From equation (89), it follows that the log real exchange rate growth must be perfectly correlated with the difference in the log growth rates of aggregate per capita consumption of respective countries. However, as it can be seen from Table II, the correlation between relative consumption and the real exchange rate is negative. This illustrates the consumption-real exchange rate puzzle.

For the PIPO stochastic discount factor, with joint conditional lognormality and homoskedasticity of the variables in the Euler equations, we obtain for the log risk premium:

$$E_t[r_{M,t+1} - r_{f,t+1}] = -\frac{1}{2}var(r_{M,t+1}) + \gamma cov(r_{M,t+1}, \Delta c_{t+1}) - \frac{\gamma - \gamma^2}{2} cov(r_{M,t+1}, \Delta var_h(c_{h,t+1})) \quad (90)$$

and for the log risk-free rate:

$$\begin{aligned} r_{f,t+1} = & -\ln(\beta) - \frac{\gamma^2}{2} var(\Delta c_{t+1}) + \gamma E_t[\Delta c_{t+1}] - \frac{\gamma - \gamma^2}{2} E_t[\Delta var_h(c_{h,t+1})] \\ & - \frac{(\gamma - \gamma^2)^2}{8} var(\Delta var_h(c_{h,t+1})) + \frac{\gamma(\gamma - \gamma^2)}{2} cov(\Delta c_{t+1}, \Delta var_h(c_{h,t+1})). \end{aligned} \quad (91)$$

At economically realistic (below 10) values of the risk aversion parameter, the CRS model yields the mean log excess return, which is too low compared with the value observed in the data. Because, as follows from Table II, $cov(r_{M,t+1}, \Delta var_h(c_{h,t+1})) > 0$, we may expect the PIPO model with $\gamma > 1$ to generate (at the same values of γ) a higher log equity premium and hence to fit the observed log excess return on the market portfolio over the risk-free rate with a lower, compared with the CRS model, value of the relative risk aversion coefficient. Using the data from Table II, we obtain that the PIPO model can replicate the mean log equity premium with γ equal to 29.89, what supports the intuition above.

However, at this value of γ , the mean log risk-free rate generated by the PIPO model is much lower than the observed value, implying a subjective time discount factor, required to fit the observed log risk-free rate, close to zero.¹⁹ This is due to the fact that, as reported in Table II, $E_t[\Delta var_h(c_{h,t+1})] < 0$ and $cov(\Delta c_{t+1}, \Delta var_h(c_{h,t+1})) > 0$, implying, according to equation (91), that uninsurable risk drives down the log risk-free rate yielded by the model when $\gamma > 1$. Therefore, although the PIPO model can lower the value of risk aversion required to explain the mean log equity premium, this model tends to produce an implausibly low mean log real interest rate.

The PIPO model implies the following restriction on the currency premium

$$\begin{aligned} E_t[s_{1,t+1} - s_{2,t+1}] = & -\frac{1}{2}(var(s_{1,t+1}) - var(s_{2,t+1})) \\ & + \gamma(cov(s_{1,t+1}, \Delta c_{t+1}) - cov(s_{2,t+1}, \Delta c_{t+1})) \\ & - \frac{\gamma - \gamma^2}{2}(cov(s_{1,t+1}, \Delta var_h(c_{h,t+1})) - cov(s_{2,t+1}, \Delta var_h(c_{h,t+1}))). \end{aligned} \quad (92)$$

¹⁹KP (2009) also emphasize this problem with the subjective time discount factor in the PIPO framework.

For the data set under consideration, the difference between $cov(s_{1,t+1}, \Delta var_h(c_{h,t+1}))$ and $cov(s_{2,t+1}, \Delta var_h(c_{h,t+1}))$ is negative and hence, with $\gamma > 1$, the PIPO model yields the mean log currency premium that is lower, at the same value of γ , than the mean log currency premium in the case of complete risk sharing. The mean log currency premium increases as the risk aversion coefficient decreases. Despite this, with γ equal 29.89 the mean log currency premium generated by the PIPO model remains too low compared with the observed value and a lower value of γ (a γ of 14.31) is needed for the model to fit the observed currency premium.

In the PIPO economy, the log of the growth rate in the real exchange rate is

$$\Delta q_{t+1} = \gamma (\Delta c_{t+1} - \Delta c_{t+1}^*) - \frac{\gamma - \gamma^2}{2} (\Delta var_h(c_{h,t+1}) - \Delta var_h(c_{h,t+1}^*)). \quad (93)$$

Within the PIPO setting, agents buy contracts from insurance firms to insure against individual shocks subject to incentive constraints. Hence, there are two opposite effects: the precautionary saving effect and the incentive effect. When the coefficient of relative risk aversion is lower than 1, the precautionary saving effect dominates and therefore the home country currency appreciates with the increase in the home country uninsurable risk (i.e., the increase in the value of $\Delta var_h(c_{h,t+1})$). The higher the agent's aversion to risk, the greater the incentive effect, so that, when γ exceeds 1, the incentive effect dominates the precautionary effect, what results in the depreciation of the home country currency.

As we can see from Table II, the log of the growth rate in the real exchange rate, Δq_{t+1} , is negatively correlated with the term $(\Delta var_h(c_{h,t+1}) - \Delta var_h(c_{h,t+1}^*))$ and hence the new (compared with the CRS model) term in the right-hand side part of equation (93) can help explain volatility of the log of the growth rate in the real exchange rate only if $\gamma < 1$, while the value of the risk aversion coefficient greater than 1 is required to explain the equity premium.

To check whether the results are different for the DI model, observe that in the DI environment the restrictions on the expected equity premium, risk-free rate, currency premium, and the log of the growth rate in the real exchange rate are as follows.

The expected log excess return on the market portfolio over the risk-free rate is

$$E_t[r_{M,t+1} - r_{f,t+1}] = -\frac{1}{2}var(r_{M,t+1}) + \gamma cov(r_{M,t+1}, \Delta c_{t+1}) - \frac{\gamma + \gamma^2}{2} cov(r_{M,t+1}, \Delta var_h(c_{h,t+1})). \quad (94)$$

The equation for the log risk-free rate is

$$\begin{aligned} r_{f,t+1} = & -\ln(\beta) - \frac{\gamma^2}{2} var(\Delta c_{t+1}) + \gamma E_t[\Delta c_{t+1}] - \frac{\gamma + \gamma^2}{2} E_t[\Delta var_h(c_{h,t+1})] \\ & - \frac{(\gamma + \gamma^2)^2}{8} var(\Delta var_h(c_{h,t+1})) + \frac{\gamma(\gamma + \gamma^2)}{2} cov(\Delta c_{t+1}, \Delta var_h(c_{h,t+1})). \end{aligned} \quad (95)$$

For the expected log currency premium, we have

$$\begin{aligned}
E_t [s_{1,t+1} - s_{2,t+1}] &= -\frac{1}{2} (var (s_{1,t+1}) - var (s_{2,t+1})) \\
&\quad + \gamma (cov (s_{1,t+1}, \Delta c_{t+1}) - cov (s_{2,t+1}, \Delta c_{t+1})) \\
&\quad - \frac{\gamma + \gamma^2}{2} (cov (s_{1,t+1}, \Delta var_h (c_{h,t+1})) - cov (s_{2,t+1}, \Delta var_h (c_{h,t+1})))
\end{aligned} \tag{96}$$

and, finally, the real exchange rate equation is

$$\Delta q_{t+1} = \gamma (\Delta c_{t+1} - \Delta c_{t+1}^*) - \frac{\gamma + \gamma^2}{2} (\Delta var_h (c_{h,t+1}) - \Delta var_h (c_{h,t+1}^*)). \tag{97}$$

In the DI framework, higher home country uninsurable risk (that results in a higher value of $\Delta var_h (c_{h,t+1})$) raises the precautionary demand for both traded and non-traded goods, which makes the home country currency appreciate.²⁰ As in the case of the PIPO model, within the DI setting in the absence of uninsurable risk the equation for the real exchange rate reduces to the Backus-Smith (1993) specification.

It is observed that at γ equal to 593.68 (the value of the risk aversion coefficient, at which the CRS model fits the observed mean log excess return on the market portfolio), the mean log equity premium produced by the DI model is very low. As γ decreases, the mean log excess market portfolio return generated by the model increases, but always remains below the observed value for any non-negative value of γ , so that, in contrast with the PIPO model, there is no value of the risk aversion parameter, at which the DI model fits the observed mean log equity premium. This result is in the line with KP (2009).²¹

For the DI model, the effect of uninsurable risk on the log risk-free rate depends on the sign of the sum of the fourth through six terms in the right-hand side of equation (95). At high values of the risk aversion parameter γ , the log risk-free rate generated by the DI model is very low and increases as γ decreases. With the moments reported in Table II and β set to 1, the DI model fits the observed mean log risk-free rate at $\gamma = 1.02$. This is due to the positive effect of the fourth and sixth terms in the right-hand side of equation (95), which overweight the negative effect of the fifth term at any $\gamma > 0.54$.

The mean log currency premium generated by the DI model is implausibly large at high values of the relative risk aversion coefficient, but decreases when γ decreases. However, with the moments reported in Table II, there is no positive value of γ , at which the DI model fits the observed mean log currency premium.

As of the equation for the log of the growth rate in the real exchange rate, it may be seen from equation (96) that, because the term $(\Delta var_h (c_{h,t+1}) - \Delta var_h (c_{h,t+1}^*))$ in the right-hand side of

²⁰Since $\gamma > 0$, for any value of γ the home country currency depreciates (appreciates) in response to the increase in the foreign (home) within-country variance in the DI environment. In the PIPO model, the implication is the same if $\gamma < 1$ and is exactly reverse as long as $\gamma > 1$.

²¹When using calibration, KP (2009) find that the PIPO model can explain the observed mean equity premium with a risk aversion parameter between 5 and 6, while there is no value of the relative risk aversion coefficient that enables to explain the mean excess return on the market portfolio in the DI environment.

this equation is negatively correlated with Δq_{t+1} and the coefficient of this term is negative for any $\gamma > 0$, this new term may have significant additional explanatory power beyond the difference in the log growth rates of aggregate per capita consumption of respective countries in explaining volatility of the log of the growth rate in the real exchange rate.

The results reported above suggest that there is no pricing kernel that is able to explain simultaneously all the considered asset pricing anomalies. The PIPO model has the potential to solve the equity and currency premia puzzles, while the DI model is likely to outperform the CRS model in explaining the risk-free rate and the log of the growth rate in the real exchange rate. It is especially unlikely that there is a value of the risk aversion parameter that is the same for all the equations and that allows to explain all the puzzles simultaneously. In this situations, it is more reasonable to look for the pricing kernel that enables to explain the asset pricing anomalies as better as possible (not necessarily exactly).

Note that the approach we used above has some limitations due to the assumptions of joint conditional log normality and homoskedasticity we made to derive the restrictions on the equity and currency premia and the risk-free rate implied by each model. In this context, the GMM estimation technique seems to be more appropriate. The advantage of the GMM approach is that, when implementing this technique, it is not necessary to make any assumptions about the distributions of the variables in the Euler equations. In contrast to calibration, the GMM technique enables to estimate conditional (and not only unconditional, as in the case of calibration) Euler equations, as required by economic theory. The use of additional orthogonality conditions as well as taking into consideration time-series properties of the variables in the estimated equations and possible correlation of the error terms in the equations may dramatically affect the inferences about the relative performance of each model.²²

In the next section, we assess the empirical performance of each of the stochastic discount factors considered in this paper using the GMM estimation approach. When using the GMM technique, we look for the values of the risk aversion coefficient and the subjective time discount factor that make the sample analogs of the orthogonality conditions implied by a stochastic discount factor as close as possible to zero. This helps us to choose a pricing kernel that is the best (among the considered pricing kernels) in the sense of minimizing (at the values of the parameters that are the same for all the equations) the difference of the sample analogs of the orthogonality conditions associated with the stochastic discount factor from their theoretical values implied by economic theory.

²²It is worth noting that calibration is a special case of the GMM estimation when the set of instruments has a constant only and is therefore equivalent to the unconditional GMM.

4.3 The GMM Estimation and Testing Results

We consider three alternative environments, namely the CRS, DI, and PIPO environments. When assessing the performance of the candidate stochastic discount factors, we first estimate the parameters of each model using the GMM estimation technique and investigate whether the pricing kernel under consideration is able to make the moment conditions as close as possible to zero at economically plausible values of the agent's preference parameters. Then, we check the predictive ability of each stochastic discount factor at the values of the parameters obtained from the GMM estimation.

For each environment, we jointly estimate four equations. These equations are the real exchange rate equation and the Euler equations for the equity premium, the risk-free rate of return, and the currency premium. Hence, in contrast with KP (2007, 2009), who test the ability of the PIPO pricing kernel to explain the exchange rate puzzle (KP, 2007) and the equity premium puzzle (KP, 2009) in isolation, in each environment we address four puzzles (i.e., the consumption-real exchange rate, equity premium, risk-free rate, and currency premium puzzles) jointly in an integrated framework with the stochastic discount factor, which is the same in all the equations.

Another advantage of our approach is that instead of using calibration, as in KP (2009), for example, we estimate the models using the GMM estimation technique. As we argued above, this allows us to estimate the parameters of not only unconditional Euler equations, as this is the case with calibration, but also conditional Euler equations, as required by economic theory. The use of additional restrictions, implied by the condition that the forecast errors associated with Euler equations are uncorrelated with any variables that are in the agent's information set (a set of instruments), may significantly affect the estimation results and therefore inferences about the appropriateness of each of the considered models. Moreover, the use of the GMM estimation approach enables us to calculate the confidence intervals for the values of parameters, what is impossible when calibration is used. One more advantage of the use of the GMM is that this approach, in contrast with calibration, allows to take into consideration time-series properties of the variables in the equations as well as possible correlation of the error terms in the estimated equations.

In estimation, we use five different sets of instruments. These sets are the same as in Epstein-Zin (1991). The first set has a constant and the US aggregate per capita consumption growth rate lagged one and two periods.²³ The second set has a constant, the US aggregate per capita consumption growth rate lagged one period and the return on the market portfolio (proxied by the value-weighted return on all stocks listed on the NYSE, AMEX, and NASDAQ) lagged one

²³As argued by Hall (1988), the second lag in instrumental variables helps in reducing the effect of time aggregation. Furthermore, Ogaki (1988) demonstrates that the use of the second lag is consistent with the information structure of a monetary economy with cash-in-advance constraints.

period. The third set has a constant, the US aggregate per capita consumption growth rate lagged one period and the return on the market portfolio (proxied by the equal-weighted return on all stocks listed on the NYSE, AMEX, and NASDAQ) lagged one period. The fourth and fifth sets are our respectively second and third sets of instrumental variables lagged an additional period.

To check whether the estimation and testing results are robust to the chosen measure of consumption and the sample selection criteria, we estimate the DI and PIPO models for the period from 1982:Q1 to 1999:Q4 using both the KP (2007, 2009) and BSW data on household-level consumption expenditures. Estimating the model parameters for the sample period that extends from 1982:Q1 to 2004:Q4 as well as for the subsample corresponding to the sample period used by KP (2009) ending in 1999:Q4 enables to investigate whether the results are robust to the time period under consideration.

Below, we report the GMM estimation and testing results for each of the three considered pricing kernels.

The CRS Model. This is our benchmark model. Recall that the stochastic discount factor for this environment from the home country's perspective is given by (84) and hence, under the assumption of complete risk sharing, we jointly estimate the Euler equation for the equity premium as

$$E_t \left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\gamma} (R_{M,t+1} - R_{F,t+1}) \right] = 0, \quad (98)$$

the Euler equation for the risk-free rate as

$$E_t \left[\beta \left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\gamma} R_{F,t+1} \right] = 1, \quad (99)$$

the Euler equation for the currency premium as

$$E_t \left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\gamma} \frac{(F_t - S_{t+1})\bar{P}_t}{S_t\bar{P}_{t+1}} \right] = 0 \quad (100)$$

and the regression equation for the real exchange rate as

$$\Delta q_{t+1} = \gamma [\Delta c_{t+1} - \Delta c_{t+1}^*] + \zeta_{t+1}. \quad (101)$$

Table III reports the GMM estimation results. We find that the estimate of the relative risk aversion coefficient, that makes the moment conditions as close as possible to zero, is within a conventional range of values and statistically different from zero at any conventional level of significance. The point estimate of the subjective time discount factor is always greater than 1 and, in most cases, statistically different from 1 at the 5% significance level. The model is not rejected statistically by Hansen's test of overidentifying conditions at the 5% level of significance.

These results are robust to the used set of instruments, the proxy for the market portfolio, the time period, and the data set used in estimation.

[Table III]

The PIPO Model. Under the assumption that agents can insure consumption using the domestic financial markets, but, due to private information about agents' skill shocks and their work effort, financial intermediaries strike incentive compatible contract, which prevents complete risk sharing, the pricing kernel is given by (83) and therefore we jointly estimate the Euler equation for the excess return on the market portfolio over the risk-free rate as

$$E_t \left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\gamma} \exp \left(\frac{\gamma - \gamma^2}{2} \Delta var_h (c_{h,t+1}) \right) (R_{M,t+1} - R_{F,t+1}) \right] = 0, \quad (102)$$

the Euler equation for the risk-free rate as

$$E_t \left[\beta \left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\gamma} \exp \left(\frac{\gamma - \gamma^2}{2} \Delta var_h (c_{h,t+1}) \right) R_{F,t+1} \right] = 1, \quad (103)$$

the Euler equation for the currency premium as

$$E_t \left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\gamma} \exp \left(\frac{\gamma - \gamma^2}{2} \Delta var_h (c_{h,t+1}) \right) \frac{(F_t - S_{t+1})\bar{P}_t}{S_t\bar{P}_{t+1}} \right] = 0 \quad (104)$$

and the regression equation for the log real exchange rate growth between the home and foreign countries as

$$\Delta q_{t+1} = \gamma [\Delta c_{t+1} - \Delta c_{t+1}^*] + \frac{\gamma - \gamma^2}{2} (\Delta var_h (c_{h,t+1}^*) - \Delta var_h (c_{h,t+1})) + \epsilon_{t+1}. \quad (105)$$

Table IV shows that the estimate of the coefficient of relative risk aversion for this model is slightly lower than the estimate for the CRS model and is also statistically different from zero at any conventional level of significance. The PIPO model is not rejected by Hansen's test of overidentifying conditions at the 5% level of significance. These results are similar to the results we obtained for the CRS model. Another common feature of these two models is that in the PIPO model the subjective time discount factor is also estimated to be greater than 1, but, in contrast with the CRS model, is not statistically different from 1 in most cases.

[Table IV]

The DI Model. Because in the DI environment the stochastic discount factor is given by (82), we jointly estimate the Euler equation for the equity premium as

$$E_t \left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\gamma} \exp \left(\frac{\gamma + \gamma^2}{2} \Delta var_h (c_{h,t+1}) \right) (R_{M,t+1} - R_{F,t+1}) \right] = 0, \quad (106)$$

the Euler equation for the risk-free rate as

$$E_t \left[\beta \left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\gamma} \exp \left(\frac{\gamma + \gamma^2}{2} \Delta var_h (c_{h,t+1}) \right) R_{F,t+1} \right] = 1, \quad (107)$$

the Euler equation for the currency premium as

$$E_t \left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\gamma} \exp \left(\frac{\gamma + \gamma^2}{2} \Delta var_h (c_{h,t+1}) \right) \frac{(F_t - S_{t+1}) \bar{P}_t}{S_t \bar{P}_{t+1}} \right] = 0 \quad (108)$$

and the regression equation that relates the log real exchange rate growth to individual consumption as

$$\Delta q_{t+1} = \gamma [\Delta c_{t+1} - \Delta c_{t+1}^*] + \frac{\gamma + \gamma^2}{2} (\Delta var_h (c_{h,t+1}^*) - \Delta var_h (c_{h,t+1})) + \eta_{t+1}. \quad (109)$$

As follows from Table V, although the relative risk aversion coefficient is often estimated imprecisely, it is always in the conventional range. Likewise the CRS and PIPO models, the DI model is not rejected by Hansen's test of overidentifying conditions at the 5% significance level. However, in contrast with the previous two models, the DI model yields the estimates of the subjective time discount factor that are (except a few cases) close to but lower than 1, as required by economic theory.

[Table V]

4.4 The Predictive Ability of the Models

The goal of this section is to investigate whether the stochastic discount factors considered in this paper can fit the observed fluctuations of the real exchange rate, the equity premium, the risk-free rate of return, and the currency premium at the values of the parameters obtained in the GMM estimation above.

We use the property that for any asset i

$$E_t [SDF_{t+1} R_{i,t+1}] = 1. \quad (110)$$

By taking unconditional expectations of the both sides of equation (110) and lagging one period (to simplify notations), we obtain

$$E [SDF_t R_{i,t}] = 1. \quad (111)$$

For the excess return on the market portfolio over the risk-free rate,

$$E [SDF_t (R_{M,t} - R_{F,t})] = 0. \quad (112)$$

Because

$$E [SDF_t (R_{M,t} - R_{F,t})] = cov (SDF_t, R_{M,t} - R_{F,t}) + E [SDF_t] E [R_{M,t} - R_{F,t}], \quad (113)$$

we can write for the equity premium

$$E[R_{M,t} - R_{F,t}] = -\frac{\text{cov}(SDF_t, R_{M,t} - R_{F,t})}{E[SDF_t]}. \quad (114)$$

Equivalently, for the currency premium, we obtain

$$E\left[\frac{(F_{t-1} - S_t)\bar{P}_{t-1}}{S_{t-1}\bar{P}_t}\right] = -\frac{\text{cov}\left(SDF_t, \frac{(F_{t-1} - S_t)\bar{P}_{t-1}}{S_{t-1}\bar{P}_t}\right)}{E[SDF_t]}. \quad (115)$$

For the risk-free rate,

$$E[R_{F,t}] = \frac{1 - \text{cov}(SDF_t, R_{F,t})}{E[SDF_t]}. \quad (116)$$

To check the ability of a candidate pricing kernel to predict the mean equity premium, for example, we first calculate the average excess return on the proxy for the market portfolio²⁴ over the risk-free rate for the first 30 quarters using equation (114), when the value of the pricing kernel for each quarter within the 30-quarter sample period is calculated at the GMM estimates of the parameters reported in the previous section.²⁵ Then, we move one quarter ahead and calculate the average equity premium generated by a model over quarters 2 to 31, etc. Repeating this rolling sample procedure, we obtain 62 estimates of the average (over 30 quarters) excess returns on the market portfolio over the risk-free rate. To assess the quality of fitting, we use two different measures of goodness-of-fit. The first measure is the coefficient of correlation between the value of the mean excess return predicted by the model and the value observed in the data. The second measure used in the paper is the mean squared error (MSE) of prediction. We implement the same procedure to assess the ability of each candidate stochastic discount factor to fit the observed fluctuations of the risk-free rate (using equation (116)) and the currency premium (using equation (115)).

For the log real exchange rate growth between two countries, we directly use the associated equation and estimate for each quarter the predicted log growth in the exchange rate as the value of the right-hand side part of the equation calculated at the GMM parameter estimates. Because we can do this for each quarter within the time period from 1982:Q1 to 2004:Q4, we generate 91 predicted values (one value per quarter). After that, as for the equity premium, the risk-free rate, and the currency premium, we calculate the coefficient of correlation between the predicted and observed values as well as the MSE of prediction.

²⁴As proxies for the return on the market portfolio, we use the value- and equal-weighted returns on all stocks listed on the NYSE, AMEX, and NASDAQ.

²⁵The choice of the length of a sample period is somewhat arbitrary. We choose it to be equal 30 quarters for the following reasons. The first reason is that this length allows us to quite accurately estimate the population mean leaving the opportunity to get a sufficiently long time-series of the estimates of the mean values. The second reason is that this length is comparable with the average duration of business cycles since November 1982 until December 2004. As documented by the US National Bureau of Economic Research (NBER), there were three business cycles with troughs in November 1982, March 1991, and November 2001 with the average duration of 28.4 quarters (trough from previous trough).

Table VI reports the results. Panel A shows the testing results when the proxy for the market portfolio is the CRSP value-weighted index. In Panel B, we report the results for the CRSP equal-weighted index.

[Table VI]

When comparing the relative performance of the considered stochastic discount factors, we can see that for the both proxies for the market portfolio the PIPO model is very close to the CRS model in its ability to predict the risk-free rate. The both models predict the mean risk-free rate of return, which is highly positively correlated with the observed mean risk-free rate and is characterized by a low value of the MSE. The CRS and PIPO models both fail to accurately predict the mean equity premium. For the both models, prediction is negatively correlated with the observed value when the CRSP value-weighted index is used as a proxy for the market portfolio. The PIPO model performs even worse than the CRS model when the proxy is the CRSP equal-weighted index. Although the MSE for the mean currency premium is similar for the both models, the CRS model outperforms the PIPO model in terms of correlation between the predicted and observed mean currency premia. For the CRS model, the coefficient of correlation is much greater and hence this model is able to better predict the direction of movement of the mean currency premium. The both models fail to predict the log of the growth rate in the real exchange rate. The values generated by the both models are negatively correlated with the values observed in the data. The performance of the PIPO model is even worse than that of the CRS model. The coefficient of correlation for the PIPO model is much lower than the correlation coefficient obtained for the CRS model.

In contrast with the CRS and PIPO models, the predictive ability of the DI model is not sensitive to the used proxy for the market portfolio return. The DI model works much better than the other two models in predicting the mean excess return on the market portfolio over the risk free-rate. The generated by the DI model and observed mean values of the equity premium are always positively correlated, while they are positively or negatively correlated, depending on the used proxy for the market return, for the CRS and PIPO models. Although the MSE for the mean currency premium for the three models is practically the same, the DI model yields the predicted value that is highly positively correlated with the observed value. The DI model also significantly outperforms the CRS and PIPO models in predicting the log of the growth rate in the real exchange rate. This is the only (among the considered) model that generates the log of the growth rate, which is positively correlated with the value observed in the data. This result is not surprising given the intuition presented in Section 4.2. Perhaps the only point, at which the DI model is slightly outperformed by the CRS and PIPO models is its ability to predict the mean risk-free rate. As for the other two models, the predicted mean risk-free rate is highly positively correlated with the observed mean value, but the MSE for the DI model is

higher than for the other two models.

Figures 1-3 illustrate graphically the performance of each of the three models when the CRSP value-weighted index is used as the proxy for the market portfolio and the estimates of the parameters are those obtained for the instrumental variable set INST1.²⁶ The black line in each figure presents the observed values and the grey line presents the values generated by a model. In the figures for the mean equity premium and the mean currency premium, the left vertical axis is for the observed value and the right vertical axis is for the predicted value.

[Figures 1-3]

As can be seen from Figures 1-3, although the DI model generates the mean currency premium, which is more correlated (compared with the CRS model and, especially, the PIPO model) with the observed mean currency premium, it is still unable to replicate the highly volatile excess return on currency at an economically realistic estimate of the risk aversion parameter and hence is still unable to solve the currency premium puzzle. The evidence is also that the DI model outperforms the CRS and PIPO models in predicting the log real exchange rate growth. The predicted and observed values of the log of the growth rate in the real exchange rate are positively correlated (instead of being negatively correlated, as in the case of the CRS and PIPO models), but the observed real exchange rate is more volatile than predicted by the DI model, so that the consumption-real exchange rate anomaly still remains a puzzle. Despite the fact that, in contrast with the CRS and PIPO models, the DI model yields plausible estimates of the agent's preference parameters when the four anomalies are considered jointly in an integrated framework, the mean equity premium predicted by this model is very close to zero. This is the sign of the equity premium puzzle. It must however be noted that the predictive ability of the three candidate pricing kernels is quite good when these stochastic discount factors are used to predict the risk-free rate.

4.5 Measurement and Sampling Errors in the Consumption Data

In estimation, we use the home and foreign country first-differences of the cross-sectional variances of log consumption calculated from quarterly data on individual consumption expenditures. In this section, we investigate how the first-differences of the cross-sectional variances and therefore the GMM estimation and testing results might be affected if individual consumption is measured with error. As in KP (2007, 2009), we focus our attention on the implications of the measurement and sampling errors in individual consumption.

²⁶Because, as it is shown in Table VI, the properties of the values generated by each of the considered models only slightly depend on the used set of instruments, we report the figures for only one set, namely INST1. The results for the other sets of instrumental variables are similar.

Assume first, as in Vissing-Jørgensen (2002), that an observation error in the consumption level, $\varepsilon_{hk,t}$, is multiplicative and unbiased, i.e.,

$$C_{hk,t} = C_{hk,t}^T \varepsilon_{hk,t}, \quad (117)$$

where $C_{hk,t}$ and $C_{hk,t}^T$ are respectively the observed and true consumption of household h in country k in period t , and the observation error $\varepsilon_{hk,t}$ is independently and identically distributed across households with mean 1 and variance $var_h(\varepsilon_{hk,t})$, $\varepsilon_{hk,t} \sim IID(1, var_h(\varepsilon_{hk,t}))$,²⁷ and independent of the true consumption level $C_{hk,t}^T$.

Since the exact form of the observation error is typically unknown, following Vissing-Jørgensen (2002), assume that $\varepsilon_{hk,t}$ is independently and identically log-normally distributed (in the cross section), $ln(\varepsilon_{hk,t}) \sim IIDN(\mu_{k,t}, var_h(ln(\varepsilon_{hk,t})))$.

Because the observation error is independent of the true consumption,

$$var_h(c_{hk,t}) = var_h(c_{hk,t}^T) + var_h(ln(\varepsilon_{hk,t})) \quad (118)$$

and hence

$$\Delta var_h(c_{hk,t+1}) = \Delta var_h(c_{hk,t+1}^T) + \Delta var_h(ln(\varepsilon_{hk,t+1})). \quad (119)$$

The assumption that the measurement error $\varepsilon_{hk,t}$ is homoskedastic over time implies that $ln(\varepsilon_{hk,t})$ is stationary over time with mean $\mu_{k,t} = ln(2/\sqrt{1 + var_h(\varepsilon_{hk,t})/4})$ and variance $var_h(ln(\varepsilon_{hk,t})) = ln(1 + var_h(\varepsilon_{hk,t})/4)$. From this, it follows that $\Delta var_h(ln(\varepsilon_{hk,t+1})) = 0$ and therefore

$$\Delta var_h(c_{hk,t+1}) = \Delta var_h(c_{hk,t+1}^T) \quad (120)$$

for any t .

This suggests that the observation error of the form assumed here does not affect the first-differences of the sample cross-sectional variances of log consumption and therefore does not affect the estimation and testing results.

The situation may be different if $\varepsilon_{hk,t}$ is not homoskedastic over time and thus $ln(\varepsilon_{hk,t})$ is not time stationary. Assume that $var_h(ln(\varepsilon_{hk,t}))$ is α ($0 < \alpha < 1$) times $var_h(c_{hk,t})$. In this case,

$$\Delta var_h(c_{hk,t+1}^T) = (1 - \alpha) \Delta var_h(c_{hk,t+1}) \neq \Delta var_h(c_{hk,t+1}) \quad (121)$$

meaning that, in the presence of measurement error, the estimates of the relative risk aversion coefficient and the subjective time discount factor will both be biased if the observed (and not true) consumption levels are used to calculate the first-differences of the cross-sectional variances of log consumption.

The estimates of the cross-sectional variance of log consumption are generally subject to sample-to-sample variation (sampling error). This raises the question of the admissibility of using

²⁷See also Brav *et al.* (2002).

the sample variance instead of the population variance when estimating the model parameters. Note that in our data set in each quarter the number of observations on household consumption expenditures is between 700 and 1400. The law of large numbers implies that, when the number of households in the cross section in each period goes to infinity, the sample variance converges to the population variance (the sample variance is a consistent estimator of the population variance). It follows that, with such a large number of observations in the cross section, we may expect the sample cross-sectional variance to stay close to the population variance and hence may expect sampling error not to affect the estimation and testing results.²⁸

Because, in contrast with sampling error, the observation error in individual consumption biases the estimates of the parameters of interest when the observation error of the form assumed here is heteroskedastic over time, we assess the magnitude and direction of this bias by setting α equal to 0.3 and reestimating the DI and PIPO models with $\Delta var_h(c_{hk,t}^T) = 0.7\Delta var_h(c_{hk,t})$ for all t using the BSW data set for the period from 1982:Q1 to 2004:Q4. The estimation results are reported in Table VII. We find that, when observation error is taken into consideration, the estimation results for the both models become closer to the results obtained for the CRS model. This finding is not surprising given that, as we argued above, when the first-differences of the cross-sectional variances of log consumption converge to zero, both the DI and PIPO models converge to the CRS model. The three models coincide when the first-differences of the cross-sectional variances of log consumption equal zero for all t . This means that if observation error is large and properly taken into consideration, then the estimates of the parameters of the DI and PIPO models might be very close to (perhaps even indistinguishable from) the estimates obtained for the CRS model.

[Table VII]

In this paper, we do not directly investigate whether trimming and winsorizing affect the estimation results for the PIPO and DI models, as it is done in Kollmann (2009). However, the intuition is that, since trimming and winsorizing both reduce the volatility of the cross-sectional distribution of log consumption, we may expect them to also reduce the changes in the cross-sectional variances of log consumption in both the home and foreign countries. This case is very close to the case we considered above when estimated the PIPO and DI models with $\Delta var_h(c_{hk,t}^T) = 0.7\Delta var_h(c_{hk,t})$ and hence we may expect that trimming and winsorizing would make the estimation and testing results for the PIPO and DI models closer to the results for the CRS model.

²⁸As mentioned in KP (2007), sampling error might be a serious problem when estimating the parameters of the pricing kernel calculated as the ratio of the γ th uncentered moments of the cross-sectional distribution of consumption in periods t and $t + 1$.

5 Conclusion

This paper addressed a few extant domestic and international financial markets anomalies, namely the equity premium, risk-free rate, consumption-real exchange rate, and currency premium puzzles. We investigated the potential of two stochastic discount factors, which allow incomplete risk sharing in economies with consumer heterogeneity, to resolve these anomalies in an integrated framework. The first stochastic discount factor is the DI pricing kernel. This stochastic discount factor describes the market structure with domestically incomplete financial markets, where idiosyncratic privately observed shocks are uninsured, while sequential trade in assets enables agents to partially hedge publicly observed shocks. The second stochastic discount factor is the PIPO pricing kernel that describes the market environment in which both private and public shocks are insured subject to truth revelation constraint by agents. Assuming cross-sectional log normality of individual consumption levels, we derived the closed-form formulas for these two pricing kernels. The obtained stochastic discount factors are new to the literature.

We tested empirically both these stochastic discount factors using household-level data on consumption expenditures from the US and the UK. In contrast with KP (2007, 2009), when assessing the empirical performance of each candidate stochastic discount factor, we addressed the above-mentioned four puzzles in an integrated framework with the same pricing kernel in all the equations. Using the GMM estimation technique, we found that, when the Euler equations for the equity premium, the risk-free rate of return, and the currency premium, as well as the regression equation for the log growth in the exchange rate are estimated jointly, the asset pricing implications of the PIPO model are very close to those of the CRS model. In contrast with the CRS and PIPO models, the DI model makes the sample analogs of the orthogonality conditions as close as possible to zero at economically plausible values of both the relative risk aversion coefficient and the subjective time discount factor. When testing the predictive ability of these pricing kernels, we however found that although the DI model outperforms the CRS and PIPO models, it is unable to solve the above-mentioned puzzles when they are considered jointly in an integrated framework. This might be due to a low time-series volatility of the change in the cross-sectional variance of the log of individual consumption by means of which the agents' heterogeneity is taken into account in both the PIPO and DI pricing kernels.

This result suggests that although heterogeneity in individual consumption plays an important role in explaining asset returns, it is unable to resolve the asset pricing anomalies in an integrated framework when taken into account alone.

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Table I.
Descriptive Statistics.

Variable	Min	Median	Max	Mean	SD	Skew	Kurt	JB
A. 1982:Q1 - 1999:Q4								
C_{t+1}/C_t	0.9926	1.0056	1.0126	1.0055	0.0038	-0.6077	3.9229	6.6960
C_{t+1}^*/C_t^*	0.9893	1.0080	1.0291	1.0084	0.0088	0.1374	2.7390	0.4130
$R_{VW,t+1}$	0.7622	1.0410	1.2066	1.0373	0.0793	-0.4887	4.3604	8.0667
$R_{EW,t+1}$	0.7023	1.0253	1.3160	1.0330	0.1090	-0.0842	3.6424	1.2679
$R_{F,t+1}$	0.9998	1.0066	1.0166	1.0066	0.0037	0.4976	2.9751	2.8496
Δq_{t+1}	-0.1608	0.0020	0.1487	0.0006	0.0565	-0.1246	3.1078	0.2120
$\frac{(F_t - S_{t+1})\bar{P}_t}{S_t \bar{P}_{t+1}}$	-0.1618	-0.0052	0.1321	-0.0065	0.0558	-0.0939	2.8951	0.1331
$R_{1,t+1}$	0.7308	1.0463	1.2425	1.0387	0.0933	-0.4652	4.3440	7.6815
$R_{2,t+1}$	0.7906	1.0391	1.1809	1.0322	0.0617	-0.9079	5.5732	28.5149
$R_{3,t+1}$	0.7649	1.0412	1.3399	1.0454	0.0981	0.3794	4.5972	8.9894
$R_{4,t+1}$	0.7533	1.0433	1.2421	1.0404	0.0983	-0.3088	2.9683	1.0993
$R_{5,t+1}$	0.7600	1.0409	1.2070	1.0377	0.0935	-0.8229	4.0735	11.1003
The Kocherlakota-Pistaferri Data Set								
$\Delta var_h(c_{h,t+1})$	-0.0832	0.0021	0.0528	0.0004	0.0248	-0.4784	3.5101	3.3797
$\Delta var_h(c_{h,t+1}^*)$	-0.1373	0.0039	0.1086	0.0017	0.0392	-0.4294	4.7849	11.2804
The BSW Data Set								
$\Delta var_h(c_{h,t+1})$	-0.0505	-0.0015	0.0636	-0.0001	0.0227	0.2079	2.8841	0.5355
$\Delta var_h(c_{h,t+1}^*)$	-0.1482	-0.0008	0.1298	0.0020	0.0443	-0.4190	4.8169	11.5099
B. 1982:Q1 - 2004:Q4								
C_{t+1}/C_t	0.9926	1.0056	1.0126	1.0053	0.0036	-0.4920	3.7453	5.6509
C_{t+1}^*/C_t^*	0.9893	1.0073	1.0291	1.0082	0.0084	0.1521	2.8793	0.3971
$R_{VW,t+1}$	0.7622	1.0327	1.2066	1.0278	0.0856	-0.4832	3.5251	4.4864
$R_{EW,t+1}$	0.7023	1.0262	1.3160	1.0342	0.1157	-0.0777	3.1646	0.1899
$R_{F,t+1}$	0.9917	1.0052	1.0166	1.0050	0.0049	-0.4022	3.3181	2.7753
Δq_{t+1}	-0.1608	0.0016	0.1487	0.0016	0.0529	-0.1290	3.2866	0.5514
$\frac{(F_t - S_{t+1})\bar{P}_t}{S_t \bar{P}_{t+1}}$	-0.1618	-0.0057	0.1321	-0.0081	0.0523	-0.0643	3.0450	0.0688
$R_{1,t+1}$	0.7308	1.0397	1.2425	1.0319	0.0910	-0.3438	4.0195	5.6071
$R_{2,t+1}$	0.7906	1.0346	1.1809	1.0287	0.0672	-0.9112	4.7373	23.5081
$R_{3,t+1}$	0.6786	1.0339	1.3399	1.0282	0.1220	-0.3723	3.9002	5.0612
$R_{4,t+1}$	0.7533	1.0301	1.2421	1.0321	0.0972	-0.2595	3.0661	1.0151
$R_{5,t+1}$	0.7600	1.0372	1.2070	1.0323	0.0939	-0.6661	3.5549	7.7244
The BSW Data Set								
$\Delta var_h(c_{h,t+1})$	-0.0505	-0.0024	0.0636	-0.0006	0.0222	0.3099	2.7978	1.5764
$\Delta var_h(c_{h,t+1}^*)$	-0.1482	-0.0013	0.1298	0.0022	0.0436	-0.2116	4.5216	9.2506

Note: JB is the Jarque-Bera statistic.

Table II.
Moments of the Variable Distributions. The BSW Data Set, 1982:Q1-2004:Q4.

Variable	Mean	Variance	Covariance with		
			Δc_{t+1}	$r_{VW,t+1}$	$\Delta var_h(c_{h,t+1})$
Δc_{t+1}	0.005218	0.000013	0.000013	0.000038	0.000001
$r_{VW,t+1}$	0.024120	0.007295	0.000038	0.007295	0.000050
$r_{F,t+1}$	0.005160	0.000026	0.000003	0.000037	0.000002
$\Delta var_h(c_{h,t+1})$	-0.000606	0.000493	0.000001	0.000050	0.000515
$s_{1,t+1}$	-0.012942	0.000045	0.000005	0.000011	-0.000003
$s_{2,t+1}$	-0.006945	0.002645	-0.000019	-0.000653	0.000077

Variable	Mean	Variance	Correlation with	
			$\Delta c_{t+1} - \Delta c_{t+1}^*$	$\Delta var_h(c_{h,t+1}) - \Delta var_h(c_{h,t+1}^*)$
Δq_{t+1}	0.001623	0.002795	-0.022877	-0.175575