The Drivers of Cross Market Arbitrage Opportunities: Theory and Evidence for the European Bond Market

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Abstract: The focus of this paper is on the study of the drivers of a cross market arbitrage profit. Many papers have investigated the risk of trading arbitrage opportunities and the empirical existence of these events at the high frequency level for different markets. But none of the previous work has asked the simple question of how these events are formed in the first place. That is, what are the drivers behind the occurrence of a risk free profit opportunity? In this paper we investigate the theoretical (and empirical) implications of a cross platform arbitrage profit. Following a microstructure model we show that this event is the result of microstructure frictions in trading. We are able to decompose the likelihood of an arbitrage opportunity into three distinct factors: the fixed cost to trade the opportunity, the level of which one of the platforms delays a price update and the impact of the order flow on the quoted prices (inventory and asymmetric information effects). In the second (empirical) part of the paper, we investigate the predictions from the theoretical model for the European Bond market with an event study framework and also using a formal econometric estimation of a probit model. Our main finding is that the results found in the empirical part corroborate strongly with the predictions from the structural model. The event of an arbitrage opportunity has a certain degree of predictability where an optimal ex ante scenario is represented by a low level of spreads on both platforms, a time of the day close to the end of trading hours and a high volume of trade.

Keywords: arbitrage opportunities, negative spreads, market microstructure

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Table of Contents

Introduction

The subject of this paper is the investigation of the drivers behind an arbitrage opportunity. A pure arbitrage event exists when the market quoted prices imply the existence of a riskless profit opportunity. For our case, this is related to a cross market arbitrage. For an example, consider two distinct physical venues located one next to the other that sell and buy cars (two car dealers). Dealer number one buys a particular type of car[4] for P_1^{Bid} and sells it for P_1^{Ask} while dealer number two buys the same type of car for P_2^{Bid} and sells it for P_2^{Ask}. This is similar to the case we want to investigate where there are two alternative order books. [5]

Consider now the existence of an arbitrageur who pays enough attention to all four prices displayed by the dealers. The arbitrageur knows that if he can buy cars at a price lower than the price at which he can instantly sell it, then he will always lock in a profit. This operation is relatively riskless as long as the operation is quick enough to close the second leg of the trade (a sell) before the dealers update their prices once again. But, there are hidden costs to this operation. First there is a funding issue as the arbitrageur will have to come up with the resource for the first leg of the transaction (a buy). Second there are delivery costs since he/she will also have to transport the goods to the next shop for each transaction. Finally, there are search and transaction costs as this arbitrageur will have to check prices in each dealership and pay for commission and taxes. For simplicity, we will further assume that these costs are all insignificant.

This arbitrageur will have positive net profits for the situation where either P_1^A is lower than P_2^B or P_2^A is lower than P_1^B. The profit of this operation will be higher the higher is the divergence between the quoted prices in the two dealerships. This is the case of a negative spread event. Remember that spread is the difference between the ask and the bid price. If we aggregate this dual market by retrieving the best available quotes, that is, finding the highest bid ($P_{1,2}^{Bid} = \max\left(P_1^{Bid}, P_2^{Bid}\right)$) and the lowest ask ($P_{1,2}^{Ask} = \min\left(P_1^{Ask}, P_2^{Ask}\right)$), we will see that the arbitrage event will be true when $P_{1,2}^{Ask} - P_{1,2}^{Bid}$ (a.k.a. the spread) is negative. If a dealer continues to quote the same prices, the arbitrageur will continuously trade the arbitrage portfolio and his/her wealth will grow at a constant rate.

From the point of view of the dealers, the quoted prices are still plausible as the ask price is always higher than the bid, and therefore as they buy and sell cars there is still a positive expected compensation per round trip trade. These dealers may or may not know about the operations of the arbitrageur. Since this arbitrageur is simply buying and selling cars, he/she is still bringing business to the dealerships so his/her trading is commercially beneficial. From the price side, these managers are reactive to the number of cars that they hold in stock. The value of automobiles in this particular region is very uncertain and the value of the dealer's stocks can decrease or increase in an unpredictable way. The dealers are aware of this unpredictability and they do

[4] Prices are therefore inelastic.

[5] An order book displays the best bid price, which is the best price an aggressive seller can get at a particular point in time and also the best ask price, which is the best price an aggressive buyer can reach.

not appreciate it as their business is not speculation (they are risk averse). Every time which there is a number of cars in stock higher than a particular threshold, the owners will lower the ask prices in order to motivate the incoming of buys, which will further decrease their level of price exposure. If this decrease of price is not also true for the other shop, it may happen to trigger an arbitrage opportunity. The arbitrageur will come in and trade at the specific prices until the inventory level of the shops is once again at a comfortable level.

Therefore, as you can see, the arbitrageur is actually performing a service to the shops since he/she will always trade when the arbitrage bounds are broken. This service takes the form of helping the dealers to manage their level of exposure. The compensation for the trader is then driven by the mismanagement of inventory by the dealers. An optimal solution for the dealer's inventory problem would be to open a trading channel with his competitor. If the car dealers could trade among themselves, they would always get better prices than trading with the arbitrageur. This exchange would be beneficial for both sides as the inventory problem can occur in both venues. In this case it is likely that the trader would cease its arbitrage operations as the opportunities would no longer be observed.

This example shows some of the dynamics we can expect in a multi platform market. The point we are making from this simple situation is that there is a driver of a cross market arbitrage profit. For the scenario in question it is particularly the excessive inventory sustained by one of the shop's owners. But we could also argue that an arbitrage opportunity would be likely if one of the shops has a relative delay[6] in updating the prices of cars with respect to the other.

While the fictional case of the car arbitrage is compelling, the situation is far more complex in the real world. First there are time varying degrees of uncertainty regarding the true price of an asset. If news about a risk factor hit the market (e.g. default of Greek bonds), this will also impact the other countries. So, if price variability is changing, we can argue that the threshold over which the market makers manage their inventory exposure is also time varying.

Second, there are multiple dealers that come and go in the market, each with particular restrictions on their activity[7]. This makes the analysis far more complicated as their identity (and inventory) is not usually traceable in empirical data. Third, the dealers can hedge their inventory by trading in the opposite direction for a particular risk factor instead of being forced to improve quotes for decreasing inventory exposure. For example, the market maker for a three year fixed coupon bond of Germany can partially hedge his inventory exposure to the interest rate by trading the five (or more) year fixed coupon bond of the same country. Therefore, for the cases where there are other instruments with similar risk exposures, the empirical effects of inventory management cannot be fully assessed with trades and quotes data for a single asset.

[6] For instance we could imagine the situation where each shop's owner receives the price of cars by calling a central supplier in each 1 and 10 minutes respectively. The owners then set the bid and ask by respectively subtracting and adding half the spread from this price. Depending on how much the prices move in each time interval it can happen that the delay in updating the quotes will portray an arbitrage opportunity. This is the slow price adjustment scenario provided in (Harris, 2003).

[7] Here we refer to commercial restrictions (e.g. provide competitive quotes for a significant part of the trading hours).

Fourth, these are electronic platforms and prices are fully disseminated so that the market maker on one platform knows the prices being quoted on the other. For some cases the dealers can trade on both. The speed (and sophistication) of trading is also a factor. One of the risks in arbitrage trading is the execution of the trades. If prices are moving fast the quoted values may slip away from the arbitrage boundaries and an arbitrage trader may actually be forced to sustain a loss if he/she cannot trade at the previously quoted values. Also, the market makers in the present day are well equipped with state of art software and hardware. These are justified by competitive forces since a dealer would be better prepared to make markets if he/she can react quicker and more efficiently to the observed news and order flow. This also applies to quote control, where, given an overview of the whole market, a computer can always check whether a cross market arbitrage was created or not.

As the reader can see, the analysis of an arbitrage opportunity in real cross markets is far more complex than the two shops example in the first part of the introduction. But, while the details are different, we still have the same intuition that a cross market arbitrage opportunity is motivated by well-known microstructure effects such as the impact that a trade has on the quotes[8] and the price update delay. In this paper we are interested in analyzing what are these drivers in a more formal approach. With the support of a microstructure model we are able to show quantitatively the effects of the trade impact and also how the delay of a quote update in one of the platforms with respect to the true price can create arbitrage opportunities. In the empirical part of the research we analyze cross market arbitrage events for the Italian bond market. The results are quite positive as there is considerable evidence which corroborates the predictions from the theoretical part.

The paper is organized as follows. First we briefly present a review of the previous work on the subject, second we show the theoretical derivations behind a pure arbitrage profit for a multi platform framework. In the third part we describe the methods used to test the implications from the theory. We follow with an analysis of the results and we finish the paper with the usual concluding remarks.

Literature Review

The subject of arbitrage has attracted a significant amount of attention from the academic community. The concept of arbitrage opportunities impacts financial theories in a set of branches, from efficient market hypothesis[9] to pricing theorems[10]. A significant proportion of the literature has focused on showing that arbitrage trades are in fact risky. These are segmented into four types: fundamental, synchronization, noise trader risk and execution risk.

The first is the simplest, fundamental risk. When engaging arbitrage trades for similar (but not equal) assets, there is a risk that an exogenous factor impacts the value of the arbitrageur's portfolio. For example, consider an arbitrage for an ADR instrument

[8] For the example of the car dealers this was the inventory management effect. But trades can also impact the quotes by the information asymmetry effect, in which the intuition is that there are traders with privileged information, therefore changes in the efficient price are correlated to a portion of the order flow, which, given that the quotes are a function of efficient price, results in another source of positive covariance between quotes and the trades.

[9] See for example (Malkiel, 2003).

[10] See for example (Hull, 2002)

regarding a Brazilian company (e.g. Petrobras). The price of Petrobras in the Brazilian equity market (Bovespa) is cheap comparing to the price of the ADR traded on the NYSE. A fundamental arbitrageur with access to both markets places a long position for the undervalued Brazilian stock in Bovespa and a short for the NYSE ADR instrument. The logic is that since both assets have the same cash flows, then the observed price difference between Bovespa and NYSE is mean reverting and should eventually return to zero. It is clear that both assets have similar exposure to the same risk factors (e.g. international oil prices). But, given that the ADR is traded in the US, it implies a whole new set of local risks which cannot be hedged by the Brazilian asset. Therefore, a part of the portfolio of this arbitrage trader will be exposed to a different set of country specific risks, including exchange rate risks. For more details see (Gagnon & Karolyi, 2004).

The second type of risk in an arbitrage operation is the synchronization risk (Abreu & Brunnermeier, 2002). This is the risk that arbitrageurs face when competing for similar arbitrage opportunities. Rather than correct the market instantaneously, the uncertainty regarding the time it would take for the market to correct itself in a fundamental basis forces the arbitrage trader to time the arbitrage in order to minimize holding costs of his operation. While it is clear that the market portrays an arbitrage opportunity, the arbitrageurs simply doesn't know when the market will be back in equilibrium, resulting in a synchronization cost for the trader.

The synchronization risk is related to (but distinct[11] from) to the noise trader risk, (Shleifer & Vishny, 1997) and (Bradford De Long, Shleifer, Summers, & Waldmann, 1990). The disequilibrium of fundamentals in the market can last for a time longer than expected if noise traders are clustered on one side of the trade. These noise traders are investors with erroneous beliefs about the true price of an asset. If by chance there is a high proportion of these noisy investors on one side of the market, the traded prices may actually move further away from fundamentals[12] instead of converging, which is the expected behaviour. Therefore, the existence of noise traders creates another set of risk for an arbitrage trader. This rational trader with wealth constraints could then be forced to liquidate a position with a loss even thought his fundamental analysis is correct.

The last set of risks is the execution risk (Kozhan & Tham, 2009). Since arbitrage usually involves more than one asset, then there is a chance that the trader is not able to fill all trades required to complete the arbitrage portfolio. This risk will increase as the higher is the required amount of trades to complete the portfolio and the higher is the number of traders competing for the same arbitrage opportunities. For further details see the previously mentioned paper.

On the empirical side of arbitrage research, many studies have shown that arbitrage opportunities do exist for different markets. But, when accounting for real world costs, these opportunities do not seem very attractive. For example a significant proportion of the studies were focused on the interest rate parity in exchange rate markets. One of the first papers to show that arbitrage trades exist but are likely not profitable when accounting for transaction costs is (Frenkel & Levich, 1975). The authors demonstrated that when taking into account the delay of executed trades and

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[11] This is different from noise trader risk as it isn't necessarily originated from traders with erroneous beliefs, but also on the time that other arbitrage trader will step in the market to correct the mispricing.

[12] There is a clear example of such risk in financial history, the LTCM episode. See (Lowesntein, 2002) for details.

other transaction costs, the possible mispricing was not actually profitable for the empirical data (in this case Canada-US and US-UK exchange rate pairs).

In a more recent work, (Juhl, Miles, & Weidenmier, 2006) investigate the occurrences of covered interest parity violations in the Gold standard period (1880-1914) against the evidence for the present market. As one can expect, they find a higher number of CIP violations for the Gold standard period when compared to the present period. This is explained by the fact that slower information distribution leads to a higher lag of price update towards the fundamental value, which causes a higher proportion of arbitrage opportunities to exist.

Still in the exchange market, in the work of (Akram, Rime, & Sarno, 2008) the objective was to provide a descriptive study on the size, duration and economic significance for arbitrage opportunities in the foreign exchange market. Using tick data for major exchange rates (USD/EUR, USD/GBP and JPY/USD) for the period of February to September of 2004, the authors find that the profitable CIP deviations last for an average of a few minutes, which, as the authors argue, would be a short lived event but relatively long enough for the arbitrage portfolio to be traded. This result is consistent with the view that arbitrage opportunities do exist, but are rare and last for short time intervals. Further investigation also shows that the frequency, size and duration of these arbitrage opportunities are positively related to the volatility of the market. While the authors of this study have not discussed the reasoning behind this result, we argue that the positive dependency of arbitrage opportunities with respect to volatility is consistent with our view regarding the impact of price update delay and the inventory problem in the occurrence of arbitrage opportunities. For the first, if there is a constant delay for price updates, the higher the volatility of prices changes, higher the chances that quoted prices will portray an arbitrage opportunity. For the second, if volatility is high, the market makers will be motivated to decrease portfolio exposures by reducing inventory. This is accomplished with a drastic improvement of quotes, which may in turn also trigger an arbitrage opportunity. Therefore, the implications of the present study can explain some of the results found in (Akram, Rime, & Sarno, 2008).

Another significant part of the literature in arbitrage opportunities has devoted attention to the case of futures-cash basis. As with the case of covered interest parity, the arbitrage relationship between a futures contract and its underlying security is relatively easy to check for empirical data. The authors in (Roll, Schwartz, & Subrahmanyam, 2007) looked at the effect of liquidity in arbitrage opportunities and vice versa. For example, if the markets are not liquid enough, then the futures-cash basis can widen, giving the chance for an arbitrage opportunity, which will then attract arbitrageur traders, therefore increasing liquidity. So clearly there is a feedback effect in this process. This paper studies this effect by performing Granger-causality tests between the index-futures basis and stock market liquidity. The results found are consistent with the feedback effect and shows that both variables Granger cause each other. Further research on the case of index future arbitrage can be found in (Mackinlay & Ramaswany, 1988), (Henker & Martens, 2001), (Cummings & Frino, 2007), among many others.

As one can see, much of the literature is concerned with an ex post analysis of the economic aspects of arbitrage opportunities including profitability and riskiness. Our interest in this paper is with the ex ante implications of this event, that is, which factors are triggering arbitrage opportunities. In this perspective, we approach an arbitrage opportunity as a stochastic event driven by exogenous processes. In terms of contribution to the literature, we do not claim to be the first to point out the role of microstructure frictions in arbitrage opportunities[13]. But, we seem to be the first to provide a formal analysis of it. Next we present the theoretical part of the paper where we describe the implications of a cross market arbitrage in a microstructure model of prices and trades.

Theoretical Foundations

In this section we are interested in showing the theory behind pure inter-platform arbitrage profits. First, we set up a structural microstructure model and then we investigate the conditions for an instantaneous arbitrage profit to exist. We start our derivations with a structural model for a multiplatform trading framework. This is based on a multi-asset model[14] with inventory management and asymmetric information effects. In our case, we have one underlying asset, which is traded on two different platforms. The efficient price is the same across trading venues but inventory effects and trading costs are different. The efficient price process is given by:

$$m_t = m_{t-1} + \theta v_t + u_t \qquad (1)$$

This equation represents the true price of an asset, which will follow an extended random walk process. The main innovation in the formula is given by u_t, which is an i.i.d. random variable with zero expectation, constant variance and zero covariance at any lag. It is easy to prove that the price change for a price process with a formula such as (1) is not predictable[15]. For Equation (1), the term θv_t measures the information asymmetry effect where v_t is the unexpected order flow. The idea in this effect is that a portion of the traders have privileged information such that their order flow is correlated to the true price of an asset. For example, consider a trader with insider information on particular news for time *t*. This investor will then buy the asset before the news becomes public and will subsequently see an improvement in prices. This translates into a positive correlation between order flow the efficient price, which is being measured by parameter θ.

The structural equations for quotes, inventories and trades will have their own version for each platform. For platform 1 we have:

$$q_{1,t} = m_t - b_1 I_{1,t-1} \qquad (2)$$

$$I_{1,t} = I_{1,t-1} - x_{1,t} \qquad (3)$$

$$x_{1,t} = \lambda v_t \qquad (4)$$

$$p_{1,t} = q_{1,t} + \frac{c_1}{2} x_{1,t} \qquad (5)$$

[13] For instance see page 373 of (Harris, 2003) and the first paragraph of (Roll, Schwartz, & Subrahmanyam, 2007).

[14] See (Hasbrouck, Modeling Market Microstructure Time Series, 1996) for details.

[15] For more information regarding properties of a random walk model, see (Hasbrouck, Empirical Market Microstructure: The institutions, economics and econometrics of securities trading, 2007).

For platform 2:

$$q_{2,t} = m_{t-k} - b_2 I_{2,t-1} \tag{6}$$
$$I_{2,t} = I_{2,t-1} - x_{2,t} \tag{7}$$
$$x_{2,t} = (1-\lambda)v_t \tag{8}$$
$$p_{2,t} = q_{2,t} + \frac{c_2}{2} x_{2,t} \tag{9}$$

The above equations define the time varying evolution of the quotes, inventories, order flow and traded prices respectively, in each of the platforms. Parameters b_i, c_i for i=1,2 are all positive and λ is between zero and one. The quote equation, formula (2), is related to the efficient price at the same time plus a reaction to the inventory which the market maker is facing. The higher the value of b, the higher the increase (decrease) of the mid quote with respect to a negative (positive) level of inventory (I_t). That is, if the past inventory is positive, there will be a downwards improvements in the quotes, which will motivate the arrival of buy trades, which will then decrease the level of inventory of the dealer. Therefore the parameter b in Equations (2) and (6) represents the sensitivity of the market maker to inventory imbalances. The inventory process for the microstructure model is given by a random walk representation where the innovation is the negative of the exogenous trades, which is itself a random component. Remember that if a buy trade comes in, it is actually a sell trade from the dealer side and it will decrease his inventory, which justifies the negative sign in (3) and (7).

The order flow of the structural model is represented by v_t, an i.i.d. random component uncorrelated at any lag[16]. This is an exogenous process and is split across the trading platforms given a value[17] of λ. A proportion of this order flow is correlated to the efficient price given the information asymmetry effect. Such an effect is being measured by parameter θ. The higher is the value of θ, the higher is the proportion of informed trades in the order flow and higher is the covariance between the quote price changes and the trades. It is important to point out that given the microstructure model presented before, there are two ways that a trade can affect the quotes. The first is the inventory management in each of the platforms and the second is the information asymmetry effect. Both effects will affect quoted prices in the same direction but the temporal drivers are different as the inventory effect is

[16] Some remarks should be made with respect to the process underlying the order flow. In general it would be more realistic to assume that the order flow has a mean reverting component of the type $-\alpha_1 (q_{1,t-1} - m_{t-1})$, where α_1 is the reaction of traders to the mispricing of the asset. This is more realistic than assuming a totally random process. But, ignoring such mean reverting effects does not change the main point of this derivation, which is to show that arbitrage opportunities are the result of microstructure frictions. Given that, we chose to keep the simpler case with an exogenous process for order flow, which makes the following derivations far more tractable.

[17] This parameter is bounded by zero and one.

driven by the lagged order flow and the informational asymmetry effect is driven[18] by the contemporaneous trade.

The prices at which a trade takes place will be given by Equations (5) and (9). If one unit of a buy trade reaches the market, the actual traded price will be $p_t = q_t + \frac{c}{2}$. If one unit of a sell trade comes to the market, the traded price will be $p_t = q_t - \frac{c}{2}$. The dealer of this round trip trade then profits from the spread, which is given by the value of c. This spread is set to cover the trading costs on the market maker side (e.g. clearing, processing and management costs). The dynamic process portrayed by Equations (5) and (9) are similar to an order book structure but there are significant differences. For instance, in a real order book there are minimum volumes to be traded. Also, the price dynamic in a real order book is discrete, meaning that price changes can be measured in terms of minimum price variations or *ticks* which are defined by the exchanges. These properties are clearly not true for (5) and (9) as x_t follows a continuous distribution.

Note that from (2) to (9) we also assume that we have a different market maker with different inventory needs in each trading channel. From the structural equations in (2) and (6) we see that the second platform lags the update of the efficient price in the quote equation by a value of *k*[19], which is an integer higher than or equal to zero. This could be the case of a temporary operational failure in the affected platform, a lack of mismanagement by the market maker or perhaps a unique market structure such as a crossing market[20], which lags the update due to processing delays.

Now, the situation which we are interested in is the case of a cross platform arbitrage profit that is, buying a particular quantity ϕ on the underpriced platform and instantaneously selling[21] it in the overpriced market. As a starting point we use platform two as the venue with undervalued prices ($p_{1,t} > p_{2,t}$). Using Equations (1) – (9), we have that the profit from the cross platform arbitrage will be given by:

$$arbPL_t^{2,1} = \phi\left(p_{1,t} - p_{2,t}\right) - C \qquad (10)$$

For the last formula, $arbPL_t^{i,j}$ is the arbitrage profit for buying ϕ units in platform *i* and selling it in platform *j*. The term *C* is the fixed trading cost (in cash terms) for the round trip trade (e.g. broker fees). For simplicity we will assume that $\phi = 1$, that is,

[18] This is not exactly evident from (2), but by substituting m_t in q_t, it becomes clear that the trades will impact the quotes at a contemporaneous level.

[19] On a side note, we could also have set the process for quotes as $q_{1,t} = m_{t-j} - b_1 I_{1,t-1}$ and $q_{2,t} = m_{t-j-k} - b_2 I_{2,t-1}$ and it would not have change the results from our derivations. The effect which drives the result is the relative lag from one platform to the other, which in this case is measured by *k*.

[20] See page 97 in (Hasbrouck, Empirical Market Microstructure: The institutions, economics and econometrics of securities trading, 2007).

[21] We are assuming that each trade is certain to be executed. This is different than the setup given in (Kozhan & Tham, 2009), in which the execution of arbitrage trades is uncertain. Further research could be taken in this direction, but we start with a simple scenario.

the arbitrage strategy buys and sells one unit of the asset. Further expansion of the last equation[22] yields:

$$arbPL_t^{2,1} = \sum_{j=0}^{k-1} u_{t-j} + \theta \sum_{j=0}^{k-1} v_{t-j} - b_1 I_{1,t-1} + b_2 I_{2,t-1} - \frac{c_1 + c_2}{2} - C \tag{11}$$

The condition for an arbitrage profit will be given by $arbPL_t^{2,1} > 0$. This implies that, in order for a profitable negative spread event to exist, the condition is:

$$\sum_{j=0}^{k-1} u_{t-j} + \theta \sum_{j=0}^{k-1} v_{t-j} + b_2 I_{2,t-1} > b_1 I_{1,t-1} + \frac{c_1 + c_2}{2} + C \tag{12}$$

Now, the last formula shows that given the structural equation defined earlier, there are three distinct components to the event of a multi platform arbitrage profit. The first one is simply the lag update of order *k* for one of the trading channels. This effect is given by the sum of innovations in the efficient price (terms $\sum_{j=0}^{k-1} u_{t-j}$ and $\theta \sum_{j=0}^{k-1} v_{t-j}$ in formula (12)). The informational shocks to the common efficient price have an immediate effect on the price of the first platform but affect the price of the second platform with a delay *k*. Clearly if *k=0*, both terms vanish and hence do not impact the profit of the arbitrage. The second factor is the imbalance between inventories across the platforms, which is driven by the terms $b_2 I_{2,t-1}$ and $b_1 I_{1,t-1}$. For our case where the buy (sell) signal was generated for platform one (two), the higher (lower) the difference of reaction of market makers to their respective inventory, the higher the chances for an arbitrage profit. Exemplifying, let's say that the market maker in platform two executes a big sell trade at time *t*-1, meaning that he bought stock and increased significantly his corresponding inventory level. The dealer in platform one has a steady inventory, with low variability. The response of the market maker for platform two at time *t* will be to lower the quotes. Depending on how strong is this improvement, it might be possible that the incoming of a large sell trade at time *t*-1 indirectly triggers an arbitrage opportunity. Note also that this effect is relative to the misbalance in between platforms. The variable in question is the distance between $b_2 I_{2,t-1}$ and $b_1 I_{1,t-1}$, which is a mix between the previous inventory and also the reaction level of the market makers. The last component of the condition is the trading cost of the operation, which is measured by the average of the spreads on the different platforms plus the fixed cost of trading given by *C*. This is a clear effect as spreads are trading costs to be paid by the trader. So the higher the overall cost, the lower the chances of a profitable arbitrage opportunity.

The last result given by (12) is intuitive as it gives the conditions for the arbitrage profit to be positive. It is clear from the last formula that the model has stochastic behaviour with two components (u_t and v_t) as the source of randomness. This

[22] See appendix 1 for the details.

implies that the condition given in (12) is also driven by a random component. That being said, the interest now is to calculate the probability of a negative spread as a function of the parameters from the structural model.

We begin with the expansion of the inventory term in (12). It can be proved[23] that the term $I_{1,t-1}$ will follow:

$$I_{1,t-1} = -\lambda \sum_{j=1}^{t-1} v_{t-j} \tag{13}$$

Therefore the inventory at time *t*-1 is just the negative of the cumulative sum of trades. Substituting back in (11), we can decompose the arbitrage profit equation as follows:

$$arbPL_t^{2,1} = X + Y + Z \tag{14}$$

Where:

$$X = \sum_{j=0}^{k-1} u_{t-j} + \theta \sum_{j=0}^{k-1} v_{t-j} \tag{15}$$

$$Y = \left(\lambda b_1 - (1-\lambda) b_2\right) \sum_{j=1}^{t-1} v_{t-j} \tag{16}$$

$$Z = -\frac{c_1 + c_2}{2} - C \tag{17}$$

Equation (14) is intuitive as it explicitly breaks the arbitrage profit into different factors. Factor *X* is the lagged update at order *k* on one of the platforms. Factor *Y* is the inventory imbalance between the market makers and factor *Z* represents the trading costs associated with the arbitrage strategy (quoted spread and trading fees). Another interesting result from last equation is that the expected profit from the arbitrage operation is negative. This result is based on the properties of the disturbances and the spread. That is, $E(u_t) = 0$, $E(v_t) = 0$, $c_j, C > 0$ and $E(c_j) = c_j$ for *j*=1,2. The expected loss is then given by $E\left(arbPL_t^{2,1}\right) = -\frac{c_1 + c_2}{2} - C$. Therefore, in expectation, the operation of buying and selling in different platforms will yield a loss, which is an intuitive result.

By checking the condition of a positive profit of the arbitrage trades ($arbPL_t^{2,1} > 0$), we have the result that this event would be true for:

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[23] See Appendix 2 for derivation.

$$X + Y > -Z \tag{18}$$

Note that given the parameters of the structural model, the term Z is a constant and $X + Y$ is a stochastic process. Our interest is in calculating the closed formula for the probability that Equation (18) holds true. Given a particular distribution for $X + Y$, this probability is given by:

$$\Pr\left(arbPL_t^{2,1} > 0\right) = 1 - cdf\left(X + Y \leq -Z \mid \Omega\right) \tag{19}$$

For the last equation, the term $cdf\left(X + Y \leq -Z \mid \Omega\right)$ is the cumulative density function of $X + Y$ evaluated at *–Z,* conditional on a set of parameters defined in vector Ω. So far we do not have the distribution function for the measure in question. In order to derive that we need to explicitly set the distributions of the disturbances. For the sake of simplicity we are going to use the normal distributions[24] for u_t and v_t . That is, u_t and v_t are uncorrelated at any (cross) lag and they follow an i.i.d. normal distribution with constant variances σ_u^2 and σ_v^2, respectively.

A useful property for our case is that the sum of dependent normal random variables will also follow a normal with mean equal to the sum of the means and the variance equal to the sum of the variances plus a term for the covariance of the variables[25]. This property implies[26] that:

$$X \sim N\left(0, k\left(\sigma_u^2 + \theta^2\sigma_v^2\right)\right) \tag{20}$$

$$Y \sim N\left(0, \left(\lambda b_1 - (1-\lambda) b_2\right)^2 (t-1)\sigma_v^2\right) \tag{21}$$

And:

$$X + Y \sim N\left(0, \sigma_{X+Y}^2\right) \tag{22}$$

$$\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2 + 2\sigma_{X,Y} \tag{23}$$

$$\sigma_X^2 = k\left(\sigma_u^2 + \theta^2\sigma_v^2\right) \tag{24}$$

$$\sigma_Y^2 = \left(\lambda b_1 - (1-\lambda) b_2\right)^2 (t-1)\sigma_v^2 \tag{25}$$

$$\sigma_{X,Y} = \theta\left(\lambda b_1 - (1-\lambda) b_2\right)(k-1)\sigma_v^2 \tag{26}$$

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[24] The assumption of normality is not particularly necessary for the main conclusions of the derivations. The tractability of the Gaussian density greatly simplifies the analysis but we are confident that the main intuition from the results will also hold for more unconventional distributions.

[25] Formally, the property is that if *X* and *Y* follow normal, then *X*+*Y* will also be normal with variance given by $\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2 + 2\sigma_{X,Y}$.

[26] Derivation for the covariance of *X* and *Y* can be found in Appendix 3.

With this last result given by (20)-(26) we have the distribution for $X+Y$, for which we can easily calculate the cumulative probability function given values of b_1, b_2, λ, t, θ, k, σ_v^2 and σ_u^2. Therefore, following previous arguments, we can show that the probability of a profit by buying on platform two and instantaneously selling on platform one is given by:

$$\Pr\left(arbPL_t^{2,1} > 0\right) = 1 - \frac{1}{2}\left(1 + erf\left(\frac{-Z}{\sqrt{2\sigma_{X+Y}^2}}\right)\right) \quad (27)$$

For Equation (27), $\Pr\left(arbPL_t^{2,1} > 0\right)$ is the probability of a cross market arbitrage profit. The term $erf(.)$ is the error function. It is easy to see that this formula is simply one minus the cumulative distribution function evaluated at $-Z$ of a normal variable with zero expectation and variance given by Equation (23).

But, note that so far we had an arbitrary signal of trades on the platforms (buy order for platform two and sell order for platform one). This is counter intuitive as given the existence of an arbitrage profit, a trader is indifferent as to on which platform to buy and which to sell. That is, the measure which we are interested in is the joint probability of an arbitrage profit, irrespective of the trading platform which we make the buy and sell operations. By inverting the signs in (10), we can show that the profit by buying in platform one and selling in platform two is:

$$arbPL_t^{1,2} = -X - Y + Z \quad (28)$$

Where the terms X, Y and Z are given by Equations (15) to (17). Using the last formula, we can see that the probability of a profit in this setup is:

$$\Pr\left(arbPL_t^{1,2} > 0\right) = \Pr\left(X + Y < Z \mid \Omega\right) \cong \Pr\left(X + Y \le Z \mid \Omega\right) \quad (29)$$

Note that for the last formula we assume that the probability of $X+Y$ being exactly equal to Z is negligible. This simplifies the formula as now the term $\Pr\left(X + Y \le Z \mid \Omega\right)$ is the cumulative distribution function of a normal variable evaluated at a given point. Aggregating the result of (19) and (22) we have that the probability of an arbitrage profit (and consequently a negative spread), irrespective of the trading platform, is given by:

$$\begin{aligned}\Pr\left(arbPL_t > 0\right) &= \Pr\left(\left[arbPL_t^{2,1} > 0\right] \cup \left[arbPL_t^{1,2} > 0\right]\right) \\ &= \Pr\left(\left[X + Y > -Z\right] \cup \left[X + Y \le Z\right]\right) \\ &= 1 - \Pr\left(X + Y \le -Z \mid \Omega\right) + \Pr\left(X + Y \le Z \mid \Omega\right)\end{aligned} \quad (30)$$

For the last formula we used the property that both events are mutually exclusive - that is, if $arbPL_t^{2,1}$ is higher than zero, then this implies that $p_{2,t}$ is higher than $p_{1,t}$, therefore $arbPL_t^{2,1}$ has to be lower than zero (see Equation (10)).

By combining the results from Equation (30) with the results from Equation (27) we have the final formula for the probability of a cross market arbitrage opportunity:

$$\Pr\left(arbPL_t > 0\right) = 1 + \frac{2}{\sqrt{\pi}} \int_0^{\psi} \exp\left(-t^2\right) dt \tag{31}$$

Where:

$$\psi = \frac{-\left(2^{-1}\left(c_1 + c_2\right) + C\right)}{\sqrt{2\left[k\left(\sigma_u^2 + \theta^2\sigma_v^2\right) + \left(\lambda b_1 - (1-\lambda)b_2\right)\sigma_v^2\left(\left(\lambda b_1 - (1-\lambda)b_2\right)(t-1) + 2\theta(k-1)\right)\right]}} \tag{32}$$

For Equation (31), note that the values of $\Pr\left(arbPL_t > 0\right)$ are bounded by zero and one. The probability of an arbitrage opportunity will increase as σ_u^2, σ_v^2, k and θ increases and will decrease as the fixed costs to trade (parameters c_1, c_2 and C) increase. For the coefficients b_1, b_2 and λ, the analysis is not direct as they are interacting in the calculation of the variance of *X+Y,* which is one of the drivers of Equation (32). But, by holding λ constant we can say that the higher the difference between b_1 and b_2, the higher the chances of a cross market arbitrage opportunity. This is an intuitive result as the higher is the difference of the inventory reaction level for the dealers in the different platforms, the higher should be the chances that a change in the mid quote portrays an arbitrage profit. Another intuitive result from (30) is that if $b_1 = b_2$ and $\lambda = 0.5$ that is, if both platforms evenly share the order flow and both market makers react equally to their inventory, then it is true that $\sigma_Y = 0$ and $\sigma_{X,Y} = 0$, implying that in this case, only a possible price delay (value of *k*) can create a chance for an arbitrage opportunity.

Also, for the special case of $k = 0$ (no delay of quote update in one of the platforms), we see that $\sigma_X^2 = 0$ and $\sigma_{X,Y} = 0$, which simplifies formula (23) to $\sigma_{X,Y}^2 = \left(\lambda b_1 - (1-\lambda)b_2\right)^2 (t-1)\sigma_v^2$. This result implies that parameter θ, which is measuring covariance between order flow and the efficient price (the asymmetric information effect), only comes into play for the likelihood of an arbitrage opportunity if there is a lag in the quote update process (parameter *k*). If there is no delay for the quote update (*k*=0), then the proportion of informed traders in the market is irrelevant to an arbitrage event.

Given fixed values for $c_1 = c_2 = C = 0.25$, $\phi = 1$, $\sigma_u^2 = 0.05^2$, $\sigma_v^2 = 0.5^2$, $\theta = 0.05$, $b_1 = 0.1$, $b_2 = 0.2$, $\lambda = 0.75$ and varying values of *t* and *k*, we have the following figure for the shape of Equation (31).

Figure 1 – The Probability of Arbitrage Opportunities

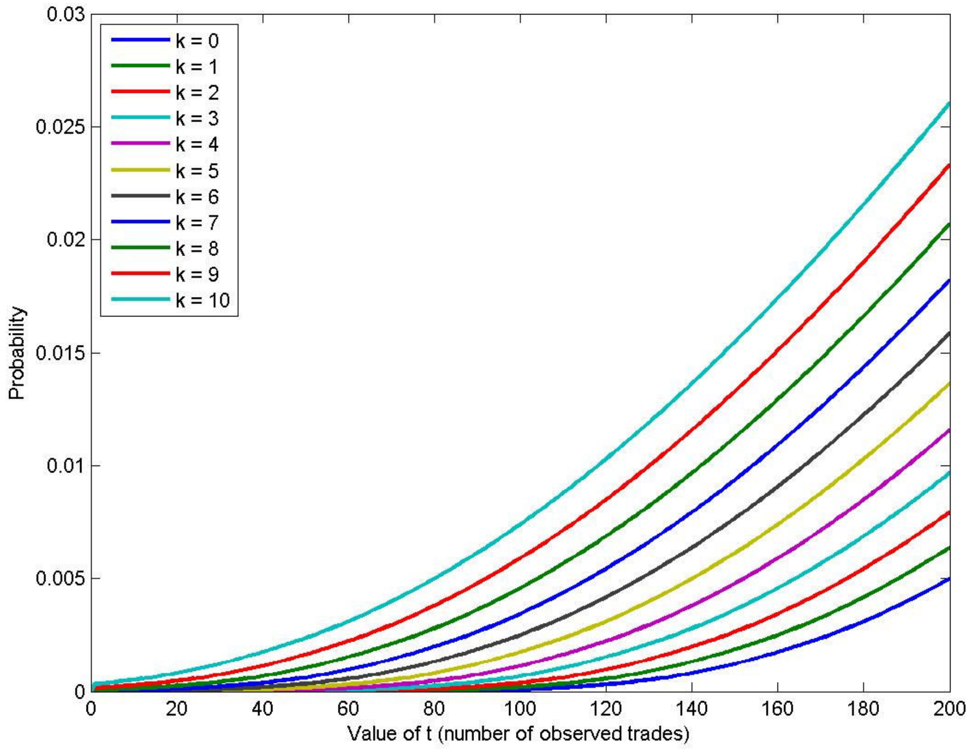
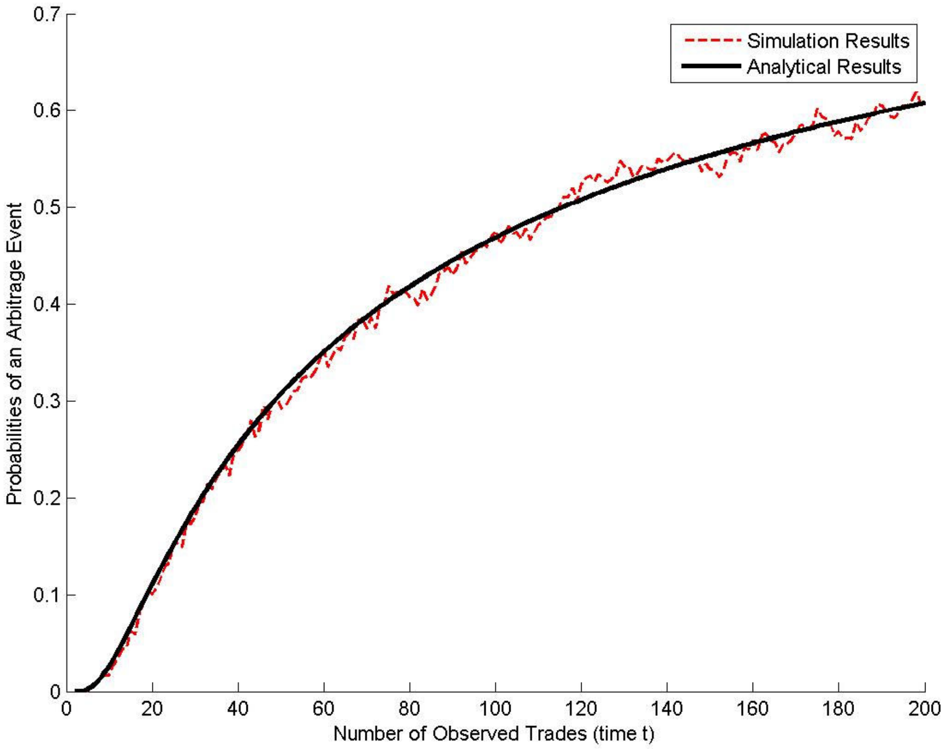


Figure 1 shows the probabilities of a profitable arbitrage position as functions of the lagged value of the efficient price update in the quote equation for one of the platforms and also the number of observed trades (value of *t*). The higher the number of trades observed, the higher the probability of the existence of a positive profit in a multi-platform arbitrage. This relation also holds true for the value of *k*, where the higher the lag update in the quote equation on one of the platforms, the higher the probability of an arbitrage profit. Note that we can also follow similar arguments for the volatilities of the random components. In general, the higher are the values of σ_v^2 and σ_u^2, the higher is the likelihood of an arbitrage opportunity.

We can further support the derivations of Equation (32) with the use of Monte Carlo simulations. The multivariate process given by formulas (2) to (9) can be simulated given values of the parameter and we can check for each time *t* whether an arbitrage opportunity is available or not[27]. This is basically a computational approach to the derivation of the probabilities of negative spreads. We performed this investigation with the following parameters: $\theta = 0.15$, $\sigma_u = 0.05$, $\sigma_v = 0.5$, $b_1 = 0.25$, $b_2 = 0.2$, $\lambda = 0.75$, $k = 1$ and $C = 0.25$. By comparing the probabilities from the simulations and the probabilities from the analytical solution we have the following picture:

[27] This is given by checking whether the buy price is lower than the sell price. Formally the condition is given by $\min\left(p_{1,t} + \frac{c_1}{2}, p_{2,t} + \frac{c_2}{2}\right) > \max\left(p_{1,t} - \frac{c_1}{2}, p_{2,t} - \frac{c_2}{2}\right)$.

Figure 2 – Simulation and Analytical Solutions for Arbitrage Opportunities



The values for Figure 2 were calculated by simulating the microstructure model for 1000 times. As one can see, the values from the analytical solution match the probabilities from the simulated equations very well. This result corroborates our mathematical derivations.

Methodology

The objective of this paper is to identify the drivers of an arbitrage profit at a microstructure level. The theoretical derivations showed that the likelihood of a negative spread is related to three factors: the average spread across platforms and the explicit transaction cost, the inventory imbalance in terms of past inventory and dealer's reaction and also the existence of a lag update for the efficient price in one of the platforms. The first two are relatively easier to test than the last one, the reasoning being that the quoted spread is observed at any point in time and, even though the inventory of each dealer is not visible, we know from Equations (3) and (7) that its driver is the incoming of trades[28] (the higher the number of observed trades, the higher the change in inventory). The difficulty in empirically studying the lag update factor is because the efficient price process is not observed and therefore it is not

[28] But, be aware that if there are informed traders in the market, order flow can also impact the quotes at a contemporaneous level. Therefore, the effect of trades in arbitrage opportunities has two dimensions, the inventory effect and the asymmetric information effect. The difference of the impact of these effects towards the likelihood of an arbitrage profit is that the last one is only true if k is higher than zero.

possible to test whether the quotes are following a particular lagged regression specification.

For the other factors, the research is a straightforward causal relationship investigation. For the paper we use two methods. The first is an event study framework and the second is the formal estimation of a probit specification. In the event study part we are interested in seeing what is the profile for the behavior of the variables in question (e.g. number of trades, average value of spreads) just prior to the event of a negative spread. Since the time of a negative spread is always observable, we can compute aggregate measures of the number of trades, the value of the spread, volatility and so forth for the time before and after the event. These values are then analyzed through a graphical representation. This simple framework of the study can give important information regarding the profile of an arbitrage profit and whether the implications from the theoretical model make sense in real world data.

While the event study is straightforward to implement, it does not provide a formal testing background for our hypothesis regarding the likelihood of an arbitrage profit. For this objective, we choose to use a formal probit specification for each of the bonds. The dependent variable will be the occurrence of a negative spread, which takes value one if an arbitrage opportunity takes place at time *t* and zero otherwise. The econometric specification will follow:

$$
\begin{aligned}
DnegSpread_t &= \Phi\left(X_t\right) \\
X_t &= \alpha + \beta.Spread_t^- + \phi.nTrades_t^- + \gamma.Volat_t^- + \theta.timeDay_t
\end{aligned} \tag{33}
$$

Where:

$DnegSpread_t$ - Dummy vector with ocurrences of a negative spread

$Spread_t^-$ - Adjusted average value of spread for time *t*

$nTrades_t^-$ - Adjusted number of trades for time *t*

$Volat_t^-$ - Volatility at time *t*

$timeDay_t$ - Time of the day (in hours)

For Equation (33), the function Φ is the cumulative normal density which ensures that the fitted values ($\Phi\left(X_t\right)$) are bounded in [0, 1]. All the models were estimated by maximum likelihood[29]. The choice of explanatory variables follows the hypothesis which we are testing in the paper, more precisely, that the behavior of spreads and inventory in each platform helps to predict the occurrence of an arbitrage profit. The spread variable is calculated by aggregating the best quotes in both platforms. That is, we find the best ask and bid prices for every point in time and then we compute the difference between, which is our measure of spread.

Note thought that the inventory level for each market maker is not observed. But, it is clear that such a measure is driven by the number of executed trades. So, the higher

[29] Special thanks to James Le Sague for providing a resourceful econometric toolbox for Matlab (http://www.spatial-econometrics.com/).

the number of observed trades, the higher the variation in the inventory of a particular dealer. Therefore we use the number of trades as our proxy for inventory behavior. But, it should also be pointed out that the order flow will impact the quotes on a different level if there is a delay in price update and informed traders are present in the market. Therefore, following our theoretical model we can argue in two ways regarding the role of the number of trades[30]. The variable $timeDay_t$ measures (seconds) the time from midnight for each of the 10 minute intervals. The idea is to test whether there is an intraday pattern for the likelihood of an arbitrage opportunity. The motivation behind this test is given in the event study part of the paper.

It also should be pointed out that in order to be able to estimate the model given by (33), we aggregate the tick by tick data by taking averages in ten minute intervals. We also adjust such an aggregation in order to avoid any bias in the estimation. For instance, if we aggregate the explanatory variables to ten minute intervals unconditionally, then it may happen that in the estimation we are using information after the occurrence of a negative spread. This would be counterintuitive as what we are studying are the drivers of an arbitrage profit which, by definition, would consist of information prior to the event. The solution we find for this problem is to aggregate the data conditionally. For the times where a negative spread event was present, we find the time it occurred within the ten minute interval and for the averages we use only events prior to it[31]. This way, for any of the variables, the probit model will not be using future information for the estimation of the parameters.

The Data

The data for this study are kindly supplied by MTS[32] in conjunction with the ICMA Centre. The main data consist of trades and quotes for fixed income instruments in European countries. While this vast database contains tick by tick data starting from April 2003 to the present, we select the year 2004. We further restrict the data to cover bonds only from Italy. This is the country with highest liquidity in trading terms[33]. For example, Italy alone represents approximately 65% of all trades for the year 2004. We further restrict the data to only bonds that are traded on the European and the local platforms[34] with issue date prior to the first day in the sample. Given this population of instruments, we group them according to their maturity with respect to the first day of the research (02/01/2004) taking into account a band of half a year. This means that a bond maturing on 01/04/2005 would have 1.24 years since 02/01/2004 and would be labeled as a one year bond. When more than one bond is found for each maturity label, we search for the one with the highest number of trades. We also remove any bond which has no occurrence of negative spreads. At the end of this selection procedure we are left with twelve Italian fixed income instruments.

[30] We also ran the probit models using the absolute of order flow imbalance in between platforms as one of the explanatory variables. The results are very comparable to the use of the number of trades.
[31] For the cases where there were no events prior to the negative spread occurrence, we use the values of the variables in the previous 10 minute interval.
[32] See http://www.mtsgroup.org/.
[33] See (MTS, 2007).
[34] Further details about the structure of the local platforms in MTS markets can be found in (Cheung, Jong, & Rindi, 2005) and (MTS, 2007).

After finding the bonds of the research, we need first to filter the high frequency data before using it in the research. The only filter we used for this research was to delete any quote change which does not imply a change in the quoted price or size for the first level of the order book. The original MTS dataset was built by saving every change in the first three levels of the order book. For this study, we are not particularly interested in the second or third best prices. The second adjustment of the dataset is the filtering of large quote prices changes. The market makers in MTS are not obliged to provide quotes in all trading hours of the day. It may happen that for some time window there are no primary dealers providing competitive quotes for a particular bond. If this event is true and a large trade comes through, it will consume the order book in the same direction of the trade. This joint event, if it happens, results in high price movements in the bid ask quotes (very low (high) for bid (ask)). Since the quoted spread is one of the variables of the research, we are particularly interested in removing such noise. The approach here is to find any price movement that is higher than an arbitrary 5% percent threshold. When these cases are found, the "inadequate" prices are substituted by the previous prices[35]. This procedure produces a smooth behavior for the quoted prices (bid and ask) and consequently a smooth behavior for the quoted spread[36].

On a side note, it is important to establish that there are some crucial differences between the structure of the theoretical model given in the theory and the structure of the MTS markets. For the theory we had two platforms where there was a unique market maker in each. For the European bond market there are multiple dealers for each instrument in each platform, which are not identifiable on a tick by tick basis. Also, some of these dealers in MTS can post parallel quotes in both platforms, that is, they can simultaneously update their prices in both trading venues. But, even with this difference, we can still argue that the conclusions from the theoretical part should hold for the MTS market since there are still inventory needs for each market maker. The fact that there are multiple dealers should scale down the inventory effect, but it will still be present. Next in Table 1, we present some statistical information regarding this dataset.

[35] The consequence of this procedure is the assumption that if there was a market maker, he/she would not price the impact of these large trades in the quotes. While this is unrealistic, it is more intuitive (and simpler) than assigning an arbitrary impact to the prices.

[36] It should be pointed out that we also test the robustness of our results by running the study without any filtering of large price moves. The final results are almost identical.

Table 1 – Descriptive Statistics of the Data

Bond Code	Bond Type	Market Code	Issue Date (dd/mm/yyyy)	Maturity Date (dd/mm/yyyy)	Maturity Label*	Number of Trades	Number of Quote Changes	Standard Deviation of Mid Quote Changes	Average Quoted Spread
IT0003248512	BTP	EBM&MTS	01/03/2002	01/03/2005	1	3,169	103,533	0.4379	0.0079
IT0003364566	BTP	EBM&MTS	15/09/2002	15/09/2005	2	9,930	183,190	0.5063	0.0142
IT0003522254	BTP	EBM&MTS	01/09/2003	01/09/2006	3	7,741	165,649	1.4676	0.0172
IT0003271019	BTP	EBM&MTS	15/04/2002	15/10/2007	4	3,404	219,814	0.4379	0.0194
IT0003532097	BTP	EBM&MTS	15/09/2003	15/09/2008	5	6,152	246,385	0.6011	0.0208
IT0001448619	BTP	EBM&MTS	01/11/1999	01/11/2010	7	3,068	294,160	0.5829	0.0226
IT0003190912	BTP	EBM&MTS	01/08/2001	01/02/2012	8	3,495	308,396	0.8475	0.0237
IT0003472336	BTP	EBM&MTS	01/02/2003	01/08/2013	10	7,386	321,943	0.9527	0.0239
IT0003625909	BTi	EBM&MTS	15/09/2003	15/09/2014	11	1,421	155,844	1.1277	0.0935
IT0003242747	BTP	EBM&MTS	01/02/2002	01/08/2017	14	2,875	289,035	0.8882	0.0449
IT0003493258	BTP	EBM&MTS	01/02/2003	01/02/2019	15	4,574	269,613	2.1821	0.0530
IT0003256820	BTP	EBM&MTS	01/02/2002	01/02/2033	30	1,452	277,524	1.2316	0.1138

* These are based on a half year maturity band. The values are calculated by rounding off the difference (in year scale) of the difference of days between the maturity date and the first day of the research (01/01/2004).

For Table 1, the bond Code column is the ISIN[37] nomenclature for the different assets. The second column shows the type of bonds where, in this case, the great majority are of the BTP type[38] which are bullet bonds[39] with different maturities paying a fixed coupon rate annually or semi-annually. For the maturities of the bonds in Table 1, we can see that they are sorted by the relative maturity from the first date of the sample (01/01/2004). We have in total twelve bonds, each in a specific maturity band. Note that the maturity of the bonds has effects on the volatility of the price changes (measured by mid quote change). For Table 1 it is clear that bonds with higher maturity also show higher variability in the price changes. This can be explained by the fact that higher maturities are more sensitive to interest rate dynamics than shorter maturities[40]. So, when news about interest rates comes to the market, the prices of bonds with higher maturities will present a stronger reaction to it, resulting in higher variability of price changes when compared to short term bonds.

In terms of the size of the dataset, which is being measured by the number of quote changes and trades in Table 1, we see that the volume of information is relatively high. Note that these values are computed after the filtering of the data[41]. This is an expected feature of tick by tick data. As a rule, we see a bigger number of quote changes with respect to the number of observed trades. On average there are approximately 52 quote changes for each trade.

Interestingly, the simple statistics presented in Table 1 already show some microstructure effects. For instance, when one looks at the average values of the quoted spreads, it is clear that they are increasing with the maturity of each country. This is intuitive since bonds with a higher maturity have higher sensitivity to interest rates. Therefore it is riskier for the market maker to provide liquidity for this particular instrument as the changes in the price can go against the volume of his inventory. Therefore, this dealer will demand a higher return for the increase in risk, which is translated in higher spreads. We also explain such higher spreads for longer maturities with the information asymmetry effect. A trader with privileged information about the interest rate will maximize his earnings by trading in the longer maturities. The market maker is aware of such an effect and will also demand higher compensation given this higher likelihood of informed traders trading in these particular instruments.

Results

We start the discussion of the results with the presentation of a simple description of the negative spreads events, in Table 2.

[37] The ISIN code is a unique international label for each bond. Those are assigned by each country's numbering agency.

[38] BTP translates to "*Buoni del Tesoro Poliennali*".

[39] These are bonds that cannot be redeemed prior to maturity.

[40] This is based on the pricing function of a bond. The higher the maturity of the bond, higher the sensitivity of the pricing function with respect to the discount rate. See (Martellini, Priaulet, & Priaulet, Fixed-Income Securities, Risk Management and Portfolio Strategies, 2003) for details.

[41] On average, the filtering removed 66% of the original dataset.

Table 2 – Statistical Description of Cross Market Arbitrage Opportunities

Bond Code	Number of Negative Spreads	Percentage of European Platform for Quote Change in Neg Spreads	Average Value of Negative Spread	Average Duration of Negative Spread (in Sec)	Average Number of Sec for Quote Change after Neg Spread	Average Number of Sec of incoming of Trade after Neg Spread	Number of times (percentage) arbitrage opportunities were traded*	Average Size of Arbitrage Position*	Average Percentage Return per trade (Basis Points)*
IT0003248512	121	0.545	-0.002	0.154	0.154	1,511.377	0.000	€ 483,921,074	0.201
IT0003364566	133	0.444	-0.003	0.551	0.551	2,335.852	0.000	€ 518,327,406	0.258
IT0003522254	25	0.480	-0.016	2.645	2.645	880.627	0.000	€ 691,961,000	1.559
IT0003271019	67	0.507	-0.014	0.360	0.360	2,231.070	0.000	€ 1,020,770,896	1.290
IT0003532097	124	0.403	-0.013	1.480	1.415	1,140.142	0.000	€ 996,150,403	1.275
IT0001448619	339	0.448	-0.014	0.925	0.700	2,237.326	0.000	€ 1,096,101,549	1.252
IT0003190912	479	0.426	-0.013	1.977	1.663	1,880.007	0.000	€ 1,069,829,123	1.212
IT0003472336	556	0.419	-0.013	2.240	1.675	1,334.610	0.000	€ 1,099,659,802	1.305
IT0003625909	352	0.003	-0.030	912.377	79.036	6,002.794	0.000	€ 381,910,440	2.955
IT0003242747	407	0.450	-0.016	1.052	0.991	2,490.744	0.000	€ 601,244,595	1.470
IT0003493258	261	0.441	-0.017	0.871	0.871	1,181.436	0.000	€ 412,144,828	1.715
IT0003256820	91	0.418	-0.025	1.853	1.853	2,786.939	0.000	€ 464,123,901	2.171

- The arbitrage strategy rule was to trade as much as possible given the offered quotes at an arbitrage event. This mean that the buy and sell operations involved the minimum volume being offered at the bid and ask prices. Again we ignore any explicit transaction cost or execution risk.
- The number of times the arbitrage opportunity was traded is calculated by checking the times where a negative spread was followed by a consecutive buy and sell trade prior to the incoming of a quote change. We also check whether the platform of the trades is consistent with the arbitrage strategy (buy in the underpriced and sell in the overpriced).

The values in Table 2 describe the events of arbitrage opportunities found in the data. It is important to note that we only count non consecutive negative spreads. So, if there was a negative spread at *t* and *t*+1, we only save the event for *t*. The intuition is that the consecutive negative spreads are just a continuation of the first occurrence, and therefore they do not constitute a new event.

For the second column of Table 2, we have the number of negative spreads found for each bond on the period of 2004. When comparing the relative size of these events with respect to the number of quote changes, it is clear that they represent a very small proportion. On average, there are approximately 246 cases of negative spreads for each bond. This implies an average rate of approximately 1787 quote changes for each occurrence of a negative spread. Therefore, inter platform arbitrage opportunities are not common in the market. The third column shows the percentage of times which the European platform originated the arbitrage event. Looking at these values we can see that on average the arbitrage events are more likely to be initiated in the local platforms. This can be explained by the fact that there is a higher volume of trades in the local platform as opposed to the European one.

The fifth column of Table 2 shows how much time an arbitrage profit lasts in the market. Again this calculation does not recognize a consecutive spread as a new event. The calculated values take into account the whole time for which an arbitrage profit was quoted in the market. A typical negative spread lasts for approximately 77 seconds for all of the bonds. But, one of the bonds is pushing this average up. When ignoring the outlier[42] (IT0003625909) we have an average duration of approximately 1.28 seconds. Remember that the MTS market is a multi dealer platform so that there are multiple dealers quoting each instrument. Therefore the duration of negative spreads in Table 2 shows an aggregate measure across dealers. For the majority of the bonds this duration is very small (e.g. fifteen percent of a second for IT0003248512), which implies that a great majority of the dealers in MTS indeed have a software control[43] against the creation of arbitrage opportunities.

The sixth column of Table 2 provides the number of seconds which it takes for the market maker to change the quotes after the negative spread. Understandably, it would be intuitive to expect that such a change happens right after the event. But, when looking at values in Table 2, we see that for some of the bonds the arbitrage opportunities last longer than the time it takes for the next quote change after the negative spread. This implies that some of the dealers still make quote changes even after the occurrence of the arbitrage profit. Note also that the duration of a negative spread (fifth column in Table 2) is much less than the average amount of time it takes for a trade to come in after the negative spread event (seventh column in Table 2, average equal to approximately 37 minutes). This implies that, on average, these arbitrage opportunities are not actually traded. This is also supported when one looks at the eighth column of Table 2, which shows the proportion of cases where the negative spread event was followed by a consecutive buy and sell trade at the underpriced and overpriced platforms, respectively.

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[42] The bond in question is clearly an outlier. Other statistics also show the same pattern.

[43] Algorithm quoting is the use of computers for controlling dealers' quotes in the market. For this particular case it represents a control from the software side which makes sure that no inter platform arbitrage opportunity is created. But note that the software control is ex post reactive, meaning that it only becomes active after the arbitrage event occurred. The argument is that if it was ex ante reactive no arbitrage opportunity would be found in the data.

The last three columns of Table 2 show simple trading statistics regarding these arbitrage opportunities. In order to construct these figures, we follow a simple strategy which is to trade a volume equal to the minimum being offered in the arbitrage bid and ask volumes. The idea is the existence of an arbitrageur with no capital constraints who takes the maximum possible size position in order to profit from the mispricing. Since there are two legs in the arbitrage strategy, the maximum possible traded volume will be given by the minimum volume at the quotes involved in the arbitrage. We further assume no transaction costs and no execution risks in the trades. From the average size of these positions, one can see that they deal with a significant amount of resources. On average across the bonds, each purchase leg of the arbitrage trade will involve more than half a billion Euros. But, these can be scaled down by decreasing the volume traded, which was assumed to be the maximum possible. When comparing the profit from the trades with respect to the size of the position, we see that the average return per trade is small (average of three basis points per trade). This result was expected as the price divergence between the platforms is usually very small.

The next part of our analysis is related to the event study framework. In this part we show what happens to the variables in question when a negative spread occurs. For this event study we set a window of one hour around the occurrence of a negative spread. First we find all the cases of negative spreads and save the data within this particular window before and after the event. We further aggregate all the variables using averages in one minute bands. While we have a picture for each of the bonds in the research, we are far more interested in reporting an overall picture for the whole data. The problem in this case is that all of the data are asset dependent. Different volatilities and microstructure effects will make the scale of each variable unique to a particular instrument, meaning that they cannot be simply aggregated cross bonds by taking an average. For the paper we choose to use a simple normalization prior to cross bond aggregation. This normalization will follow:

$$Y_t = \frac{y_t - E(y_t)}{\sigma_y} \tag{34}$$

For Equation (34), the term y_i is the variable in question (e.g. volatility, number of trades etc). The calculation of the averages ($E(y_t)$) and standard deviations (σ_y) will take into account the whole period. So, this normalized variable related to the information of how much the values are situated from the expectation. This formula is then applied for each of the bonds and then averages are taken cross bonds in the calculation of the aggregate measures. Starting the presentation of empirical results, next, Figures 2 to 5 we report the main results of the event study.

Figure 3 – Event Study for Normalized Spreads

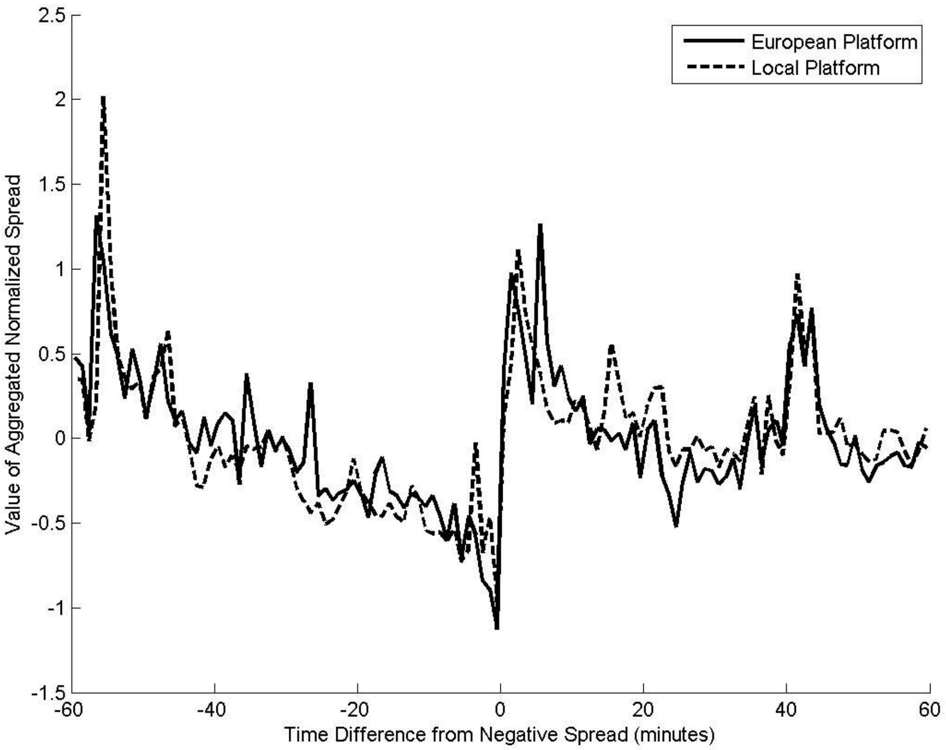


Figure 4 – Event Study for Normalized Volatility

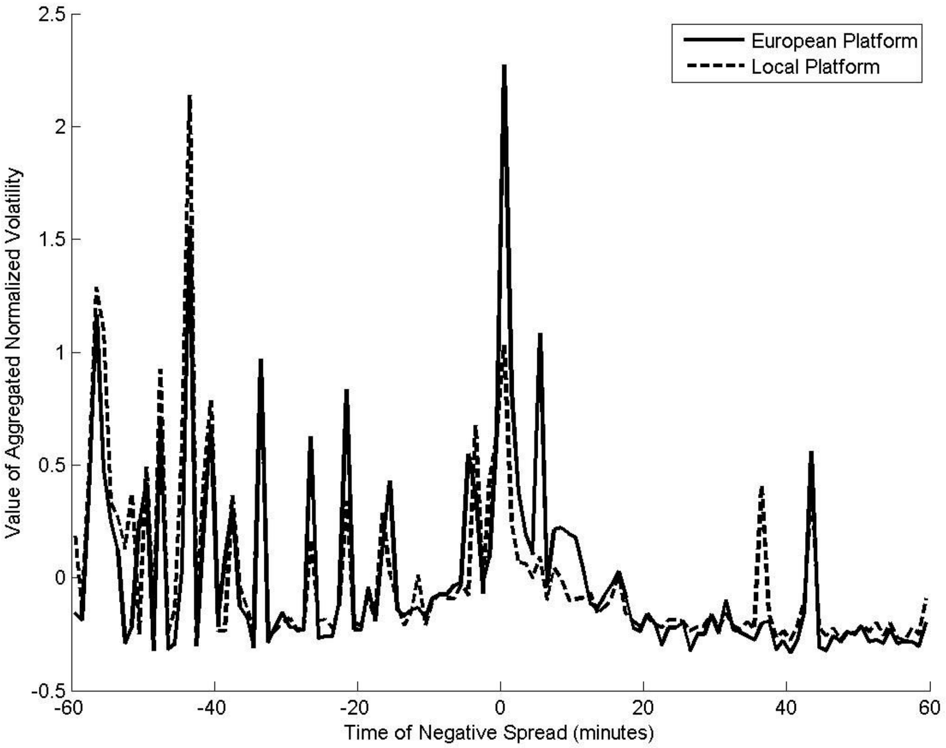


Figure 5 – Event Study for Normalized Number of Quote Changes

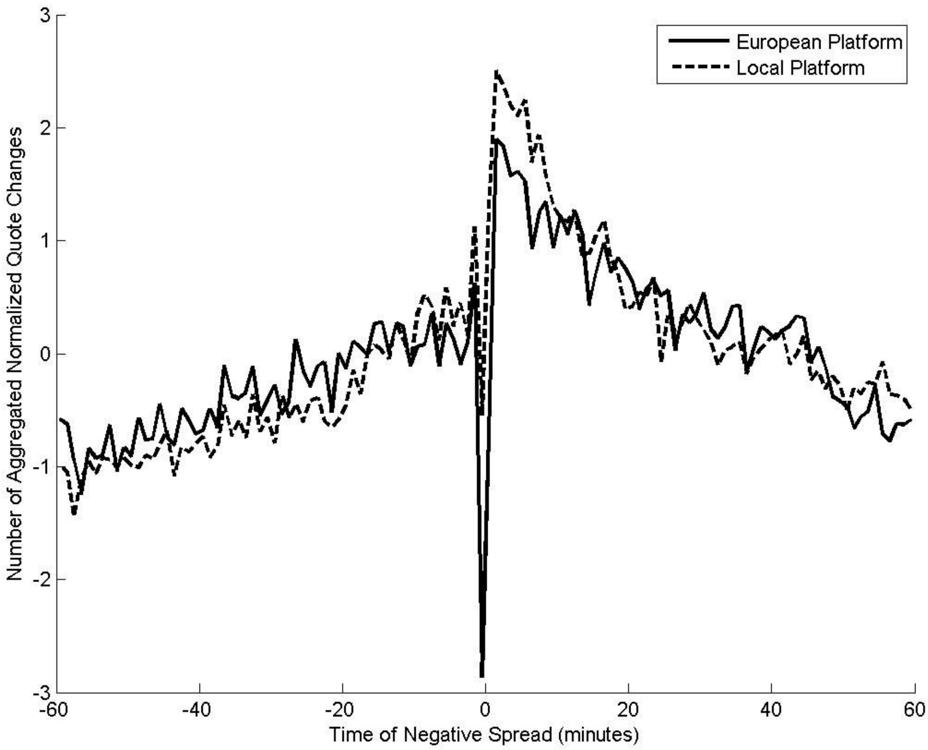
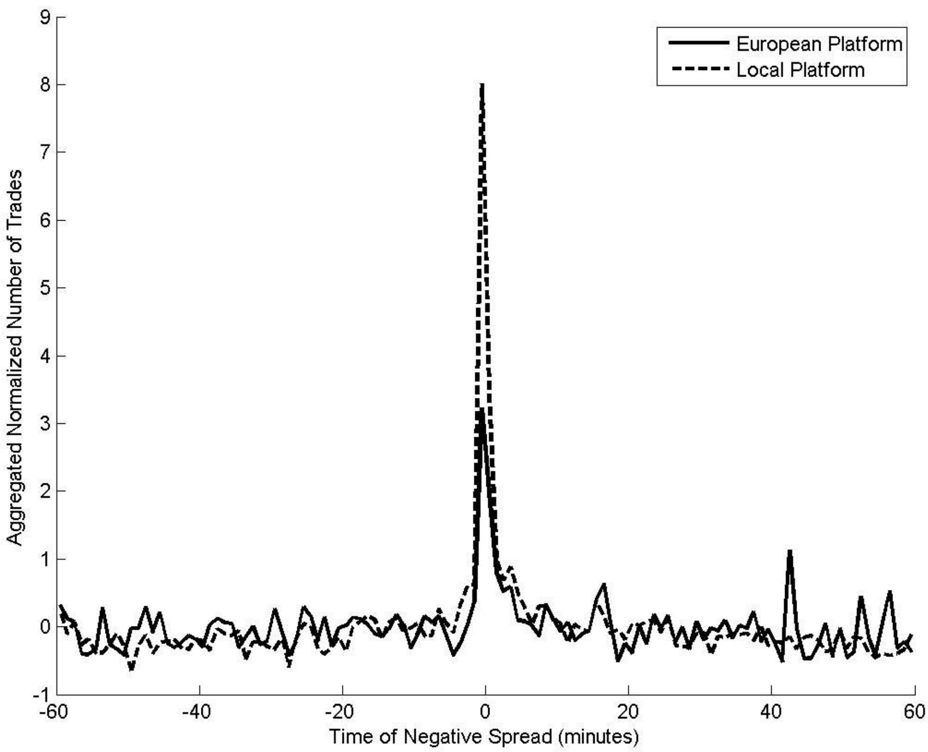


Figure 6 – Event Study for Normalized Number of Trades



The illustrations in Figure 3 to Figure 6 show the average behavior of each of the aggregated measures. The *x* axis represents the sixty minutes before and after the occurrence of a negative spread. The *y* axis measures how the averages of normalized values behave for the whole set of bonds in each platform. These are normalized distances from the sample's average so that the values in the previous figures show how different the values of the variables in question are from their respective expectations.

The first picture, Figure 3, shows the behavior of the normalized spread. Prior to the occurrence of an arbitrage profit, the values of the spread are, in general, lower than the average in both platforms. Half a hour before the arbitrage event, the spreads are on average equal to the mean but descend very fast up to time zero, which is when the negative spread occurs. After that, the spreads quickly change to normal levels. Note that this is true for the local and European platforms.

For the second picture, Figure 4, we have the values for normalized volatility. For this case it seems that prior to the event of a negative spread, the values for normalized variability are relatively unstable, oscillating between high and low values. In general, these values are mostly higher than zero, implying that prior to a negative spread event there is a relatively high variation in mid quote changes. After time zero, the normalized volatility shoots up but this is simply the effect of the quote change after the arbitrage profit. From this picture of normalized volatility, we can say that the absolute changes in quotes after the arbitrage event are, normally, higher than their expectation. That is, the quote change which nulls the arbitrage opportunity is relatively strong when compared to its unconditional distribution. This can be explained by the software control from the market maker side.

The third picture, Figure 5, shows the aggregate number of quote changes prior to (and after) the negative spread event. It seems that one hour before period zero, the number of quote changes is relatively lower than the average. Then they follow a clear trend towards normal levels. After the arbitrage event, the number of quote changes goes up radically. Clearly this is the quick reactive response to the arbitrage opportunity, which also corroborates the evidence for algorithm quoting[44] given in Table 2.

The last picture, Figure 6, shows the average normalized number of trades. While for times away from the zero mark it seems that the number of trades in both platforms is relatively normal, for the times near to the arbitrage events, the number of trades reaches 4 standard deviations from the mean in the local platform and 1.5 for the European. Clearly these are abnormal values when compared to the whole sample's distribution. This implies that just prior to an arbitrage event there is an abnormal number of trades on both platforms. This measure is also abnormal after the event and this can be explained by the fact that the improvement of quotes from the market maker side, which was one of the drivers of the arbitrage opportunity, has attracted trades. Therefore we would expect a high trading activity before and after the negative spread event, which is exactly the picture portrayed by Figure 6.

This result corroborates the theoretical implication in the previous section of the paper. Remember that in the theoretical model, one of the factors behind the likelihood of an arbitrage profit is the inventory imbalances in between the platforms. If the two market makers have opposite excess inventories, they will improve quotes

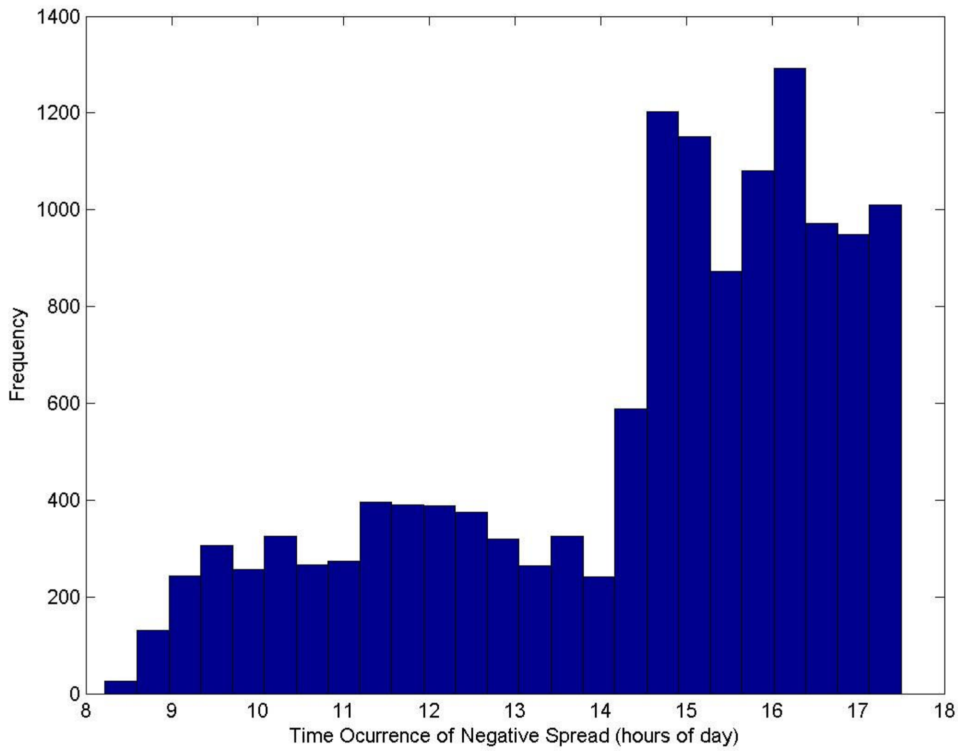
[44] This could be either the computer driven cancelation or a change of the quotes.

in opposite direction in order to induce trades. When such an event happens, it might be possible that the improved quotes portray an arbitrage opportunity. Now, remember that the dealer's inventory is not observed but we know that its driver is the number of trades. The higher the number of trades, the higher the inventory change in any direction. The picture for the normalized number of trades in Figure 2 clearly corroborates such an argument. But be aware that following the microstructure model we can also explain the impact of the order flow as the effect of informed trading and price delay. We cannot differentiate these effects within the analysis of Figure 6.

One of the conclusions to draw from the results of the event study (Figure 3 to Figure 6) is that market makers respond strongly and quickly to the occurrence of an arbitrage profit. This is portrayed by Figure 3 to Figure 5 where right after the occurrence of a negative spread we observe an abnormal number of quote changes, a high value for normalized volatility and also an abrupt change in the value of the normalized spreads. When comparing the results for the event study with respect to the theoretical derivations in first part of the paper, we see that they do corroborate the case of the effect of trades (inventory imbalance and price delay) and the level of spreads. In general, the occurrence of an arbitrage opportunity will be preceded by an unusually small value of the spread and an abnormally high number of trades.

An interesting piece of information from this study is the pattern for the intraday time of occurrence of a negative spread. Next, in Figure 7, we present the frequency of arbitrage opportunities with respect to the time of the day.

Figure 7 – Histogram for Intraday Time of Occurrence of Arbitrage Profits



The values in Figure 7 show an almost linear relationship between the occurrence of the negative spread and the time of the day. This was calculated based on the negative spreads occurrence for all of the bonds. The closer we are to the end of the day, the higher the chances of a negative spread. A possible explanation for the previous picture is that when dealers reach the end of the day, they have sufficient reasons for breaking even, that is, to finish the day with a zero (or insignificant) amount of inventory[45]. The reasoning being that overnight news is a risk factor for the market maker. A market maker who does not care about end of day inventory will demand higher compensation for this particular risk by increasing spreads during trading hours. But, remember that for the MTS market there are multiple dealers. A dealer who manages his inventory and can end the day flat is not exposed to overnight inventory risk will be able to offer more competitive quotes[46].

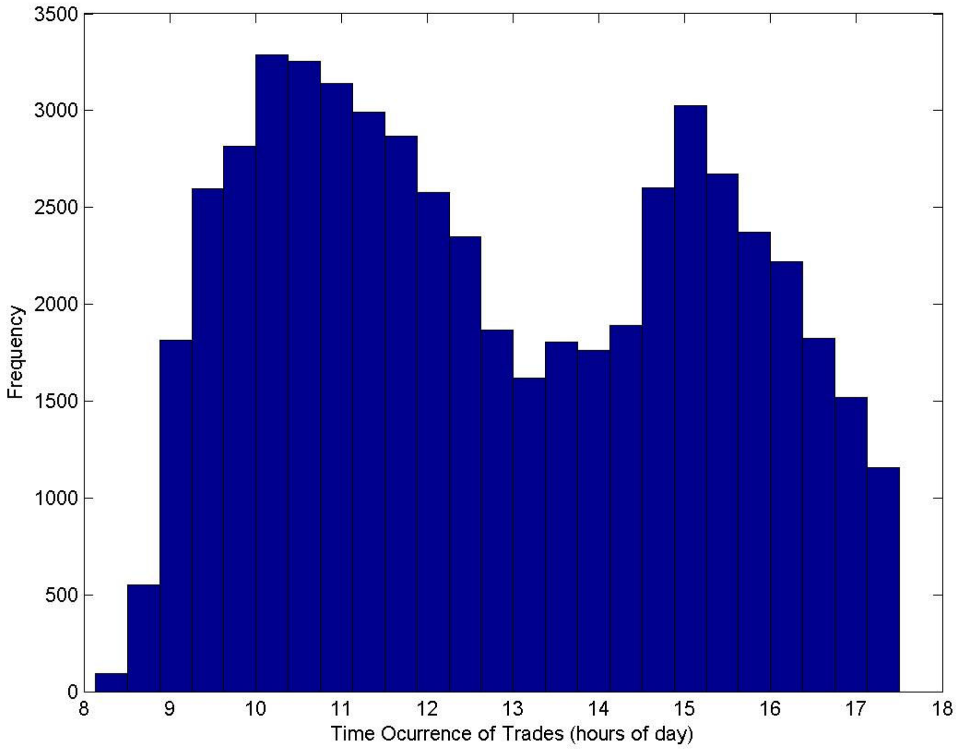
Following this logic within a competitive market dynamics, it is clear that each market maker has an incentive to end the day with flat (or insignificant) inventory level. Now, remember that one of the parameters that drive the likelihood of the arbitrage profit event was the reaction of the dealers towards its past inventory (see parameter b in Equation (2)). As we reach the end of the day, following the logic given before, the dealers have more incentive to improve quotes with respect to their inventory level, that is, they increase the value of b. As long as this increase is not also observed on the other platform (see Equation (16)), this effect will increase the likelihood of an arbitrage profit.

An alternative explanation would be that the shape of Figure 7 is simply the shape of the intraday pattern for the number of trades. Remember that the higher the number of trades, the higher the inventory change and therefore the higher the chances of an arbitrage profit. If the number of trades increases within the day, then it would be expected that the number of arbitrage opportunities also increases. But, for our data, this is clearly not the case.

[45] Empirical evidence of this inventory management strategy can be found in Hasbrouck (2007).

[46] This relation will be dependent on the requirements for disposing of inventory. In our argument, we assume that the cost of disposing of inventory is lower than the required compensation for the overnight inventory risk.

Figure 8 – Intraday Pattern for Number of Trades



As we can see from Figure 8, the trading activity follows an inverted "W" pattern over the trading hours with a high number of trades at around 10:00 and 15:00. Therefore, this alternative explanation does not hold for our case. Close to the middle of the day (13:00) there is a decrease in the trading activity and this could explain the same gap for the pattern in Figure 7, where the decrease in number of trades also decrease the likelihood of a arbitrage opportunity. But, for the rest of the times of the day, it is clear that the shape of Figure 7 cannot be explained by the shape of Figure 8. Therefore, we maintain our argument that the pattern of frequency of arbitrage opportunities towards the time of the day is the result of inventory management from the dealer's side, which cannot be explained by the information asymmetry effect.

Next, we show the results from the estimation of the formal probit specification.

Table 3 – Results from the Estimation of Probit Model

Bond Code	Value of Alpha	Value of Beta	Value of Phi	Value of Gamma	Value of Theta	Log Likelihood	McFadden pseudo-R^2	Correct Signals Percentage	Incorrect Signals Percentage	Box-Ljung Q stat Resid	Box-Ljung Q stat sqr Resid
IT0003248512	-2.06***	-1.00***	0.25***	-1.71	-0.01	-423.86	0.29	20.95%	4.76%	49.46***	38.90***
IT0003364566	-1.60***	-2.39***	0.10***	41.63***	0.03*	-545.99	0.24	2.40%	7.20%	34.05***	37.50***
IT0003522254	-3.07***	-0.21	0.15***	0.06	0.01	-134.75	0.18	0.00%	0.00%	22.09***	16.80***
IT0003271019	-2.42***	-0.41**	0.16***	43.63**	0.02	-316.60	0.16	3.45%	1.72%	110.25***	76.14***
IT0003532097	-2.81***	-0.44***	0.15***	45.09***	0.07***	-483.04	0.23	12.15%	7.48%	17.62***	13.55**
IT0001448619	-2.15***	-0.43***	0.20***	7.85***	0.06***	-1,107.05	0.22	20.55%	2.40%	86.86***	83.66***
IT0003190912	-2.24***	-0.42***	0.20***	20.15***	0.07***	-1,412.84	0.24	23.96%	2.69%	131.89***	121.60***
IT0003472336	-1.97***	-0.31***	0.17***	34.58***	0.04***	-1,521.36	0.28	28.84%	4.36%	163.24***	132.63***
IT0003625909	-0.65**	-0.49***	0.18***	74.50***	0.08***	-340.74	0.89	92.87%	3.74%	457.97***	1,327.42***
IT0003242747	-1.87***	-0.39***	0.19***	15.86***	0.08***	-1,228.10	0.26	21.07%	2.53%	109.23***	143.77***
IT0003493258	-2.09***	-0.29***	0.18***	13.92***	0.06***	-775.72	0.32	29.41%	4.07%	120.57***	142.86***
IT0003256820	-2.16***	-0.13***	0.25***	5.97***	0.05**	-382.42	0.22	11.25%	1.25%	56.55***	43.43***

- All standard errors are calculated with White's Robust Covariance Matrix.
- The McFadden pseudo-r^2 measures the relative improvement in the likelihood of an unrestricted model against a restricted model with just the intercept parameter.
- The "Correct Signals percentage" column calculates the percentage of negative spreads events which were correctly forecasted by the probit model.
- The "Incorrect Signals percentage" column calculates the percentage of negative spreads events which were mistakenly forecasted by the probit model.
- The Box-Ljung Q stat is measuring the autocorrelation up to order 5 in the residuals (squared or not) from the model.
- The estimated probit model is given by:

$$DnegSpread_t = \Phi\left(\alpha + \beta.Spread_t^- + \phi.nTrades_t^- + \gamma.Volat_t^- + \theta.timeDay_t\right)$$

Where:

$DnegSpread_t$ - Dummy vector with ocurrences of negative spread

$Spread_t^-$ - Adjusted average value of spread for time t

$nTrades_t^-$ - Adjusted number of trades for time t

$Volat_t^-$ - Volatility at time t

$timeDay_t$ - Time of the day (in hours)

The values in Table 3 show how the different factors impact on the likelihood of an arbitrage profit. Remember that in the estimation we only use past information for the models[47]. In the aggregation procedure, we adjusted the variables so that they do not use data after the arbitrage profit event. For the adjusted spread coefficient, we see that its parameter (β) is negative for all cases and statistically significant for approximately 92% of the bonds (11 out of 12), meaning that the lower the value of the spread, the higher the likelihood of an arbitrage profit.

For the value of ϕ, which is the coefficient for the adjusted number of trades, we see that its value is always positive and statistically significant. We had similar results from the aggregate picture in the event study (Figure 3 to Figure 6) and from the information of Table 3, it seems that the relationship also holds strongly for the bonds on an individual basis. Therefore, the higher the number of observed trades in the past, the higher the probability of occurrence of an arbitrage profit.

For the parameter γ, which measures the relationship between the likelihood of a negative spread and the pre-event volatility of the log returns, we see that its value is mostly positive, indicating that the higher the volatility of the mid quote price, the higher the probability of the occurrence of an arbitrage profit. This positive property is also found for the θ parameter, which measures the dependency of the negative spread arbitrage with respect to the time of the day. But, it is clear that the effect is weaker as for only approximately 75% of the cases (9 out of 12) is the theta parameter positive and statistically significant.

When looking at the fitting performance of the models, which is being measured by the McFadden pseudo R squared and the percentage of correct predictions[48], we see that the performance of the models is relatively acceptable when considering that we are dealing with high frequency data. With the exception of one bond (IT0003625909), the values of McFadden R squared values range from 0.16 to .29. When analyzing the value of correct and incorrect percentage predictions, we observe that the model performed relatively well, with the number of correct predictions significantly larger than the number of incorrect signals. Again, we find an outlier value for the previously mentioned bond, which correctly predicted 94.52% of the negative spreads occurrences.

In general the results from Table 3 are as expected. The signs of the parameters are all consistent with the theoretical predictions given in the first part of the paper. That is, the optimal scenario for the occurrence of an inter dealer arbitrage profit is given by low spreads across platforms, the incoming of a high number of trades, an increase in the volatility and also a time period close to the end of the day. Therefore, empirically, we can argue that the effect seen in the abstract microstructure model applies to the European bond market.

[47] This is the reasoning behind the use of the minus symbol on the parameter's names.
[48] The number of correct predictions is simply the number of times which the model correctly forecasted an arbitrage opportunity. We use a threshold of 50% in such calculation. Similar calculations are conducted for the value in the column "Incorrect signals percentage" but, in this case, the interest is the number of incorrect signals generated by the model.

Conclusions

The objective of this paper was to investigate the drivers behind the occurrence of a cross market arbitrage opportunity. Within a theoretical framework supported by a structural model, we were able to show that theoretically the likelihood of a multi platform arbitrage profit at any point in time is a function of the average level of the spreads, the inventory reaction imbalance in between the market makers of the different trading platforms and also the relative delay of the efficient price update in the mid quote price process.

We investigated the existence of these negative spreads for a vast dataset consisting of twelve bonds from the most liquid European bond market, Italy. Simple statistics show that these inter platform arbitrage opportunities are relatively rare, they usually last for less than one and a half seconds and are not traded upon. The event study which we applied showed that prior to the occurrence of an arbitrage opportunity, the spreads are in a relatively low level and there are a relatively large number of observed trades in both platforms. Also, after the event the market makers react quickly, which increases the number of quote changes significantly. This quick reaction, which in some cases takes an average of only 15% of a second suggests that the reaction from the market maker is software related (algorithm quoting) for most of the cases. The results from the event study were corroborated in the formal probit econometric model. The direction of the effects were all respecting the predictions from the theoretical microstructure model.

The contribution of this paper resides in describing the drivers of arbitrage opportunities at a microstructure level. We show empirically that the predictions from the microstructure model are supported in the observed high frequency dataset. This sheds theoretical and empirical evidence that that arbitrage opportunities are the result of microstructure frictions in the trading process. While we constrained our analysis to inter platform arbitrage opportunities, the setup given in the theoretical part of the paper can easily be extended to other types of arbitrage with minor modifications. These and other considerations with respect to the microstructure model underlying the derivations are left for future research.

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APPENDIX

Appendix 1 - Expansion of the Arbitrage Profit Equation

The profit from instantaneously buying one unit ($\phi = 1$) on platform 2 and selling it instantaneously on platform 1 is given by:

$$arbPL_t^{2,1} = p_{1,t} - p_{2,t} - C \qquad \text{[A. 1.1]}$$

Expanding Equation [A. 1.1] by following equations (1) to (9) and setting $x_{1,t} = -1$ (a sell in platform 1) and $x_{2,t} = 1$ (a buy in platform 2) we have:

$$arbPL_t^{2,1} = q_{1,t} - \frac{c_1}{2} - \left(q_{2,t} + \frac{c_2}{2} \right) - C \qquad \text{[A.1.2]}$$

Substituting for $q_{1,t}$ and $q_{2,t}$:

$$arbPL_t^{2,1} = m_t - m_{t-k} - b_1 I_{1,t-1} + b_2 I_{2,t-1} - \frac{c_1}{2} - \frac{c_2}{2} - C \qquad \text{[A.1.3]}$$

We can now expand $m_t - m_{t-k}$ to show that it follows a weighted sum of both innovations. Starting with:

$$m_t - m_{t-k} \qquad \text{[A.1.4]}$$

Substituting for m_t:

$$m_{t-1} + \theta v_t + u_t - m_{t-k} \qquad \text{[A.1.5]}$$

Substituting for m_{t-1}:

$$m_{t-2} + \theta v_{t-1} + u_{t-1} + \theta v_t + u_t - m_{t-k} \qquad \text{[A.1.6]}$$

The pattern is already clear. Each substitution for m_{t-d} will give out $m_{t-d-1} + \theta v_{t-d} + u_{t-d}$. Therefore, recursively going back to time *t-k* we can show that:

$$m_t - m_{t-k} = \sum_{j=0}^{k-1} u_{t-j} + \theta \sum_{j=0}^{k-1} \nu_{t-j} \qquad [A.1.7]$$

Using the last result in Equation [A.1.3] yields:

$$arbPL_t^{2,1} = \sum_{j=0}^{k-1} u_{t-j} + \theta \sum_{j=0}^{k-1} \nu_{t-j} - b_1 I_{1,t-1} + b_2 I_{2,t-1} - \frac{c_1 + c_2}{2} - C \qquad [A.1.8]$$

Appendix 2 - Expansion of the Inventory Equation

The term $I_{1,t-1}$ is given by:

$$I_{1,t-1} = I_{1,t-2} - x_{1,t-1} \qquad [A.2.1]$$

By substituting for the term $I_{1,t-2}$ we have:

$$I_{1,t-1} = I_{1,t-3} - x_{1,t-2} - x_{1,t-1} \qquad [A.2.2]$$

Substituting again for $I_{1,t-3}$:

$$I_{1,t-1} = I_{1,t-4} - x_{1,t-3} - x_{1,t-2} - x_{1,t-1} \qquad [A.2.3]$$

It is clear now that recursively substituting the inventory at time *t*-1 will be given by:

$$I_{1,t-1} = I_{1,0} - \sum_{j=1}^{t-1} x_{1,t-j} \qquad [A.2.4]$$

For the last equation, $I_{1,0}$ is the inventory level at the beginning of the period and is equal to zero. Using this property and substituting for $x_{1,t}$, we have:

$$I_{1,t-1} = -\lambda \sum_{j=1}^{t-1} v_{t-j} \qquad [A.2.5]$$

Therefore, for platform 1, the inventory level at time *t*-1 can be represented as the negative cumulative sum of v_{t-j}, weighted by λ.

Appendix 3 – Derivation of the Covariance between *X* and *Y*

We are interested in calculating the covariance between the terms *X* and *Y*, which are given as follow:

$$X = \sum_{j=0}^{k-1} u_{t-j} + \theta \sum_{j=0}^{k-1} v_{t-j} \qquad [A.3.1]$$

$$Y = \left(\lambda b_1 - \left(1-\lambda\right) b_2\right) \sum_{j=1}^{t-1} v_{t-j} \qquad [A.3.2]$$

The covariance between both processes is calculated as:

$$\sigma_{X,Y} = E\left([X - E(X)][Y - E(Y)]\right) \qquad [A.3.3]$$

Substituting [A.3.1] and [A.3.2] in [A.3.3] and using the property that $E(X) = 0$ and $E(Y) = 0$ yields:

$$\sigma_{X,Y} = E\left(\left[\sum_{j=0}^{k-1} u_{t-j} + \theta \sum_{j=0}^{k-1} v_{t-j}]\right]\left[\left(\lambda b_1 - \left(1-\lambda\right) b_2\right) \sum_{j=1}^{t-1} v_{t-j}\right]\right) \qquad [A.3.4]$$

Notes now that u_t and v_t are uncorrelated at any lag. Therefore, the term $\sum_{j=0}^{k-1} u_{t-j}$ in [A.3.4] is redundant in the derivations as any interaction of u_t and v_t will be set to zero given that $E\left(u_{t-i} v_{t-j}\right) = 0$ for any *j* and *i*. This simplifies last formula to:

$$\sigma_{X,Y} = E\left(\left[\theta \sum_{j=0}^{k-1} v_{t-j}]\right]\left[\left(\lambda b_1 - \left(1-\lambda\right) b_2\right) \sum_{j=1}^{t-1} v_{t-j}\right]\right) \qquad [A.3.5]$$

For Equation [A.3.5], see that some of the terms of both sums are the same. We can further simplifies last formula with:

$$\sigma_{X,Y} = E\left(\left[\theta v_t + \theta \sum_{j=1}^{k-1} v_{t-j}]\right]\left[\left(\lambda b_1 - \left(1-\lambda\right) b_2\right) \sum_{j=1}^{t-1} v_{t-j}\right]\right) \qquad [A.3.6]$$

Now, by assessing the property that $E\left(v_{t-i}v_{t-j}\right)=0$, we can disregard any interaction of v_{t-i} with v_{t-j} for $j \neq i$ since they will equal to zero once the expectations operation is applied. This operation yields:

$$\sigma_{X,Y} = E\left(\left[\theta\sum_{j=1}^{k-1} v_{t-j}]\right]\left[\left(\lambda b_1-(1-\lambda)b_2\right)\sum_{j=1}^{k-1} v_{t-j}\right]\right) \qquad [A.3.7]$$

Now, using the result that $E\left(u_{t-i}^2\right)=\sigma_u^2$ we can further simplify [A.3.7], leading us to the final result:

$$\sigma_{X,Y} = \theta\left(\lambda b_1-(1-\lambda)b_2\right)(k-1)\sigma_v^2 \qquad [A.3.8]$$

Appendix 4 – Derivation of Cross Market Arbitrage Probability

The result which we are interested in is the closed form solution for the probability of an arbitrage profit, irrespective of the trading platform. This will be given by:

$$\Pr\left(arbPL_t > 0\right) = 1 - \Pr\left(X + Y \leq -Z \mid \Omega\right) + \Pr\left(X + Y \leq Z \mid \Omega\right) \qquad [A.4.1]$$

For last formula, the probability functions are the cumulative density function of a normal variable (*X*+*Y*) evaluated at *Z*. This will be given by:

$$\Pr\left(X + Y \leq -Z\right) = \frac{1}{2}\left(1 + erf\left(\frac{-Z}{\sqrt{2\sigma^2_{X+Y}}}\right)\right) \qquad [A.4.2]$$

Substituting [A.4.2] in [A.4.1] we have:

$$\Pr\left(arbPL_t > 0\right) = 1 - \frac{1}{2}\left(1 + erf\left(\frac{-Z}{\sqrt{2\sigma^2_{X+Y}}}\right)\right) + \frac{1}{2}\left(1 + erf\left(\frac{Z}{\sqrt{2\sigma^2_{X+Y}}}\right)\right) \qquad [A.4.3]$$

Simplifying last equation and noticing that $erf\left(-x\right) = -erf\left(x\right)$ we have that:

$$\Pr\left(arbPL_t > 0\right) = 1 + erf\left(\frac{Z}{\sqrt{2\left(\sigma^2_{X+Y}\right)}}\right) \qquad [A.4.4]$$

Where:

$$\sigma^2_{X+Y} = k\left(\sigma^2_u + \theta^2\sigma^2_v\right) + \left(\lambda b_1 - \left(1-\lambda\right)b_2\right)\sigma^2_v\left[\left(\lambda b_1 - \left(1-\lambda\right)b_2\right)\left(t-1\right) + 2\theta\left(k-1\right)\right]$$

$$Z = -\left(2^{-1}\left(c_1 + c_2\right) + C\right)$$

Expanding the error function in [A.4.4] we have the final result:

$$\Pr\left(arbPL_t > 0\right) = 1 + \frac{2}{\sqrt{\pi}}\int_0^{\psi} \exp\left(-t^2\right)dt \qquad [A.4.5]$$

Where:

$$\psi = \frac{-\left(2^{-1}\left(c_1 + c_2\right) + C\right)}{\sqrt{2\left[k\left(\sigma_u^2 + \theta^2 \sigma_v^2\right) + \left(\lambda b_1 - (1-\lambda) b_2\right)\sigma_v^2\left(\left(\lambda b_1 - (1-\lambda) b_2\right)\left(t-1\right) + 2\theta\left(k-1\right)\right)\right]}}$$