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Emenike, Kalu O.

Dept. of Banking and Finance, University of Nigeria Enugu Campus, Enugu State Nigeria

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Emenike Kalu O.

Department of Banking and Finance, University of Nigeria, Enugu Campus, Enugu State, Nigeria E-mail: <u>emenikekaluonwukwe@yahoo.com</u> Tel: +2348035526012

Abstract

There is quite an extensive literature documenting the behaviour of stock returns volatility in both developed and emerging stock markets, but such studies are scanty for the Nigerian Stock Exchange (NSE). Modelling volatility is an important element in pricing equity, risk management and portfolio management. For these reasons, this paper investigates the behaviour of stock return volatility of the NSE returns using GARCH (1,1) and the GJR-GARCH(1,1) models assuming the Generalized Error Distribution (GED). Monthly All Share Indices of the NSE from January 1999, to December 2008, provided the empirical sample for investigating volatility persistence and asymmetric properties of the series. The results of GARCH (1,1) model indicate evidence of volatility clustering in the NSE return series. Also, the results of the GJR-GARCH (1,1) model show the existence of leverage effects in the series. Finally, the Generalized Error Distribution (GED) shape test reveals leptokurtic returns distribution. Overall results from this study provide evidence to show volatility persistence, fat-tail distribution, and leverage effects for the Nigeria stock returns data.

Key words: Modelling, Volatility, Stock Returns, GARCH Models, Nigerian Stock Exchange **JEL Classification:** C22, C52, G10

1. Introduction

Numerous studies have documented evidence showing that stock returns exhibit phenomenon of volatility clustering, leptokurtosis and Asymmetry. Volatility clustering occurs when large stock price changes are followed by large price changes, of both signs, and small price changes are followed by periods of small price changes. Leptokurtosis means that the distribution of stock returns is not normal but exhibits fat-tails. In other words, Leptokurtosis signifies high probability for extreme values than the normal law predict in a series. Asymmetry, also known as leverage effects, means that a fall in return is followed by an increase in volatility greater than the volatility induced by an increase in returns. This implies that more prices wander far from the average trend in a crash than in a bubble because of higher perceived uncertainty (Mandelbrot, 1963; Fama, 1965; Black, 1976). These characteristics are perceived as indicating a rise in financial risk, which can adversely affect investors' assets and wealth. For instance, volatility clustering makes investors more averse to holding stocks due to uncertainty. A greater risk premium in order to insure against the increased uncertainty. A greater risk premium results in a higher cost of capital, which then leads to less private physical investment.

Modelling volatility is an important element in pricing equity, risk management and portfolio management. Stock prices reflect all available information and the quicker they are in absorbing accurately new information, the more efficient is the stock market in allocating resources. Modelling volatility will improve the usefulness of stock prices as a signal about the intrinsic value of securities, thereby, making it easier for firms to raise fund in the market. Also, detection of stock returns volatility-trends would provide insight for designing investment strategies and for portfolio management. Hence, it is important to understand the behaviour of the NSE returns volatility.

The main objective of this paper is to investigate the behaviour of stock return volatility in Nigeria. This will involve examining NSE return series for evidence of volatility clustering, fattails distribution and leverage effects as they provide essential information about the riskiness of assets in the market. The paper used the Generalized Autoregressive Conditional Heteroscedasticity (GARCH 1, 1) model to capture the nature of volatility, the Generalized Error Distribution (GED) to capture fat-tails and the Glosten, Jagannathan and Runkle (1993) modification to GARCH (1, 1), known as GJR-GARCH (1,1) model to capture leverage effects. This paper proceeds as follows: coming after Section 1 is Section 2, which provides a brief review of the relevant literature. Section 3 provides data and methodology. Section 4 provides discussion of empirical findings and Section 5 concludes.

2 Brief Review of Relevant literature

The studies of Mandelbrot (1963), Fama (1965) and Black (1976) highlight volatility clustering, leptokurtosis, and leverage effects characteristics of stock returns. Engle (1982) introduced the autoregressive conditional Heteroscedasticity (ARCH) to model volatility by relating the conditional variance of the disturbance term to the linear combination of the squared disturbances in the recent past. Bollerslev (1986) generalized the ARCH model by modeling the conditional variance to depend on its lagged values as well as squared lagged values of disturbance. Since the works of Engle (1982) and Bollerslev (1986), various variants of GARCH model have been developed to model volatility. Some of the models include EGARCH originally

proposed by Nelson (1991), GJR-GARCH model introduced by Glosten, Jagannathan and Runkle (1993), Threshold GARCH (TGARCH) model due to Zakoian (1994). Following the success of the ARCH family models in capturing behaviour of volatility, Stock returns volatility has received a great attention from both academies and practitioners as a measure and control of risk both in emerging and developed financial Markets.

Concerning the effectiveness of the ARCH family models in capturing volatility of financial time series, Hsieh (1989) found that GARCH (1,1) model worked well to capture most of the stochastic dependencies in the time series. Based on tests of the standardized squared residuals, he found that the simple GARCH (1,1) model did better at describing data than a previous ARCH(12) model also estimated by Hsieh (1988). Similar conclusions were reached by Taylor (1994), Brook and Burke (2003), Frimpong and Oteng-Abayie (2006) and Olowe (2009). In a like manner, Bekaert and Harvey (1997) and Aggarwal et al.(1999) in their study of emerging markets volatility, confirm the ability of asymmetric GARCH models in capturing asymmetry in stock return volatility. Thus, ARCH family models are good candidates for modelling and estimating volatility in emerging stock markets. In literature, also, studies like Campbell and Hentschel (1992), Braun *et al* (1995) and LeBaron (2006) provide evidence that stock returns has time-varying volatility.

Although the GARCH model has been very successful in capturing important aspect of financial data, particularly the symmetric effects of volatility, it has had far less success in capturing extreme observations and skewness in stock return series. The Traditional Portfolio Theory assumes that the logarithmic stock returns are independent and identically distributed (IID)

normal variables which do not exhibit moment dependencies, but a vast amount of empirical evidence suggest that the frequency of large magnitude events seems much greater than is predicted by the normal distribution (Harvey and Siddique, 1999; Verhoeven and McAleer, 2003; diBartolomeo, 2007). According to Mandelbrot (1963), extreme events are far too frequent in financial data series for the normal distribution to hold. He argues for a stable Paretian model, which has the uncomfortable property of infinite variance. Fama (1965) provides empirical tests of Mandelbrot's idea on daily US stock returns and finds fat- tails. Moreover, investors view upside and downside risks differently, with a preference for positively skewed returns, implying that more than the first two moments of returns may be priced in equilibrium (see Lai, 1991; Satchell, 2004). This has lead to the use of non-normal distributions such as: Student-*t*, GED, asymmetric Student-*t* and asymmetric GED to model the empirical distribution of conditional returns (Theodossiou, 1998, 2001; Olowe, 2009).

In Nigeria, the few published studies on modelling volatility of stock returns, include: Ogum, Beer and Nouyrigat (2005), Jayasuriya (2002), Okpara and Nwezeaku (2009). Jayasuriya (2002) use asymmetric GARCH methodology to examine the effect of stock market liberalization on stock returns volatility of fifteen emerging markets, including Nigeria, for the period December 1984 to March 2000. The study reports, among others, that positive (negative) change in prices have been followed by negative (positive) changes indicating a cyclical type behavior in stock price changes rather than volatility clustering in Nigeria. In contrast to Jayasuriya (2002), Ogum, Beer and Nouyrigat (2005) investigate the emerging market volatility using Nigeria and Kenya stock return series. Results of the exponential GARCH model indicate that asymmetric volatility found in the U.S. and other developed markets is also present in Nigerian, but Kenya shows evidence of significant and positive asymmetric volatility, suggesting that positive shocks increase volatility more than negative shocks of an equal magnitude. Also, they show that while the Nairobi Stock Exchange return series indicate negative and insignificant risk-premium parameters, the NSE return series exhibit a significant and positive time-varying risk premium. Finally, they report that the GARCH parameter (β) is statistically significant indicating volatility persistence in the two markets. Okpara and Nwezeaku (2009) examine the effect of the idiosyncratic risk and beta risk on the returns of 41 randomly selected companies listed on the NSE from 1996 to 2005. They employed a two-step estimation procedures, firstly, the time series procedure is used on the sample data to determine the beta and idiosyncratic risk for each of the companies; secondly, a cross-sectional estimation procedure is used employing EGARCH (1,3) model to determine the impact of these risks on the stock market returns. Their results reveal, among others, that volatility clustering is not quite persistent but there exists asymmetric effect in the Nigerian stock market. They concluded that unexpected drop in price (bad news) increases predictable volatility more than unexpected increase in price (good news) of similar magnitude in Nigeria.

From the brief review of literature above, it is glaring that ARCH family of models has, extensively, been used to model volatility. While simple GARCH (1,1) is good enough to capture volatility clustering, it cannot capture fat-tails and asymmetry. Asymmetric model such as EGARCH, GJR-GARCH, have been specifically developed to capture asymmetry. Also, while there is disagreement on volatility clustering in Nigeria, all agree that leverage effects exist. This paper, therefore, contributes and extends the existing literature on modelling stock returns volatility in Nigeria using more recent data.

3. Data and Methodology

3.1 Data

The data for this study consist of the Monthly All Share Index (ASI) of the NSE. The ASI is a value weighted index made up of the listed equities on the Exchange. The period under study begins from January 1985 and ends on December 2008. This yields a total of 288 time series observations. The data were obtained from the NSE and transformed to Market returns as individual time series variables. Market returns are proxied by the log difference change in ASI of the NSE thus:

Where, R_{mt} is Monthly returns for period_t. P_t and P_{t-1} are the All Share 1ndices for Months *t* and *t*-*1*. Ln is Natural Logarithm.

3.1.1 Descriptive Statistics

Table 1 shows the descriptive statistics of the NSE return series. The average monthly return is 1.96%. The monthly standard deviation is 5.3%, reflecting a high level of volatility in the market. The wide gap between the maximum (0.240374) and minimum (-0.240798) returns gives support to the high variability of price change in the NSE. Under the null hypothesis of normal distribution, J-B is 0. The J-B value of 366.45 deviated from normal distribution. Similarly, skewness and kurtosis represent the nature of departure from normality. In a normally distributed series, skewness is 0 and kurtosis is 3. Positive or negative skewness indicate asymmetry in the series and less than or greater than 3 kurtosis coefficient suggest flatness and peakedness, respectively, in the returns data. The skewness coefficient of -0.358 is negatively skewed. Negative skewness implies that the distribution has a long left tail and a deviation from

normality. Kurtosis coefficient of 5.4 shows that the empirical distribution of the kurtosis is clearly not normal but peaked.

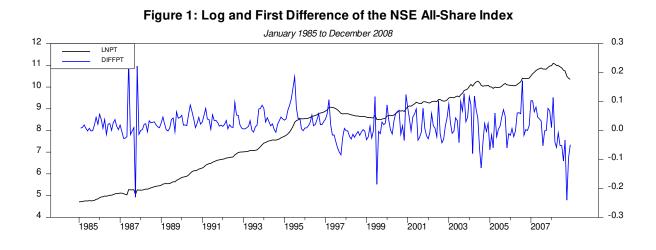
On the whole, the NSE return series do not conform to normal distribution but display negative skewness and leptokurtic distribution. These results are, however, based on the null hypothesis of normality and provide no information for the parametric distribution of the series.

Table 1: Descriptive Statistics of the NSE Return Series

Mean	0.019665	Variance	0.002817	Jarque-Bera	366.450883
Maximum	0.240374	Skewness	-0.357697	Sig. of J-B	0.000000
Minimum	-0.240798	Kurtosis	5.489279	Std Dev	0.053075
Sample. Janu	ary 1985 to Dec	ember 2008			

Sample: January 1985 to December 2008

Figure 1 presents the pattern of level data and return series of the NSE for the period under review. The level data show no tendency to return to its mean indicating the need for differencing. But the first differenced return series show sign of returning to its mean suggesting that the series are weakly stationary. From figure 2, we see that the NSE stock returns distribution is peaked confirming the evidence of non-normal distribution in Table 1. Peaked distribution is a sign of recurrent wide changes, which is an indication of uncertainty in the price discovery process.



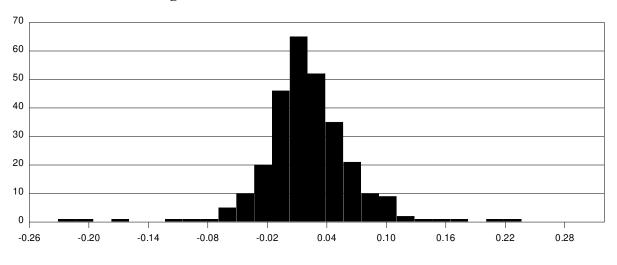


Figure2. Bar Chart of the NSE Return Series

Sample: January 1985 to December 2008

3.2 Methodology

To capture stock returns volatility clustering, leptokurtosis and leverage effects on the NSE return series, the GARCH (1, 1), and the GJR-GARCH (1,1) models were used. The GARCH (1, 1) is a generalization of the ARCH (q) model proposed by Engle (1982) as a way to explain why large residuals tend to clump together, by regressing squared residual series on its lag(s).

However, empirical evidence shows that high ARCH order has to be selected in order to catch the dynamics of the conditional variance. Bollerslev (1986) proposed the Generalized ARCH (GARCH) model as a solution to the problem of high ARCH orders. The GARCH reduces the number of estimated parameters from an infinite number to just a few. According to Brook and Burke (2003), the lag order (1, 1) is sufficient to capture all the volatility clustering that is present in a data.

To model leverage effects characteristics of the NSE, the GJR-GARCH (1,1) model was used. It assumes that the impact of the squared error term of the conditional variance is different when the error term is positive or when it is negative. GJR therefore introduces an indicator function that takes the value 0 when the conditional variance is positive and 1 when negative. The leverage term usually arises when the unconditional returns are skewed, resulting in a positive (negative) *d* estimate when the returns are negatively (positively) skewed, on average. The GJR-GARCH (1,1) model is very similar to the Threshold GARCH (TGARCH) model of Zakoian (1994) but the latter, models the conditional standard deviation instead of the conditional variance.

The longitudinal returns of stock prices have been found not to be described by normal distribution (Verhoeven and McAleer, 2003). To capture the non-normal density function of the NSE return series, the GED was used. The GED is a powerful alternative in cases where the assumption of conditional normality cannot be maintained. The GED has a shape parameter, which determines its kurtosis, and a scale parameter, which determines the variance given the shape parameter. The GED can assume a normal distribution, a leptokurtic distribution (fat tails)

or even a platykurtic distribution (thin tails). Thus, GED allows for a test of the hypothesis that the GARCH process innovations are Independent and identically distributed (IID) normal process. The GARCH (1, 1) modelling process involves two steps. The first step involves specifying a model for the mean return series: the second step involves modelling the conditional variance of the residuals. The GARCH (1, 1) which was used in this study is estimated as:

$$R_{t} = \theta + \mu_{t}$$
(2)
$$\mu_{t} \backsim (0, \delta^{2}_{t})$$

$$\delta^{2}_{t} = \alpha_{0} + \alpha_{1}\mu^{2}_{t-1} + \beta_{1} \delta^{2}_{t-1}$$
(3)

Although the simple GARCH (1,1) model captures symmetric behaviour of volatility, a vast amount of empirical evidence suggest that time-varying asymmetry is a major component of volatility dynamics (Hsieh, 1991). Hence, to avoid misspecification of the conditional variance equation, the GJR leverage term is included. The GJR-GARCH (1,1), model proposed by Glosten, Jagannathan and Runkle (1993), is estimated thus:

To examine the empirical distributional shape of the NSE return series, the GED specified by RATS7 User's Guide (2007:419) is estimated as follows:

$$S = \frac{\exp\left[(-|\mathbf{x}|/b)^{2/c}/2\right]}{b(2^{c/2+1})\Gamma(1+c/2)}$$
(5)

Where in the second equation, R_t is the mean return equation, θ is a constant, and μ_t is the error term; in the third equation, δ^2_t is the conditional variance equation (i.e. the volatility at time *t*), α_0 is the constant, and the α_1 and β_1 refers to a first order ARCH term (i.e., news about volatility from the previous period) and a first order GARCH term (i.e., persistent coefficient), in the fourth equation, *I* is an indicator function and d_1 is the leverage effects parameter and in the fifth equation, *c* is the shape parameter which controls the shape of the tails, whereas *b* is the scale. The conditional variance equation (3) postulates that volatility in the current period (i.e. month *t*) is not only related to the squared error term in the previous term but also on its conditional variance in the previous time period (i.e. month *t*-1).

The essence of estimating the mean return equation (2) with Ordinary Least Square, in the first step, is to obtain the residuals from the regression with which to test for ARCH and GARCH features in the second step. The second step essentially involves regressing the squared residual series and conditional variance on their lags. Under the null hypothesis of no GARCH effects (i.e. no volatility clustering in the NSE series), parameters α_0 and α_1 should be higher than 0 and β_1 should be positive to ensure that conditional variance δ_1^2 is non-negative. The sum of parameters α_1 and β_1 is a measure of the persistence in the volatility shocks taking values between 0 and 1. The more this sum tends to unity, the greater the persistence of shocks to volatility, which is known as volatility clustering. For equation (4), a positive value of the asymmetry parameter d_1 means that negative residuals tend to increase the variance more than positive ones (RATS version 7 user guide, 2007: 420). For the GED, the shape parameter c equals to 1 reflects normal distribution of the NSE return series. c > 1 indicate evidence of a fat-

tail density and c < 1 suggests a thin-tail one. The GED is, therefore, leptokurtic when 1 < c < 2. The parameters are estimated using RATS econometric software, version 7.

4. Empirical Findings and Discussion

This section presents the empirical results and the discussion of the findings. The models are estimated using Maximum Likelihood estimators under the assumption of Generalized Error Distribution (GED). The choice of GED is due to the presence of excess kurtosis in the NSE returns data. The log likelihood is maximized using the Broyden, Flectcher, Goldfarb and Shanno (BFGS) iterative algorithm in RATS₇ to search for optimal parameters.

The results presented in Table 2 show that the coefficient of the ARCH effect (α_1) is statistically significant at 1% significance level. This indicates that news about volatility from the previous *t* periods has an explanatory power on current volatility. Similarly, the coefficient of the lagged conditional variance (β_1) is significantly different from zero, indicating volatility clustering in NSE return series. The sum of ($\alpha_1 + \beta_1$) coefficients is unity, suggesting that shocks to the conditional variance are highly persistent. This implies that wide changes in returns tend to be followed by wide changes and mild changes tend to be followed by mild Changes. A major economic implication of this finding for investors of the NSE is that stock returns volatility occurs in cluster and that it is predictable.

From Table 2, we also notice that asymmetry (gamma) coefficient d_1 is positive. The sign of the gamma reflects that a negative shock induce a larger increase in volatility greater than the positive shocks. It also implies that the distribution of the variance of the NSE returns is left

skewed, implying greater chances of negative returns than positive returns. The positive asymmetric coefficient is indicative of leverage effects evidence in Nigeria stock returns.

The shape parameter (*c*) determines how the variation of the conditional returns is distributed about the location. It estimates the distributional pattern of a series. The coefficient of the shape parameter is greater than one (i.e. c > 1), indicating evidence of a leptokurtic distribution in Nigeria stock returns. This result corroborates the results of Table 1 and Figure 2 which largely show that the Nigeria stock return distribution is leptokurtic.

Parameters	Coefficient	Std Error	T. Statistics	Significance
Mean	0.0210142324	0.0017912314	11.73172	0.00000000
Constant (α_0)	0.0001939255	0.0000872886	2.22166	0.02630641
ARCH (α_1)	0.5926944763	0.1698780733	3.48894	0.00048494
GARCH (β_1)	0.4565508418	0.0671807280	6.79586	0.00000000
$(\alpha_1 + \beta_1)$	1.049245304			
Asymmetry (d)	0.1750700137	0.2351751097	0.74442	0.45661996
Shape (c)	1.5644739655	0.1756330631	8.90763	0.00000000

Table 2: Results of GARCH (1,1), GJR- GARCH (1,1) Models, and GED Shape Test

The adequacy of the fitted GARCH (1,1) model is confirmed by concord of the estimated parameters with *a priori* expectations. Theory expects parameters α_0 and α_1 to be higher than zero (0), and β_1 to be positive to ensure that the conditional variance (δ^2_t) is non-negative. From Table 2, the parameters α_0 and α_1 are more than 0 at 1% marginal significance level, and β_1 is positive. Thus, the GARCH (1,1) seems quite good for explaining the behaviour of stock returns volatility in Nigeria.

5. Conclusion

This paper investigated the volatility of stock market returns in Nigeria using GARCH (1,1) and the GJR-GARCH (1,1) models. Volatility clustering, leptokurtosis and leverage effects were examined for the NSE returns series from January 1985, to December 2008. The results from GARCH (1,1) model show that volatility of stock returns is persistent in Nigeria. The result of GJR-GARCH (1,1) model shows the existence of leverage effects in Nigeria stock returns. Also, the shape parameter estimated from GED reveals evidence of leptokurtosis in the NSE returns distribution. Finally, volatility persistence in NSE return series is clearly indicated in the unity of the ARCH and GARCH parameter estimates.

Overall results from this study provide evidence to show volatility clustering, leptokurtic distribution and leverage effects for the Nigeria stock returns data. These results are in tune with international evidence of financial data exhibiting the phenomenon of volatility clustering, fattailed distribution and leverage effects. The results also support the evidence of volatility clustering in Nigeria provided by Ogum, et al. (2005); existence of leverage effects in Nigeria stock returns provided by Okpara and Nwezeaku (2009), but disagree with their conclusion that stock returns volatility is not quite persistent in Nigeria.

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