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Abstract

We analyze the liquidity component in a derivative transaction where both counterparties can default, and the effect of a counterparty's default probability on his funding costs and benefits. The analysis shows that the value of a transaction is influenced not by the total cost of funding of a counterparty, but only by that component of the cost of funding corresponding to his bond-CDS basis spread, and this regulates which trades are possible in the market. Moreover, we find that the DVA can be represented as a funding benefit for the borrower, alternatively to the market standard that considers it a benefit coming from the borrower's own default risk.

1 Introduction

The pricing of funding liquidity and the pricing of counterparty risk are closely related. Companies usually compute a spread for funding costs that includes a compensation for their own risk of default. However, interactions between the two are still poorly understood.

Consider first the debt value adjustment (DVA), introduced for example in [2]. The DVA term is required for the agreement on the fair price when the counterparty computes the credit value adjustment (CVA). Ignoring DVA would make the derivatives business of a bank less competitive. However the standard definition of DVA has some unpleasant features. DVA is an asset, the value of which increases the more its owner approaches default. This appears to lead to a distortion of financial choices and of financial communications. As Algorithmics puts it, "can you profit now from your own future default?"[1]. In fact DVA

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is an asset which is not readily realizable, as this would require the institution that recognizes it to sell protection on itself. In this work we argue that when funding costs are properly accounted for, an asset equivalent to DVA is obtained without incurring in the above unpleasant features.

Funding costs can be introduced following the market standard based on the recent breakthrough [5], which however does not consider explicitly counterparty risk adjustments. In this work we argue that introducing liquidity by a modification of the discounting rate, as in the market standard, together with CVA and DVA, can lead to double-counting of assets that can be realized only once.

In this work we try and provide some cornerstones of a unified consistent framework for liquidity and credit risk adjustments. We give an explicit representation to the funding strategy associated with any derivatives deal, including the effect of default events on the cost of this strategy. One finding is that, consistently with [5], different liquidity costs lead to an asymmetric market where net borrowers cannot be exchanged with net lenders, although this does not preclude the possibility to reach an agreement on price in the derivatives market. In fact, an agreement may be struck at price that generates day-one profits for both counterparties. A second finding is that the consistent incorporation of liquidity and credit risk, that goes beyond [5], implies that the crucial variable determining the cost of liquidity and discriminating between lenders and borrowers is not the bond spread or the CDS spread, but the bond-CDS basis.

Finally we show that a possible solution to the DVA puzzle, recovering symmetry and making DVA a replicable asset, is to evaluate DVA as a funding benefit. In fact, even if a company does not take its own risk of default explicitly into account, accounting for funding benefits can generate an asset equal to DVA but more meaningful in terms of derivative replication.

2 The setting

We consider a deal in which entity B (borrower) enters at time 0 into the commitment to pay a fixed amount K at time T to party L (lender) without the exchange of collateral. This simple payoff allows us to focus on liquidity and credit costs without unnecessary complications. Moreover, this payoff is the derivative equivalent of a zero-coupon bond issued by B or a deposit from L to B, so that the results of the analysis can be compared with the practice for well-established market benchmarks.

We assume that party X, with $X \in \{B, L\}$, has a recovery rate R_X and that the risk free interest rate that applies to maturity T has a deterministic value r. As usual, r represents the time-value of money on which the market agrees, excluding considerations on liquidity costs or credit risk (it is an approximation for the OIS rate).

In our setting, X makes funding in the bond market and is a reference entity in the CDS market. We have the following information:

1. instantaneous deterministic CDS spread π_X. This can be written

$$\pi_X = \lambda_X \, \mathsf{LGD}_X \tag{1}$$

where λ_X is the deterministic default intensity and $\mathsf{LGD}_X = 1 - \mathsf{R}_X$ is the loss given default of entity X. If recovery is null, we have $\mathsf{LGD}_X = 1$ and the CDS spread coincides with λ_X, so that $\Pr(\tau_X > T) = e^{-\pi_X T}$. Clearly $\pi_X \geq 0$.

2. instantaneous deterministic bond-CDS basis spread γ_X. The sum $s_X = \pi_X + \gamma_X$ is the cost of funding for X as measured in the bond market. The spread γ_X is associated with the marketability of the bonds of X, creating a link between funding liquidity, which is the focus of this work, and market liquidity. The spread γ_X can in some cases be negative, but this is recorded mainly for certain sovereigns, so we assume for now $\gamma_X \geq 0$.

Our aim is to describe the net present value V_X (at time zero) of all cash-flows generated by the transaction for the party X, by consistently accounting for liquidity and counterparty risk. We proceed as follows: we first consider the current standard approach to DVA, then we attempt to introduce liquidity costs by adjusting the discount rate, and show that this path would lead to double accounting of the funding benefit associated with the DVA. Then we introduce our approach that includes risky funding and discuss some interesting implications. We start with the assumption $\mathsf{R}_X = 0$, but in Section 5.4 we extend the results to the case of positive recovery.

3 Standard DVA: Something is missing?

Let's start from the market standard for CVA and DVA, presented for example in [2]. The above transaction has a net present value for party L equal to

$$\begin{aligned} V_L &= e^{-rT} K - \mathsf{CVA}_L - P \\ V_L &= 0 \quad \Rightarrow \quad P = e^{-rT} K - \mathsf{CVA}_L \end{aligned} \tag{2}$$

where P is the premium paid by the lender at time 0 and

$$\begin{aligned} \mathsf{CVA}_L &= \mathbb{E}[e^{-rT} K 1_{\{\tau_B \leq T\}}] = e^{-rT} K \, \mathbb{Q}[\tau_B \leq T] \\ &= e^{-rT} K \left[1 - e^{-\pi_B T}\right] \end{aligned}$$

At the same time party B sees a value

$$\begin{aligned} V_B &= -e^{-rT} K + \mathsf{DVA}_B + P \\ V_B &= 0 \quad \Rightarrow \quad P = e^{-rT} K - \mathsf{DVA}_B \end{aligned} \tag{3}$$

with

$$\mathsf{CVA}_L = \mathsf{DVA}_B$$

This guarantees the symmetry $V_B = V_L = 0$ and the possibility for the parties to agree on the premium P of the deal.

This approach does not consider explicitly the value of liquidity. In fact, in exchange for the claim, at time 0 party B receives a cash flow from party L equal to P, so while party L has to finance the amount until the maturity of the deal at its funding spread s_L, party B can reduce its funding by P. So party B should see a funding benefit, and party L should see the fair value of its claim reduced by the financing costs.

The absence of the liquidity term for L can be explained by assuming $s_L = 0$. This implies $\pi_L = 0$. Can we explain the absence of the liquidity term for B by assuming $s_B = 0$? No, because this implies $\pi_B = 0$, which in turn would cancel the DVA term. Thus B has a funding cost above r by at least $s_B = \pi_B > 0$, that seems to be missing in the above formula. In the next sections we analyze if it is really missing.

4 Standard DVA plus liquidity: Something is duplicated?

We introduce liquidity costs along the lines of [5] but applied to the above defaultable payoff:

$$\begin{aligned} V_L &= \mathbb{E}\left[e^{-(r+s_L)T} K 1_{\{\tau_B > T\}}\right] - P \qquad (4) \\ &= \mathbb{E}\left[e^{-rT} e^{-\gamma_L T} e^{-\pi_L T} K 1_{\{\tau_B > T\}}\right] - P \\ &= e^{-rT} e^{-\gamma_L T} e^{-\pi_L T} K e^{-\pi_B T} - P \end{aligned}$$

And analogously

$$\begin{aligned} V_B &= -\mathbb{E}\left[e^{-(r+s_B)T} K 1_{\{\tau_B > T\}}\right] + P \qquad (5) \\ &= -\mathbb{E}\left[e^{-rT} e^{-\pi_B T} e^{-\gamma_B T} K 1_{\{\tau_B > T\}}\right] + P \\ &= -e^{-rT} e^{-\pi_B T} e^{-\gamma_B T} K e^{-\pi_B T} + P \\ &= -e^{-rT} e^{-2\pi_B T} e^{-\gamma_B T} K + P \end{aligned}$$

To compare this result, including CVA, DVA and liquidity from discounting, with results on DVA obtained in the previous section 3, it is convenient to reduce ourselves to the above situation where L is default free and with no liquidity spread, while B is defaultable and has the minimum liquidity spread allowed in this case: $s_L = 0$, $s_B = \pi_B > 0$.

We have

$$\begin{aligned} V_L &= e^{-rT} K e^{-\pi_B T} - P \\ V_B &= -e^{-rT} e^{-2\pi_B T} K + P \end{aligned}$$

There are two bizarre aspects in this representation. First, even in a situation where we have assumed no bond-CDS basis, two counterparties do not agree on the simplest transaction with default risk. A day-one profit should be accounted by borrowers in all transactions with CVA. This belies years of market reality.

Secondly, the explicit inclusion of the DVA term results in the duplication of the funding benefit for the party that assumes the liability. The formula implies against all evidence that the funding benefit is remunerated twice. If this were correct then a consistent accounting of liabilities at fair value would require pricing zero-coupon bonds by multiplying *twice* their risk-free present value by their survival probabilities. This also belies years of market reality.

5 Solving the puzzle

In order to solve the puzzle, we do not compute liquidity by the adjusted discounting of (4) and (5), but generate liquidity costs and benefits by modelling explicitly the funding strategy. The approach we take is that companies capitalize and discount money with the risk-free rate r. Then they add or subtract the adjustments for credit risk and liquidity costs. This allows us to investigate more precisely where credit/liquidity gains and losses are financially generated.

We now take into account that the above deal has two legs. If we consider for example the lender L, one leg is the "deal leg", with net present value

$$\mathbb{E}\left[-P + e^{-rT}\,\Pi\right]$$

where Π is the payoff at T, including a potential default indicator; the other leg is the "funding leg" with net present value

$$\mathbb{E}\left[+P - e^{-rT}\,F\right]$$

where F is the funding payback at T, including a potential default indicator. When there is no default risk or liquidity cost involved, this funding leg can be overlooked because it has a value

$$\mathbb{E}\left[+P - e^{-rT}\,e^{rT}P\right] = 0.$$

Instead, in the general case the total net present value is

$$\begin{aligned} V_L &= \mathbb{E}\left[-P + e^{-rT}\,\Pi + P - e^{-rT}\,F\right] \\ &= \mathbb{E}\left[e^{-rT}\,\Pi - e^{-rT}\,F\right]. \end{aligned}$$

Thus the premium at time 0 cancels out with its funding, and we are left with the discounting of a total payoff including the deal's payoff and the liquidity payback. An analogous relationship applies to the borrower, as detailed in the next section.

5.1 Risky Funding with DVA for the borrower

The borrower B has a liquidity advantage from receiving the premium P at time zero, as it allows him to reduce its funding requirement by an equivalent amount P. The amount P of funding would have generated a negative cashflow at T, when funding must be paid back, equal to

$$-P\,e^{rT}\,e^{s_B T}\,1_{\{\tau_B>T\}} \tag{6}$$

The outflow equals P capitalized at the cost of funding, times a default indicator $1_{\{\tau_B>T\}}$. Why do we need to include a default indicator $1_{\{\tau_B>T\}}$? Because in case of default, under the assumption of zero recovery, the borrower does not pay back the borrowed funding and there is no outflow.

Thus reducing the funding by P corresponds to receiving at T a positive amount equal to (6) in absolute value,

$$\begin{aligned} & P\,e^{rT}\,e^{s_B T}\,1_{\{\tau_B>T\}} \\ = \;& P\,e^{rT}\,e^{\pi_B T}\,e^{\gamma_B T}\,1_{\{\tau_B>T\}} \end{aligned} \tag{7}$$

to be added to what B has to pay in the deal:

$$-K\,1_{\{\tau_B>T\}}.$$

Thus the total payoff at T is

$$1_{\{\tau_B>T\}}P\,e^{rT}\,e^{\pi_B T}\,e^{\gamma_B T}-1_{\{\tau_B>T\}}K$$

Taking discounted expectation,

$$\begin{aligned} V_B &= e^{-\pi_B T}\,P\,e^{\pi_B T}\,e^{\gamma_B T}-K\,e^{-\pi_B}\,e^{-rT} \\ &= P\,e^{\gamma_B T}-K\,e^{-\pi_B}\,e^{-rT} \end{aligned} \tag{8}$$

Compare with (5). Now we have no unrealistic double accounting of default probability. Notice that

$$V_B=0 \quad\Rightarrow\quad P=K\,e^{-\pi_B}e^{-\gamma_B T}\,e^{-rT}.$$

Assume, as in (3), that $\gamma_B=0$ so that

$$P=K\,e^{-\pi_B}\,e^{-rT}. \tag{9}$$

and compare with (3). We can conclude that in this case the standard computation from Section 3 is correct, as taking into account the probability of default in the valuation of the funding benefit removes any liquidity advantage for the borrower. On the other hand, charging liquidity costs by an adjusted funding spread as in Section 4 cannot properly account for the funding benefits that accrue to a defaultable company.

We observe that in writing the payoff for the borrower we have not explicitly considered the case in which the deal is interrupted by the default of L, that is $\tau_L < \min(\tau_B, T)$. According to standard derivative agreements this event forces the solvent borrower to settle the value of the claim at the close-out date. The subject of mathematically representing the close-out value of defaulted claims inside the payoff of a derivative with liquidity and counterparty risk is still wanting a deep treatment. However, again based on standard derivative documentation, B would normally be able to deduct from the close-out amount any costs incurred in replacing the transaction with an identical one with a new counterparty, letting the deal value V_B unaltered. Omitting in the above the case $\tau_L < \min(\tau_B, T)$ is equivalent to assuming that, in a close-out for the default of L, B can replace the transaction at no cost in the market. Thanks to this V_B does not contain any terms that depend on the default risk of the lender, consistently with the reality of bond and deposit markets.

5.2 Risky funding with CVA for the Lender, and the conditions for market agreement

If the lender pays P at time 0, he incurs in a liquidity cost. In fact he needs to borrow P until T. At T, L will give back the borrowed money with interest, but only if he has not defaulted. Otherwise he gives back nothing, so the outflow is

$$\begin{aligned} & P\, e^{rT}\, e^{s_L T}\, 1_{\{\tau_L > T\}} \\ = \; & P\, e^{rT}\, e^{\gamma_L T}\, e^{\pi_L T}\, 1_{\{\tau_L > T\}} \end{aligned} \tag{10}$$

while he receives in the deal

$$K\, 1_{\{\tau_B > T\}}$$

The total payoff at T is therefore

$$-P\, e^{rT}\, e^{\gamma_L T}\, e^{\pi_L T}\, 1_{\{\tau_L > T\}} + K\, 1_{\{\tau_B > T\}}.$$

Taking discounted expectation

$$\begin{aligned} V_L &= -P\, e^{\gamma_L T}\, e^{-\pi_L T}\, e^{\pi_L T} + K\, e^{-rT}\, e^{-\pi_B T} \\ &= -P\, e^{\gamma_L T} + K\, e^{-rT}\, e^{-\pi_B T} \end{aligned} \tag{11}$$

If we impose the matching condition

$$V_L \geq 0,$$

then

$$P \leq K\, e^{-rT}\, e^{-\gamma_L T}\, e^{-\pi_B T}$$

It is interesting to note that the lender does not charge the borrower for the component of the cost of funding, namely π_L, which is associated with its own risk of default, because this is cancelled by the fact that funding is not given back in case of default. This is exactly symmetric to the fact that for the borrower the inclusion of the DVA eliminates the liquidity advantage associated with π_B.

For reaching an agreement in the market we need

$$V_L \geq 0, V_B \geq 0$$

which, recalling (8), implies

$$\begin{aligned} K\, e^{-rT}\, e^{-\gamma_L T}\, e^{-\pi_B T} &\geq P \geq K\, e^{-\pi_B}\, e^{-rT}\, e^{-\gamma_B T} \\ e^{-\gamma_L T} &\geq P \geq e^{-\gamma_B T} \end{aligned}$$

An agreement can be found whenever

$$\begin{aligned} e^{-\gamma_L T} &\geq e^{-\gamma_B T} \\ \gamma_B &\geq \gamma_L \end{aligned}$$

This solves the puzzle, and shows that the liquidity cost that must be charged to the counterparty of an uncollateralized derivative transaction is just the bond-CDS basis, rather than the bond spread or the CDS spread. This is in line with what happened during the liquidity crisis in 2007-2009, when the bond-CDS basis exploded.

5.3 The accounting view for the borrower: risk-free funding without DVA

One of the most controversial aspects of DVA relates to its application in the context of fair value accounting, in the sense that allowing a borrower to condition future liabilities on survival may create a distorted perspective in which our default is our lucky day. On the other hand, companies recognize a profit when their cash obligations trade at a discount in the bond market, thus the same must apply to derivatives that generate an obligation.

In the above we have seen that, when we include the risk of default, the cost of funding is associated with the bond-CDS basis γ_X rather than with the full bond spread s_X. In this section we show that, if the borrower does not condition its liabilities upon survival, namely he does not recogniza a DVA, but accounts for the funding benefit at the full bond spread s_B, then an alternative DVA term emerges. Accordingly, let B pretend, for accounting purposes, to be default free, and modify the payoff accordingly. The premium P paid by the lender gives B a reduction of the funding payback at T corresponding to a cashflow at T

$$Pe^{rT}e^{s_B T},$$

where there is no default indicator because B is treating itself as default-free. This cashflow must be compared with the payout of the deal at T, which is

$$-K$$

again without indicator, *i*e without DVA. Thus the total payoff at T is

$$P e^{rT} e^{s_B T} - K$$

By discounting to zero we obtain

$$V_B = P\, e^{s_B T} - K\, e^{-rT} = P\, e^{\pi_B T}\, e^{\gamma_B T} - K\, e^{-rT}$$

So the borrower B recognizes on its liability a funding benefit that actually takes into account its own risk of default π_B, plus additional liquidity costs γ_B. If we set $\gamma_B = 0$ like in (3) and (9), we obtain the same equilibrium premium as in (3) and (9),

$$P = e^{-rT}\, e^{-\pi_B T}\, K,$$

thereby matching the premium computed by the lender that includes the CVA/DVA term. But now this term is accounted for as a funding benefit and not as a benefit coming from the reduction of future expected liabilities thanks to default.

5.4 Positive recovery extension

In this section we look at what happens if we relax the assumption of zero recovery. The discounted payoff for the borrower is now

$$\begin{aligned}
&1_{\{\tau_B > T\}} e^{-rT}\, P\, e^{\pi_B T}\, e^{\gamma_B T}\, e^{rT} \\
&+1_{\{\tau_B \le T\}} e^{-r\,\tau_B}\, \mathsf{R}_B\, e^{-r\,(T-\tau_B)}\, P\, e^{\pi_B T}\, e^{\gamma_B T} e^{rT} \\
&-1_{\{\tau_B > T\}} e^{-rT}\, K \\
&-1_{\{\tau_B \le T\}} e^{-r\,\tau_B}\, \mathsf{R}_B\, e^{-r\,(T-\tau_B)}\, K
\end{aligned}$$

where the recovery is a fraction R_X of the present value of the claims at the time of default of the borrower, consistently with standard derivative documentation. Notice that B acts as a borrower both in the deal and in the funding leg, since we represented the latter as a reduction of the existing funding of B. Simplifying the terms and taking the expectation at 0 we obtain

$$\begin{aligned}
V_B &= \mathbb{Q}\{\tau_B > T\}\, P\, e^{\pi_B T}\, e^{\gamma_B T} + \mathbb{Q}\{\tau_B \le T\}\, e^{-r\,T}\, \mathsf{R}_B\, P\, e^{\pi_B T}\, e^{\gamma_B T} e^{rT} \\
&\quad -\mathbb{Q}\{\tau_B > T\}\, e^{-rT}\, K - \mathbb{Q}\{\tau_B \le T\}\, \mathsf{R}_B\, e^{-r\,T}\, K \\
&= P\, e^{\pi_B T}\, e^{\gamma_B T}\, [S_B(T) + \mathsf{R}_B\,(1 - S_B(T))] \\
&\quad -e^{-r\,T}\, K\, [S_B(T) + \mathsf{R}_B\,(1 - S_B(T))] \\
&= [1 - \mathsf{LGD}_B\,(1 - S_B(T))]\, \left(P\, e^{\pi_B T}\, e^{\gamma_B T} - e^{-r\,T}\, K\right)
\end{aligned} \tag{12}$$

where $S_B(T) = \mathbb{Q}\{\tau_B > T\}$ is the survival probability of the borrower. Using (1), we can write the first order approximation

$$1 - e^{-\pi_B\,T} \approx \mathsf{LGD}_B\, \left(1 - e^{-\lambda_B\,T}\right)$$

which allows us to approximate (12) as

$$\begin{aligned} V_B &\approx e^{-\pi_B T} \left(P\, e^{\pi_B T}\, e^{\gamma_B T} - e^{-r\,T}\, K \right) \\ &= P\, e^{\gamma_B T} - e^{-\pi_B T}\, e^{-r\,T}\, K \end{aligned}$$

We have thus shown that (8) is recovered as a first order approximation in the general case of positive recovery rate.

Similar arguments apply to the value of the claim for L, that acts a lender in the deal and as a borrower in the funding leg. For L, (11) is recovered as a first order approximation of

$$\begin{aligned} V_L = &- \left[1 - \mathsf{Lgd}_L\, (1 - S_L(T))\right]\, P\, e^{\pi_L T}\, e^{\gamma_L T} \\ &+ \left[1 - \mathsf{Lgd}_B\, (1 - S_B(T))\right]\, e^{-r\,T}\, K \end{aligned}$$

6 Conclusions

In this article we have laid the groundwork of a consistent framework for the joint pricing of liquidity costs and counterparty risk. By explictly modelling the funding components of a simplified derivative where both counterparties can default, we have shown how bilateral counterparty risk adjustments (CVA and DVA) can be combined with liquidity/funding costs avoiding unrealistic double counting effects. We have shown that DVA has a meaningful representation in terms of funding benefit for the borrower, but also that default probability affects the funding cost of the lender in a similar way. The lender's cost of funding includes a component that is associated with his own risk of default, but this component cancels out with the default benefit, so that only the liquidity cost represented by his own bond-CDS basis spread contributes as a net funding cost to the value of a transaction for the lender. We have shown that the comparison between the bond-CDS basis of lender and borrower is crucial to assess if a trade is convenient for both counterparties. The extension of these results to more general derivative payoffs, where a counterparty can shift between a net borrower position or a net lender position depending on market movements, is a crucial topic for future research.

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