Modeling the fraud-like investment founds by Petri nets

Ciuiu, Daniel

Technical University of Civil Engineering, Bucharest, Romania, Romanian Institute for Economic Forecasting

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MODELING THE FRAUD-LIKE INVESTMENT FOUNDS BY PETRI NETS

Daniel Ciuiu[1, 2]
[1] Technical University of Civil Engineering, Bucharest, Romania
[2] Romanian Institute for Economic Forecasting, Bucharest, Romania
e-mail: dciuiu@yahoo.com

Abstract: *In this paper we model the fraud-like investment founds using place-transition Petri nets.*

We will also classify the business using regression line in order to find the possible fraud-like investment founds. In these regression lines we compute analytical the mark of a place in function of some other elements of the Petri net, and next we express this value in function of the same elements using regression. From the identity of the coefficients we find a ratio between two weights of arcs.

We make also a $C++$ *program where the marks and transitions are implemented as classes for Petri nets, and, using the heritage mechanism we extend the Petri net to Petri net with priorities.*

Key words: *Petri nets, fraud-like investment founds, objects programming.*

1. INTRODUCTION

Definition 1 ([6]). *It is called Petri net the triplet* $N = (S, T, F)$*, where:*

1) S *and* T *are disjoint sets.*

$F \subset S \times T \cup T \times S$ *is a binary relation*.

Definition 2 ([6]). *Let* $N = (S, T, F)$ *be a Petri net.*

1) N *is nonempty if* $S \cup T \neq \Phi$ *.*
2) N *is finite if* $X = S \cup T$ *is finite.*
3) N *is pure if for any* $x \in X = S \cup T$ *we have* ${}^{\bullet}x \cap x^{\bullet} = \Phi$*, where* ${}^{\bullet}x = \{y \in X | (y, x) \in F\}$ *and* $x^{\bullet} = \{y \in X | (x, y) \in F\}$.
4) N *is simple if for any* $x, y \in X$ *such that* ${}^{\bullet}x = {}^{\bullet}y$ *and* $x^{\bullet} = y^{\bullet}$ *we have* $x = y$ *.*

The element $x \in X$ *is isolated if* ${}^{\bullet}x \cup x^{\bullet} = \Phi$.

Definition 3 ([6]). *It is called place-transition Petri net the quintuple* (S, T, F, K, W)*, where:*

1) (S, T, F) *is a Petri net.*
2) $K : S \to \mathbb{N}^* \cup \{\infty\}$ *is the capacity function of the Petri net.*
3) $W : F \to \mathbb{N}^*$ *is the weight function of the Petri net.*

In this case S *is called the places set, and* T *is called the transitions set.*

If the functions K and W are constant 1, S is the conditions set, T is the events set, and the obtained Petri net is a condition-event Petri net.

Let $\Sigma = (S, T, F, K, W)$ a place-transition Petri net and $t \in T$ one of its transitions. We denote by t^-, t^+ and Δt the functions $t^-, t^+ : F \to \mathbb{N}^*$ and $\Delta t : F \to \mathbb{Z}^*$ such that $t^-(s) = W(s,t)$, $t^+(s) = W(t,s)$ and $\Delta t = t^+(s) - t^-(s)$.

Definition 4 ([6]). *Let $\Sigma = (S, T, F, K, W)$ be a place-transition Petri net. It is called mark of the net a function $M : S \to \mathbb{N}^*$ such that for any $s \in S$ we have $M(s) \le K(s)$.*

Graphically the places of a place-transition Petri net are represented by circles, the transitions by rectangles and the arcs (the elements of F) by oriented lines. The capacities different of ∞ are written between parentheses after the places labels, and the weights different of 1 are written on the corresponding lines. The marks are represented by points in the places where they are positive. If a mark is large we represent only a point and its value.

Definition 5 ([6]). *Let (S, T, F, K, W) be a place-transition Petri net.*

1) A transition $t \in T$ is enabled to fire at the mark M (or it has concession at the mark M) if for any $s \in^\bullet t$ we have $M(s) \ge W(s,t)$ (the resources of the precedents are large enough), and for any $s \in t^\bullet$ we have $M(s) + W(t,s) \le K(s)$ (if we add the resources produced by t to its successors we do not exceed the capacity).

2) The mark M' is produced by firing of the transition t at the mark M if for any $s \in^\bullet t$ we have $M'(s) = M(s) - W(s,t)$ and for any $s \in t^\bullet$ we have $M'(s) = M(s) + W(t,s)$, and for the other $s \in S$ we have $M'(s) = M(s)$.

The first part of the above definition is the enabling rule and the second part is the firing rule. We denote by $M[t\rangle_\Sigma$ the fact that the transition t is enabled to fire at the mark M and by $M[t\rangle_\Sigma M'$ he fact that the mark M' is produced by firing of the transition t at the mark M. We denote also by $T(\Sigma, M) = \{t \in T \mid M[t\rangle_\Sigma\}$. If there is no confusion about the Petri net Σ can be omitted.

D efinition 6([6]). *Let Σ be a place-transition Petri net and M a mark of it.*

1) $w \in T^*$ is a sequence of transitions from M if there exist the marks $M_0 = M$, $M_1, \ldots, M_n$ such that $w = t_0 t_1 \ldots t_{n-1}$ and $M_i [t_i\rangle_\Sigma M_{i+1}$. We denote this by $M[w\rangle_\Sigma$.

2) The mark M' is accessible from M if there exists a sequence of transitions w as above such that $M' = M_n$. We denote this by $M[w\rangle_\Sigma M'$.

In the above definition we accept also the empty sequence λ: we have $M[\lambda\rangle_\Sigma$ and $M[\lambda\rangle_\Sigma M$.

Definition 7([6]). *It is called marked place-transition Petri net or place-transition Petri system the pair $\gamma = (\Sigma, M_0)$ where Σ is a place-transition Petri net and M_0 an initial mark of Σ.*

Definition 8([6]). *A marked place-transition Petri net is without contact if* $(\forall M \in [M_0])(\forall t \in T)(\forall s \in S)(M(s) \ge W(s,t) \Rightarrow M(s) + W(t,s) \le K(s))$.

Therefore if a transition is not enabled to fire at a given mark this happens only because of the lack of resources, not because of overtake a capacity.

If the transitions are produced sequentially we have a sequential evolution of the place-transition Petri net. If some transitions are produced in the same time we have a parallel evolution of the net.

Definition 9([6]). *Let* Σ *be a place-transition Petri net without contact,* M *a mark of it and* $A \subseteq T$*.*

1) A is a set of transitions parallel enabled to fire at the mark M (in Σ) if $\sum_{t\in A} t^{-} \le M$.

2) The mark M' is produced by parallel firing of the set of transitions A at the mark M (in Σ) if $M' = M + \sum_{t\in A} \Delta t$.

Remark 1. The above definition is given only for Petri nets without contact, but this definition can be extended by the condition $M + \sum_{t\in A} t^{+} \le K$ (condition to not overtake the capacities).

Definition 10([6]). *Let* $\Sigma = (S, T, F, K, W)$ *a place-transition Petri net with* $S = \{s_1,...,s_m\}$ *and* $T = \{t_1,...,t_n\}$*. The incidence matrix of* Σ *is the* $m \times n$ *matrix* I_Σ *such that* $I_\Sigma(i,j) = \Delta t_j(s_i)$ *for any* $i = \overline{1,m}$ *and* $j = \overline{1,n}$*.*

Theorem 1([6]). *Let* Σ *be a place-transition Petri net and two marks of it,* M_1 *and* M_2 *represented as* $m-$*vectors. The mark* M_2 *is accessible from* M_1 *if and only if there exists a* $n-$*vector* f *such that* $M_2 = M_1 + I_\Sigma \cdot f$*.*

From the proof of the above theorem we know (see [6]) that f_i is the number of appearances of t_i in w such that $M_1[w\rangle_\Sigma M_2$.

Definition 11([6]). *Let* $\Sigma = (S, T, F, K, W)$ *be a place-transition Petri net with the incidence matrix* I_Σ*.*

1) *The vector with* m *integer components* J *is an* $S-$*invariant if* $J^T \cdot I_\Sigma = 0$*.*
2) *The support of the* $S-$*invariant* J *is the set* $P_J = \{s_i \in S | J_i \ne 0\}$*.*
3) *The* $S-$*invariant* J *is nonnegative if* $J \ge 0$*.*
4) *The* $S-$*invariant* $J > 0$ *is minimal if there exists not an* $S-$*invariant* J' *such that* $0 < J' < J$*.*
5) *The place-transition Petri net generated by the* $S-$*invariant* J *is the Petri net* $\Sigma' = (S', T', F', K', W')$ *where*
   a) $S' = P_J$*.*
   b) $T' = {}^\bullet S' \cup S'^\bullet$*.*
   c) $F' = F \cap ((S' \times T') \cup (T' \times S'))$*.*
   d) $K' = K|_{S'}$*.*
   e) $W' = W|_{F'}$*.*

From the existence of positive $S-$invariants we can conclude that we can give weights to the places by a vector g such that for any marks *M* and *M'* accessible from *M* we have $g^T \cdot M = g^T \cdot M'$ (see [6]). Therefore for any initial mark M_0 (which represents the initial resources of the modeled system) the weighted resources of the part of the system represented by P_J remains constant. If the $S-$invariant is minimal then the weights are minimal for the involved places.

The set of $S-$invariants is a $\mathbb{Z}-$module i.e. it has the properties of a vector space, but instead of a field we have only a ring, namely $\mathbb{Z}$.

D efinition 12([6]). *Let* $\Sigma = (S, T, F, K, W)$ *be a place-transition Petri net with the incidence matrix* I_Σ*.*

1) *The vector with* n *integer components* J *is a* $T-$*invariant if* $I_\Sigma \cdot J = 0$*.*
2) *The support of the* $T-$*invariant* J *is the set* $P_J = \{t_i \in T | J_i \ne 0\}$*.*
3) *The* $T-$*invariant* J *is nonnegative if* $J \ge 0$*.*
4) *The* $T-$*invariant* $J > 0$ *is minimal if there exists no* $T-$*invariant* J' *such that* $0 < J' < J$*.*

5) *The place-transition Petri net generated by the* T – *invariant* J *is the Petri net* $\Sigma' = (S', T', F', K', W')$ *where*

a) $T' = P_J$.

b) $S' = {}^{\bullet}T' \cup T'^{\bullet}$.

c) $F' = F \cap ((S' \times T') \cup (T' \times S'))$.

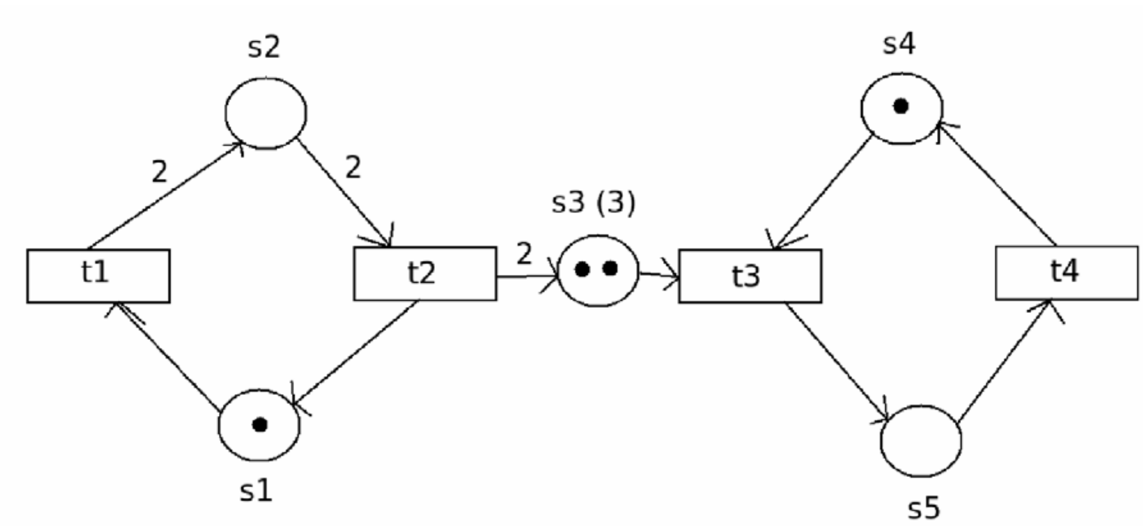
d) $K' = K|_{S'}$.

e) $W' = W|_{F'}$.

Suppose there exists a positive T – invariant *J* and for a given mark *M* there exists a sequence of transitions from *M* that contains the transitions of P_J with the corresponding multiplicities of *J*, and only these transitions. In this case the mark *M* can be reproduced after a finite number of transitions (we apply theorem 1). The minimality of a T – invariant means that the mark is reproduced after a minimum number of appearances of the involved transitions. If there exists no sequence of transitions from *M* as above for a minimal T – invariant *M* can not be reproduced after a finite number of transitions.

The set of T – invariants is also a $\mathbb{Z}$ – module.

Example 1([6]). *Consider the following producer-consumer model*:



Picture 1: *Petri net for a producer-consumer model*

In the above Petri net the interpretation of the elements is as follows.

- s_1 *is a signal that the producer is ready to produce.*
- s_2 *is a signal that the producer is ready to send the products.*
- s_3 *is a buffer (capacity is* 3*).*
- s_4 *is a signal that the consumer is ready to receive the products.*
- s_5 *is a signal that the consumer is ready to consume.*
- t_1 *is the production activity.*
- t_2 *is the sending to buffer activity.*
- t_3 is the receiving from buffer activity.
- t_4 is the consumption activity.

The transitions t_1 *and* t_3 *are parallel enabled to fire at the initial mark* $M_0 = (1,0,2,1,0)^T$. *We notice that after the fire of* t_1 t_2 *is enabled to fire only after we reduce the mark of* s_3 *(the buffer) by firing* t_3 *(receiving from buffer). Therefore the producer can not produce as many items he wants while the consumer does not empty the buffer* *by consumption. After firing t_3 this transition is no more enabled to fire: it must be fired first t_4: effective consumption.*

The incidence matrix is $I_\Sigma = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 2 & -2 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$.

The $S-$*invariants are* $J = (2 \cdot x_2, x_2, 0, x_4, x_4)^T$ *with the minimal* $S-$*invariants* $(2,1,0,0,0)^T$ *and* $(0,0,0,1,1)^T$*. We notice that we can take* $g = (2,1,0,1,1)^T$*, which can be considered as an equilibrium between offer and demand.*

The $T-$*invariants are* $J = (x_1, x_1, x_1, x_1)^T$ *with the minimal* $T-$*invariant* $(1,1,2,2)^T$*. Therefore if we fire two times the sequence* t_3, t_4 *we empty the buffer, and after we fire* t_1 *and* t_2 *we reproduce the initial mark.*

We present now some extensions of the Petri nets.

Definition 13([6]). *A Petri net with priorities is a couple* $\gamma = (\Sigma, \rho)$ *where* Σ *is a Petri net and* ρ *is a partial order relation on the transitions set* T*. The signification of the order relation* ρ *is that if* $t_1 \rho t_2$ *the transition* t_2 *has higher priority in firing than* t_1*.*

A transition t *is* $p-$*enabled to fire at the mark* M *(in* γ*) if* $M[t\rangle_\Sigma$ *and for any* t' *such that* $M[t'\rangle_\Sigma$ *we have not* $t \rho t'$*. We denote this by* $M[t\rangle_{\gamma,p}$*.*

A mark M' *is* $p-$*produced by firing of the transition* t *at the mark* M *if* $M[t\rangle_{\gamma,p}$ *and* $M[t\rangle_\Sigma M'$*. We denote this by* $M[t\rangle_{\gamma,p} M'$*.*

Definition 14([6]). *A Petri net controlled by queues is a couple* $\gamma = (\Sigma, Q)$ *where* Σ *is a Petri net and* Q *is the set of queues with transitions that appear only once in the queue.*

Let $\gamma = (\Sigma, Q)$ *a Petri net controlled by queues,* M *a mark of* Σ *and* $q \in Q$ *a queue with above properties.*

The transition t *is* $Q-$*enabled to fire at* (M, q)*, and we denote it by* $M[t\rangle_{\Sigma,Q}$*, if* $M[t\rangle_\Sigma$ *and* t *is the first transition enabled to fire in* q*.*

If M' *is a mark of* Σ *and* $q' \in Q$ *is a queue with above properties we say that* (M', q') *is* Q_i-*produced by firing of the transition* t *at* (M, q)*, and we denote it by* $(M, q)[t\rangle_{\Sigma,Q_i}(M', q')$*, if* $(M, q)[t\rangle_{\Sigma,Q_i}(M', q')$*,* $M[t\rangle_\Sigma M'$ *and* q' *is obtained from* q *as follows:*

(a) *We remove* t *from* q

(b) *We add to the end of* q *all the transitions enabled to fire at the mark* M' *that are not already in the queue (in an arbitrary order)*

(c) *With the transitions from the obtained queue by* (a) *and* (b) *that are not enabled to fire at the mark* M' *we do the following step depending on* $i = \overline{1,3}$*:*

$(c1)$ *remain in the queue until a possible removing (when it becomes possible at the step* (a)*)*

$(c2)$ *they are removed from the queue*

$(c3)$ *they are removed from the queue from the beginning to the first transition enabled to fire at the mark* M'.

Definition 15([6]). *Let Σ be a Petri net, M a mark of it and $A \subseteq T$. A is a maximal set of transitions parallel enabled to fire at M (in Σ) if it is a set of transitions parallel enabled to fire at M and for any $t \in T - A$ the set $A \cup \{t\}$ has no more this property.*

Definition 16([6]). *Let Σ be a Petri net, M a mark of it, $A \subseteq T$ and $t \in T$.*

1) t *is max-enabled to fire at (M, A) in Σ, and we denote it by $(M, A)[t\rangle_{\Sigma,\max}$, if:*

a) $M[t\rangle_{\Sigma}$.

b) $t \in A$.

2) (M', B) *is max-produced by firing of the transition t at (M, A) in Σ, and we denote it by $(M, A)[t\rangle_{\Sigma,\max}(M', B)$, if:*

a) $(M, A)[t\rangle_{\Sigma,\max}$.

b) $M[t\rangle_{\Sigma} M'$.

c) $B = \begin{cases} A - \{t\} \text{ if } A - \{t\} \neq \Phi \\ C \text{ if } A - \{t\} = \Phi \end{cases}$, where C is any arbitrary set of transitions parallel enabled to fire at M'.

Definition 17([6]). *A nondeterministic finite automaton is a quintuple $A = (Q, \mathrm{Inp}, \mathrm{Out}, \delta, q_0)$, where Q, Inp and Out are nonempty finite sets representing the set of the states, the set of the entries and respectively the set of exits, $\delta: Q \times \mathrm{Inp} \to P(\mathrm{Out} \times Q)$ is the transition function and $q_0 \in Q$ is the initial state.*

Definition 18([6]). *A Petri net controlled by finite automata, shortened APTN, is a couple $\gamma = (\Sigma, A)$ where Σ is a place-transition Petri net and $A = (Q, \mathrm{Inp}, \mathrm{Out}, \delta, q_0)$ is a nondeterministic finite automaton such that:*

1) $\mathrm{Inp} = \mathbf{P}(T)$, $\mathrm{Out} = T$;

2) For any $q \in Q$ and $U \in \mathbf{P}(T) - \{\Phi\}$ we have $\delta(q, U) \neq \Phi$ (unlocking by A);

3) For any $q \in Q$, $U \in \mathbf{P}(T) - \{\Phi\}$ and $(t, q') \in \delta(q, U)$ we have $t \in U$ (consistency in decision).

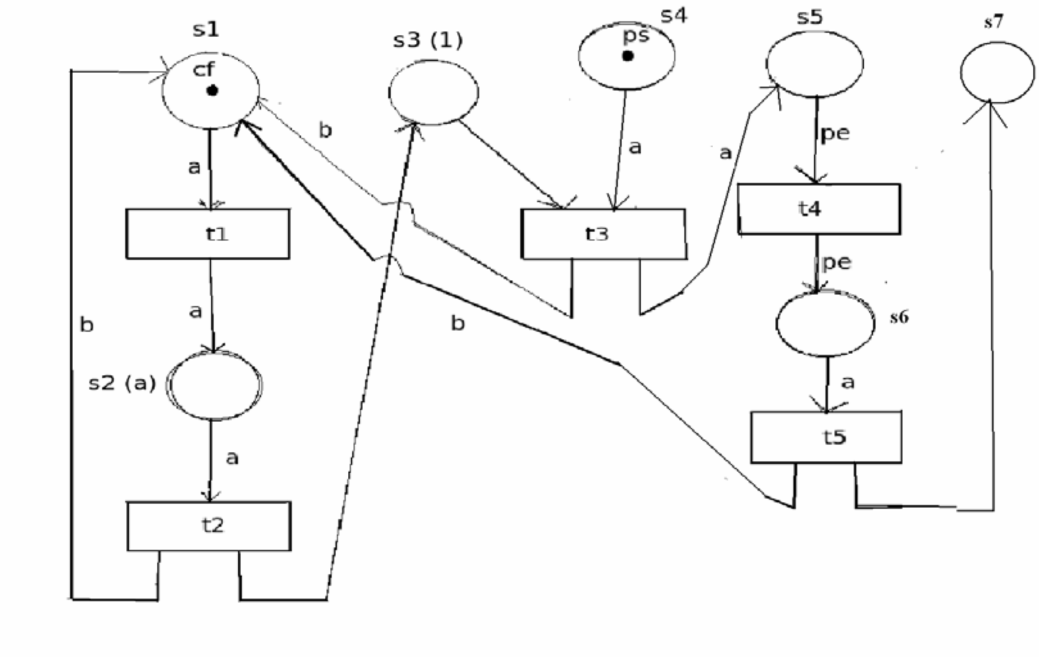
Definition 19([6]). *Let $\gamma = (\Sigma, A)$ be an APTN, q a state of A, M a mark and t a transition of Σ.*

1) t is $a-$enabled to fire at (M,q) (in γ), and we denote this by $(M,q)[t\rangle_{\gamma,a}$, if there exist a state q' of A such that $(t, q') \in \delta(q, T(M))$.

2) (M', q') is $a-$produced by firing of the transition t at (M,q) (in γ), and we denote this by $(M,q)[t\rangle_{\gamma,a}(M', q')$, if $(t, q') \in \delta(q, T(M))$ and $M' = M + \Delta t$.

2. THE MODEL

Consider the following place-transition Petri net system:



Picture 2: *Petri net for a fraud-like investment found*

In the above Petri net the interpretation of the elements is as follows.

- s_1 is the fictive capital.
- s_2 is the sum that will be invested in a fictive investment (capacity is a).
- s_3 is a signal to possible investors (capacity is 1).
- s_4 is the expected benefit for the organizer.
- s_5 is the first deposit of the organizer (the place where the money from investors are deposited).
- s_6 is the place from where the organizer simulates that the game is not over.
- s_7 is the final account of the organizer.
- t_1 is the extracting operation from the fictive capital.
- t_2 is the fictive investment.
- t_3 is the effective investment.
- t_4 is the copying of the final benefit.
- t_5 is the collapse of the found.

Remark 2. In the above model we have $b > a$ (in fact $b >> a$), $a \mid pe$ and $pe \le ps < pe + a$. ps is the target benefit of the organizer, pe is the real benefit of the organizer, a is the average rate of subscripted money from population and $\frac{b}{a} - 1$ is the false benefit rate of the game.

We can add to the above Petri net the places s_8 and s_9, and the transitions t_6 and t_7 analogous to the places s_6 and s_7, and the transitions t_4 and t_5: the sequence s_7, t_6, s_8, t_7, s_9 is identical to the sequence s_5, t_4, s_6, t_5, s_7. Analogously we can add to the Petri net any number of such sequences containing two places and two transitions.

The above place-transition Petri net is nonempty, finite, pure and simple, and it has no isolated elements. The initial mark is

$$M_0 = (cf, 0, 0, ps, 0, 0, 0)^T . \qquad (1)$$

The incidence matrix is

$$I_{\Sigma}=\begin{pmatrix} -a & b & b & 0 & b \\ a & -a & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -a & 0 & 0 \\ 0 & 0 & a & -pe & 0 \\ 0 & 0 & 0 & pe & -a \\ 0 & 0 & 0 & 0 & a \end{pmatrix}. \qquad (2)$$

The $S-$invariants obtained by solving the linear system $I_{\Sigma}^{T}\cdot J=0$ (the transposed relation from definition 11) are of the form

$$J=\left(J_1, J_1, (a-b)J_1, \left(2\frac{b}{a}-1\right)J_1+J_5, J_5, J_5, J_5\right)^{T}. \qquad (3)$$

We notice that the $S-$invariant J is nonnegative only if $J_1=0$, obtaining in this case the minimal $S-$invariant $J=(0,0,0,1,1,1,1)^{T}$. An interesting thing is that the Petri net generated by this minimal $S-$invariant is the chain of places and transitions $s_4\to t_3\to s_5\to t_4\to s_6\to t_5\to s_7$: the investment found consists by tacking money from the investors (place s_4) and deposit to the place s_5, and then moving to the place s_6 and finally to the place s_7.

The only $T-$invariant is with all the components equal to 0 ($P_J=\Phi$ for any $T-$invariant J). This means that no mark (including the initial one) can be reproduced after a finite number of steps.

Firstly the only transition enabled to fire is t_1 and it produces the mark $M_1=(cf-a, a, 0, ps, 0, 0, 0)^{T}$. Next the only transition enabled to fire is t_2 and it produces the mark $M_2=(cf-a+b0, 1, ps, 0, 0, 0)^{T}$. Now there exist two enabled to fire transitions which are also parallel enabled to fire: t_1 and t_3. Even the Petri net evolution is sequential using first the transition t_1 or the transition t_3, even it is parallel the final mark is $M_3=(cf-2\cdot a+b, a, 0, ps-a, a, 0, 0)^{T}$. This above sequence ($t_2$ and next the subsequence t_1 and t_3) is applied until we obtain the mark $M_4=\left(cf-pe-a+2\cdot\frac{b}{a}\cdot pe, a, 0, ps-pe, pe, 0, 0\right)^{T}$. In this moment there are two transitions parallel enabled to fire: t_2 and t_4. If we do not apply at the final step t_1 the obtained mark is $M_5=\left(cf-pe+2\cdot\frac{b}{a}\cdot pe, 0, 0, ps-pe, pe, 0, 0\right)^{T}$, and t_2 is replace by t_1 between the above parallel enabled to fire transitions. The parallelism holds on until we apply the transition t_2 and next the transition t_1, or we apply the transition t_4 and we repeat the transition t_5 until the mark of s_6 is 0. When we apply t_2 the transition t_3 is not enabled to fire because we have $ps-pe<a$. After we apply t_1 the only transition enabled to fire remains t_4 or t_5 (if t_4 was applied). Next we apply t_5 until the mark of s_6 is 0. If the mark of s_6 is 0 the only transition enabled to fire is t_2, and next t_1. In all the above cases we obtain the final mark $M_6=\left(cf-2\cdot a+b+\frac{3\cdot b-a}{a}\cdot pe, a, 1, ps-pe, 0, 0, pe\right)$ and none of the above transitions is now enabled to fire: the game failed.

Because at any time all the enabled to fire transitions are parallel enabled to fire we can consider the model of Petri net under maximum strategy: when the mark is the initial one $A=\{t_1\}$. For this reason we can consider also the model of Petri net controlled by finite automata: we take

$$Q=\{q_0, q_1\}, \qquad (4)$$

$$\delta(q_0, U) = \{(t, q_0) | t \in U\} \text{ if } t_4, t_5 \notin U, \quad (5)$$

$$\delta(q_0, U) = \{(t, q_1) | t \in U\} \ if \ t_4 \in U \ or \ t_5 \in U, \ and \quad (5')$$

$$\delta(q_1, U) = \{(t, q_1) | t \in U\}. \quad (5'')$$

Of course, we can consider above $Q = \{q_0\}$ and the corresponding δ, but we have used two states for the automaton to point out the stages of the investment found: the collection of money from the investors (transitions t_1, t_2 and t_3) and the simulation of the continuity of the found (transitions t_4 and t_5).

The above marks M_4 and M_6 are obtained by solving the equations

$$M_4 - M_0 = (cf' - cf, a, 0, ps - pe, pe, 0, 0)^T = I_\Sigma \cdot f' \ and \quad (6)$$

$$M_6 - M_0 = (cf'' - cf, a, 1, ps - pe, 0, 0, pe)^T = I_\Sigma \cdot f'' \quad (6')$$

with the variables cf', f_1', f_2', f_3', f_4' and f_5', respectively cf'', f_1'', f_2'', f_3'', f_4'' and f_5''.

If we denote the final mark of s_1 by Y and the total invested sum pe by X we obtain

$$Y = c + \left(3 \cdot \frac{b}{a} - 1\right) \cdot X, \quad (7)$$

where $c = cf - 2 \cdot a + b$ and if we add elements as in remark 2 the coefficient 3 of $\frac{b}{a}$ in (7) increases by 1 for each set of two places and two transitions**.**

If we classify investment founds using the regression line 7 (see [4]) the fraud-like ones will be classify in classes with large coefficients of the explanatory variable X. For only one class these points can be also outliers.

3. APPLICATIONS

We define in the $C++$ header "petri.h" two classes: "tranz" and "marc". Class "tranz" has the transitions as objects, and the integer properties "nrloc" (number of places of the Petri net), "indice" (the index of the transition), "pred" (the integer vector of weights from its precedents) and "succ" (the integer vector of weights to its successors). The methods are the constructor of the class and "citire". In the constructor all the weights are initialized with 0, and "citire" is a void method with two integer arguments (the number of places and the index of the transition) that reads the real weights.

Class "marc" has the marks as objects and the integers properties "nrloc" (the same as for "tranz"), "nrtrp" (number of transitions enabled to fire), "val" (the integer vector representing the current mark of the Petri net), "cap" (the integer vector representing the capacities of the nodes) and "ltrp" (the integer vector representing the list of transitions enabled to fire). The methods are analogous to class "tranz": in constructor the capacities are initialized with -1 (with the signification of infinite capacity) and the marks with 0. The method "init", having an integer argument (the number of places) is analogous to the method "citire" of class "tranz": we use it to read the capacities and the initial mark. This is the reason that we have called it "init" instead of "citire".

We have defined also two operators with an argument transition and returning the pointer *this. First is the operator "*=" which tests if the transition is enabled to fire at the current mark and, if it is, it increase "nrtrp" by 1, adds the transition index to "ltrp" and writes that the transition is enabled to fire on screen. The second operator is "+=", it has the argument a transition enabled to fire, replaces the current mark with the produced mark and it writes on the screen the new mark.

Because in the main program we apply the operator "*=" using the transition index from 1 to the number of transitions and then it is fired the transition with the minimum index the Petri net of our model is in fact a Petri net with priorities: the order relation ρ is a total relation decreasing on the transition index. This order relation is not essential because whenever there exist two transitions enabled to fire the final mark does not depend on the order of these transitions. We can extend using this relation the Petri net with priorities by transitions and arcs from s_1, s_2 and s_3 to empty these places. Of course, these transitions have lower priorities then the transitions from t_1 to t_5 and their priorities decrease from the transition of s_1 to the transition of s_3. Because the locations s_2 and s_3 have the marks a and respectively 1 we can consider only one transition to empty them with the weights equal to the marks. Tacking into account the final mark of s_1 we can consider two transitions: first with the weight of the arc $cf - 2 \cdot a + b$ and second one with the weight of the arc $3 \cdot b - a$ (we take into account that $a \mid pe$).

To define a Petri net with priorities we do not need to consider the above total relation: it is enough to consider only a partial order relation between the transitions that can be parallel enabled to fire at a given moment. Therefore the partial relation is such that t_1 has higher priority than t_3 and t_4, and t_2 has higher priority than t_4. In our $C++$ program we define the class "priortranz" that is derived from "tranz" and it has in addition the properties "nrsucc", the integer number of the transitions with higher priority and "lsucc" the list of these transitions. We define also the class derived from "marc", namely "priormarc" using the same heritage mechanism. The constructor of "priortranz" call first the parent constructor, and next initializes the list of successors. the method "citire" is defined in a similar way. The operator "*=" for the derived classes checks first if the transition is enabled to fire using the parent operator, and if it is checks if the given transition has no other transition enabled to fire with higher priority.

Tacking into account the way we have built the list of transitions enabled to fire using the operator "*=" we can also consider that the Petri net is a Petri net controlled by queues in the regime $c2$: the first queue consists in s_1 and at any time the queue contains all the transitions enabled to fire and only them.

Example 2. *Consider in the Petri net of the previous section:*

1) $cf = 100$, $ps = 99$, $a = 3$ and $b = 11$.

2) $cf = 400$, $ps = 1500$, $a = 7$ and $b = 30$.

Solution:

1) We have $a \mid ps$, and from here we obtain $pe = ps = 99$ and the final mark $(1095, 3, 1, 0,\ 0, 0, 99)^T$.
2) In this case we have not $a \mid ps$: $1500 = 214 * 7 + 2$. We obtain $pe = 214 * 7 = 1498$ and the final mark is $(18178, 7, 1, 2, 0, 0, 1498)^T$.

All the above final marks are in the form M_6 and verify (7).

Example 3. *Consider* 21 *existing investment founds in April* 2000 *(Rapoarte lunare ale Asociatiei Administratorilor de Fonduri în* 1996|1997|1998|1999|2000|2001|2002|2003|2004|2005|2006|2007|2008|2009*, [15]). Because we have not the data on the total invested sum, consider as explanatory variable the number of investors (the hypothesis is that the above sum is proportional to this number). As resulting variable we consider the row "VAN" in the Excel table, expressed in millions lei. The results are in the following table.*

Table1: *The 21 investment founds in April 2000*

Found	*Active Clasic*	*Active Dinamic*	*Active Junior*	*ALPHA*	*ARDAF*	*Armonia*
X	521	938	481	453	4532	874
Y	769	2388	839	300191	2796	1193
Found	*Capital Plus*	*FCEx*	*FDI Galați*	*FIDE*	*FIG*	*FNA*

X	8900	2011	229	560	14548	18306
Y	16112	39082	17820	11436	251426	121019
Found	*FNI*	*FON*	*Fortuna Classic*	*Fortuna Gold*	*FVG*	*Stabilo*
X	301331	92	22359	88	2587	651
Y	3412516	59160	37329	7291	3340467	23378
Found	*Tezaur*	*FMT*	*UNOPC*			
X	94	556	379322			
Y	5077	46656	4394222			

We apply the $C++$ program of classification using polynomial regression from Ciuiu, 2007. The degree of the polynomial is 1 (regression lines). If we consider two classes we obtain the first class $Y = 185505.77381 + 10.89722X$ and the second class $Y = 5143278.71429 - 10691.14286X$, containing the investment founds Active Junior and ALPHA. The other investment founds are in the first class.

If we consider three classes we obtain the first class $Y = -8665.80285 + 1324.75936X$ with the investment founds FON, Fortuna Gold and FVG, the second class $Y = 5143278.71429 - 10691.14286X$ with the investment founds Active Junior and ALPHA, and the third class $Y = -19712.58989 + 11.52103X$ with the other investment founds.

We notice that if we increase the number of classes from 3 to 10 it remains one of the classes containing only the investment founds Active Junior and ALPHA. The coefficient of X for the regression line corresponding to the class containing FNI is 11.74369 for 4 classes, 11.7062 for 5 classes, 12.58743 for 6 classes, 11.28776 for 7 or 8 classes, respectively 11.34757 for 9 or 10 classes.

4. CONCLUSIONS

In recession time, because of the acute lack of goods, there appear many organizers of such fraud-like investment founds. They promise gains that are not sustainable even in a period of economic boom. A simple Petri net model of this investment founds was presented in this paper. The model can be extended by a transition that models the payment of the taxes to the state's budget and the place of it, to maintain the appearance of the honesty of the found. Of course, the possible Petri net model of a honest investment found must be stochastic (see [11,14]) to model the risk of the found: we can not have sure gains.

Models using classical place-transition Petri nets and extensions of them are used for the inventory of the products of a factory for selling them to customers by a given number of retailers (see [11]) or raw materials for a printing house (see [3]), for modeling and performance evaluation of hardware/software partitioning (see [8]), or in manufacturing modeling: modeling and evaluation of manufacturing systems (see [14]), modeling and evaluation of its software (see [5]) or deadlock control of flexible manufacturing systems (see [13]). An economic plan for production, supply, quality control and selling in a drugs factory is modeled in [9] by a colored, stochastic, timed and hierarchical Petri net. In this paper was presented also an economic model: the tokens from the places of the Petri net represent sums of money.

Two regressions using the Petri net elements as explanatory and resulting variables (nonlinear ones, not linear as in this paper) were used in [11] to optimize the performances of the modeled system. Using a stochastic Petri net there is defined first a probability distribution for firing three transitions in conflict (which share the same resources): p_1 for the first, p_2 for the second and $1 - p_1 - p_2$ for the last one. Next there are considered p_1 and p_2 as explanatory variables (in fact, because of nonlinearity the real explanatory variables are nonlinear functions of p_1 and p_2), and the resulting variable $C -$ the total inventory cost. Using one of the obtained regressions $C = f(p_1, p_2)$ the optimal cost is obtained in both cases for the minimum point of f .

The operators "*=" and "+=" are defined in the header "petri.h" tacking into account that the multiplication has higher priority than the sum: in any Petri net (common or an extension of it) we must check first if a transition is enabled to fire at a given mark and only if it is indeed enabled to fire we it is fired to obtain a new mark.

All the properties and methods from the classes "tranz" and "marc" are public to have full access to them (including the main program, in which we read some properties as "nrloc" for marks and transitions and we write some other ones as "indice" for the transitions enabled to fire). An open problem is to check which of the properties and methods must remain public and which can become private, or at least protected.

In [3] it is used the software CPN Tools and in [5] there are used the softwares "PED" and "FUNlite Petri net simulator" for the Petri nets. But our header allows us to make classes for extensions of the Petri nets as colored Petri nets (see [3,9,14]), stochastic Petri nets (see [11,9,14]) or timed Petri nets (see [8]) using the heritage mechanism. In the heritage mechanism for the Petri nets with priorities the parent classes are declared in the header "petri.h" virtual for using multiple heritage. In fact, in practice the used Petri nets are not only simple extensions of Petri nets: for instance we can use a temporal Petri net with priorities.

An open problem is to use Petri nets or their extensions for other economic models. For instance we can check if it is a connection between the $S-$invariants and equilibrium equations. Another open problem connected to this paper is to use some other extensions of Petri nets for modeling the fraud-like investment founds or other frauds, like pyramid games for instance. Firstly we can try to use stochastic Petri nets (with simulation of random elements) to model the random elements of the system and hierarchical Petri nets to model the structure of the system. Using timed Petri nets we can also take into the model the time intervals of the operations in the modeled system.

For the colored Petri nets (see [3,9,14]) the first step is to build the $AS-IS$ model, the next step is to evaluate its performances, to try some changes scenarios (see [9]) to improve the performances of the system and finally to build the $TO-BE$ model. An interesting question is if we can go in reverse order: from the $TO-BE$ model to the $AS-IS$ model. If it is possible we can use colored Petri nets for other models of frauds and even of informal economy (see [2]): the $TO-BE$ model will be the false model and the $AS-IS$ model will be the real model.

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