Transaction Cost, Technology Transfer and Mode of Organization

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Abstract

We develop a monopolistically competitive model for a closed economy without contract incompleteness. We show that if superior technology is not allowed to be transferred, integration would be the best mode of organization given that the transaction cost of intermediate input is sufficiently small. However, transferability of technology calls for adding the dimension of factor intensity of input. We then prove that integration could be the better option only when input production technology is capital-intensive. Thus we validate the empirical claim of Antras (2003) from a perspective other than incomplete contract.

Key words: Transaction Cost, Technology Transfer, Outsourcing, Organization of Production, Intra-firm Trade.

JEL classification: D23, O14, L23, F12.

I. INTRODUCTION

Choice of organization of firm has come up as the most interesting topic of research in industrial organization and trade theory in the recent past. Intra-industry trade was the major point of concern before firms' organizational choice issue has garnered the attention. We know that intra-industry trade takes place between similar countries whereas trade is of Heckscher-Ohlin type if it is between dissimilar countries. Substantial amount of literature is there to corroborate the claim. However, the focus has been shifted recently to intra-firm and inter-firm trade. Hence the issue of choice of mode of organization has been taking the front seat. In what follows, the current research has been tilted towards the choice between outsourcing (inter-firm trade), be it domestic or international and integration (intra-firm) of production organization. Empirical papers suggest that integration is more profitable if production of input turns out to be capital-intensive. Zeile (1997) has shown that roughly more than 30% of world trade is taking place within the firm boundaries, i.e. intra-firm. Another remarkable phenomenon is the biasness of capital-intensive industries or inputs towards intra-firm trade and inter-firm if otherwise. Drawing the seminal work of Grossman and Hart (1986), Antras (2003) nicely elaborated how and why contract incompleteness could help explaining the empirical findings. He developed a property rights model of firm boundaries and satisfactorily explained why intra-firm trade is heavily concentrated in capital-intensive industries. There is a growing literature which put emphasize on the integration vs. outsourcing debate. Grossman & Helpman (2002, 2004, 2005), Antras (2005a, 2005b), Antras and Helpman (2004), Grossman and Rosi-Hansberg (2008) etc. are some other interesting papers in this tradition. The frameworks they have used are so parsimonious to have generated some particularly sharp theorems in economics. In another paper Acemoglu, Antras and Helpman (2007) introduced the idea of technology adoption in a world with both complete and incomplete contracts. But their focus was not the organization of production. Their attempt was to show how the degree of contractual incompleteness and the technological complementarities among intermediate inputs affect the choice of technology.

In this paper we build on a monopolistically competitive model with final good and an intermediate input to argue the same that have been highlighted in Antras (2003). However, unlike Antras (2003) we do frame our model in a world without contract incompleteness and we also abstract from any sort of information asymmetry whatsoever. We shall start with the basic reason of outsourcing. It is the cost difference. The underlying implication is that if factor returns are identical across firms, the stand alone firm (who gets the outsourced work) must have superior technology over their 'affiliated' counterpart. This makes outsourcing a lucrative option for the final good producer. We further assume that shipment of input from supplier to final good producer requires some transaction cost which is absent when output and input producing firms are integrated. These issues are a bit touched upon in Markusen (1984) and Brainard (1997). We also presuppose that superior technology is not transferable from stand-alone supplier to final good producer. If the better technology could be transferred free of cost the final good producing firm would have never thought of opting for vertically integrating the mode of production organization. We then extend our model to introduce technology transferability, but at certain cost. In doing so we presume that transferring capital or capital-technology is relatively easier than that of labor. This argument was put forth by Dunning (1983), Milgrom and Roberts (1993). This assumption is quite sensible in that managing workers, be it skilled or unskilled, calls for a physical presence of authority in the production site. Here it is worth mentioning that in a paper Pack and Saggi (2001) showed how vertical transfer of technology through international outsourcing can lead to benefit both by reducing the double marginalization problem.

The basic result that we derive in this paper is that a lower transaction cost for input makes integration less profitable when technology can not be transferred. And allowing for technology transfer indicates higher profit for integrating firm if input production is capital-intensive.

The rest of the paper is schematized as follows. In section II we develop the environment to formulate the model in a closed economy. Section III describes the behavior of final good producing firm with both transferability and non-transferability of efficient technology. Industry equilibrium is shown in section IV. The last section provides with some concluding remarks. Nevertheless, the relevant mathematical details are relegated to the Appendix.

II. ENVIRONMENT

Here we build up a closed economy framework along the lines proposed by Antras (2003), Grossman and Helpman (2002). The economy is characterized by the existence of two factors of production namely capital *(K)* and labor *(L)*. These factors are employed to produce a continuum of varieties of a single commodity *X*. We assume the love-for-variety utility function following Dixit and Stiglitz (1977). Different varieties of good X are considered as differentiated goods to the prospective buyers. Therefore the utility function for the representative consumer is given by the following. n is the total number of varieties produced in the economy.

$$U = \left(\int_0^n x(i)^\alpha \; di\right)^{1/\alpha} \tag{1}$$

Note that *x(i)* implies the variety of good *X* and α is the measure of substitutability between two varieties of final consumable good *X* namely *x(i)* and x*(j)*. Each variety requires a specifically tailored intermediate input, *m*. Transformation of intermediate input into final good requires no further processing and hence no further cost. Input could be procured from two sources. One is integrated firm (implying intra-firm trade) and other is stand-alone supplier (implying inter-firm trade). Both the integrated firm and independent firm can supply same quality input but at different costs. Stand-alone supplier has the cost advantage over integrated firm since it has access to superior technology.

For simplicity let us assume that the technology of intermediate input production for the stand-alone firm (supplier) be Cobb-Douglas.

$$m_O(i) = \left(\frac{K_m(i)}{\beta}\right)^{\beta} \left(\frac{L_m(i)}{1-\beta}\right)^{1-\beta} \tag{2}$$

m denotes one unit of intermediate input and O stands for outsourcing (stand-alone supplier). We further assume that one unit of m is required to produce one unit of final good without adding any extra cost. And β is the factor intensity of m, i.e. $\beta = \left(\frac{K}{L}\right)_m$.

The input production technology for the integrated firm is denoted by

$$m_I(i) = \left(\frac{\gamma K_m(i)}{\beta}\right)^{\beta} \left(\frac{\gamma L_m(i)}{1-\beta}\right)^{1-\beta} \quad ; \quad \gamma > 1 \tag{3}$$

Here γ represents the inefficiency parameter for the integrated supplier.

However, let us assume that there is a fixed cost of production associated with input. We further assume that the fixed cost is same for all modes of organization. Let it be $f_c r^{\beta} w^{1-\beta}$. This assumption is made in line of Antras (2003). The cost function here resembles the same factor intensity like variable cost.

Since the input is specific to a particular variety of final good our framework is, essentially, a bi-lateral monopoly. There are large number of input suppliers and final good producers. Per variety producer is one. The input is absolutely useless if it is not bought by the respective final good producer. This phenomenon transforms the competitive environment into bilateral monopoly. Assumption of large number of varieties helps ruling out the strategic behavior on part of the intermediate input producer.

In this paper our prime focus would be to check the role of technology transferability in determining the mode of organization. Thus we shall assume away any sort of contract incompleteness just to avoid the issues of asymmetric information related hold-up problem or allocation of residual rights of control. These two were the main driving forces in Antras (2003).

III. FIRM BEHAVIOR

III. A. Regime-I

We first consider the non-transferability of technology. Final good producer *(G)* has to choose between integration *(I)* and outsourcing *(O)*. *G* is a profit maximizer. The stand-alone input supplier *(S)* also maximizes its own profit. In fact, *S* is non-existent in case of integration. Integration implies a single vertically integrated unit of production even if there is an input supplier.

To solve for the relevant variables let us assume that the total income or total expenditure of the representative consumer is symbolized by *E*.

Therefore, $E = \int_0^n p(i)\, x(i)\; di$ (4)

From utility maximization principle one gets,

For variety *i*, $x(i) = \left(\frac{\lambda\, p(i)}{\alpha}\right)^{\frac{1}{\alpha-1}}$ (5)

For variety *j*, $x(j) = \left(\frac{\lambda\, p(j)}{\alpha}\right)^{\frac{1}{\alpha-1}}$ (6)

λ is the standard Lagrange multiplier.

From equation (5) and (6) we get,

$$\frac{x(i)}{x(j)} = \left(\frac{p(i)}{p(j)}\right)^{\frac{1}{\alpha-1}} \quad (7)$$

In what follows, the elasticity of substitution between two different varieties is expressed by σ.

$$\sigma = \frac{1}{1-\alpha} \quad (8)$$

Demand for *x(j)* could be represented by

$$x(j) = \frac{E}{p(j)^{\sigma} \int_0^n p(i)^{1-\sigma}\; di} \quad (9)$$

Let us define the index of prices of all varieties as

$$P = \left(\int_0^n p(i)^{1-\sigma}\;\; di\right)^{\frac{1}{1-\sigma}}$$

Here it is important to note that *p(i)* = *p(j)* for symmetric assumption. Hence equation (9) can be reproduced as

$$x(j) = E.P^{\sigma-1}.p(j)^{-\sigma}$$

Since the number of firms are taken to be a continuum, the value of $E.P^{\sigma-1}$ could be considered as given. Let us call it F.

So, $$x(j) = F.p(j)^{-\sigma}; \quad \text{where } F = E.P^{\sigma-1} \qquad (10)$$

By virtue of representative consumer and symmetric assumptions all varieties must have same price are consumed in equal amounts. This implies

$$p(j) = p(i) = p\ (say)$$

and

$$x(j) = x(i) = x\ (say)$$

Solving for the profit maximizing prices for all varieties (considering equation (2) as the relevant production function) we have,

$$p(i) = \frac{r^{\beta}w^{1-\beta}}{\alpha} \qquad (11)$$

Equation (11) states the text-book kind of mark-up pricing for goods which are supplied in a monopolistically competitive market. Therefore, for given wage rate (w) and rental rate (r), the profit of the final good producer *(G),* when inputs are procured from a stand-alone supplier, is given by

$$\pi_{G,O}(i) = F.\left(r^{\beta}w^{1-\beta}\right)^{1-\sigma}.\alpha^{\sigma}\left[\frac{1}{\alpha} - 1\right] - f_c r^{\beta}w^{1-\beta}$$

$$\text{Or, } \pi_{G,O}(i) = F.\left(r^{\beta}w^{1-\beta}\right)^{1-\sigma}.\alpha^{\sigma}\left[\frac{1}{\sigma-1}\right] - f_c r^{\beta}w^{1-\beta} \qquad (12)$$

Following the same procedure we can arrive at the profit for G when input supplier is an affiliated unit of G. This is

$$\pi_{G,I}(i) = F.\left(r^{\beta}w^{1-\beta}\right)^{1-\sigma}.\alpha^{\sigma}\left[\frac{1}{\sigma-1}\right]\gamma^{1-\sigma} - f_c r^{\beta}w^{1-\beta} \qquad (13)$$

Comparing (12) and (13) we find that $\pi_{G,O}(i) > \pi_{G,I}(i)$ (since $\sigma > 1$). Therefore we can propose that:

Proposition 1: *When technology transfer is not possible outsourcing is invariably the preferred mode of organization compared to integration.*

This result is quite apparent. If there is no other cost than the cost of production, inefficiency parameter will hinder the integration possibility. Now to make things more closer to reality let us introduce a transaction cost or trading cost for the intermediate input produced by unaffiliated supplier. Let the cost be μ. μ is the per unit trading cost. Evidently this cost is not needed for integrating firms. Hence there would be tug of war between inefficiency-cost and trading-cost. Outsourcing would no more be an obvious mode of organization. Under this circumstance the profit function for G when non-integration is opted for is : $\pi_{G,O}(i) = p(i)x(i) - \left(r^{\beta}w^{1-\beta}\right).x(i).\mu - f_c r^{\beta}w^{1-\beta}$ where μ is a top-up over cost of production and $\mu > 1$. In what follows, equation (12) boils down to

$$\pi_{G,O}(i) = F.\left(r^{\beta}w^{1-\beta}\right)^{1-\sigma}.\alpha^{\sigma}\left[\frac{1}{\sigma-1}\right]\mu^{1-\sigma} - f_c r^{\beta}w^{1-\beta} \qquad (12\,')$$

Let us start from a situation where the values of equation (12 ′) and (13) are same. For constant inefficiency parameter (no option for technical progress), if μ goes up, the value of $\pi_{G,O}(i)$ will become relatively less.

Corollary 1.1: *Firms' choice between integration and outsourcing crucially depends on the values of γ and μ .*

A reduction in transaction cost implies higher profits for non-integrating firms. Nevertheless, if transaction cost is sufficiently high the final good producer may not opt for outsourcing even if it needs to do with inefficient production otherwise. This idea essentially corroborates the recent shift in mode of organization from integration to outsourcing as an aftermath of information technology revolution. Perhaps the prime reason was the huge slash in transaction cost that took place in information technology sector.

III. B. Regime-II

In the last section we considered a regime where technology was not transferable. Now we will think about a regime shift from non-transferability to transferability of input production technology. Technology is transferable but it requires some cost. As discussed earlier, we further assume that the cost of technology transfer is decreasing in $\left(\frac{K}{L}\right)$ ratio. Therefore the cost associated with technology transfer significantly depends on whether intermediate input production technology is capital intensive or labor intensive.

The cost of production of input *(m)* when technology is transferred from stand-alone supplier *(S)* to final good producer *(G)* takes the following form.

$$C = (w.L + r.K)\left(1 + \phi(\beta)\right) + f_c r^{\beta}w^{1-\beta} \qquad (14)$$

$\phi(\beta)$ follows the properties: $\phi(\beta) > 0$ *and* $\phi'(\beta) < 0$. The reason behind the negativity of the first order derivative of $\phi(\beta)$ is discussed in the introduction.

We are abstracting from any mixed combinations of outsourcing, integration without technology transfer (TT) and integration with TT. We will focus on three extreme cases: first where G entirely outsources input production; second on integration with own technology and the third is the integration with transferred technology from S.

Equation (12′) of Regime-I remains unchanged while equation (13) resembles the profit of integrating *G* when technology is not transferred. We need to calculate afresh the profit of integrating *G* with transferred technology. Let us denote it by $\pi_{G,I,TT}(i)$.

The profit maximizing price of the integrating firm with efficient technology is

$$p(i) = \frac{r^{\beta}w^{1-\beta}}{\alpha}\left(1 + \phi(\beta)\right) \tag{15}$$

Substituting the new equilibrium price in the profit equation we get,

$$\pi_{G,I,TT}(i) = F.\left(r^{\beta}w^{1-\beta}\right)^{1-\sigma}.\alpha^{\sigma}\left[\frac{1}{\sigma-1}\right]\left(1 + \phi(\beta)\right)^{1-\sigma} - f_c r^{\beta}w^{1-\beta} \tag{16}$$

A careful investigation of equation (16) reveals that an increase in β would lead to a fall in $\phi(\beta)$ and hence the value of $\pi_{G,I,TT}(i)$ will be larger. One can easily use the thought process to compare as to what mode of organization gives higher profit (compare (12′) and (16)). This is evident from the evaluation that if the input technology is biased in favor of capital, technology should be transferred from *S* to *G*. In fact, a labor-saving technological progress can induce firm to transfer the technology and to integrate the entire production spectrum. In what follows, we can write down the following proposition.

Proposition 2: *Integrating mode of organization is preferred over outsourcing when input is capital intensive and conversely when intermediate-input uses relatively more labor.*

IV. INDUSTRY EQUILIBRIUM

Absence of any barriers to entry ensures zero profit for all firms in every mode of organization. Let us first consider the case of regime-I with outsourcing. Therefore, equation (12′) boils down to

$$F.\left(r^{\beta}w^{1-\beta}\right)^{1-\sigma}.\alpha^{\sigma}\left[\frac{1}{\sigma-1}\right]\mu^{1-\sigma} - f_c r^{\beta}w^{1-\beta} = 0 \tag{17}$$

We know that, $p(i) = r^{\beta}w^{1-\beta}.\frac{\mu}{\alpha}$

From (10) we have $F = E.P^{\sigma-1}$

$$\text{Or, } F = E.\frac{1}{\int_0^n p(i)^{1-\sigma}di}$$

$$\text{Or, } F = E.\frac{1}{np(i)^{1-\sigma}}$$

$$\text{Or, } F = E.\frac{p(i)^{1-\sigma}}{n} \tag{18}$$

Plugging (18) and the value of $p(i)$ into (17) one gets

$$n = \frac{E}{f_c r^{\beta} w^{1-\beta}} \left[\left(r^{\beta} w^{1-\beta}\right)^{\sigma-1} \left(r^{\beta} w^{1-\beta}\right)^{1-\sigma} . \mu^{\sigma-1} . \; \alpha^{1-\sigma} . \alpha^{\sigma} \left[\frac{1}{\sigma-1}\right] \mu^{1-\sigma} \right]$$

$$Or, \;\; n = \frac{E}{f_c r^{\beta} w^{1-\beta}} \left[\left(r^{\beta} w^{1-\beta}\right)^{\sigma-1} \left(r^{\beta} w^{1-\beta}\right)^{1-\sigma} . \mu^{\sigma-1} . \; \alpha^{1-\sigma} . \alpha^{\sigma} \left[\frac{1}{\alpha} - 1\right] \mu^{1-\sigma} \right]$$

$$Or, \;\; n = \frac{E}{f_c r^{\beta} w^{1-\beta}} [1-\alpha] \qquad (19)$$

This is the total number of varieties in the industry. It is apparent from (19) that n depends only on E, α and fixed cost of production. This expression does not contain anything like μ, γ *and* ϕ. Therefore, for all different modes of organization total number of varieties of X must be identical.

V. CONCLUDING REMARKS

In this paper we have developed a simple model of monopolistic competition with love-for variety preference. Final good production requires specific intermediate input. Final good producer can either integrate or outsource the input production unit. We have assumed away any genus of contract incompleteness, whatsoever. However, stand-alone input supplier is relatively efficient than integrated supplier. We have shown that in absence of technology transfer lowering the transaction cost of input induces outsourcing. Whereas, when technology is transferred at a certain cost outsourcing would be the preferred mode of organization if and only if the intermediate input is labor-intensive.

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Appendix

Integrating final good producer *(G)* maximizes the following profit function in absence of technology transfer.

$$\pi_{G,I}(i) = p(i).x(i) - r^{\beta}w^{1-\beta}.x(i).\gamma - f_c r^{\beta}w^{1-\beta} \quad \text{(A.1)}$$

We further know that, $x(i) = E.P^{\sigma-1}.p(i)^{-\sigma}$ (A.2)

From (A.2) one gets, $\frac{x(i)}{\frac{\partial x(i)}{\partial p(i)}} = (-)\frac{p}{\sigma}$ (A.3)

The first order condition implies

$$p(i) - r^{\beta}w^{1-\beta}.\gamma = \frac{p(i)}{\sigma}$$

$$\text{Or, } p(i)\left(1-\frac{1}{\sigma}\right) = r^{\beta}w^{1-\beta}.\gamma$$

$$\text{Or, } p(i) = r^{\beta}w^{1-\beta}.\frac{\gamma}{\alpha} \quad \text{(A.4)}$$

Plugging (A.2) and (A.4) in (A.1), the profit equation reduces to

$$\pi_{G,I}(i) = F.\left(r^{\beta}w^{1-\beta}\right)^{1-\sigma}.\alpha^{\sigma}\left(\frac{1}{\alpha}-1\right).\gamma^{1-\sigma} - f_c r^{\beta}w^{1-\beta} \quad \text{(A.5)}$$

This is the same equation that we have in text as equation (13).

Following the same technique we can derive the profit maximizing equilibrium price for *G* when input is sourced from stand-alone supplier.

$$p(i) = r^{\beta}w^{1-\beta}.\frac{\mu}{\alpha} \quad \text{(A.6)}$$

In what follows, the profit for *G* with outsourced intermediate input becomes,

$$\pi_{G,O}(i) = p(i).x(i) - r^{\beta}w^{1-\beta}.x(i).\mu - f_c r^{\beta}w^{1-\beta}$$

$$\pi_{G,O}(i) = F.\left(r^{\beta}w^{1-\beta}\right)^{1-\sigma}.\alpha^{\sigma}\left(\frac{1}{\alpha}-1\right).\mu^{1-\sigma} - f_c r^{\beta}w^{1-\beta} \quad \text{(A.7)}$$

This equation is identical with (12′) of the main text.

However, with technology transfer (TT) the total cost function for producing intermediate input would be:

$$C = r^{\beta}w^{1-\beta}.x(i).\left(1+\phi(\beta)\right) + f_c r^{\beta}w^{1-\beta} \quad \text{(A.8)}$$

Therefore the profit equation becomes,

$$\pi_{G,I,TT}(i) = p(i).x(i) - r^{\beta}w^{1-\beta}.x(i).\left(1+\phi(\beta)\right) - f_c r^{\beta}w^{1-\beta} \quad \text{(A.9)}$$

Above equation gives us the new profit-maximizing equilibrium price as

$$p(i) = r^{\beta}w^{1-\beta}.\frac{(1+\phi(\beta))}{\alpha} \quad \text{(A.10)}$$

Plugging (A.2), (A.8) and (A.10) into (A.9) one gets the equation identical with (16) of the main body of the paper.

$$\pi_{G,I,TT}(i) = F.\left(r^{\beta}w^{1-\beta}\right)^{1-\sigma}.\alpha^{\sigma}\left(\frac{1}{\alpha}-1\right).\left(1+\phi(\beta)\right)^{1-\sigma} - f_c r^{\beta}w^{1-\beta} \quad \text{(A.11)}$$