Incomplete Contracts: Foundations and Applications

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Summary:

More than twenty years have elapsed since Oliver Hart's Fisher-Schultz lecture on the topic of incomplete contracts. Incomplete contract theory (ICT) has become a rigorous and widely used approach in dealing with various issues. It's applications include firm theory (hierarchies, ownership and control rights, authority,etc.), international trade (judicial quality as comparative advantage, intra-firm trade, etc.), scope of organizations (including the government, see Hart et. al. 1997) and many others. However, it's theoretical foundations have been seriously debated since its first emergence, and even today, the debate is not coming to an end. We will review several significant works on the foundations on ICT, and from comparing their differences in assumptions, methodology and results, we could get some merit on the critical disagreement over these issues, and from these critical disagreements, we could also capture the central ideas for future research on this field. The critical comments on Hart and Moore's 2008 paper about reference point may also suggest that ICT desperately need a solid foundation.

Key Words:

incomplete contracts, unforeseen contingencies, unverifiable, renegotiation, property rights, transaction costs, complexity, implementation mechanism

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I. Introduction

Every normal being would not refute the fact that real world contracts are sometimes quite 'incomplete'. We have witnessed numerous cases when written contracts are silent. Some terms are hard to enforce or the meaning of which is unclear. In these circumstances, contractual incompleteness may cause disputes and disagreements after the initial one is signed.

This phenomena could be observed both from the national level (such as 'promoting the long-range security and well-being of the United States'[2] or 'establishing 'just and reasonable rates') to firm level and personal level (such as 'for the mutual benefits of the two parties' etc.), where the arbitration of a court is needed in case of conflicts. The costs of incomplete contracts are obvious, the patent cost (Tirole 1999) is proved to be an inefficient way to fully exploit the benefits from an invention or an idea, but since the value of an invention could not be fully specified by a contract or the enforcement cost is so formidable, it is still used as a second best choice. Contracts are sometimes silent, but indeed, we need not to be too pessimistic about this stylized fact. If everything in the world could be resolved by written contracts, then the Soviet Empire could not have perished. Since in a complete contract world, power is useless, contracts could even dominate every aspect we could imagine, so communism or capitalism would make no difference in terms of economic efficiency. This is the point from which the study of institutions and transaction costs begin to thrive.

In the Arrow-Debreu world, the organizational form or any aspect of the institutions is irrelevant to market transactions. A general equilibrium system generates a unique value for all the variables and the solution of which is independent from all other elements other than preferences, utilities and choice.

After Corse's enunciation of the idea that 'the existence of coordination costs on the market justifies resorting to various coordination mechanisms in a decentralized economy(Coarse 1937), economists' endeavors to explain on what is coordination cost, what is transaction cost, how does it come about, have been increasingly significant. There are namely two schools which had a profound influence on contemporary contract theory. [3]One is the property right school, which yielded essential elements of theories of the firm and contracts (Alchian and Demsetz 1972), and

[2] Conducts of the US department of State

[3] See Brousseau and Glachant (2002)

suggested that 'only a reallocation of property rights can overcome agent's propensity to be opportunistic' (Klein, Crawford and Alchian 1978). The other one represented the Carnegie behaviorist school in the 1960s. Based on results from previous studies, Klein et al. (1978) and Williamson (1979) proposed that even if a contract exists between separate buyers and sellers, each party could still be plagued by opportunistic and inefficient behavior in situations which there are large amount of surplus to be divided ex post, which, will lead inevitably to under-investment problems by both parties ex ante.

The first model to formulate contractual incompletemness was set up by Grossman and Hart (1986), and further developed by Hart and Moore (1988, 1990), to provide an rigorous explanation of the make or buy decision and also open the black-box and gain insight about what is happening inside a firm. Their approach, and nearly all the related work on contractual incompleteness before 1999 relied upon three critical assumptions: the ex ante investments could not be verified, the future contingencies could not be foreseen, the ex post payoffs are not verifiable. Under these assumptions, Grossman, Hart and Moore (see also Hart and Moore 1988, 1990) illustrated the case when inefficiencies could also occur given perfectly symmetric information between the contracting parties. Parties would refuse to invest optimally since they face the hold-up problem by the other party. In this case, first best could not be achieved even we suppose that the ex post negotiation is frictionless and ex post optimal could always be achieved.

The foundations of incomplete contracts based on unforeseen contingencies and unverifiable terms have been heatedly debated since complete contract theorists, such as Maskin and Tirole constructed a complete contract in terms of unforeseen contingencies. In 1999, the Review of Economic Studies published five papers, all about the same topic, adjacently in the same volume, including the significant works of Maskin, Moore, Hart, Tirole and Segal. Indeed, we could reach the conclusion in later sections that as long as one of the three critical assumptions is violated, agents could design a complete contract given their unlimited ability to perform calculations and unbounded rationality. Maskin and Tirole (1999a) questioned the foundation of the long established Grossman-Hart-Moore framework based on symmetric information, unforeseen contingencies and unverifiable terms. Segal(1999) introduced the concept of complexity as well as its effects on the implementation mechanisms when ex post renegotiation is allowed, and from a revised GHM model lay upon this, Hart and Moore (1999) argues that Maskin and Tirole's irrelevance theorem have only captured a special case while under a contracting environment which is complex enough, indescribability does affect the ex post payoffs in terms of incentive distortions and limited describability ex post.

The dispute also involves the concept and underlying assumptions of individual rationality. In both the initial GHM model and the revised version accomplished by Segal, Hart and Moore, agents are assumed to have unbounded rational. But whether this implies that agents have the ability to construct an onto function between the numerous objects and a set of numbers, whether they have the ability ex post to describe large number of state contingent actions ex post etc. is still under debate. Tirole's (Tirole 1999) literature review on the foundations of incomplete contracts pushed this dispute on the foundations of ICT to its climax.

Due to the foundational weaknesses of the previous incomplete contracting models, as well as the other difficulties and problems related, Hart and Moore proposed the reference point approach to provide an alternative explanation of why contracts are incomplete. In a typical GHM or HMSmodel the parties could bargain costlessly ex post, and the focus is on ex ante investment inefficiencies. Hart and Moore have argued (Hart and Moore 2005,2008) that, while such an approach can yield useful insights about optimal asset ownership, it is unlikely to be helpful for studying the internal organization of large firms. Specifically, in a world of Coasian bargaining, it is hard to see why important aspects of organizational form such as authority, hierarchy and delegation matter. Why would the parties not simply bargain ex post on every aspect while using monetary side-payments?

The reference point approach (Hart and Moore 2008) took the advantage of several psychological terms and the meaning of aggrievement and a term so-called 'shading cost' and provided an alternative mean to model the authority corporate structure as well as shedding light on the internal decision making processes within a firm (Hart and Holmström 2010).

This paper would give a brief review of the significant papers in discussing the theoretical foundations of incomplete contracts, and will later provide a mechanism undermining Hart and Moore's reference point approach. The paper is organized as follows: Section II would bring a brief review to the classical Grossman-Hart-Moore framework. Section III would introduce the MT Mechanism and the central point of the dispute. Section IV would focus on renegotiation as we will go through Hart and Moore (1988) and discuss about how the different assumptions on the court's verifiability ex post and processes of renegotiation would affect the contract's incompleteness. We will also come back to Segal and MT's disagreements on renegotiation and how renegotiation could affect the SPE in the MT mechanism (in terms of incentives and describability). Section V lays out the reference point framework proposed by Hart and Moore (2008) and Section VI will provide an example to discuss about this new framework's foundational weaknesses. Section VII would give out several critical remarks and suggestions for further research.

II. Unforeseen contingencies and unverifiable information

In a typical Grossman-Hart-Moore model, two agents (a buyer and a seller) engaged in a three periods relationship. In time 0, the agents meet and contract, and then they undertake a relationship specific investment. In time 1, some variables about the state of the world is verified and the agents make further decisions about the transaction which is about to take place in time 2, which is also called, fill in the specific details of the initial contract. Note that the relation could also break down if renegotiation failed. In the final period, the transaction take place (if none of the agents chooses to default at time 1), and the payoffs of each agent is realized and the relationship comes to an end. Several fundamental assumptions are essential to this framework; we will list them together with the expressions we will use in our model:

1) There is symmetric information all round, unlike the previous incentive theoretical work on adverse selection and moral hazard. All the information, including investments, utilities, etc could be observable by both parties.

2) The agents determine the nature of the game including the control over assets, especially the allocation of residual control rights in case of 'missing contracts'. The ex ante investment $e=(e_1,e_2)\in\Omega=\Omega_1\times\Omega_2$, ($e_i\in\Omega_i$, Ω is a closed compact subset of R^2), is not verifiable and are relationship specific, so that they could not be written in an enforceable contract and the specific properties the agents gain from the investments worth much less if the relationship breaks down.

3) There are numerous states, denoted by $\Theta=\{\theta_t, t\in T\}$, in time 1 that are unforeseeable in the contracting period and the numerous contingencies could not be specifically described in a written contract. The scenario to justify this assumption could be seen in Grossman and Hart (1986), Hart

Moore (1988, 1990) and Tirole (1999) as the cost of writing a formidable contract to specify all the future contingencies. The state is contingent to a random variable ε , and the effort levels of both agents could also affect the probability distribution between different states at time 1, so $\theta = (e, \varepsilon)$. The utility function under state θ is: $u^{\theta} = (u_1^{\theta}, u_2^{\theta})$.

4) In time 1, the state of the world is realized, and the agents could renegotiate to determine the state one action, which is drawn from action set $A = X^{\theta} \times Y$, X^{θ} is the set of state dependent actions (such as which good to provide and what varieties to perform,etc.) and Y is the set of state independent actions (such as transfer payments). In this preliminary model, we simplify matters by assuming that the state contingent action is the choice between the set of n tradable widgets, namely $X^{\theta} = \{a_0^{\theta}, a_1^{\theta}, a_2^{\theta}, ..., a_n^{\theta}\}$ (a_0 represents no trade). In this buyer-seller model, we assume that the state contingent utility functions when a_i is traded are: ($i \in \{0,1,2,...n\}$)

$$u_1^{\theta}(a_i^{\theta}, y_1) = v_i^{\theta} + y_1 = v_i^{\theta} - y \tag{1}$$

$$u_2^{\theta}(a_i^{\theta}, y_2) = -c_i^{\theta} + y_2 = y - c_i^{\theta} \tag{1'}$$

The elements in the state contingent vector (v, c) denotes the valuation for the buyer and the cost for the seller respectively. $y = -y_1 = y_2$ denotes the monetary transfer from the buyer to the seller. For simplicity, we will further assume that vector $v = (v_0, v_1, ..., v_n)$ is independent of e_2 and $c = (c_0, c_1, ..., c_n)$ is independent of e_1 . We will relate to the ex ante investments as human capital investment, and the ex post decisions as exploitation of physical capital.

5) There is no trading costs in ex post renegotiation, so, effective bargaining between the agents could always lead to an ex post efficient result, as directly deduced from the Coarse Theorem. (We could see that this assumption is loosed in Hart and Moore's later work on their newly introduced reference point approach.) We simply assume the agents engaged in a Nash Bargaining and determines their share of the residual by their relative bargaining powers and default options (which are specified by the ex ante contract).

6) The payoffs are private benefits which is also not verifiable and hence, not contractible. The payoff functions for the buyer and seller are $P_1 = u_1^{\theta}(a_i, y_1) - e_1, P_2 = u_2^{\theta}(a_i, y_2) - e_2$ under

state θ and the state contingent action a_i, while a minus sign to the effort level represents the agent's disutility level.

7) Agents know the distribution of ε ex ante, and could perform all sorts of complex computations at zero cost.

8) Agents could rescind the ex ante contract ex post in case of mutual agreements.

Assuming that the agents could foresight perfectly at time 0 about the probabilistic distribution of the contingencies to occur at time 1, they could sign the second best contract at time 0 through effective and costless bargaining, this could be in a form of a lump sum transfer from the gainer to the loser. They will determine the most efficient ex ante arrangements through backward induction, and by calculating the equilibrium solution in every sub-game. In Hart and Moore (1990), the optimal allocation of multiple assets between multiple agents and its implications about how it could affect ex ante investment levels and ex post bargaining are clarified. Here, we will restrain our discussion to the 2×2 case.

We will assume that the ownership of an asset increases the agent's ex post bargaining power, and will endogenize the bargaining process in section IV. So, in time 1, the buyer and seller choose the widget to trade. Given e and ε, and hence a unique $\theta = \theta(e, \varepsilon)$, the first best choice in time 1 is:

$$w^\theta \in \arg\max_i (v_i^\theta - c_i^\theta) \qquad (2)$$

The maximum surplus is given by:

$$M^\theta = v_{w^\theta}^\theta - c_{w^\theta}^\theta \qquad (2')$$

The first best investment of both agents is given by:

$$e^* = (e_1^*, e_2^*) \in \arg\max_{e \in \Omega} E_\varepsilon M^\theta - e_1 - e_2 \quad (3)$$

We will assume that the default option is no trade, fixing the buyer's bargaining power at λ, the payoff functions are: $P_1 = \lambda M^\theta - e_1, P_2 = (1-\lambda) M^\theta - e_2$ respectively. (e_1^{NE}, e_2^{NE}) constitutes a Nash Equilibrium of the non-cooperative ex ante investment game if and only if:

$$e_1^{NE} \in \arg\max_{e_1} E_\varepsilon P_1(e_1, e_2^{NE}, \varepsilon) \qquad (4)$$

$$e_2^{NE} \in \arg\max_{e_2} E_\varepsilon P_2(e_1^{NE}, e_2, \varepsilon) \qquad (4')$$

We could see that both the buyer and the seller internalizes only part of its investment contributions to ex post surplus and, intuitively, lead to under-investment relative to the first best. The following proposition (Milgrom and Shannon 1994) formalizes this result:

Proposition 1. For any $e^{NE} = (e_1^{NE}, e_2^{NE})$ and any e^*. Suppose that $\partial E_\varepsilon M(e,\varepsilon)/\partial e_i \geq 0$ on Ω for i=1,2. Then:

1) If agent $j,(j \in \{1,2\})$ has only one investment choice and $e_i^* \in \Omega_i$, then $e_i^{NE} \leq e_i^*$. $i \in \{1,2\}, i \neq j$

2) If there exists an ex post efficient trade $w^\theta = \arg\max_w (v_w^\theta - c_w^\theta)$, which is independent of e, and $e^* \in \operatorname{int}\Omega$, then $e_i^{NE} \leq e_i^*$. $(i \in \{1,2\})$

Given this condition, we could also draw the conclusion that an agent's incentive to invest ex ante relates positively to his bargaining power ex post. Hence, the buyer would invest more when he controls over the seller's physical asset relative to separate ownership, and the seller would further under-invest. The opposite result is also true under the seller controlled case.

As a supplement to Grossman and Hart, Hart and Moore (1990) further concerned about the property right solution under the incomplete contract scenario by using the Shapley Value in specifying the agent's contribution to a given trading group, and hence enunciate how the ownership of assets could alleviate the hold-up problem. In the extended version, not only the ex ante investments matter in the optimal allocation of assets, but also the indispensability of an agent in ex post trade. The example of the tycoon, skipper and chef[4] illustrated that asset ownership should depend on the individual agent's indispensability, either in ex ante investment, ex post trade, or due to some specific characteristics which arises from the nature of the asset(s), such as complementary, etc. With an indispensable agent gains control over an asset, it would reduce the number of parties which is included in the ex-post renegotiation process, and therefore, would raise the proportion of the investing agent's private benefit in respect to gross social benefit, give the agent greater incentives to invest, regardless of whether the owner of the asset has an investment role or not.

Other applications of this framework including the ownership and decision right allocation inside a firm or an organization, the optimal financial structure (equity debt ratio) of an individual firm

[4] See Hart and Moore (1999 JPE)

(see Hart: Firms, Contracts, and Financial Structure); the proper scope of organizations, including governments (Hart et. al 1997); the explanation of judicial system as a comparative advantage in international trade (Nunn 2007); the proportion of intra-industry trade in different sectors of the economy according to the factor intensity of the production function (Antràs 2003), etc.

These applications mainly focus on either the ex ante relationship specific investments which brings out the hold-up problem, or the non-contractibility of certain terms, which makes the allocation of control rights over assets (the structure problem) relevant to efficiency of outcomes.

III. The Maskin-Tirole Mechanism and Disputes Over the ICT's Foundation

The most lethal attack on the initial GHM model is launched by Maskin and Tirole (1999a), when they begin to question ICT's theoretical foundation. The GHM explanation of why contracts are incomplete relies heavily on unverifiable information and unforeseen contingencies. If one of the assumptions about e, θ, u is loosen (say, when e or u is verifiable or a state contingent contract could be written ex ante), first best could easily be achieved by designing a formal ex ante contract based on either one of them.

Maskin and Tirole challenges the GHM's foundation by providing a basic argument that indescribability does not matter in respect to agents' ex post payoffs, which means, given a complete contract when all the future states are foreseeable, agents could also design a corresponding contract under unforeseen contingencies, which would unequivocally leads to the same payoffs. The key point in designing the MT mechanism is to how to verify those variables which could not be directly implemented by a written contract. They extended the implementation mechanism developed by Moore (1992), Moore and Repullo (1998) in an unforeseeable states scenario. We will consider in this section the case when there is no ex post renegotiation, and loose the assumption in the next section. We start with defining several key concepts which are relevant to our discussion. (We will continue to use the variables defined the previous section.)

Definition 1:

1) Two states θ_1 and θ_2 are **equivalent** if $\left|X^{\theta_1}\right| = \left|X^{\theta_2}\right|$, and there exists a bijection $\psi : X^{\theta_1} \to X^{\theta_2} : \psi(x) = x'$ and $\alpha_i(>0), \beta \in R$, such that

$u_i^{\theta_1}(x, y_i) = \alpha_i u_i^{\theta_2}(x', y_i) + \beta_i$ ------------(5)

for all $x \in X^{\theta_1}, y_i \in Y_i, i = 1,2$. We will denote the state equivalence as: $\theta_1 \sim \theta_2$. The definition implies that the states are equivalent if and only if the von Neumann-Morgenstern (noted by vNM below) preferences are the same under the two states subject to a positive affine transformation.

2) Two states θ_1 and θ_2 are weakly equivalent if the state contingent utility possibility sets are the same, noted by $\theta_1 \sim_w \theta_2$.

3) A complete contract $f : \Theta \to A$ is **welfare neutral** if for any $\theta_1 \sim \theta_2$:

$u_i^{\theta_1}(f(\theta_1)) = \alpha_i u_i^{\theta_2}(f(\theta_2)) + \beta_i$ -----------(6)

which implies that the vNM utility functions are the same modulo a positive affine transformation under any two states which are equivalent.

4) Similarly, we call a complete contract is strong welfare neutral if the conditions above satisfies for any two states which are weakly equivalent.

5) If $\alpha_1 = \alpha_2$ holds in (6), we will say that the vNM utility function is state independence of the ratios of marginal utilities of money.

6) The ex ante effort $e = (e_1, e_2)$ is unidentified if for any two equivalent states $\theta_1 \sim \theta_2$ and ex ante effort e, e^* :

$\Pr(\theta_1 | e, \theta_1, \theta_2) = \Pr(\theta_1 | e^*, \theta_1, \theta_2)$ -----------------(7)

Remarks: In the initial version of the GHM framework, the welfare neutrality (condition 2) always holds, as well as condition 5 and 6. Condition 5 holds if the utility function could be expressed in a separable form of x and y . Condition 6 implies that all the information about the agents' investment levels is contained solely in the vNM utility function of the realized state.

Definition 2: A number based contract $f^* : V \to M \times Y$, if V is the set of all the possible number based payoff functions $v = (v_1, v_2) : \{1,2,..., m\} \times Y \to R$. Assuming that all the state contingent physical actions in X , though could not be described in a contract, could be corresponded to a set of integers by a state dependent one-to-one mapping $\sigma^\theta : M^\theta \to X^\theta$ under state θ . σ is called the deciphering key of the contract. So, $\theta = \theta(X^{\theta}, \nu^{\theta}, \sigma^{\theta})$ for every $\theta \in \Theta$, which means for each determined feasible state, there exists a unique set of state contingent actions, a number based utility function, and a deciphering key which is ex ante specified by the two agents (the deciphering key could not be enforced directly by court).

Definition 3: A number based contract f^* is welfare neutral if whenever $\theta_1 \sim \theta_2$:

$\nu_i^{\theta_1}(f^*(\theta_1)) = \alpha_i \nu_i^{\theta_2}(f^*(\theta_2)) + \beta_i$ -------(6')

for $i = 1,2$. The definition of strong welfare neutral is defined correspondingly.

Definition 4: A number based contract f^* corresponds to a complete contract f if for $i = 1,2$; and all $\nu \in V$, $\theta \in \Theta$, such that $\nu^{\theta} = \nu$:

$\nu_i(f^*(\nu)) = u_i^{\theta}(f(\theta))$ ------------(8)

A second step is to specify a number based contract by pre-specifing the ex post utilities they would like to implement, when describing the physical actions become infeasible. The key question now remains are how to reveal the true utility function ν and the deciphering key σ when the state of nature is realized. Here, we are also referring to the key assumption that the ex ante actions, the utility functions under various states of nature are unverifiable. So, one mechanism which could lead to the realization of the real utility functions is by a multi-stage implementing game, which under the unique SPE, the true deciphering key and number based utility functions could be revealed.

We will need to assume that the court could verify ex post, whether a named action is feasible or not. This is not contradictory the assumptions of the GHM, since it is only due to the numerous possible contingencies and state dependent action that the courts could not verify, by selecting a single element within the action set, the court could have the ability to justify its feasibility. The no trade action $a_0 = (x_0, y)$ is always feasible, so, it could be described ex ante.

To define an implementation mechanism, a necessary condition is that f and f^* are **welfare neutral** (see definition 1 above).

Definition 5: If the state is observable by both parties and neither σ nor ν is verifiable, a implementation mechanism χ implements the number based contract f^* if for all $\theta \in \Theta$, and all the subgame perfect Nash equilibrium $s = (s_1, s_2) \in S = S_1 \times S_2$ in state θ ,

$u_i^\theta(\chi(s)) = v_i^\theta(f^*(v))$ ---------(9)

To make our argument concise, we will first assume that the utility function of both agents are quasi-linear and there is no bound in respect to lump sum transfers, as has already been shown by MT that these two assumptions do not hinder us from getting a generalized result. We will design the game between the two agents like this:

Stage0: Agent 1 will name an action x and agent 2 could choose to challenge or not. If he accepts, they will choose (x, y), where y has been ex ante determined. If he challenges, he will pay a small amount ε to a third party (for example, court) and the game proceeds to stage 1.

Stage1: Agent 1 will announce all the feasible actions at that certain state X^θ ; a number based utility function $v = (v_1, v_2)$; and a deciphering key $\sigma : M^\theta \to X^\theta$

Stage2: Agent 2 could choose to accept or challenge. If he accepts, the game is played according to agent 1's announcement and the number based contract. If he challenges, 1 is required to pay a huge fine F to 2, and the game proceeds to stage 3.

Stage3: Agent 2 could challenge on

1) The action set agent 1 has proposed, either by pointing out an action in X^θ which is not feasible, or by naming out an action which is feasible but is not included in X^θ .

2) The number based utility function v_1 by pointing out that $v_1(k)$ is not true for a given integer k .

Agent 1 is then given the following two options:

(a) Acting $\sigma(k)$

(b) Acting x^0 with a transfer $v_1(k) + \varepsilon$

If agent 2 believes that agent 1 has under stated $v_1(k)$, he could set $\varepsilon < 0$, and the challenge is successful if agent 1 chooses b. Otherwise, if he believes that agent 1 has over stated $v_1(k)$, then, he could set $\varepsilon > 0$, and the challenge is successful if agent 1 chooses a. If the challenge is unsuccessful, agent 2 will be requested to transfer $2F$ to a third party.

3) The number based utility function v_2 by naming out another function v_2^* different from v_2 and transfers F to a third party. Agent 1 could choose to accept or counter-challenge. If he accepts, the game is implemented according to (v_1, v_2^*). If he challenges, agent 2 is asked to transfer F to agent 1 and play the mechanism under 2 with relative roles shifting.

Given the number based contract is Pareto Optimal, and the game is played from stage1 to 3, the best outcome agent 1 could get is the one which is specified by the contract, and so, truth telling by agent 1 and accepting the proposal for agent 2 constructs the unique subgame perfect Nash equilibrium of this game. Any misstate by agent 1 would lead to the challenge by agent 2, since it is impossible for both agents to be better off through collusion given this optimal contract (the two agents are playing a zero sum game).

Considering that the naming out the feasible action set could be an tedious task, we will further add stage 0 into the game and the subgame perfect Nash equilibrium indicates that agent 1's advantage from acting first is no more than ε and the game will never reaches stage1.

These results still hold if we assume that there is an upper and lower bound for y_i, which real life contracts always have, and simultaneously loosing the assumption that the utility function is quasi-linear. This would bring us to theorem 2 (the irrelevance theorem)

Theorem 2: Assume that there is a describable action x_0 (no trade) for all states and an upper bound to $|y_i|$ $(i = 1,2)$, f^* corresponds to a complete contract f which is welfare neutral and Pareto optimal, f^* could be implemented in a SPE when states are indescribable, which means, indescribability does not matter given the previous assumptions.

The proof of the theorem includes

1) For all $\theta \in \Theta$, there exists a successful mechanism χ^{θ}.

2) A minor revise to the initial challenging mechanism, which transfers are unbounded.

The challenging mechanism shown above and the aside from the huge penalty, the mechanism here implements $(x_0, y_{\min})$, which satisfies: $u_i^{\theta}(x_0, y_{\min}) < u_i^{\theta}(x_0, y)$ given an unsuccessful challenge, and would let agent 2 name an action he desires and implement if he challenges successfully. This result also holds when agent 1 is making a counter challenge. From this point, huge amount of transfer is avoided, while providing one of the agents to report the real state of the world and the other to accept in the unique SPE of the extensive form game.

Indescribability of future contingencies is still irrelevant to agents' payoffs in the case when the realized action set X^{θ} could not be described with quasi-linear number based utility function and unbounded transfers. The assumption of welfare neutrality in this case, should be strengthened as 'strong welfare neutrality'. A similar design of the challenging mechanism under these assumptions may involve a query option in stage 2, where he was fined a small amount ε . The details of which would be discussed together with Segal's work.

We will then return to the welfare neutrality assumption and examine the conditions under which it could be loosed. To substitute the assumption of welfare-neutrality with condition 5 and 6, we need the following lemma:

Lemma 1: If conditions 5 and 6 hold, then for any complete contract f_0 , there exists a welfare neutral contract f , such that:

$$\sum_{\theta\in\Theta} p(\theta|e)u^{\theta}(f_0(\theta)) = \sum_{\theta\in\Theta} p(\theta|e)u^{\theta}(f(\theta)) \text{ ---------(10)}$$

From this conclusion, we could conceive a welfare-neutral complete contract equivalently, and from theorem 2, there exists a number based contract f^{*} in indescribable states which corresponds to f , and hence f_0 . So, a contract is payoff irrelevant to the describability of states if it satisfies state independence of ratios of monetary utilities of money and effort is unidentified, together with the other key assumptions in theorem 1 aside from welfare neutrality.

IV. Ex post Renegotiation

Ex post renegotiation also plays a vital role in incomplete contracts literature. Grossman and Hart (1986) treated the process of renegotiation as a simple Nash Bargaining between two agents, where the bargaining power been exogenously given and the reserves of both parties depend on the ownership structure of the physical assets. This brief treatment, although captures the essential point of the cause and consequences of the contract's incompleteness, does not provide us any insight into the bargaining process itself. The internal mechanism for renegotiation is also relevant, if not indispensable, to determine two parties' payoffs in equilibrium.

1. Renegotiation and the assumptions about the court

Hart and Moore (1988) is the first to discuss systematically about the renegotiation process under an incomplete contracting environment. The role and ability of the court is vital in concerning the renegotiation mechanism. The court in a typical HM framework is being assumed that:

1) Could not observe the ex ante investment and the agents' utility functions directly, as the incomplete contract literature always assumes.

2) Could not verify the reason for a certain ex post action. (such as, who is responsible for the absence of efficient trade)

3) Could discern the no trade action x_0 and all the state independent actions, such as monetary transfers, etc.

4) Some messages the parties reveal. We will always assume that messages could not be forged, which means, an agent could not have claimed to receive a message that does not exist. We will first consider the case that agents could deny a message which he receives, which means, messages could be sealed if they are not desirable to the receiver. This assumption is key to the difference between various renegotiation literatures.

We will further assume that a third party could not be included in a contract and the agents could rescind the contract if both of them agrees. An ex ante contract specifies the relative powers in ex post renegotiation (residual rights) and the default option which includes a describable action ex ante (though the intrinsic properties of that action is completely unknown), and a state independent transfer.

The following theorem lists several special cases when first best could be achieved.

Theorem 3: In the context shown in section II, first best could be achieved using nonverifiable message scheme if any one of the following conditions hold:

1) v is independent of e_1;

2) c is independent of e_2;

3) There exists a widget w which $w^\theta \in \arg\max_w (v_w^\theta - c_w^\theta)$ and a transfer price t, which satisfies: $v_w^\theta \geq t \geq c_w^\theta$ for all $\theta \in \Theta$;

4) If there is only one possible widget (which means the ex post design is a binary variable),

and $u = (v, c)$ is independent of ε .

The proof of the theorem is intuitive, in case 1 and 2, first best could be achieved through the common property right solution, by allocating the asset ownership to indispensable or important agents. In case 3, when trade over a certain widget is always efficient, and the price specified always satisfies the agent's rationality constraint, trade will always occur at that price, so there is no room for renegotiation to be profitable. In case 4, if there is no uncertainty, either both parties would want and expect trade, or both parties prefer no trade. This would induce the first best investment to assure trade or no investments ex ante for both parties. Note that there is no externality in case 4, since when trade is efficient and parties could conceal the messages they receive, they could always make the last minute offer in a certain trading period, the renegotiation outcome is

1) At the pre-specified t if $c \le t \le v$;
2) At c if $v \ge c \ge t$;
3) At v if $t \ge v \ge c$.

Given these results, the ex post gains from the investment always fully go to the investing agent. However, in more general cases, first best could not be achieved as the externality problem arises with ex post renegotiation, as shown in Hart and Moore (1988). Even if messages are verifiable (we will omit the details in this brief review), the first best could not be achieved while second best could be achieved using a unverifiable message scheme, under which the investment levels are strictly less than their first best counterparts.

2. Renegotiation and the MT mechanism, does it really matter?

When ex post renegotiation is allowed in the MT mechanism, the conditions for indescribability to be irrelevant is stricter. First, we must assume that the parties do not have the ability to commit not to renegotiate when the contractual results are inefficient ex post. So, the contract is not publicly registered. Secondly, the renegotiation process generates a Pareto improvement to the initial state, so as to satisfy the parties' participation constraint.

As to proceed with our argument, we need to conceive a formal framework of renegotiation in advance. Denote the renegotiation function as $h: A^{\theta} \to A^{\theta}$, which is a mapping between action spaces in state θ , an initial outcome $a=(x,y)$ becomes $h(a)=(x^*,y^*)$ after negotiation. Hence, given a state θ ,the action a satisfies: $a=h\circ\chi(\theta)$ (χ as the message game ex post, which leads to the no renegotiation outcome). h has the following properties due to our previous assumptions:

1) h is a single valued function;
2) $h(a|\theta)$ is strongly Pareto efficient for every $\theta\in\Theta$ and $a\in A^\theta$;
3) $u_i^\theta(h(a|\theta))\geq u_i^\theta(a)$.

A result contingent and useful is proved by Maskin and Moore (1999) as a basic theorem for renegotiation.

Theorem 4: A contract f is implementable with renegotiation function h satisfying the assumptions above, if and only if for any $\theta_1,\theta_2\in\Theta$, there exists an action a which satisfies:

$u_i^{\theta_i}(f(\theta_i))\geq u_i^{\theta_i}(h(y|\theta))$ ------------(11) for $i=1,2$.

If we assume x^* is the Pareto dominate outcome in state θ , then, we have:

$$h(\chi(\theta))=(x^*,y+\Delta y),\Delta y=(\Delta y_1,\Delta y_2)$$

and Δy is contingent to the state, especially on the initial contracting result x . We make further assumptions that the utility function takes the form: $u_i^\theta(x,y_i)=U_i(w_i^\theta(x)+y_i)$, which also satisfies Grossman and Hart(1986), Hart and Moore(1988,1990,1999), Segal(1999).

The third key assumption to our model is that the renegotiation process is payoff relevant, which means that the point reached is entirely determined by the payoffs at the inefficient outcome and by the vNM utility functions. In order to derive the irrelevance theorem under renegotiation, we need to enunciate the concept of state equivalence and welfare neutral with renegotiation.

Definition 6:

1) Two states θ_1,θ_2 are renegotiation equivalent if there exists a bijection $\psi:X^{\theta_1}\to X^{\theta_2}$, such that $\Delta y_i^{\theta_1}(x)=\Delta y_i^{\theta_2}(\psi(x))$ for all $x\in X^{\theta_1}$, $i=1,2$.

2) A contract f is renegotiation welfare neutral if for any two states θ_1,θ_2 which are renegotiation equivalent, $f(\theta_1)=a^*,f(\theta_2)=a^{**},y^{\theta_1}=y^{\theta_2}$ always hold. (f is a mapping to the final

outcome of the contract, including renegotiation.)

Given the utility function's form, we could generate the above results that two equivalent states should also be equivalent after renegotiation, and this could only be achieved by restraining on the possible transfers associated with the renegotiation mechanism. It is not hard to see from the definition above that renegotiation equivalent is weaker than equivalent, so renegotiation welfare neutral is stronger than welfare neutral specified in section III.

The key problem lies in the challenging mechanism we previously described to induce the agents to reveal the true state of the world in a SPE, during which truth telling satisfies the agents' incentive compatibility constraint. When renegotiation is allowed however, the huge amount of transfer to a third party so as to punish the deviating agent may not occur, since agent 1 might collude with agent 2 not invalidate a false challenge, therefore, provides agent 2 the incentive to challenge strategically even if agent 1 reports the true state of the world.

To tackle with the incentive problems arise in the implementation mechanism, Maskin and Tirole further assumes that the agents are risk averse, by assigning a random variable to the possible monetary transfers, from agent 2 to agent 1 for an unsuccessful challenge. Given the agents are risk averse, it is possible to design a lottery which will make agent 1's certainty equivalent (CE) 0 while remaining the agent 2's CE negative. Hence, by penalizing agent 2 to prevent him from false challenges and meanwhile, prevent giving agent 1 the incentive to make a valid challenge fail.

3. Incentive Compatibility and Renegotiation

In respond to Maskin and Tirole's critical comments, Segal (1999) countered by proposing an alternative foundation for ICT by incorporating the concept of 'complexity', and by reforming the preliminary works of Grossman, Hart and Moore, he clarified that when the trading environment become complex enough, and agents could not commit not to renegotiate, a message game ex post could not generate first best, and in extreme cases, an ex ante contract's provide no better incentive for the agents to invest than the null contract.

Complexity could cause the contract's incompleteness in two ways: First, the incentive constraints of the agents in a message game, which was ex ante designed to implement the utilities which is not directly verifiable, was hard to satisfy given a bounded transferring level and an increasing number of possible ex post actions. Secondly, the limited describability of possible actions by agents ex post could also limit the contract's effectiveness. He argues that the MT mechanism could implement a number based contract, which was corresponding to a complete contract when the possible action space is small. But the validity of the mechanism would be under question when more complex environment is concerned and therefore could induce incentive problems and implementation difficulties. In the extreme case, an ex ante contract could be equivalent to no contract, under which agents' incentive to invest is diminished compared to first best, as illustrated by theorem 1.

The setup assumptions are regular ones in traditional ICT literature and that of Maskin and Tirole. We will refer to the case in section II, as the trade over n widgets. At each state, determined by the ex ante investment and a random variable, each widget would have a cost c for the seller to produce, and a value v for the buyer to consume. Let us suppose that there is only one widget at each state whose valuation and cost is relevant to the parties' ex ante investment, and assuming that this is the efficient one to trade. The other widgets are non-responsive to investments. To rule out the possibilities that an ex ante non-contingent clause could enhance welfare, we assume that widgets are indistinguishable ex ante and all trades are equally likely to be efficient, and that for any non-contingent contract, each party asymptotically receives one half of the its investment contribution, which is equivalent to a null contract case. To illustrate the meaning for n, we need to assume that the number of inefficient trade increases with n, and that by assuming that only one widget is responsive to investments and efficient ex post, we have clarified a case satisfying these assumptions.

If an ex ante contract suggests that under state θ, widget w to be traded with price t before renegotiation, and the efficient trade surplus is M^{θ}, by trading w^*. Setting the buyer and seller's bargaining power to be1/2, the agents' welfare after renegotiation could be written as:

$u_1 = -t + 0.5(v_w + c_w) + 0.5M^{\theta}$ ------------(12)

$u_2 = t - 0.5(v_w + c_w) + 0.5M^{\theta}$ -------------(12')

To gain a better position for negotiation, the buyer would like to maximize $q_w = v_w + c_w$ in a contractual outcome before renegotiation, and the seller desires the opposite. We name the widgets whose $q_w > q_{w^*}$ as 'goldplated widgets', which is denoted by G; and whose $q_w < q_{w^*}$ as 'cheap imitation widgets', as denoted by C. The result of the implementation game ex post should depend solely on the agents' reports and actions since little additional information could be gained through verifiable terms. With the revelation principal, we could confine our discussion to the message game, in which two agents separately report the state of the world θ_B, θ_S, and $\chi(\theta_B, \theta_S)$ is the result implemented from this report. In the case specified here, we could simplify θ as a list of description of the various widget and the valuation and cost of the efficient widget as (N, v, c). The transfer level t is the critical variable to control for our construction of this mechanism, and a t irrelevant to N_B and N_S is equivalent to no-contract, since it does not provide the agents with incentives to report the true state of the world. Contrary to Maskin and Tirole, Segal assumes that the agents are risk neutral, and following this, Hart and Moore was able to prove that with renegotiation, indescribability could be a cause for the contract's incompleteness.

Back to the discussion for incentives, the critical point is how to deal with the disagreements between N_B and N_S, since if two agents agree with each other which widget to trade, the mechanism would implement trade on that very widget, and given these assumptions above, we could prove that it is a Nash Equilibrium. Following the report of one agent in cases of disputes certainly would not achieve first best in a general case, since the other agent would under-invest ex ante due to the lack of power ex post (the other agent could choose the state he desires, and by renegotiation, this could be achieved even with Maskin's mechanisms). Define the vital disagreement between two reports as:

1) For a certain widget, the buyer reports normal, and the seller reports goldplated;

2) For a certain widget, the buyer reports cheap imitated, and the seller reports normal.

Using results of theorem 4, we could derive the following incentive constraints:

$$-t + \frac{v_S - c_S}{2} \leq v_S - t(N_S) \text{--------------(13)}$$

$$t + \frac{v_B - c_B}{2} \leq t(N_B) - c_B \text{---------------(13')}$$

Which follows:

$t(N_S) - t(N_B) > 0$, since the buyer would be inclined to over report the number of cheap imitated widget and the seller's incentive is quite the opposite. Therefore, the transfer should depend on the levels of vital disagreements between the agents' reports. For each level, agents must be provided with enough incentives not to deviate from truth telling, and hence, let reporting the true state becomes the unique SPE of the extensive game.

But given transfers are bounded (in everyday contracts, this usually hold), the incentives to provide between two levels is diminished as n goes to infinity(the environment is complex enough), and the results of the contract converges to non-contract, which would case under-investment and hold-ups.

Hart and Moore (1999) further confirms that when renegotiation is allowed and agents are risk neutral, complexity would lead to contractual incompleteness even if we allow for unbounded monetary transfers.

4. Describability and renegotiation

The paragraph above specifies the case when non-contractibility and unforeseen contingencies do matter when the contracting environment is complex enough, but the methodology itself is too narrow ranged and could not be applied to a more general setting. Suppose for example that the number of responsive widget is proportion to the total number of widget or the number of gold-plated widget is ex ante known (or when the number of cheap imitated widget is ex ante known), we could derive a complete contract within the above framework. In order to seek for an alternative setting, we will focus then on the describability of widgets ex post.

The MT mechanism has proved that indescribability ex ante do not affect the possible contractual results ex post if the actions is describable and thus contractible ex post and when renegotiation is not allowed. Assuming perfect contractibility ex post is somewhat unrealistic as pointed out by Segal (1995), since the complex nature implies that it is unrealistic even to describe all the possible actions ex post. In contrast, we could assume that the parties could at most describe a given number M of actions (state dependent) in the action space, which is named by Segal as 'limited describeability'. Recall that Maskin and Tirole (1999a) had specified the case when the describing all actions in under a certain state is costly, and from a subgame perfect implementation, they could design a mechanism which could implement first best without enunciating the complex action space, since agents would never be forced to do so in a subgame perfect Nash Equlibrium. But when the describability is limited so that describing it fully is impossible even when the state of the world is realized, contracts could also be incomplete.

Let us suppose that every agent could describe at most k widgets ex post. From the revelation principle, we could model the contract and the renegotiation process as a message game, where every agent send out a message to describe the state of the world (the description of is not necessarily complete), denoted by M_B and M_S , where $f: M_B \times M_S \to X$ [5]is the contract specified mechanism to decide about the ex post actions. $X_0 \in X$ as the set of widgets which is not described under the equilibrium strategy of the ex post message game. So, $|X_0| \geq n-2k$, and intuitively, the buyer would suggest the best widgets while the seller would suggest the worst ones. The message game could only rule out a given number of 'radical' widgets, but the responsive widget may still remain unverified, even with the case we specified in the beginning of this chapter. When the number of widget is large enough ($n \to \infty$), the ex ante contract provides no extra incentive for ex ante investments comparing with null contract.

Theorem 5: In an environment described above with the standardized assumptions in Segal (1999) and assuming that the parties' messages could not distinguish between the undescribed widgets[6]. The difference between the change in party i's payoff $(i = B, S)$ and half of the change in total surplus could be controlled by the maximum value of these three terms:

1) The changes in the valuations of the k best widgets.
2) The changes in the valuations of the k worst widgets.
3) Change in the valuations for the average widget.

All of the three terms converge to zero as n goes to infinity[7], which implies that the average widget is very likely to be non-responsive ones. The ex ante contract converges to null contract.

V. The Reference Point Approach

Realizing the foundational weaknesses for the initial GHM model as well as several other important aspects in explaining firm scope, firm boundaries, as well as modeling the internal

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5 The state independent action Y could be ignored here without loss of generality

6 The conditions could be formalized, using permutations between widgets. See (C1) and (C2) in Segal (1999)

7 The concept of convergence could be seen in <A course in probability> by Kai Lai Chung and also we will impose the concept of Hausdorff distance when we refer to the distance between sets.

structures and hierarchies of a firm, Hart and Moore advovated the reference point approach in providing an alternative foundation for contractual incompleteness, as well as capturing the exercise of authority in a firm. The GHM model as well as the revised Hart Moore and Segal model assume that the ex post conflict of the agents could be resolved by frictionless renegotiation, the incompleteness is due to the insufficient investment ex ante caused by the hold-up problem. While this could partially explain certain aspects in reality, we rarely observe negotiation in a firm between hierarchies, for example, between the CEO and the department managers, between the manager and employees. Authority seems to be the key aspect in the decision making process in a hierarchical organization. An employment contract could specify about the wage of an employee, but does not specify every aspect of the task the employee has to undertake. The boss could exercise his authority and give the employee orders, the employee would have to obey or otherwise being fired. But irresponsible actions by employees may sometimes occur, the employee could act consummately according to the spirit of the contract, or could he act perfunctorily within the letter of the contract. The court could only enforce perfunctory performance but not consummate performance. The reverse is also true for the boss (or principal), in which he could act consummately or perfunctorily to his subordinates.

There are psychological assumptions in the reference point approach: the cost between perfunctory performance and consummate performance is very little, or we could view them as almost the same. An agent would tend to perform consummately if she feels she is being well treated, and revenge on the other party in case she feels she is unfairly treated. The degree of revenge is linearly correlated with the degree of "mistreatment", which also depends on the reference of fairness in the agent's mind. We assume that the initial contract signed ex ante provides the sole reference point for the parties ex post (the possibility of other sources of reference points is ruled out for simplicity), but the parties may have different opinions about what is the right contractual outcome. We will further assume that each party feels entitled to the best contractual outcome permitted by the contract while not violating the other parties' participation constraint. If the ex post outcome is determined by a strategy, then the parties would be feeling entitled to the best outcome provided by all the possible mix strategies. Renegotiation of the contract is equivalent to the ' no contract' case, in which each party feels entitled for the best possible outcome from the relationship.

The assumptions in the model are the standard ones of the GHM except for

1) No ex ante investments are undertaken.

2) Ex post actions could not be fully verified, the court could only enforce perfunctory behaviour.

3) Parties do not feel entitled to outcomes outside the contract, but may have different views of what they are entitled to within the contract. More specifically, each side interprets the contract in a way that is most favorable to him. When he does not get the most favored outcome within the contract, he feels aggrieved and shades by performing in a perfunctory rather than a consummate fashion, creating deadweight losses. Denote $u_S, u_B, a_S, a_B, \sigma_S, \sigma_B$ as the contractual payoffs, aggrievement levels, and the deadweight loss cost by shading of the seller and buyer respectively. The proportion of shading is denoted by $\theta, (0 < \theta < 1)$. The buyer and seller's net payoffs could be written as:

$U_B = u_B - \sigma_S - \max\{\theta a_B - \sigma_B, 0\}$ ---------(14)

$U_S = u_S - \sigma_B - \max\{\theta a_S - \sigma_S, 0\}$ ---------(14')

These expressions indicate that the optimal choice of the buyer and the seller is to shade by $\sigma_B = \theta a_B, \sigma_S = \theta a_S$. So U_B and U_S could be expressed by $\theta, u_S, u_B, a_S, a_B$. We will cite some concrete examples to see how shading costs work and the use of contracts to lower shading costs. Assume that in the trading period, the cost and valuation is certain for the seller and the buyer and assume them to be 0 and 10. If there is no contract, the buyer would feel entitled to the price 0 and the seller to the price 10. No matter what the negotiation trading price is, the net shading cost always equal to 10θ. By signing a contract specifing the trading price at any number between 0 and 10, nobody would feel aggrieved due to the reference provided. But there are still cases in which a contract is incomplete, which means, shading costs could not be exterminated.

First, we will introduce several technical assumptions of the reference point approach to make the shading cost well-defined:

1) A party conjures with mixed strategies in the contractualmechanism (and with correlated strategies if the mechanism has simultaneous moves).

2) A party imagines a commitment to trade on the part of the other party, whatever the outcome of the randomization.

Now let us suppose in period 1, there are three possible states, remember that the states could not be foresee or verified ex ante (in the contracting period 0).

State (S)	1	2	3
Value (V)	9	20	10
Cost (C)	0	10	9

Trade is always efficient given this condition, so the first best is achieved if trade always occur and there is no shading cost. If no renegotiation is allowed, a contract which ensures trade would have to specify a number no larger than 9 and a number no smaller than10 as possible trading prices. In this contract however, when state 3 occurs, the buyer would feel entitled to 9 and the seller 10. The overall shading cost would be θ . So, first best could not be achieved under Hart and Moore's perspective (we will later provide a contract which could be approaching first best in this circumstance). This is a trade-optimal[8] circumstance with future uncertainties, and there is no common trade within different states. We could reach the following conclusions:

Theorem 6: A simple contract achieves the first-best if

(i) only v varies;

(ii) only c varies;

(iii) the smallest element of the support of v is at least as great as the largest element of the support of c.

Otherwise, in a trade-optimal circumstance with uncertainties, every ex ante contract could not fully exterminate the deadweigh shading cost at all possible states.

VI. Reference point approach: Solid or not?

After the first paper on the reference point approach is published and economists began to do empirical tests on the shading cost argument, some key questions still remain: Did the reference point approach provide a rigorous foundation for ICT? Did the aggrievement shading cost argument could provide a basis for contractual incompleteness when future states are uncertain and difficult to foreseen precisely ex ante? Under the assumptions of Hart and Moore (2008) as well as their specification on shading, we will design a nearly complete contract[9] under the

[8] Trade optimal indicates that trade is always efficient, v is always greater than c.

[9] Nearly complete means that the we could lower the transaction cost to any level (below any $\varepsilon > 0$) if we adjust our contracting parameter to a certain range.

circumstance of uncertainty as well as no common trading price between different states.

We will maintain the assumptions used in the previous section, and reconsidering the 3-states circumstance:

State (S)	1	2	3
Value (V)	9	20	10
Cost (C)	0	10	9

We will also assume that there are no outside options for both the buyer and the seller, and from the coefficients shown in the diagram (Hart Moore 2008 QJE appendix), trade is always efficient.

We will design the ex ante contract like this:

In period 1, the buyer would be asked to report which state is actually in, the seller could choose to challenge or agree. If he agrees, the parties would trade at the following prices: p=9 in state 1 and 3, p=10 in state 2.

If the seller challenges when the buyer reports S3:

1) B is punished by $100.

2) Let B buy the good at p=11, if he buys, S will be punished by $1, and the game ends. Otherwise, the game continues.

3) Let B buy the good at p=9.5, if he buys, S will be punished by $100. Otherwise, S will be rewarded by $9.5.

The payoffs:

When B reports S3	Challenge (Buyer, Seller)	Agree (Buyer, Seller)
State 1	-100, 9.5	0, 9
State 2	-91, 0	11, -1
State 3	-99.5, -99,5	1, 0

If the seller challenges when the buyer reports S2:

1) B is punished by $100.

2) Let B buy the good at p=11, if he buys, S fails and would be punished by $100. If not, S will be rewarded by $11.

The payoffs:

When B reports S2	Challenge (Buyer, Seller)	Agree (Buyer, Seller)
State 1	-100, 11	-1, 10

State 2	-91, -99	10, 0
State 3	-100, 11	0, 1

If the seller challenges when the buyer reports S1:

1) B is punished by \$100.

2) Let B buy the good at p=11, if he buys, S will be punished by \$1, and the game ends. Otherwise, the game continues.

3) Let B buy the good at p=9.5, if he buys, the game ends. If not, S will be punished by \$100.

The payoffs:

When B reports S1	Challenge (Buyer, Seller)	Agree (Buyer, Seller)
State 1	-100, -100	0, 9
State 2	-91, 0	11, -1
State 3	-99.5, 0.5	1, 0

We will calculate the shading cost in each state, the definition of 'aggrieve' is the same as Hart and Moore (2008). We could easily see that the only subgame perfect Nash Equilibrium is the buyer reports the true state in period 1 and the seller challenges if B lies and agrees if B is telling the truth.

In state 1, B will report S1, and the subsequent payoffs for B and S are 0 and 9 repectively. S will not feel aggrieved since it will violate B's participation constraint if he chooses to report S1 and S2 with positive probability in a mixed strategy. B will not feel aggrieved since 0 is his highest payoff.

In state 2, B will report S2, and the subsequent payoffs for B and S are 10 and 0 respectively. S will not be aggrieved since 0 is her highest possible payoff. B will not feel aggrieved since if he reports S1 or S3, S would definitely challenge to keep up with her participation constraint.

In state 3, B will report S3, and the subsequent payoffs for B and S are 1 and 0 respectively. S will be aggrieved since she would expect B to report S3 with prob 100/101 and S2 with prob 1/101. Her expected payoff under this strategy would be 11/101. She will be aggrieved by 11/101 and the shading cost is $11\theta/101$, which θ denotes the shading coefficient. B will not feel aggrieved since 1 is his highest payoff.

The expected shading cost under this contract is: $11\theta P_3 / 101$.

If we raise B's punishment levels and assume that B would be punished by \$1000 if he is being challenged, then the total shading cost would be $11\theta P_3 / 1001$. When we raise this number, the total expected shading cost would be approaching 0, which means, we could design a nearly complete contract under this state.

VII. Critical Remarks and Suggestions for Future Research

We will begin with our remarks by repeating the three classical arguments on the sources for contractual incompleteness (Tirole 1999), and focus on the robustness and solidness of these foundations.

1) *Enforcement Costs:* When it is difficult to verify parties' ex post valuations for differ-ent trades, the complete long-term contract specifying the efficient ex post trade in all states of the world may be infeasible. It has been argued that whenever the parties' valuations are mutually observable ex post but not verifiable, it may be optimal to negotiate the trade after these valuations are observed, rather than rigidly contract the trade ex ante.

2) *Unforeseen contingencies:* "Parties cannot define ex ante the contingencies that may occur (or actions that may be feasible) later on. So, they must content themselves with signing a contract such as an authority or ownership relationship that does not explicitly mention those contingencies, or with signing no contract at all."

3) *Writing Costs:* If specifying contractual contingencies is costly, the optimal contract should trade off the loss from contractual incompleteness against the cost of adding contractual clauses.

We will see that none of the three provides a solid foundation for contractual incompleteness.With respect to enforcement, by using the subgame perfect implementation suggested by Moore and Repullo (1988), first best investment could be implemented, this result is further confirmed by Moore (1992). Unforeseen contingencies have been proved to be irrelevant to parties' payoffs under the welfare neutral assumptions when we rule out renegotiation as well as renegotiation welfare neutral and risk aversion when the possibility of renegotiation exists, as shown in section 3. As to the writing cost argument, we still do not have a rigorous model to justify why agents would shrink from specifing important aspects even in important economic situations, and we are not convinced by the current behaviour model's explanation.

Ever since 1999, the focus of contractual incompleteness lies on the role of renegotiation and the complexity of the contracting environment. In the models of Hart and Moore (1988), Aghion et al. (1994) and Noldeke and Schmidt (1995), etc. renegotiation outcome is given by the 'Outside Option Principle'. By setting a high penalty for breach, a contract can make one party's outside option bind, so that the other party receives the entire renegotiation surplus. This allows to implement the first-best in one-sided investment models, and to achieve a substantial improvement over the no-contract outcome in two-sided models. Maskin and Tirole's result holds perfectly when parties could commit not to renegotiate, while Hart Moore and Segal indicated that in a complex contracting environment with parties lacking the commitment ability, all the aforementioned sources of transaction costs may be important ingredients in explaining contractual incompleteness.

The questions remained are:

1)Do the agents have the ability to commit not to renegotiate under any circumstance?

2)Is renegotiation really costless? How to model renegotiation cost?

Maskin and Tirole cited the examples when the no-renegotiation commitment is possible (such as signing a contract publicly, etc.), and according to their results, such an ability would imply completeness in contracts even when the states are not foreseeable, which also implies the elimination of transaction costs in this symmetric information circumstance. From the 1999 literatures, an appropriate concluding remark should be: renegotiation, together with other elements, (unforeseen contingencies, unverifiable information,etc.) seems to be the possible cause of contract incompleteness. The tempting question is, (pointed out by Segal 1999) why there is no law forbidding renegotiation, or why do parties still signing contracts in such a fashion which make the commitment more difficult while the other options are still available? Unfortunately, none of the five literatures in RES1999 gave a satisfactory explanation on this vital point.

Contrast to previous literatures, which assumed that ex post renegotiation is costless, Hart and Moore 2008 formalized the fact that parties could commit not to renegotiate, given that renegotiation has certain costs (shading cost). Unless the gain from renegotiation overwhelm the dead-weigh loss from shading, parties would prefer to maintain the terms specified in an initial contract. The shading cost approach create the trade off between flexibility and shading. This framework provided a reasonable explaination for long-term employment contracts, which fix wages in advance , and the employer maintained the residual control rights could be optimal since it lowers the shading cost.

In respect to Hart and Holmström's 2010 work base on the reference point approach, it succeeded in figuring out the case when firms do not always maximize profits, in addition to the treat of authority as shown before. However, two modeling aspects need to be refined: first, in terms of delegation, when information is symmetric between all agents, the boss take back the authority of one manager, who's private benefit is lowered in an efficient coordination, in this way, the possibilities for efficient coordination could be higher, even if shading cost could not be fully exterminated under certain cases. Secondly, the assumptions for the increased aggrievement level under reversals is not natural, the exogenous given $\theta^* > \theta$ is unfounded, and nearly all the results under delegation was founded under this assumption. A endogenous model to explain the increase in aggrievement level could shed light on how reversals could be costly as well as the commitment ability for the boss to give up her rights of control.

Besides, the reference point approach have at least two other advantages other than foundational issues. First, aggrievement provides a robust explanation for an initial choice of ownership. Under the previous GHM or HMS model, a contingent ownership (or decision right) structure may proved to be first best (Aghion and Bolton), since one could choose the structure ex post without affecting any of the terms. Secondly, in a dynamic model with uncertainty, one would expect to see the continous transfer of decision rights in absence of shading costs. The reference point approach justifies a sort of inertia in property ownership (as well as other forms of power or authority) which is frequently observed in the real world.

The reference point approach is, by every mean, a significant and pioneering work by formulating psychological effects into ICT literature by a well defined variable named: aggrievement level, and the associated shading cost. But whether the reference point approach could provide a rigorous foundation for ICT is still unknown, since with the current assumptions and methodologies, we could design an 'almost complete contract' as shown in section 6. Whether the individual rationality constraint should be used in confining the reference, or by what form should it be applied needs to be seriously considered. Otherwise, we could design a contract with huge penalties which would make an individual's probabilitistic weight on a deviation strategy small enough to lower total shading cost to any $\varepsilon > 0$, given unbounded penalty and the parties' entitlements defined in Hart and Moore 2008 given a specific contract.

Given that the reference point approach still has its flaws in providing a solid foundation for ICT, the search for a rigorous foundation is still going on. Tirole (2007) tried on the bounded rationality case in his recent working paper. The transaction cost theory giving up the assumption of savage rational, but the key issue is, how to model people's behaviour in a Simon[10] rational world giving heterogeneous agents and uncertain behaviours? How to model renegotiation or whether or not renegotiation is necessary in such a different world from neo-classical economics? Bounded rationality approaches are appealing, but questions remained to be solved for them to provide a solid foundation for incomplete contract theory.

Future theoretical studies on the foundations of contractual incompleteness may try on these directions as shown below:

1) By incorporating elements such as norms, reputation, etc. into current reference point models, and consider about the outside references other than a contract. These elements, if their way of influencing agents' incentives and behaviours are well-defined, could nevertheless make significant contributes to ICT as well as firm theory and organizational structure.

2) Several key assumptions needs to be justified. Such as in ICT literature, the judge is supposed to have bounded rational as he could not verify some of the variables. But why postulating that agents have perfect rationality escapes certain limitations? It would be even more reasonable to assume bounded rationality for all players engaged in the game, as proposed by TCT[11] theorists.

3) Incorporating certain TCT assumptions and ideas such as Simon rationality and institutional failure, etc.

[10] This concept was first advocated by Simon (1947, 1976) as agents have non-savage rational.

[11] The new institutional transaction cost theory

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