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**Abstract**

This paper adapts in a simple static context the Rogoff's (1985) analysis of monetary policy delegation to a conservative central banker to the robust control framework. In this framework, uncertainty means that policymakers are unsure about their model, in the sense that there is a group of approximate models that they also consider as possibly true, and their objective is to choose a rule that will work under a range of different model specifications. We find that robustness reveals the emergence of a precautionary behaviour in the case of unstructured model uncertainty, reducing thus government's willingness to delegate monetary policy to a conservative central banker.

**Keywords** *:* Robust control, Monetary policy delegation, Central bank conservativeness.

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# 1 Introduction

In a seminal paper Rogoff (1985) suggested that, by appointing a conservative central banker with a more important inflationary aversion than the whole society, social welfare could be improved with respect to discretionary equilibrium. This proposition, adopted by the majority of countries during the last two decades, reduces the inflation bias arising from discretionary monetary policies (Kydland and Prescott, 1977 and Barro and Gordon, 1983). Thus the choice of optimally inflationary aversion would find the socially best balance between the output stabilisation and prices in response to supply shocks hitting the economy.

However, in this framework policymakers are assumed to know the true model of the economy and observe accurately all relevant macroeconomic variables. Uncertainty arises from the unknown future realisations of the supply shocks, assumed to be modelled according to some stochastic process whose properties is known. Unfortunately, the reality is much more complex. The policymaker's choice is made in the fare of tremendous uncertainty about the true structure of the economy, the impact policy actions have on the economy, and even about the current state of the economy. This complexity means that a certain degree of subjectivity enters into the actual decision making when deciding upon optimal monetary policy. In other words, the policymaker is unsure about his model, in the sense that there is a group of approximate models that he also considers as possibly true.

This raises the question of how a monetary policy rule should be selected in the face of uncertainty about the correct model of the economy. In fact, solutions to the expected value problem by standard optimal control methods do not deliver the best average performance if they are applied to an incorrect model. Because uncertainty is pervasive, it is important to understand how alternative policies work when the policymaker employs a model of the economy that is incorrect in unknown ways. Therefore, the resulting problem is one of robust control, in the sense of Hansen and Sargent (2004), where the objective is to choose a rule that will work under a range of different model specifications. The notion that policy decisions may be more robust if based on systematically distorted model of the economy is a key implication of the recent research on robustness control or uncertainty aversion literature (Onatski and Williams, 2003; Giordani and Söderlind, 2004; Leitemo and Söderström, 2004; Walsh, 2004).

This paper adapts robust control approach to the problem of monetary policy delegation to a conservative central banker in a simple static context in order to illustrate the basic intuition behind this new approach to uncertainty. In this context, it is particular important to give an answer to the question whether model uncertainty can affect the government's optimal commitment to fight inflation, as well as his will to delegate the conduct of monetary policy to a conservative central banker. In this framework, we find that robustness of a monetary policy rule reveals the emergence of a precautionary behaviour in the case of policymaker's unstructured model uncertainty, reducing thus willingness to delegate monetary policy to a conservative central banker.

The rest of the paper is organised as follows. Section 2 sets up a one-period model

of monetary policy. Section 3 de-rives the discretionary equilibrium under robust control. Section 4 derives the optimal degree of central banker conservativeness. Section 5 summarises the main conclusions.

# 2 The model

In this section, we apply the basic idea of robust control to a simple static model of monetary policy delegation originally developed in Rogoff (1985) in which policymaker sets inflation according to the following augmented Phillips curve:

$$x = \pi^e - \pi + \epsilon + h \tag{1}$$

where $x \equiv u - u^*$ is the gap between the unemployment rate $u$ and the natural rate of unemployment $u^* > 0$, $\pi$ is the inflation rate, $\pi^e$ is the rationally expected inflation rate, $\epsilon$ is a random variable with mean zero and variance 1, and $h$ is an additional deterministic disturbance component which introduce ambiguity of the model. The two disturbances terms and have different properties. The term $\epsilon$ is assumed to be a random error with a prior known stochastic properties, whilst $h$ represents in the spirit of robust control (Hansen and Sargent, 2004) a totally ambiguous model misspecification error, in the sense that the policymaker is not able to assign any prior probability distribution to $h$. The model with $h = 0$ represents the reference model, while the models with $h \neq 0$ represent candidate models surrounding the reference model. The size of the distortion term $h$ must be bounded as the policymaker has some information on the process. Hence, we assume that the magnitude of the square of the specification error verifies:

$$h^2 \leq \eta^2 \tag{2}$$

where the parameter $\eta^2$ bounds the square of the government's specification error $h^2$. Restriction (2), together with equation (1), defines a set of models that the government considers as being possible outcomes in the sense that the policymaker does not know exactly the position of the Phillips curve in the space $(x, \pi^e - \pi)$.

The government's preferences are described by the following standard quadratic utility function:

$$U_s = -\frac{1}{2}\left(u^2 + \pi^2\right) \tag{3}$$

where the government is assumed to stabilise unemployment $u$ and inflation $\pi$ around their target values, which are for simplicity fixed to zero. According to Rogoff (1985), monetary policy is delegated by the government to a "conservative" central banker, whose decision making problem is presented by the following quadratic utility function:

$$U_{cb} = -\frac{1}{2}\left[u^2 + (1+\phi)\pi^2\right] \tag{4}$$

where $\phi > 0$ is the degree of conservativeness chosen by the government and is the optimal extra (relative) weight the central banker sets on inflation stabilisation[1].

# 3 Discretion under robust control

According to the robust control approach, in order to hedge against the model ambiguity, the policymaker makes a particular subjective assessment of $h$ . In other words, he chooses the worst case ($h \neq 0$) at any given $\pi^e$ and then designs corresponding monetary policy rule $\pi$ which maximises the utility at given $h$. In order to introduce such subjective assessment of $h$ into the decision making problem, we replace the standard quadratic utility function (4) by an "uncertainty aversion" utility function and we seek a solution to the following problem:

$$\max_{\pi} \min_{h} U_\theta = -\frac{1}{2}\left[u^2 + (1+\phi)\pi^2\right] + \frac{\theta}{2}h^2 \tag{5}$$

The design of a robust policy rule becomes now a max-min problem, where the optimal level of inflation is found by maximizing $U_\theta$ , with $h$ being chosen to minimize $U_\theta$ subject to the linear constraint (1). $\theta$ is a fixed penalty parameter which reflects both the government and the central banker's desired degree of robustness. In other words, we assume that the government and the central banker share the same doubts about the accuracy of the model of the economy. $\theta > 1$ can be interpreted as a Lagrangian multiplier on constraint (2). The value $\theta = 1$ is the breakdown point to be discussed later. The value for $\theta$ would be endogenous in the constrained Lagrangian, and it would be associated to the specific $\eta$ value used in the constraint (2). The way the problem is written here, $\theta$ is chosen directly and the constraint is adapted accordingly. Note also that larger values of $\theta$ imply smaller sets of models so that $\theta$ is an indicator of the precautionary behaviour of the authorities. In other words, the more $\theta$ is close to one, the more the government is insuring about the accuracy of the model it uses. In the opposite case, as $\theta \to +\infty$, the government believes that its model is a good approximation of the true model of the economy. In the limit case where $\theta = +\infty$, there is no misspecification and the government is convinced that the model it uses is the true one. From the first order conditions for $\pi$ and $h$ in the problem (5), we obtain respectively the reaction functions:

$$\pi(\theta) = \frac{\theta}{\theta + (1+\phi)(\theta-1)}(u^* + \epsilon + \pi^e) \tag{6}$$

$$h(\theta) = \frac{(1+\phi)}{\theta + (1+\phi)(\theta-1)}(u^* + \epsilon + \pi^e) \tag{7}$$

where $\pi(\theta)$ gives the central banker's (robust) best reaction function for setting $\pi$ as a function of $\pi^e$, while $h(\theta)$ determines the worst case model, given $\pi^e$ and the central banker's setting $\pi(\theta)$ . Then, using equation (6) and assuming rational expectations of the private sector ($\pi^e = E\pi$) yields:

[1] Rogoff (1985) demonstrates that the optimal extra (relative) weight the central banker sets on inflation is finite and strictly positive.

$$\pi^e(\theta) = \frac{\theta u^*}{(1+\phi)(\theta-1)} \tag{8}$$

By substituting equation (8) into equation (6), the time-consistent rational expectations equilibrium inflation rate is:

$$\pi(\theta) = \frac{\theta u^*}{(1+\phi)(\theta-1)} + \frac{\theta\epsilon}{\theta+(1+\phi)(\theta-1)} \tag{9}$$

From equation (9), it is clear that as $\theta \to 1$, the model breaks down as the equilibrium inflation rate tends to infinity. Finally, substituting equations (7), (8) and (9) into equation (1), we obtain the equilibrium rate of unemployment:

$$u(\theta) = \frac{\theta u^*}{(\theta-1)} + \frac{\theta(1+\phi)\epsilon}{\theta+(1+\phi)(\theta-1)} \tag{10}$$

Using equations (9) and (10), we can see that when $\theta \to +\infty$ there is no concern for model misspecification as $h(\infty) = 0$ and thus the standard rational expectation model arises. The equilibrium inflation rate and unemployment gap are given by

$$\pi(\infty) = \frac{u^*}{(1+\phi)} + \frac{\epsilon}{1+(1+\phi)} \tag{11}$$

$$u(\infty) = u^* + \frac{(1+\phi)\epsilon}{1+(1+\phi)} \tag{12}$$

Using equations (9) to (12), we establish the following proposition:

**Proposition 1** *If the approximating model is true (i.e., $\theta = +\infty$, and therefore $h(\infty) = 0$), so that the policymaker's concern about misspecification is misplaced, the central banker sets both inflation and unemployment higher than if he knew the model for sure. In other words, when the approximating model is correct, robust policies sacrifice macroeconomic performance.*

**Proof.** This result straightforwardly follows from the respective comparison of equations (9) and (11) and equations (10) and (12). ■

However, it can be easily shown that as the specification error increases (i.e., as the parameter $\theta$ decreases) the deterioration of the macroeconomic performances are lower under the robust policy (see also Hansen and Sargent, 2004, chap. 5).

# 4 The optimal conservativeness

We now consider the government's optimal appointment of a conservative central banker. As the government chooses optimally the central banker, $\phi$ must be solved endogenously in order to maximise the government's expected utility. Our objective is to analyse how the precautionary behaviour of the government (captured by $\theta$) affects the optimal choice about the characteristics of the central banker (i.e., his

degree of conservativeness $\phi$). Thus, using equations (9) and (10) into the govern-ment utility function (3), we derive the govern-ment's expected utility as a function of $\phi$:

$$E(U_s) = -\frac{1}{2}\theta^2 \left[ \frac{1 + (1+\phi)^2 (u^*)^2}{(1+\phi)^2(\theta-1)^2} + \frac{1 + (1+\phi)^2\epsilon}{[\theta + (1+\phi)(\theta-1)]^2} \right] \tag{13}$$

Then we derive equation (13) with respect to $\phi$ and we obtain the following first-order condition :

$$\frac{\partial E(U_s)}{\partial \phi} = 0 \quad \Rightarrow \quad f(\phi;\theta) = \frac{(1+\theta\phi)(1+\phi)^3(\theta-1)^2}{[\theta + (1+\phi)(\theta-1)]^2} - (u^*)^2 = 0 \tag{14}$$

Using equation (14), we establish the following proposition.

**Proposition 2** *The more government desires a robust monetary policy rule suitable to the uncertainty about the true structure of the economy, the less he needs to appoint a conservative central banker with a high degree of inflationary aversion.*

**Proof.** From equation (14) and remembering that $\theta > 1$, we can write:

$$\frac{\partial f(\phi;\theta)}{\partial \phi} = \frac{\theta(1+\phi)^2(\theta-1)^2 \left[\theta(1+\phi) + 3(1+\theta\phi) + (1+\phi)^2(\theta-1)\right]}{[\theta + (1+\phi)(\theta-1)]^4} > 0 \tag{15}$$

and

$$\frac{\partial f(\phi;\theta)}{\partial \theta} = -\frac{(1+\phi)^3(\theta-1)\left[(2+\phi)^2(\theta-1) - 2(1+\phi)\right]}{[\theta + (1+\phi)(\theta-1)]^4} < 0 \tag{16}$$

$$\text{if} \quad \theta > 1 + \frac{2(1+\phi)}{(2+\phi)^2} \tag{17}$$

Note that inequality (16) holds only if condition (17) is verified. However, condition (17) is not restrictive since it is verified as soon as $\theta > 1.5$ . In fact, as $\phi > 0$ it is obvious that $2(1+\phi)/(2+\phi)^2 \leq 0.5$. For example if $\phi = 1$, condition (17) becomes $\theta > 1.444$ and if $\phi = 2$, condition (17)becomes $\theta > 1.375$. In other words, more the central banker is independent and conservative and more the condition (17) can be verified. Applying the implicit function rule, we obtain :

$$\frac{\partial \phi}{\partial \theta} = -\frac{\partial f(\phi;\theta)/\partial \theta}{\partial f(\phi;\theta)/\partial \phi} > 0 \tag{18}$$

■

According to the Proposition 2, the government's robust choice of $\phi$ reveals the emergence of a precautionary behaviour in the case of uncertainty about the true structure of the economy, reducing its willingness to delegate monetary policy to a "conservative" central banker. In other words, the more uncertain the government's model is, the more reluctant the government should be to appoint a conservative

central banker. The intuition behind this proposition is that, as the structure of the economy is uncertain, it becomes optimal for the government to accept a central banker characterised by a low degree of conservativeness in order to implement a more flexible monetary policy, which will work well over a larger set of possible alternative models.

# 5 Conclusion

This paper adapts the robust control approach, in the sense of Hansen and Sargent (2004), to the Rogoff's (1985) seminal analysis of monetary policy delegation to a conservative central banker, when there is misspecification uncertainties about the model of the economy. We show that the government's robust choice reveals the emergence of a precautionary behaviour in the case of uncertainty about the true structure of the economy, reducing its willingness to delegate monetary policy to a conservative central banker with a high degree of inflation aversion. The intuition behind this result is that it becomes optimal to appoint a central banker with a low degree of conservativeness in order to implement a flexible monetary policy.

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