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Abstract

This paper presents a mixture multiplicative error model with a time-varying probability between regimes. We model the implied volatility derived from call and put options on the USD/EUR exchange rate. The daily first difference of the USD/EUR exchange rate is used as a regime indicator, with large daily changes signaling a more volatile regime. Forecasts indicate that it is beneficial to jointly model the two implied volatility series: both mean squared errors and directional accuracy improve when employing a bivariate rather than a univariate model. In a two-year out-of-sample period, the direction of change in implied volatility is correctly forecast on two thirds of the trading days.

JEL Classification: C32, C53, G13

Keywords: Implied Volatility, Option Markets, Multiplicative Error Models, Forecasting.

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1 Introduction

Implied volatility (IV) forecasting in foreign exchange markets is a relevant issue for numerous market participants. Not only are accurate implied volatility forecasts valuable for option traders and market makers who price derivatives, they can benefit investors facing risk management issues in international portfolios. As argued by Pong et al. (2004), option traders may possess additional information about future events on top of what is provided by historical data, making implied volatilities potentially more accurate forecasts of future volatility than those based on history alone.

The traditional vein of foreign exchange IV literature investigates how well IV forecasts future realized volatility. Pong et al. (2004) report that historical forecasts contain substantial incremental information beyond that included in IV for short forecast horizons of exchange rate realized volatility, crediting this result to the use of high-frequency data. Jorion (1995) and Xu and Taylor (1995) provide the opposite result, with IV from foreign exchange options yielding more accurate forecasts. In another study using high-frequency returns, Martens and Zein (2004) conclude that for options on USD/JPY futures, long-memory forecasts from fractionally integrated models and implied volatility both possess information that the other does not contain. Guo (2000) links volatility forecasting to the option market by investigating returns from trading foreign exchange options, with option strategies based on conditional volatility predictions stemming from either IV or a GARCH model specification. This study concludes that trading is more profitable when IV is used to forecast future volatility.

The level and behavior of asset prices and their volatilities can be very different when comparing times of relative calm to times of distress or panic in the market. Both exchange rate returns and their volatility have been found to exhibit regime-switching properties. It is therefore easy to surmise that the implied volatility of options on foreign exchange could also possess the same property. If a time series follows two or more distinct regimes, good model fit and forecasting performance will require a specification that allows for different conditional means and error distributions for each regime.

Studies that document regime shifts in exchange rates include Engel and Hamilton (1990), Bekaert and Hodrick (1993), and Engel and Hakkio (1996). Engel and Hamilton (1990) observe that exchange rate returns vary more when the U.S. dollar is appreciating, and that a regime-switching model outperforms a random-walk model as a forecaster. Bekaert and Hodrick (1993) estimate a two-regime model and find that the variances of forward premiums are nine to ten times larger in the more volatile regime. In Engel and Hakkio (1996), the difference between variances of exchange rate returns in the stable and volatile states is even larger.

Bollen et al. (2000) estimate a Markov-switching model with two regimes for log exchange rate changes and their variances, but with mean and variance regimes allowed to switch independently. This modification allows for high volatility in times of both appreciation and depreciation of an exchange rate. The model produces more accurate variance forecasts than competing models, and estimation results indicate that the standard deviations of exchange rate returns are two to three times higher in the more volatile regime. Klaassen (2002) shows that a regime-switching GARCH model outperforms a single-regime GARCH model in terms of mean squared forecast errors.

Both regime-switching and mixture models introduce separate regimes for times of business as usual and times of higher volatility. In essence, a model with various regimes is an alternative to the GARCH class of models: volatility clustering is now modeled through changes in the variance of the underlying data generating process, with switches in regime corresponding to these changes (see Bollen et al. (2000)).[1] Mixture multiplicative error models with two regimes have been found to fit time series of the realized volatility of exchange rates (Lanne (2006)) and the implied volatility of Nikkei 225 index options (Ahoniemi (2007) and Ahoniemi and Lanne (2007)).

In this study, we present an extension of a mixture multiplicative error model that allows for a time-varying probability between regimes, with the mixing probabilities depending on an observed variable that functions as a regime indicator. Our data set consists of the implied volatilities calculated from call and put options on the USD/EUR exchange rate, and USD/EUR exchange rate returns serve as our indicator variable. Although implied volatilities calculated from call and put options with the same strike prices and maturity dates should be equal, in practice we observe that that is rarely the case. Differences in call and put IVs can arise due to differing demand pressures and limits to arbitrage, as explained in e.g. Garleanu et al. (2006), Bollen and Whaley (2004), and Figlewski (1989).

The aim of this study is to find a good model fit with time-varying regime probabilities and to generate forecasts of the IV of USD/EUR currency options that succeed both in predicting the direction of change of implied volatility and its value. We model call and put IV separately with two-regime and three-regime model specifications, but also estimate a bivariate specification that jointly models both call and put implied volatilities. In this latter model, we include cross terms that allow call-side IV to affect put-side IV, and vice versa. The bivariate specification proves to be valuable, as the directional accuracy and mean squared errors of forecasts both improve with the bivariate model. The correct direction of change in IV is forecast on two thirds of the 514 trading days in the two-year out-of-sample period. Two regimes are sufficient in the bivariate model, but it is not evident whether two or three regimes should be preferred in the univariate setting.

This paper proceeds as follows. Section 2 describes the univariate and bivariate multiplicative error models with time-varying regime probabilities. Section 3 presents the data, the model estimation results, and diagnostics. Section 4 discusses forecast results and their evaluation, with performance measured with directional accuracy and mean squared errors. Section 5 concludes the paper.

2 The Model

In this section, we describe univariate and bivariate mixture multiplicative error models, and present an extension with multiple mixture components and time-varying mixing probabilities. Henceforth, this new model will be called the TVMEM model.

Multiplicative error models are suitable for modeling any time series that always receives non-negative values. This particular study focuses on volatility, but the same type of model could be used in, for example, duration or trading volume applications.

[1] The literature is rife with examples of GARCH modeling of exchange rate series, Baillie and Bollerslev (1991) and Hansen and Lunde (2005) among them. Rapach and Strauss (2008) find strong evidence of structural breaks in GARCH models for seven exchange rate series of the U.S. dollar against other currencies, with substantial differences in unconditional variance between regimes. In their out-of-sample forecast exercise, models with structural breaks provide lower mean squared errors than competing models. Klaassen (2002) and Haas et al. (2004) present Markov-switching GARCH models with promising results for exchange rate series.

Engle (2002) first suggested applying MEM models to volatility modeling, pointing out that with a MEM model, there is no need to take logarithms of the data. Since then, promising results with MEM models in volatility applications have been obtained by e.g. Ahoniemi (2007), Ahoniemi and Lanne (2007), Engle and Gallo (2006), and Lanne (2006). In a standard multiplicative error model volatility v_t is specified as

$$v_t = \mu_t \varepsilon_t, \qquad t = 1, 2, ..., T \tag{1}$$

where ε_t is a stochastic non-negative error term, and the conditional mean μ_t is specified as

$$\mu_t = \omega + \sum_{i=1}^{q} \alpha_i v_{t-i} + \sum_{j=1}^{p} \beta_j \mu_{t-j}. \tag{2}$$

As the mean equation is structured in the same way as in GARCH models, the parameter values must comply with the restrictions for GARCH models outlined in Nelson and Cao (1992) in order to ensure positivity. In the simplest setting, or with a first-order model, all parameter values must be non-negative. With higher-order models, non-negativity is not always required.[2]

We extend the basic MEM model by considering a mixture-MEM model. In a mixture model, the data is allowed to follow i regimes with $i \geq 2$. This type of model specification lends itself particularly well to financial time series data, as financial markets typically alternate between periods of high and low volatility. With possibly different mean equations and error distributions for each mixture component, the variations in the market in calm and more volatile time periods can be more thoroughly captured. When estimating a mixture-MEM model, one must also estimate the probability of each regime (π_i) along with the other parameters.

Lanne (2006), Ahoniemi (2007), and Ahoniemi and Lanne (2007) all estimate a mixture-MEM model with a fixed mixing probability. In other words, one value of π_i is estimated in their models, and the probability of each regime is thus the same every day. We extend the basic mixture-MEM model by allowing the probability of the regimes, π_i, to vary over time. In practice, the values of the parameters that determine π_i are estimated from the data, and a value for π_i is generated for every time period.

The regime probability varies in time according to a regime indicator. We reason that when modeling implied volatility, the returns of the options' underlying asset would work well as an indicator: a large change in the value of the underlying can be taken as a signal of higher volatility in the market. We later confirm the functioning of such an indicator graphically by plotting the IV data and the regime probabilities together (see Section III 3.2). It is also important to note that absolute values of the returns are used when estimating a model with two regimes: this allows both large positive and negative returns to be properly accounted for. When estimating a model with three regimes, the first differences are used as such, with one threshold separating large negative shocks, and a second threshold separating large positive shocks. In a two-regime model, we expect the largest probability mass to fall into the first regime of calmer days, whereas with three regimes, the middle regime should capture the largest number of trading days.

[2]For example, in a model with $p = 1$ and $q = 2$, the constraints are $\omega \geq 0$, $\alpha_1 \geq 0$, $0 \leq \beta < 1$, and $\beta\alpha_1 + \alpha_2 \geq 0$.

When using an indicator variable to determine the thresholds between regimes, strict thresholds that allow for no gray area around the value of the threshold are often estimated. However, we introduce an additional error term, η_t, into the indicator function

$$I(c_{i-1} + \eta_t \leq y_{t-d} < c_i + \eta_t) \tag{3}$$

with $\eta_t \sim NID(0, \sigma_\eta^2)$ and independent of ε_t. This idea was previously applied to autoregressive models by Lanne and Saikkonen (2003). The indicator function divides the observations onto both sides of the thresholds, but with the addition of the unobservable white noise term η_t, variation around the threshold value is introduced. In other words, it is not clear which regime the model is in when the indicator variable receives a value that is close to the threshold. Also, a switch in regime does not always occur when the value of the indicator variable crosses the threshold. This element brings additional flexibility into the model, which is beneficial as models with regime switches based on a strict threshold may not always be realistic (Lanne and Saikkonen (2003)). The volatility of η_t, σ_η, is later estimated as one of the parameters of our model and used as follows when determining the daily regime probabilities $\pi_{i,t-d}$

$$\pi_{i,t-d} = \begin{cases} 1 - \Phi((y_{t-d} - c_1)/\sigma_\eta), & i = 1 \\ \Phi((y_{t-d} - c_{i-1})/\sigma_\eta) - \Phi((y_{t-d} - c_i)/\sigma_\eta), & i = 2, \ldots, m-1 \\ \Phi((y_{t-d} - c_{m-1})/\sigma_\eta), & i = m \end{cases} \tag{4}$$

where $\Phi(\cdot)$ denotes the cumulative distribution function (cdf) of the standard normal distribution, m is the number of mixture components (regimes), σ_η denotes the standard deviation of η_t and satisfies $\sigma_\eta > 0$, and $c_1, \ldots, c_{m-1}$ denote the values of the threshold parameters and satisfy $c_1 < \ldots c_{m-1}$. The larger the volatility parameter σ_η, the larger is the range around the estimated threshold where a regime switch is probable. One common value of σ_η is estimated for all thresholds. In practice, the time index $t - d$ is always $t - 1$ in our application, as further lags would not be relevant in a financial application.

We assume that the error term of our TVMEM model follows the gamma distribution. The gamma distribution has the benefit that it is very flexible and nests e.g. the exponential and χ^2 distributions. Also, earlier work by Lanne (2006), Ahoniemi (2007) and Ahoniemi and Lanne (2007) shows that the gamma distribution is suitable for volatility modeling purposes. Our requirement that the error terms have mean unity means we must impose the restriction that the shape and scale parameters of the gamma distribution are reciprocals of each other. Under this restriction, the conditional density of v_t with two mixture components (regimes) is

$$\begin{aligned} f_{t-1}(v_t;\theta) &= \pi_1 \frac{1}{\mu_{1t}\Gamma(\lambda_1)\delta_1^{\lambda_1}} \left(\frac{v_t}{\mu_{1t}}\right)^{\lambda_1 - 1} \exp\left(-\frac{v_t}{\delta_1 \mu_{1t}}\right) + \\ &\quad (1-\pi_1)\frac{1}{\mu_{2t}\Gamma(\lambda_2)\delta_2^{\lambda_2}} \left(\frac{v_t}{\mu_{2t}}\right)^{\lambda_2 - 1} \exp\left(-\frac{v_t}{\delta_2 \mu_{2t}}\right) \end{aligned} \tag{5}$$

where θ is the vector of parameters, $\Gamma(\cdot)$ is the gamma function, λ_1 and λ_2 are the shape parameters of the gamma distribution, and δ_1 and δ_2 are the scale parameters.

As a further extension of the univariate TVMEM model, we also consider a bivariate model that allows us to jointly model the implied volatilities garnered from call and put options. As the evidence in Ahoniemi and Lanne (2007) indicates, there can be clear added value to modeling call and put IVs jointly. In the bivariate case, the mean equations include cross terms that allow call-side IV to affect put-side IV, and vice versa. The mean equations in our bivariate TVMEM model are

$$\mu_{mt}^{C} = \omega_m^{C} + \sum_{i=1}^{q_C} \alpha_{mi}^{C} v_{C,t-i} + \sum_{i=1}^{r_C} \psi_{mi}^{C} v_{P,t-i} + \sum_{j=1}^{p_C} \beta_{mj}^{C} \mu_{m,t-j}^{C}$$

and

$$\mu_{mt}^{P} = \omega_m^{P} + \sum_{i=1}^{q_P} \alpha_{mi}^{P} v_{P,t-i} + \sum_{i=1}^{r_P} \psi_{mi}^{P} v_{C,t-i} + \sum_{j=1}^{p_P} \beta_{mj}^{P} \mu_{m,t-j}^{P}$$

where μ_{mt}^{C} and μ_{mt}^{P} are the conditional means of call-side and put-side implied volatility, respectively, and the ψ's are the coefficients for lagged cross terms.

There are a number of bivariate gamma distributions with gamma marginals (see Yue et al. (2001) for a review). From the available bivariate distributions, we select the specification suggested by Nagao and Kadoya (1970), as it is a tractable alternative. The density function in this case can be written as

$$f_{\varepsilon_1,\varepsilon_2}\left(\varepsilon_{1t},\varepsilon_{2t};\boldsymbol{\theta}\right) = \frac{\left(\tau_1\tau_2\right)^{(\lambda+1)/2}\left(\varepsilon_{1t}\varepsilon_{2t}\right)^{(\lambda-1)/2}\exp\left\{-\frac{\tau_1\varepsilon_{1t}+\tau_2\varepsilon_{2t}}{1-\rho}\right\}}{\Gamma\left(\lambda\right)\left(1-\rho\right)\rho^{(\lambda-1)/2}} I_{\lambda-1}\left(\frac{2\sqrt{\tau_1\tau_2\rho\varepsilon_{1t}\varepsilon_{2t}}}{1-\rho}\right), \tag{6}$$

where ρ is the Pearson product-moment correlation coefficient, and $I_{\lambda-1}(\cdot)$ is the modified Bessel function of the first kind. The ρ's reflect the correlation of shocks between the two time series and are constrained to be between zero and unity. The fact that ρ cannot be negative should not be restrictive, as the underlying asset of the options in our application is the same. In the bivariate application, both error distributions have a distinct scale parameter. However, the shape parameter is the same for both time series, which makes the scale parameters also equal: with the parametrization of Equation 6, the scale and shape parameters must be equal (rather than reciprocals) to ensure mean unity. Given the earlier evidence in Ahoniemi (2007) and the results for univariate models in this paper, this requirement is not restrictive: while the shape (and scale) parameters tend to differ clearly between regimes, they are very close in value between the call and put IV time series.

For the conditional density function of $\mathbf{v}_t = (v_{1t}, v_{2t})'$, we use the change of variable theorem and make the substitution $\tau_1 = \tau_2 = \lambda$ to obtain

$$\begin{aligned} f_{t-1}\left(v_{1t},v_{2t};\boldsymbol{\theta}\right) &= f_{\varepsilon_1,\varepsilon_2}\left(v_{1t}\mu_{1t}^{-1}, v_{2t}\mu_{2t}^{-1}\right)\mu_{1t}^{-1}\mu_{2t}^{-1} \\ &= \frac{\lambda^{(\lambda+1)}\left[v_{1t}v_{2t}\mu_{1t}^{-1}\mu_{2t}^{-1}\right]^{(\lambda-1)/2}\exp\left\{-\frac{\lambda\left(v_{1t}\mu_{1t}^{-1}+v_{2t}\mu_{2t}^{-1}\right)}{1-\rho}\right\}}{\Gamma\left(\lambda\right)\left(1-\rho\right)\rho^{(\lambda-1)/2}} \times \\ &\quad I_{\lambda-1}\left(\frac{2\lambda\sqrt{\rho v_{1t}v_{2t}\mu_{1t}^{-1}\mu_{2t}^{-1}}}{1-\rho}\right)\mu_{1t}^{-1}\mu_{2t}^{-1}. \end{aligned} \tag{7}$$

The conditional log-likelihood function for the two-regime case can thus be written as

$$l_T(\boldsymbol{\theta}) = \sum_{t=1}^{T} l_{t-1}(\boldsymbol{\theta}) = \sum_{t=1}^{T} \ln\left[\pi_{i,t-d} f_{t-1}^{(1)}(v_{1t}, v_{2t}; \boldsymbol{\theta}_1) + (1-\pi_{i,t-d}) f_{t-1}^{(2)}(v_{1t}, v_{2t}; \boldsymbol{\theta}_2)\right],$$

where $f_{t-1}^{(1)}(v_{1t}, v_{2t}; \boldsymbol{\theta}_1)$ and $f_{t-1}^{(2)}(v_{1t}, v_{2t}; \boldsymbol{\theta}_2)$ are given by (7) with $\boldsymbol{\theta}$ replaced by $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$, respectively. All the TVMEM models described in this section can be estimated with the method of maximum likelihood (ML). The asymptotic properties of the ML estimators are not known; however, assuming stationarity and ergodicity, it is reasonable to apply standard asymptotic results in statistical inference.

3 Estimation Results

3.1 Data

The data set used in this study includes two time series of at-the-money (ATM), 30-day maturity implied volatility data: one calculated from call options on the USD/EUR exchange rate, and the other from put options (see Figure 1). Each observation is interpolated from the implied volatilities of four different options: two nearest-to-the-money strikes are used, one above and one below the current spot rate, and maturities are selected from both sides of the 30-day interval. As noted by Ederington and Guan (2005), at-the-money options are typically the most liquid, have the smallest bid-ask spreads, and are most commonly used in empirical research. The use of ATM options helps in avoiding the effects of the volatility smile. Also, at-the-money options have the highest sensitivity to volatility (Bollen & Whaley (2004)), and IV is more likely to equal the mean expected volatility over the remaining life of the option when ATM options are used (Day and Lewis (1992)). A 30-day maturity is commonly used in established IV indices such as the VIX index.

The implied volatilities of the options are calculated using the Black-Scholes extension for currency options, namely the Garman-Kohlhagen model.[3] The data is obtained from Datastream, and includes daily observations for the time period 1.8.2000-31.12.2007. Days when public holidays fall on weekdays are omitted from the data set. The full sample covers 1,904 trading days, 1,390 of which are treated as the in-sample. The last two years, or 514 days, are left as an out-of-sample period and used for forecast assessment. Our data set also includes the WM/Reuters daily closing spot rates for the EUR/USD exchange rate, which are obtained from Datastream. The reciprocals of these rates are used, as the underlying asset of the options is the USD/EUR rate. The raw IV data is multiplied by 100 before performing any estimations, and log exchange rate returns are also multiplied by 100.

The correlation of the call (USD/EUR C) and put (USD/EUR P) implied volatility time series is 84.4%. Basic descriptive statistics for the series are provided in Table 1, and autocorrelation functions in Figure 2. Table 1 shows that the values for the two implied volatilities clearly differ: although there is no great difference in the means, the maximum values of the two time series are far apart: 22.5 for USD/EUR C and 18.5 for USD/EUR P. The maximum value of USD/EUR C is higher, but its minimum value is also lower. Both time series are slightly skewed to the right.

[3] Campa et al. (1998) report that in the over-the-counter currency option market, the convention is to quote prices as Garman-Kohlhagen implied volatilities.

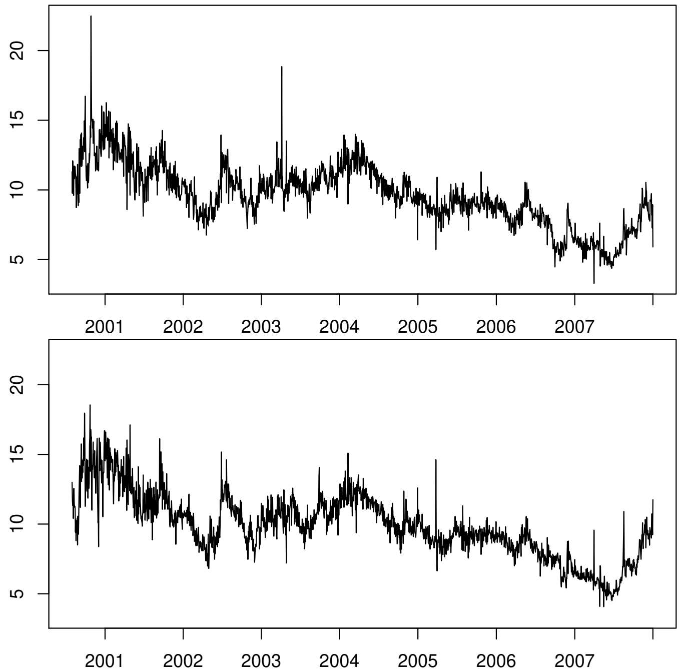

Figure 1: USD/EUR exchange rate call option implied volatility (upper panel) and put option implied volatility (lower panel) 1.8.2000 - 31.12.2007.

	USD/EUR C	USD/EUR P	USD/EUR return
Maximum	22.481	18.549	3.321
Minimum	3.296	4.075	-2.266
Mean	9.604	9.872	0.024
Median	9.583	9.790	0.018
Standard deviation	2.225	2.308	0.578
Skewness	0.144	0.225	0.113
Kurtosis	3.536	3.169	4.052

Table 1: Descriptive statistics for USD/EUR C, USD/EUR P, and the log first differences of the USD/EUR exchange rate for the full sample of 1.8.2000 - 31.12.2007. Raw IV data and log exchange rate returns are all multiplied by 100.

Table 1 also provides descriptive information on the USD/EUR exchange rate return series. The largest positive daily return in our full sample is 3.3%, and the largest negative return is -2.3%. The return distribution is more kurtotic than the IV distributions. The autocorrelation functions of USD/EUR C and USD/EUR P reveal that there is high persistence in the data, although augmented Dickey-Fuller tests indicate that both series are stationary (a unit root is rejected at the one-percent level for USD/EUR C and the five-percent level for USD/EUR P).

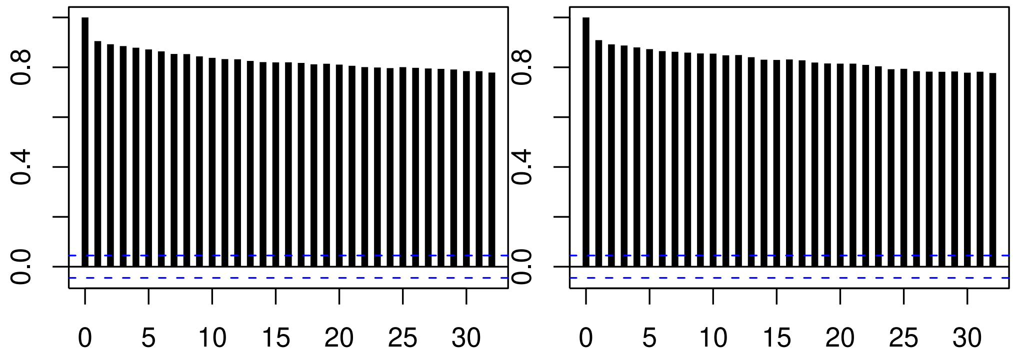

Figure 2: Autocorrelation function of USD/EUR exchange rate call option implied volatility (left panel) and put option implied volatility (right panel).

3.2 Model Estimation

We estimate three competing model specifications: two-regime and three-regime univariate models as well as a two-regime bivariate model.[4] In the two-regime models, the regime indicator variable is the absolute value of the lagged log return of the USD/EUR exchange rate, and in the three-regime models, we use the lagged log first difference as such. Thus, in the two-regime case we hypothesize that both large positive and large negative shocks have the same dynamics, whereas in the three-regime case, both tails of the return distribution are allowed to provide separate mean equations and error distributions. Kim and Kim (2003) corroborate our view that exchange rate returns could function well as a regime indicator by providing evidence that the implied volatility of currency options on futures is higher when currency futures returns are large. They also note that both positive and negative movements in the currency futures price contribute to increases in IV. However, there is evidence that IV reacts differently to positive and negative return shocks, at least in equity markets. In a study on S&P 500 index options, Bakshi and Kapadia (2003) look at implied volatility changes after large positive and large negative index returns. After a large negative return shock, the relative change in IV was 10.6% on average, and after a large positive return shock, the average change was -1.5%.

Table 2 shows the parameter values for the two-regime univariate models. The estimate of the threshold parameter c_1 is very similar for both call and put implied

[4]Estimation of a four-regime univariate model shows that three regimes is sufficient: the first threshold reaches its lower limit of 2.26, which is the smallest exchange rate return in our sample. Thus, no observations would be left for a fourth regime. Similarly, the results for a three-regime bivariate model indicated that a two-regime model is sufficient in the bivariate case: none of the coefficients of the two high-volatility regimes were statistically significant.

volatilities: 1.83 for the call IV model and 1.81 for the put IV model. The volatility parameter σ_η equals 0.6 for both models. In other words, the time-varying regime probability, which is modeled by using exchange rate returns as a regime indicator, behaves in a very similar fashion for both call-side and put-side IV. The range within which a regime switch is likely to occur is centered on 1.8% returns of the exchange rate. In a hypothetical model with $\sigma_\eta = 0$, when the absolute return is greater than 1.8%, the data is assumed to be drawn from the second regime. However, given our estimated range around the threshold, the regime switch does not automatically occur when the absolute exchange rate return exceeds the estimated threshold value. The standard errors of σ_η are very small, which lends support to this specification over the alternative where $\sigma_\eta = 0$.

The error distributions of the two regimes are also nearly identical for two-regime univariate call and put IV models, as the estimated shape parameters are very close in value for both time series. Figure 3 depicts the error distributions of the two-regime call IV model (the figure is qualitatively the same for the put IV model). The residuals of the models are more concentrated around their mean in the first, more common regime, and more wide-tailed in the more volatile regime.

The constant terms are, somewhat surprisingly, higher in the more common regime for both the call and put models. In fact, there are no constants in the second regime as their values are estimated to be zero (in practice, 1e-5, the lower limit). As there are no radical differences in the α or β coefficients either, the largest difference between the two regimes arises from the difference in error distributions.

	USD/EUR C		USD/EUR P	
Log likelihood	-1669.52		-1713.31	
c_1	1.826**	(0.178)	1.806**	(0.190)
σ_η	0.601**	(0.093)	0.600**	(0.093)
λ_1	145.72**	(0.003)	145.67**	(0.003)
ω_1	0.142**	(0.054)	0.304**	(0.056)
α_{11}	0.333**	(0.032)	0.238**	(0.019)
α_{12}	-0.080	(0.045)	-	
β_{11}	0.733**	(0.034)	0.727**	(0.022)
λ_2	24.46**	(7.870)	25.90**	(8.440)
ω_2	-		-	
α_{21}	0.393	(0.269)	0.288	(0.228)
β_{21}	0.625*	(0.257)	0.715**	(0.225)

Table 2: Estimation results for two-regime univariate TVMEM models. Standard errors calculated from the final Hessian matrix are given in parentheses. ** indicates statistical significance at the one-percent level, and * at the five percent level. Note that for σ_η, zero is on the boundary of the parameter space so that statistical significance cannot be interpreted in the standard way. Constants are excluded from the second regime as their estimated value was at the lower limit (1e-5). Second lags of IV values are excluded on the basis of likelihood ratio tests with the exception of the more common regime for USD/EUR C.

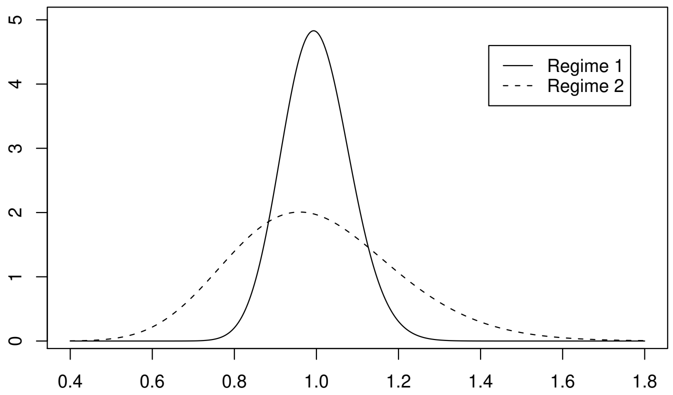

Figure 3: Estimated density of error terms of two-regime univariate model for USD/EUR exchange rate call option implied volatility.

We next estimate a univariate model specification with three regimes. We wish to check whether the use of more than two regimes is necessary, and in the absence of a formal test for the optimal number of regimes, explore the issue by estimating both alternative specifications. Parameter values for the three-regime models are presented in Table 3. The estimated threshold values are now distinctly different for USD/EUR C and USD/EUR P, with the regime of large negative shocks receiving clearly more weight for call IV (greater value of c_1 for call side). On the other hand, the regime of large positive shocks receives more weight with put IV (smaller value of c_2 for put side). These large weights coincide with gamma distribution shape parameters that are similar in size to those of the most common regimes. In other words, a more dispersed error distribution occurs only when the threshold value leaves a relatively small amount of days into a regime (in practice, the positive-shock regime of USD/EUR C and the negative-shock regime of USD/EUR P).

The estimate of the volatility parameter σ_η is larger for USD/EUR C in the three-regime model, with the same value of σ_η applying to both thresholds. As with the two-regime models, constants are highest in the most common regime, and omitted due to the lower limit being reached in three of the four remaining cases. The highest persistence in the models emerges in the negative-shock regime for USD/EUR C and the positive-shock regime for USD/EUR P, with β coefficient values of 0.89.[5] In general, the parameter values of all three regimes differ clearly from one another for both the call and put IV models, so that the use of three regimes rather than two seems justified. However, we will calculate forecasts from the two-regime models as well, as we are interested in discovering the best model specification for forecasting purposes.

The coefficients for the two-regime bivariate model specification are shown in Table 4. The threshold value c_1 is similar to that of univariate two-regime models, but the volatility parameter σ_η is somewhat smaller. Due to the distributional properties of the bivariate gamma distribution, the upper bound of the shape parameter is lower than in the univariate case, making the error distribution slightly less concentrated around unity than in the univariate case (see Figure 4 for the densities of the residuals with the

[5]The sum of the α and β coefficients is slightly greater than unity in nearly all cases for the more volatile regimes, indicating explosive dynamics. However, a visual inspection of very long simulated data series shows that the data process with our models is not explosive. Alos, the means and medians of the simulated data series are close in value to those of the original data set.

	USD/EUR C		USD/EUR P	
Log likelihood	-1676.77		-1705.64	
c_1	-0.932	(0.676)	-1.714**	(0.103)
c_2	1.997**	(0.230)	1.201**	(0.278)
σ_η	0.989**	(0.144)	0.641**	(0.072)
λ_1	146.47**	(0.070)	20.28**	(6.660)
ω_1	0.009	(0.058)	-	
α_{11}	0.109	(0.058)	0.412	(0.305)
β_{11}	0.889**	(0.059)	0.591	(0.303)
λ_2	145.67**	(0.450)	147.30**	(0.003)
ω_2	0.251*	(0.121)	0.386**	(0.072)
α_{21}	0.370**	(0.062)	0.264**	(0.022)
β_{21}	0.606**	(0.069)	0.691**	(0.027)
λ_3	22.81**	(7.610)	145.47**	(1.450)
ω_3	-		-	
α_{31}	0.462	(0.288)	0.109**	(0.034)
β_{31}	0.549	(0.282)	0.892**	(0.035)

Table 3: Estimation results for three-regime univariate TVMEM models. Standard errors calculated from the final Hessian matrix are given in parentheses. ** indicates statistical significance at the one-percent level, and * at the five percent level. Note that for σ_η, zero is on the boundary of the parameter space so that statistical significance cannot be interpreted in the standard way.

bivariate model). The correlation of shocks, ρ, is at its lower bound for both regimes, which yields the somewhat surprising interpretation that the shocks of USD/EUR C and USD/EUR P are quite uncorrelated with each other.[6] The constant term is higher in the higher-volatility regime for USD/EUR P, which is in line with the result we originally expected to see in all cases. The more volatile regime is more persistent, with β coefficients clearly higher in the second regime for both call and put IV. Cross terms are significant only in the more common regime.

Based on the coefficients, call-side IV affects put-side IV slightly more than the other way around. This result is in contrast to results achieved earlier for equity index option implied volatility (see e.g. Ahoniemi and Lanne (2007), Bollen and Whaley (2004)), as in those markets, trading in put options tends to dominate developments in the overall market. With currency options, the demand of hedgers is not as clearly concentrated on put options, so the demand and supply balances in the markets of exchange rate call and put options may be more even than for equity index call and put option markets.[7]

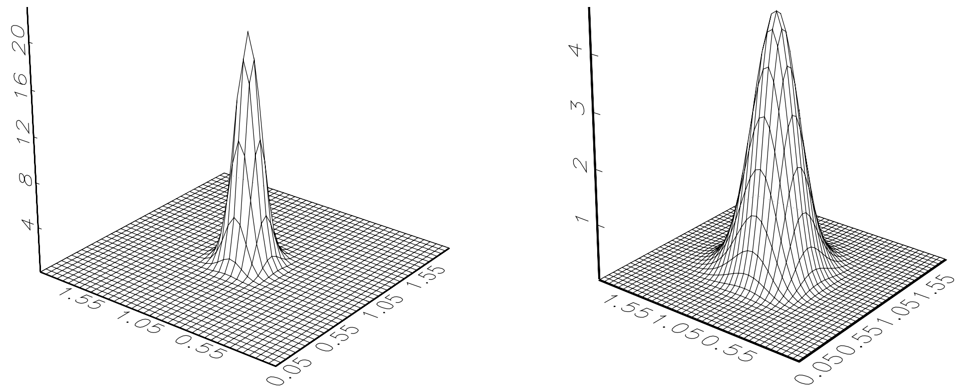

Figure 4: Estimated density of error terms of two-regime bivariate model: first regime in left panel and second regime in right panel.

Figures 5 and 6 plot the estimated regime probabilities and IV data in the same graphs. This setting allows us to see how the probabilities vary over time, but also to verify that the regime indicator is functioning properly: the regime probability, though calculated solely from exchange rate returns, will preferably have a stronger reaction on days when there is a large change in the value of IV. The regime probability should also react when the level of IV is relatively high. As can be seen from Figures 5 and 6, this relation holds quite well, so we are satisfied with the performance of the exchange rate returns as our regime indicator.[8]

[6]This may be a consequence of the fact that the underlying asset of the options in this study is an exchange rate, which naturally involves two currencies. Investors with positions in either the USD or the EUR may have differing exposures in the option market, with demand for calls and demand for puts originating from different sets of investors. Depending on which currency a shock affects more, the shock can thus affect call and put options (and their implied volatilities) differently.

[7]The significance of Friday and Monday dummy variables for lagged IV observations was also investigated. Earlier evidence indicates that there is a weekly pattern in IV (see e.g. Ahoniemi and Lanne (2007), Kim and Kim (2003), Harvey and Whaley (1992)). In particular, Kim and Kim (2003) use exchange rate options on futures data and discover that IV is low on Mondays and tends to rise by Wednesday, remaining high for the rest of the week. No weekday effects were found for the data sample used in this study, however.

[8]As regards the necessity of time-varying regime probabilities, it must be noted that with this partic-

	Bivariate, 2 regimes	
Log likelihood	-3323.84	
c_1	1.551**	(0.080)
σ_η	0.412**	(0.043)
λ_1	127.29	(-)
ρ_1	0.01	(-)
ω_1^C	0.312**	(0.114)
α_{11}^C	0.325**	(0.034)
ψ_{11}^C	0.238**	(0.029)
β_{11}^C	0.401**	(0.056)
ω_1^P	0.036	(0.085)
α_{11}^P	0.369**	(0.038)
ψ_{11}^P	0.270**	(0.035)
β_{11}^P	0.364**	(0.062)
λ_2	28.72**	(6.65)
ρ_2	0.01	(-)
ω_2^C	-	
α_{21}^C	0.138	(0.101)
ψ_{21}^C	-	
β_{21}^C	0.868**	(0.097)
ω_2^P	0.833	(0.939)
α_{21}^P	0.248	(0.208)
ψ_{21}^P	-	
β_{21}^P	0.696**	(0.224)

Table 4: Estimation results for the bivariate TVMEM model. Standard errors calculated from the final Hessian matrix are given in parentheses. Standard errors of (-) indicate that a parameter value is at a predefined limit. λ_1 reaches its upper limit, beyond which the likelihood function is no longer numerically tractable. Both ρ_1 and ρ_2 reach their lower limit of 0.01, which was set at slightly above the theoretical lower limit of zero. ** indicates statistical significance at the one-percent level. Note that for σ_η, zero is on the boundary of the parameter space so that statistical significance cannot be interpreted in the standard way.

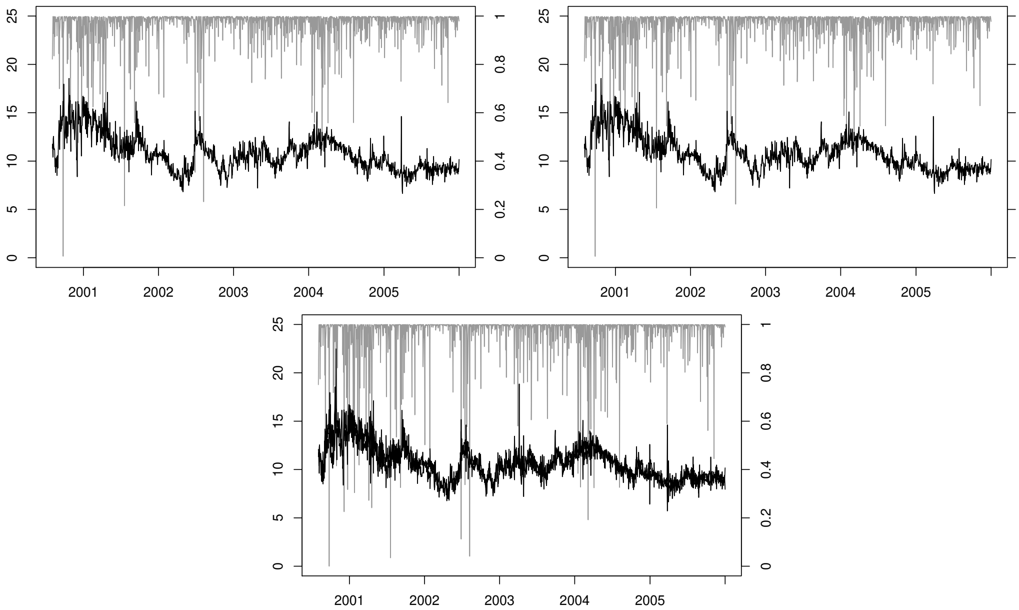

Figure 5: Implied volatility on left axis and probability of first regime on right axis. Two-regime model for USD/EUR C on left in upper row, two-regime model for USD/EUR P on right in upper row, and two-regime bivariate model in lower row.

Figure 5 depicts the probabilities of the first, more common regime together with the original IV data for all three two-regime models (univariate and bivariate). The average probability of the more common regime is 96.4% for USD/EUR C, 96.2% for USD/EUR P, and 96.0% for the bivariate model. Looking at the average probability alone, all models would seem to leave a relatively small share of days for the more volatile regime. However, the graphs show that on days of large moves in the market, the probability of the more common regime falls significantly, even to close to zero.

The probabilities obtained from the three-regime models are detailed in Figure 6. A more pronounced difference between the call and put IV time series now emerges, as was already evident from the threshold values presented in Table 3. With call-side IV, the regime of negative shocks receives an average probability of 20.8%, the regime of positive shocks has an average probability of 4.5%, with 74.7% left for the most common regime. For put-side IV, the corresponding averages are 2.7%, 9.1%, and 88.2%.

3.3 Diagnostics

Due to the fact that our models have two or three mixture components and thus do not produce conventional (Pearson) residuals, we cannot perform standard diagnostic checks for the models' goodness-of-fit. As residuals cannot be obtained in any straightforward way, we use probability integral transforms of the data to evaluate the models' in-sample fit through autocorrelation diagnostics. This approach was suggested by Diebold et al. (1998) and extended to the multivariate case by Diebold et al. (1999). For a univariate model, the probability integral transforms are calculated as

ular data set, a fixed regime probability that would not vary over time was not sufficient. For example with the bivariate specification, a fixed-probability model was not able to distinguish separate regimes from the data.

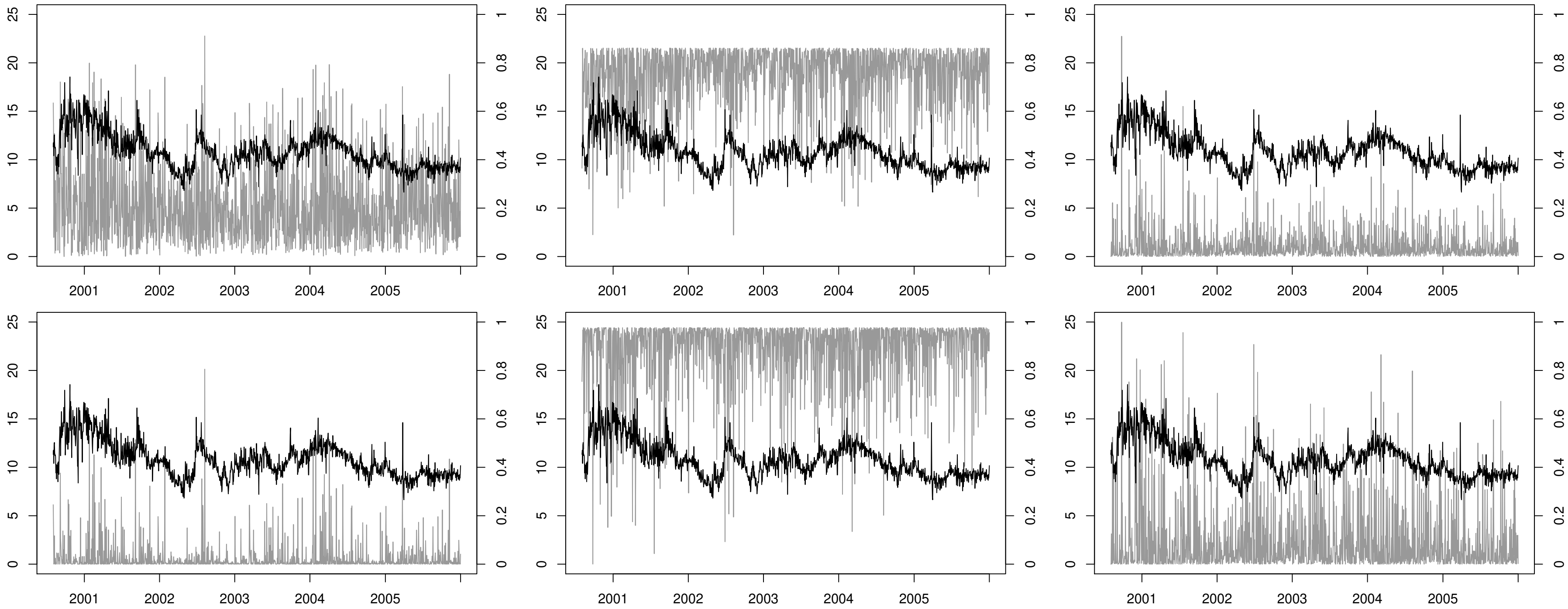

Figure 6: Three-regime model for USD/EUR C (top row) and USD/EUR P (bottom row): implied volatility on left axis and probability on right axis. Probability of first regime in left panel, probability of second regime in middle panel, and probability of third regime in right panel.

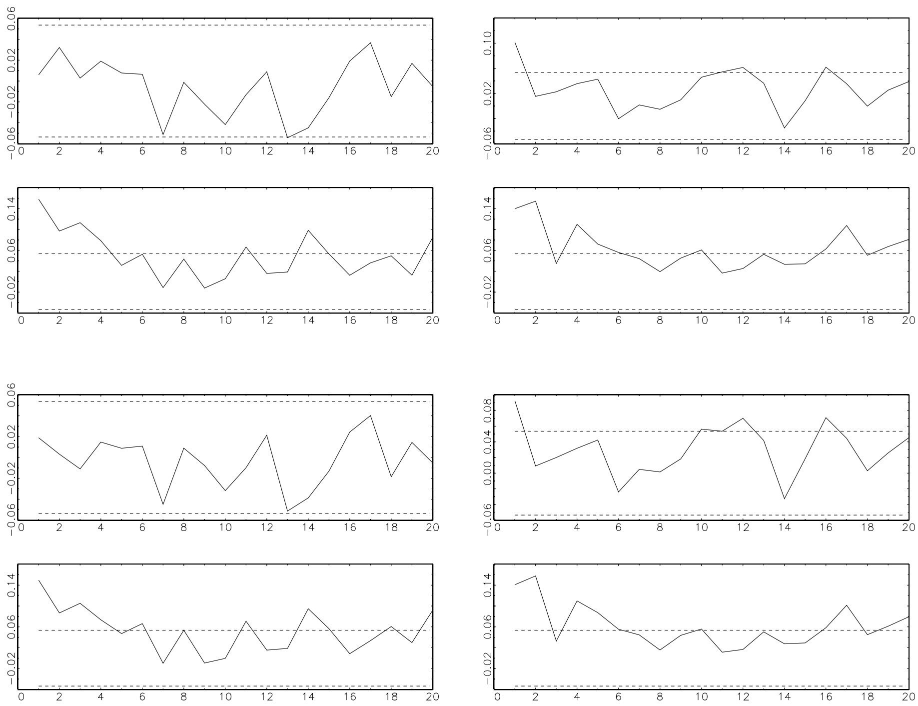

Figure 7: Diagnostics for two-regime model of USD/EUR C (top left), two-regime model of USD/EUR P (top right), three-regime model of USD/EUR C (bottom left), and three-regime model of USD/EUR P (bottom right). Autocorrelation functions of demeaned probability integral transforms in the upper panels and of their squares in the lower panel. The dotted lines depict the boundaries of the 95% confidence interval.

$$z_t = \int_0^{v_t} f_{t-1}(u)du \tag{8}$$

where $f_{t-1}(\cdot)$ is the conditional density of the data with the chosen model specification. This procedure transforms each data point into a value between zero and unity. Following Diebold et al. (1999), we calculate four series of probability integral transforms for the bivariate case: z_t^C and z_t^P (based on the marginal densities of call and put IV), and $z_t^{C|P}$ and $z_t^{P|C}$ (call IV conditional on put IV, and vice versa).

Graphs of the autocorrelation functions of demeaned probability integral transforms and their squares are presented in Figure 7 for the two-regime and three-regime univariate models and in Figure 8 for the bivariate model. The autocorrelations of the demeaned probability integral transforms in Figure 7 are extremely satisfactory for USD/EUR C, but leave some to be desired for USD/EUR P.[9] In line with previous research, there clearly seems to be some remaining autocorrelation in the squares of the demeaned probability integral transforms (this same finding has been made for volatility data in Ahoniemi and Lanne (2007) and Lanne (2006, 2007)). This autocorrelation in squares is a signal of conditional heteroskedasticity in the data, so it may be that the multiplicative

[9]Note that the confidence bands of the autocorrelations are not exactly valid, as estimation error is not taken into account. However, this most likely leads to too frequent rejections, so the diagnostics might in fact be slightly better than those shown here.

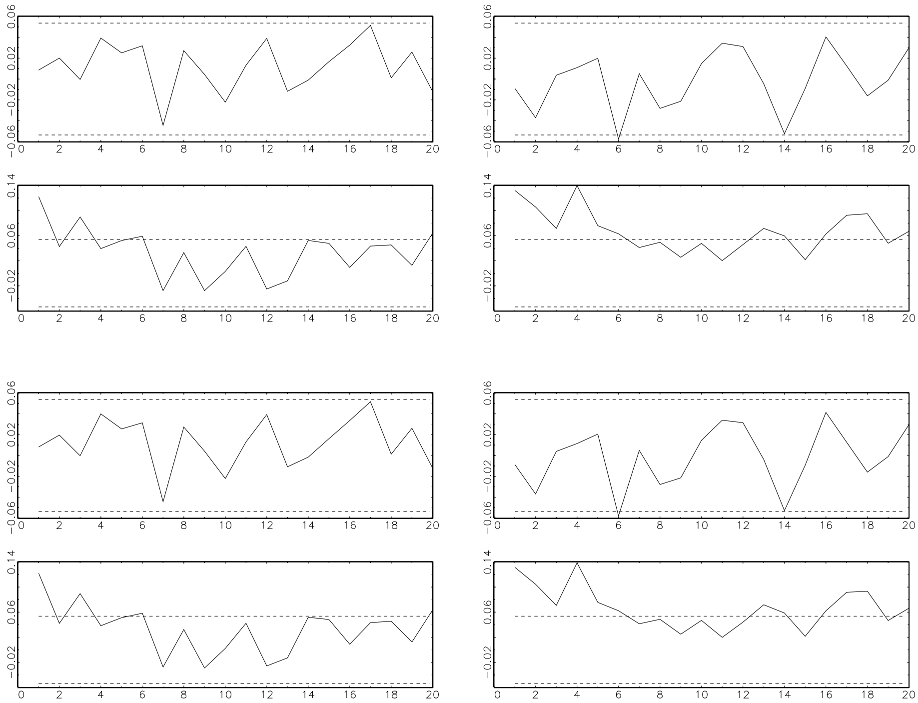

Figure 8: Diagnostics for two-regime bivariate model. Top left: z_t^C, top right: z_t^P, bottom left: $z_t^{C|P}$, bottom right: $z_t^{P|C}$. Autocorrelation functions of demeaned probability integral transforms in upper panel and of their squares in lower panel. The dotted lines depict the boundaries of the 95% confidence interval.

model structure is not sufficient in capturing the time-varying volatility of USD/EUR exchange rate option implied volatility.[10] We witness an improvement when turning our attention to the bivariate model diagnostics in Figure 8. All autocorrelations of demeaned probability integral transforms now nearly fall within the confidence bands.

4 Forecasts

In this section, we proceed to calculate forecasts from the three models presented above. Daily one-step-ahead forecasts are obtained for the out-of-sample period of 1.1.2006-31.12.2007, amounting to 514 forecasts. Forecasts are calculated using the parameter values presented in Section III3.2. Forecast evaluation involves two separate paths: directional accuracy and mean squared errors (MSE). We are interested in the direction of change of implied volatility due to volatility's important role in option pricing. Option traders can potentially profit if they have the correct view on the direction of IV. We use MSEs to assess the relative performance of the models in generating point forecasts, which are of interest in e.g. portfolio risk management.

Table 5 summarizes the forecast results. The results clearly indicate that the model of choice is the bivariate model: both directional accuracy and MSEs improve when

[10]As Corsi et al. (2008) note, realized volatility models may be subject to heteroskedastic errors due to time-varying volatility in the realized volatility estimator. This evidence can possibly also be extended to implied volatility.

switching from a univariate to a bivariate model. For USD/EUR C, the number of days with a correct prediction of sign improves from 301 to 326 when employing the bivariate model, providing a correct sign on 63.4% of trading days. For USD/EUR P, the result is even more impressive: an improvement from 318 to 346, which translates to the correct sign on 67.3% of trading days in the out-of-sample period. Any result above 50% could potentially yield profits to a trader. Mean squared errors are also lowest with the bivariate model specification.

	USD/EUR C			USD/EUR P		
	Correct sign	*%*	*MSE*	*Correct sign*	*%*	*MSE*
Univariate, 2 regimes	295	57.4%	0.358	318	61.9%	0.383
Univariate, 3 regimes	301	58.6%	0.357	313	60.9%	0.389
Bivariate, 2 regimes	**326**	**63.4%**	**0.335**	**346**	**67.3%**	**0.303**

Table 5: Correct directional forecasts (out of 514 trading days) and mean squared errors for forecasts from both univariate and bivariate TVMEM models. The best values within each column are in boldface.

When comparing the two-regime and three-regime univariate models, it is not obvious which model specification is superior. For USD/EUR C, the three-regime model is a better forecaster of the direction of change, but for USD/EUR P, the two-regime model dominates. The rank of the models is also the same when using MSE as the criterion. In general, it can be noted that the TVMEM models forecast up too often: the models make more mistakes by predicting a move up when the true direction was down than vice versa. This tendency could exaggerate forecast performance in periods of consistently rising IV, but in our particular out-of-sample period, there are 243 moves up and 271 down for call IV, and 249 moves up and 265 down for put IV. The best balance by far is achieved with the bivariate model for USD/EUR P, with 279 forecasts up and 235 down. The bivariate model is also the most balanced for USD/EUR C, but the hits are more off: 358 up forecasts and 156 down forecasts.

We next run two types of tests in order to determine the statistical significance of the models' differences in predictive ability. First, we calculate Pesaran-Timmermann test statistics for the directional forecasts.[11] This test allows us to verify that the sign predictions of the forecast series outperform a coin flip. The null hypothesis of predictive failure can be rejected at all relevant levels of significance for all of our models, as the p-values of the test statistic are all less than 0.00001.

The Diebold-Mariano test (from Diebold and Mariano (1995)) is next used to see whether the improvement in mean squared errors achieved with the bivariate model is statistically significant. When comparing the MSE from the bivariate model to the best univariate model (three regimes for USD/EUR C and two regimes for USD/EUR P), the null hypothesis of equal predictive accuracy can be rejected at the one-percent level for USD/EUR P. However, for USD/EUR C, the test delivers a p-value of 0.16. Therefore, in statistical terms, the bivariate model specification does not deliver a better MSE than the best univariate model.

[11]This test statistic is presented in Pesaran and Timmermann (1992).

Compared to earlier studies that explore directional forecast accuracy, these results fall short of only those of Ahoniemi and Lanne (2007), who calculate the correct direction of change for Nikkei 225 index option implied volatility on at best 72% of trading days. In other work on IV modeling, Ahoniemi (2008) obtains the correct sign for the VIX index on 58.4% of trading days with an ARIMA model, and Harvey and Whaley (1992) achieve an accuracy of 62.2% (56.6%) for the implied volatility of S&P 100 index call (put) options. Brooks and Oozeer (2002), who model the implied volatility of options on Long Gilt futures, report a correct sign prediction on 52.5% of trading days. The fact that put IV is more predictable than call IV for USD/EUR currency options contrasts the results of Harvey and Whaley (1992), whereas Ahoniemi and Lanne (2007) have the same result.

In another comparison to earlier work, we choose to look at relative MSEs. For this purpose, we calculate root mean squared errors (RMSE). The average value of USD/EUR C in the out-of-sample period is 7.12, and the best RMSE is some 8.1% of that. For USD/EUR P, the corresponding percentage is 7.5%. In the bivariate model for Nikkei 225 index option implied volatilities of Ahoniemi and Lanne (2007), call-side RMSE amounted to 9.6% of the average out-of-sample value, and put-side IV had a relative RMSE of 10.2%. Therefore, the point forecasts generated by the TVMEM model seem to be quite accurate.

5 Conclusions

Existing research has documented that exchange rate returns and volatilities appear to be drawn from several regimes. It is therefore plausible that the modeling of implied volatilities of currency options would benefit from allowing for two or more regimes. The results of this paper lend support to that assumption: for the IV of options on the USD/EUR exchange rate, a mixture multiplicative error model can be successfully fit to the data. Moreover, the regime probabilities vary in time, which proves to be a valuable feature of the model. Both two and three-regime models are viable in a univariate setting, but when the IV time series are jointly modeled, two mixture components are sufficient. The daily returns of the underlying exchange rate series function well as a regime indicator: days of large moves in the exchange rate seem to coincide with days of high levels of IV and days of large shifts in IV.

The bivariate model specification emerges as the model of choice for a forecaster. Both directional accuracy and point forecasts improve when moving from a univariate to a bivariate setting. The bivariate model predicts the direction of change in implied volatility correctly on 63% (67%) of trading days for call (put) options. This information can be particularly useful for traders in currency options, as the hit ratio is well over 50%. Also, at-the-money options, on which the data is based, are particularly sensitive to changes in volatility. Whether or not the forecast results could be profitably exploited in an out-of-sample option trading exercise is left for future research.

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