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Nyberg, Henri

University of Helsinki, HECER

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Henri Nyberg
University of Helsinki and HECER

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HECER – Helsinki Center of Economic Research, P.O. Box 17 (Arkadiankatu 7), FI-00014 University of Helsinki, FINLAND, Tel +358-9-191-28780, Fax +358-9-191-28781,
E-mail info-hecer@helsinki.fi, Internet www.hecer.fi

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Abstract

In the empirical finance literature findings on the risk-return tradeoff in excess stock market returns are ambiguous. In this study, we develop a new QR-GARCH-M model combining a probit model for a binary business cycle indicator and a regime switching GARCH-in-mean model for excess stock market return with the business cycle indicator defining the regime. Estimation results show that there is statistically significant variation in the U.S. excess stock returns over the business cycle. However, consistent with the conditional ICAPM, there is a positive risk-return relationship between volatility and expected return independent of the state of the economy.

JEL Classification: C32, E32, E44, G12

Keywords: Regime switching GARCH model, GARCH-in-mean model, probit model, stock return, risk-return tradeoff, business cycle

Henri Nyberg

Department of Political and Economic Studies
Economics
University of Helsinki
P.O. Box 17 (Arkadiankatu 7)
FI-00014 University of Helsinki
FINLAND

e-mail: henri.nyberg@helsinki.fi

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1 Introduction

In this study, we consider the risk-return tradeoff in the monthly U.S. excess stock market returns. Previous empirical results are ambiguous on whether there is a positive relationship between risk and expected return as postulated by the Intertemporal Capital Asset Pricing Model (ICAPM) of Merton (1973). Previously, the risk-return tradeoff has typically been examined by means of the GARCH-in-mean (GARCH-M) model originally proposed by Engle, Lilien and Robins (1987). The general idea of the GARCH-M model is that the conditional variance is included in the conditional mean equation and its coefficient is interpreted as to measure the strength of risk aversion.

The main empirical contribution of this paper is to allow the state of the economy to have an effect on the risk-return tradeoff. This is in line with the conditional ICAPM (for details, see Merton, 1973; Guo and Whitelaw, 2006, among others) where macroeconomic state variables proxying investment opportunities (i.e., real economic activity) are also assumed to be important in asset pricing. The idea of the conditional ICAPM is supported by the empirical findings of Chen (1991), Whitelaw (1994) and Pesaran and Timmermann (1995), among others, who have shown that there is significant variation in the excess stock returns related to business cycle fluctuations.

In the previous studies cited above, the dependent variable is typically "continuous" taking any real number. However, many economic and financial applications involve also discrete variables, such as binary variables, with only a limited number of possible outcomes. For instance, binary time series models have been used to predict the business cycle recession and expansion periods (see, e.g., Estrella and Mishkin, 1998; Bernard and Gerlach, 1998; Nyberg, 2010a) and the sign of future stock returns (see, e.g., Leung, Daouk, and Chen, 2000; Rydberg and Shephard, 2003; Nyberg, 2010b).

The novel idea of this paper is to construct a new regime switching model combining a binary time series model for a business cycle indicator and a regime switching model for the excess stock market return. The model is applied to U.S. data with the regime in the regime switching model being based on the NBER (National Bureau of Economic Research) business cycle indicator. In the previous ICAPM literature, various financial variables, such as the default spread, term spread, Treasury Bill rate, and dividend-price ratio (see, e.g., Ghysels, Santa-Clara and Valkanov, 2005; Bali and Engle, 2008, and the references therein) have been employed as proxies of the state of the economy instead of a binary business cycle indicator. We use the autoregressive probit model of Kauppi and Saikkonen (2008) to predict the state of the business cycle, whereas the excess stock return is assumed to follow a regime switching GARCH-M model. The latter is augmented with a qualitative response (QR) variable, and hence the model is referred to as the "QR-GARCH-M" model.

Our empirical results show that there is indeed evidence of statistically significant regime switching behavior in the U.S. excess stock returns over the business cycle. The estimated coefficients of the QR-GARCH-M model reveal that the risk aversion coefficient is positive and statistically significant in both business cycle regimes. This finding contradicts, for example, the evidence provided by Kim and Lee (2008), but is consistent with the positive risk-return relationship implied by the conditional ICAPM. Risk aversion appears to be higher in the recession regime indicating that the investors are demanding a higher risk premium during recession. As an example, Kim and Lee (2008) find evidence on the positive relation only in the expansion regime in their regime switching GARCH-M model. Furthermore, in accordance with previous studies the conditional variance of returns turns out to be higher during recession periods.

The rest of the paper is organized as follows. In Section 2, we briefly introduce the idea of risk-return tradeoff and show how it has been examined empirically with the GARCH-M model. The QR-GARCH-M model is introduced in Section 3. Empirical findings on the risk-return tradeoff are presented in Section 4. Section 5 concludes.

2 GARCH-M Model, ICAPM and Business Cycles

The tradeoff between risk and expected stock market return has been examined extensively in the theoretical and empirical finance literature. The ICAPM of Merton (1973) suggests a positive relationship between expected return and risk. French, Schwert and Stambaugh (1987) and Campbell and Hentschel (1992) were among the first to find empirical evidence of such a positive relationship. However, overall the empirical evidence seems to be mixed because several authors have also found a statistically insignificant or even negative relation (see, e.g, Glosten, Jagannathan and Runkle, 1993, and the references therein).

In empirical studies on the risk-return tradeoff, the GARCH-M (GARCH-in-mean) model of Engle *et al.* (1987) has been used to capture the potential link between time-varying expected return and risk measured by the conditional variance of returns. The main idea in the GARCH-M model is to allow the conditional variance h_t to have an impact on the conditional mean of the return r_t. This is stated formally as

$$r_t = \psi + \delta h_t + h_t^{1/2}\epsilon_t, \tag{1}$$

where $\epsilon_t \sim \mathrm{IID}(0,1)$. Often the conditional variance is assumed to follow the GARCH(1,1) model

$$h_t = \omega + \beta h_{t-1} + \alpha u_{t-1}^2, \tag{2}$$

where $u_t = r_t - \psi - \delta h_t$ and $\omega > 0$, $\beta \geq 0$, and $\alpha > 0$.[1] The inclusion of h_t in the mean equation (1) is called a "volatility feedback" effect. In the ICAPM the parameter δ is interpreted as the coefficient of relative risk aversion (Merton, 1980). A positive coefficient means that risk-averse investors require a higher expected return (a higher risk premium) when the risk is higher.

In the previous literature, Glosten *et al.* (1993), inter alia, have concluded that, despite the simplicity of the GARCH-M model (1), it should be extended to capture the risk-return tradeoff accurately. They suggested a "modified" GARCH-M model, whereas Chauvet and Potter (2001), for example, introduced a dynamic factor model to the market risk premium. Both of these studies find a negative relationship between risk and expected return. On the other hand, Ghysels *et al.* (2005) find evidence of a positive relationship by using a model based on the mixed data sampling (MIDAS) approach. In addition, Lanne and Saikkonen (2006) point out that in many empirical studies the intercept ψ is included in the mean equation (1) although, based on the

[1] In this study, we employ the conditional variance h_t instead of its standard deviation $\sqrt{h_t}$ in the mean equation (1). Both of these alternatives have been used in the literature. Overall, the results turn out to be more or less the same irrespective of this selection.

ICAPM, it is not theoretically justified. They find that the inclusion of an unnecessary intercept term makes the estimated risk aversion coefficient δ in (1) unstable and statistically insignificant. However, they find a positive and statistically significant estimate in the case of the U.S. stock returns when they exclude the intercept from the mean equation.

As discussed above, many previous studies have considered the ICAPM in a simplified form by ignoring a "hedge component" which captures investors' preferences to hedge against investment opportunities (see Merton, 1973, and Guo and Whitelaw, 2006). If the hedge component is included in the model, the model is often referred to as the conditional ICAPM. In previous empirical work, the conditional ICAPM is typically used by including various macroeconomic variables in the estimated model in order to reflect the state of real economic activity. Recently, for example, Guo and Whitelaw (2006) and Bali and Engle (2008) have shown that there is indeed a positive relation between risk and expected return conditional on macroeconomic factors. In addition to these studies, Chen (1991), Whitelaw (1994) and Pesaran and Timmermann (1995), among others, have emphasized the role of business cycle fluctuations in determining the conditional mean and conditional variance of excess stock returns.

In this paper, we consider a new model where a binary indicator variable defines the state of the economy in terms of expansion and recession periods. This is in contrast to the previous literature where various macroeconomic variables are typically employed as predictors in the model for the conditional mean of returns (see, e.g., Bali and Engle, 2008, and the references therein). The probability of the recession is obtained from a probit model and the conditional distribution of the excess stock returns is modeled using a regime switching GARCH-M model where the business cycle indicator defines the regime. Kim and Lee (2008) have considered a closely related model based on an augmented GARCH-M model where the unobserved state of the economy is modeled by using a Markov switching model. They find the binary valued business cycle indicator to be a statistically significant predictor in the conditional mean equation and interpret these findings as evidence in favor of business cycle specific risk aversion.

As Kim and Lee (2008) point out, the state of the business cycle cannot be identified for sure in real time. Due to the informational lags and revisions between the initial and final values of macroeconomic variables, there is also a substantial delay in the values of the business cycle indicator. Thus, investors have to base their investment decisions on expectations of the state of the economy. Therefore, instead of using the hindsight of the business cycle recession and expansion periods, the business cycle indicator should be modeled simultaneously with stock returns.

3 QR-GARCH-M Model

3.1 Background

As discussed above, the main interest in this study is a new type of extension of the GARCH-M and regime switching GARCH models (hereafter RS-GARCH models). The benchmark GARCH-M model defined in (1) and (2) is augmented by regime switching dynamics where the regime is determined by the value of a binary time series. For simplicity, we refer to this model as the "QR-GARCH-M" model. This model can be seen as a special case of a general mixture model where the value of the qualitative response variable, now a binary variable, and continuous variables are modeled simultaneously within the same model.

In our empirical application, the binary variable y_t is the U.S. business cycle indicator provided by the National Bureau of Economic Research (NBER). The continuous variable r_t is the monthly U.S. excess stock market return. It is constructed by subtracting one-month risk-free return from the nominal stock market return.[2] We construct the QR-GARCH-M model in a similar way to the corresponding Markov switching models (see, e.g., Hamilton and Susmel, 1994; Hamilton and Lin, 1996; Perez-Quiros and Timmermann, 2001; Kim and Lee, 2008). This means, in particular, that the stock return is dependent on the contemporaneous state of the business cycle. Thus, we consider a structural model, where the NBER business cycle indicator has an effect on the contemporaneous U.S. excess stock return, but not vice versa. However, the lagged return r_{t-1} can be used as a predictor of the business cycle phase. These assumptions imply that y_t can be seen as weakly exogenous for r_t.

[2] Details on the data set are given in Section 4.1.

In addition to the similarity to the previously considered models, our structural model with the aforementioned weak exogeneity assumption, can be justified by empirical findings presented in the recession forecasting literature. First, there is evidence that the stock market return has predictive power for the coincident and future state of the economy (see, e.g., Estrella and Mishkin, 1998; Nyberg, 2010a). Thus, as a leading indicator of future recession and expansion periods of the economy, it is reasonable to use the lagged stock return instead of the contemporaneous return in the model for the state of the business cycle. Second, Schwert (1989, 1990) and Fama (1990), among others, have provided empirical evidence that the state of the economy is a key predictor of the stock market return (see also, e.g., Chauvet and Potter, 2000).

3.2 Model

Consider two time series y_t and r_t, $t = 1, 2, ..., T$, which define a bivariate model, where the former is a binary variable and the latter is a continuous real-valued variable. For notational convenience, we collect these variables in the vector

$$\boldsymbol{z}_t = (y_t \quad r_t)'. \tag{3}$$

The framework put forth in this paper can be used in various empirical applications. In this study, we concentrate on business cycles and excess stock returns which can be seen in the notation employed in the paper.

In the QR-GARCH-M model, we assume the weak exogeneity assumption introduced in Section 2. In other words, we consider a structural form of the model in which the contemporaneous value of y_t has an effect on the variable r_t, but not vice versa. Thus, as will be seen in Section 3.3, the QR-GARCH-M model is based on a mixture distribution, where the conditional density of r_t is dependent on the value of the binary variable y_t. In our application, this means that the probit model for the business cycle indicator can be treated independently of the regime switching model of the excess stock return.

Let us first consider the model for the binary variable. Conditional on the information set Ω_{t-1}, y_t follows a Bernoulli distribution

$$y_t|\Omega_{t-1} \sim B(p_t). \tag{4}$$

Here p_t is the conditional expectation of y_t, $E_{t-1}(y_t)$, which is equal to the conditional probability of the outcome $y_t = 1$. Thus,

$$p_t = E_{t-1}(y_t) = P_{t-1}(y_t = 1) = \Phi(\pi_t), \tag{5}$$

where $\Phi(\cdot)$ is a standard normal cumulative distribution function and π_t is a linear function of variables, such as lagged values of $\boldsymbol{z}_t$ and explanatory variables $\boldsymbol{x}_t$, included in information set Ω_{t-1}. In other words, the expression (5) defines a univariate probit model.

To complete the model specification of y_t, the linear function π_t should be determined. In the previous literature, the most commonly used model is a "static" probit model

$$\pi_t = w + \boldsymbol{x}_{t-1}^{'}\boldsymbol{b}, \tag{6}$$

where w is an intercept term, the vector $\boldsymbol{x}_{t-1}$ contains the explanatory variables and $\boldsymbol{b}$ is a vector of parameters. In this study, we concentrate on an extension of the static model suggested by Kauppi and Saikkonen (2008) (see also Rydberg and Shephard, 2003). Specifically, we add a lagged value of π_t to the right hand side of (6), which results in the model

$$\pi_t = w + a\pi_{t-1} + \boldsymbol{x}_{t-1}^{'}\boldsymbol{b}, \tag{7}$$

where $|a| < 1$. Due to the first-order autoregressive structure in the variable π_t we refer to this model as the "autoregressive" model. Kauppi and Saikkonen (2008) augment model (7) with the lagged value y_{t-1}. However, we concentrate on model (7) because, as discussed in Section 2, values of the recession indicator become available with a considerable delay.

In the QR-GARCH-M model, r_t follows a regime switching GARCH-M model where the regime is defined by the value of the business cycle indicator. The regime switching structure of the conditional mean (cf. (1)) can be expressed as

$$r_t = (1 - y_t)\Big(\psi_0 + \delta_0 h_{0t} + h_{0t}^{1/2}\epsilon_t\Big) + y_t\Big(\psi_1 + \delta_1 h_{1t} + h_{1t}^{1/2}\epsilon_t\Big), \tag{8}$$

where $\epsilon_t \sim \text{IID}(0,1)$ and the outcome of $y_t = j$ ($j = 0$, $j = 1$) defines the regime and the parameters. Similarly to this specification of the conditional mean, the regime dependent volatility process, denoted by h_{jt}, follows a regime switching GARCH(1,1) model

$$h_{jt} = \omega_j + \beta_j h_{j,t-1} + \alpha_j u_{j,t-1}^2, \tag{9}$$

where $j = 0, 1$, and $u_{j,t-1}$ is the lagged value of

$$u_{jt} = r_t - (1 - y_t)\Big(\psi_0 + \delta_0 h_{0t}\Big) + y_t\Big(\psi_1 + \delta_1 h_{1t}\Big). \tag{10}$$

Model (9) can be rewritten as

$$h_{jt} = (1 - y_t)h_{0t} + y_t h_{1t}, \tag{11}$$

where h_{0t} and h_{1t} can be obtained from the expression (9). As in the GARCH(1,1) model (2), we impose the restrictions $\omega_j > 0$, $\beta_j \geq 0$ and $\alpha_j > 0$ which imply the conditional variance always be positive.

It is assumed that the error term ϵ_t in (8) is independent of both y_t and the variables included in information set Ω_{t-1}. Thus, later on in this section and also in Section 3.3., we use the augmented information set $\{\Omega_{t-1}, y_t\}$. However, it should be pointed out than when constructing forecasts the value y_t is not included in the information set. Therefore, in Section 3.4, we see that y_t will be replaced by the conditional expectation (5).

Compared with the benchmark GARCH-M model, expressions (8) and (9) emphasize the fact that the excess stock return is dependent on the binary variable. In the risk-return relationship, the flexible dynamics of the QR-GARCH-M model allows the risk aversion (risk premium) coefficient δ_j and the intercept term ψ_j to be business cycle specific in the mean equation (8). This regime switching structure of the model implies state dependent time-varying investment opportunities determined by the conditional ICAPM. Furthermore, the parameters in the model for the conditional variance are also dependent on the business cycle regime indicating that the conditional variance follows a different model in business cycle recession and expansion periods.

3.3 ML Estimation

In the QR-GARCH-M model parameters can conveniently be estimated by the method of maximum likelihood (ML). In general, the conditional density function of $\boldsymbol{z}_t$, conditional on information set $\{\Omega_{t-1}, y_t\}$, can be written as

$$g_{t-1}(\boldsymbol{z}_t; \boldsymbol{\theta}) = f_{t-1}(r_t | y_t = j; \boldsymbol{\theta}) P_{t-1}(y_t = j; \boldsymbol{\theta}), \tag{12}$$

where $j = 0$ or $j = 1$, $f_{t-1}(r_t|y_t = j; \boldsymbol{\theta})$ is the conditional density function of r_t, and $P_{t-1}(y_t = j; \boldsymbol{\theta})$ is the conditional probability of the outcome $y_t = j$. The vector of parameters, $\boldsymbol{\theta} = (\boldsymbol{\theta}_1' \quad \boldsymbol{\theta}_2')'$, contains all the parameters of the model. Hereafter, we assume that the parameters included in the vector $\boldsymbol{\theta}_2$ are related to the model specified for the binary variable. Because of the exogeneity assumption discussed in Section 3.1, the conditional density function (12) can be written as

$$g_{t-1}(\boldsymbol{z}_t; \boldsymbol{\theta}) = f_{t-1}(r_t | y_t = j; \boldsymbol{\theta}_1) P_{t-1}(y_t = j; \boldsymbol{\theta}_2). \tag{13}$$

This shows that the conditional probability $P_{t-1}(y_t = j; \boldsymbol{\theta}_2)$ is constant with respect $\boldsymbol{\theta}_1$ indicating that $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ can be estimated separately by ML.

After the distribution of the error term ϵ_t and the linear function π_t (see (5)) in probit model have been specified, the log-likelihood function can be constructed. The normality assumption of the error term has been rejected in asset return data in a large number of previous studies. This is typically related to the excessive kurtosis and fatter tails of the unconditional distribution of returns compared with the normal distribution (see, e.g., Franses and van Dijk, 2000, 9–19). Therefore, in this study, the error term ε_t is assumed to follow the Student's t distribution with ν degrees of freedom ($\nu > 2$). This appears to be the most commonly used alternative to the normal distribution. The parameter ν is estimated along with the other parameters of the model (i.e. hereafter $\boldsymbol{\theta} = (\boldsymbol{\theta}_1' \quad \boldsymbol{\theta}_2' \quad \nu)'$).

Assume that we have observed the time series y_t and r_t, $t = 1, 2, ..., T$, with initial values treated as fixed constants. The conditional density function of observation $\boldsymbol{z}_t$, $g_{t-1}(\boldsymbol{z}_t; \boldsymbol{\theta})$, is given in (13). Thus, the log-likelihood function over the whole sample, given the initial values, can be written as

$$l_T(\boldsymbol{\theta}) = \sum_{t=1}^{T} l_t(\boldsymbol{\theta}) \quad = \quad \sum_{t=1}^{T} \log(g_{t-1}(\boldsymbol{z}_t; \boldsymbol{\theta})), \tag{14}$$

where $\boldsymbol{\theta} = (\boldsymbol{\theta}_1' \quad \boldsymbol{\theta}_2' \quad \nu)'$.[3] As the error term ϵ_t follows the Student's t distribution, the log-likelihood function of observation $\boldsymbol{z}_t$, given the expressions (5), (7), (8) and (9), is

$$l_t(\boldsymbol{\theta}) = \log\Big[\frac{\Gamma((\nu+1)/2)}{\sqrt{\pi}\,\Gamma(\nu/2)} \frac{(\nu-2)^{-1/2}}{\sqrt{h_{jt}}} \Big(1 + \frac{u_{jt}^2}{h_{jt}(\nu-2)}\Big)^{-(\nu+1)/2} p_t^{y_t}(1-p_t)^{1-y_t}\Big], \quad (15)$$

where $\Gamma(\cdot)$ is the Gamma function, and u_{jt} and h_{jt} are given in (10) and (11), respectively. Due to the regime switching dynamics of the model the value of $l_t(\boldsymbol{\theta})$ is dependent on the realized value of y_t. For example, if $y_t = 1$,

$$l_t(\boldsymbol{\theta}) = \log\Big[\frac{\Gamma((\nu+1)/2)}{\sqrt{\pi}\,\Gamma(\nu/2)} \frac{(\nu-2)^{-1/2}}{\sqrt{h_{1t}}} \Big(1 + \frac{u_{1t}^2}{h_{1t}(\nu-2)}\Big)^{-(\nu+1)/2} p_t\Big],$$

where $u_{1t} = r_t - \psi_1 - \delta_1 h_{1t}$ and $h_{1t} = \omega_1 + \beta_1 h_{j,t-1} + \alpha_1 u_{j,t-1}^2$. Finally, the maximum likelihood estimate $\hat{\boldsymbol{\theta}}$ is obtained by maximizing the log-likelihood function (14) by numerical methods.

Explicit stationarity conditions of the QR-GARCH-M model are unknown although stationarity conditions for the univariate autoregressive probit model (7) and the GARCH-M model (1) are available. Meitz and Saikkonen (2008), among others, have considered stationarity conditions for GARCH and GARCH-M models. Furthermore, by recursive substitution of the autoregressive probit model (7), we obtain the following representation

$$\pi_t = \omega \sum_{j=1}^{\infty} a^{j-1} + \sum_{j=1}^{\infty} a^{j-1} \boldsymbol{x}_{t-1-j+1}' \boldsymbol{\beta}, \quad (16)$$

which shows that π_t depends on the whole lagged history of the explanatory variables $\boldsymbol{x}_t$. Hence, if the explanatory variables are stationary and $|a| < 1$, π_t is also stationary.

At the moment there is no formal proof of the asymptotic distribution of the maximum likelihood estimate $\hat{\boldsymbol{\theta}}$. Nevertheless, under reasonable regularity conditions, such as the stationarity of r_t, $\boldsymbol{x}_t$ and π_t, and correctness of the model specification, it is reasonable to assume that the ML estimator $\hat{\boldsymbol{\theta}}$ is asymptotically normal, that is,

$$T^{1/2}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \xrightarrow{L} N(\mathbf{0}, \mathcal{I}(\boldsymbol{\theta})^{-1}), \quad (17)$$

[3] The initial value of $g_0(\boldsymbol{z}_1)$ is obtained by setting $\pi_0 = (\bar{\boldsymbol{x}}_{t-k}' \boldsymbol{b})/(1-a)$ (cf. (7)) and $h_0 = (1-\bar{y})\Big(\omega_0 + \beta_0 \widehat{\mathrm{var}}(r_t)\Big) + \bar{y}\Big(\omega_1 + \beta_1 \widehat{\mathrm{var}}(r_t)\Big)$ (cf. (9)), where a bar is used to signify the sample mean and $\widehat{\mathrm{var}}(r_t)$ is the sample variance of r_t.

where $\mathcal{I}(\boldsymbol{\theta}) = \operatorname{plim} T^{-1} \sum_{t=1}^{T} (\partial l_t(\boldsymbol{\theta})/\partial \boldsymbol{\theta})(\partial l_t(\boldsymbol{\theta})/\partial \boldsymbol{\theta})'$. As the specified distribution of ϵ_t may not be correct, the maximum likelihood estimate $\hat{\boldsymbol{\theta}}$ can also be interpreted as a quasi-maximum likelihood estimator. In that case, the asymptotic distribution is

$$T^{1/2}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \xrightarrow{L} N(\mathbf{0}, \mathcal{I}(\boldsymbol{\theta})^{-1} \mathcal{J}(\boldsymbol{\theta}) \mathcal{I}(\boldsymbol{\theta})^{-1}), \tag{18}$$

where $\mathcal{J}(\boldsymbol{\theta}) = \operatorname{plim} T^{-1} \sum_{t=1}^{T} (\partial^2 l_t(\boldsymbol{\theta})/\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}')$. Robust standard errors are obtained from the diagonal elements of the asymptotic covariance matrix (18) where $\mathcal{I}(\boldsymbol{\theta})$ and $\mathcal{J}(\boldsymbol{\theta})$ are replaced by their sample analogues. Further, Wald and likelihood ratio (LR) tests for the components of the parameter vector can be applied in the usual way.

3.4 Forecasting

As discussed in Section 3.2, although in expressions (8) and (11) the value of the binary variable y_t determines the regime, its value is unknown at time $t-1$. In estimation, we can use the extended information set $\{\Omega_{t-1}, y_t\}$ to obtain parameter estimates. However, when constructing forecasts in the QR-GARCH-M model we have to replace y_t with the conditional expectation $p_t = E_{t-1}(y_t)$ given in (5). In our empirical application, when the binary variable is the business cycle indicator, this expectation can be interpreted as the expected probability of recession that investors had at real time (see Section 3.1).

Because of the structural specification of the model, the conditional expectation of y_t is independent of r_t. As the error term ϵ_t is independent of both the information set Ω_{t-1} and the contemporaneous value y_t by assumption, the conditional expectation of r_t given the information set Ω_{t-1}, $E_{t-1}(r_t)$, is a mixture of the conditional expectations of the two GARCH-M regimes. The mixing proportion between the regimes is determined by the conditional expectation $E_{t-1}(y_t)$. According to the law of iterated expectations, the conditional expectation of r_t is

$$\begin{aligned} E_{t-1}(r_t) &= E_{t-1}\Big[E(r_t|\Omega_{t-1}, y_t)\Big] \\ &= E_{t-1}\Big[(1-y_t)\Big(\psi_0 + \delta_0 h_{0t} + h_{0t}^{1/2} E_{t-1}(\epsilon_t|\Omega_{t-1}, y_t)\Big) \\ &\quad + y_t\Big(\psi_1 + \delta_1 h_{1t} + h_{1t}^{1/2} E_{t-1}(\epsilon_t|\Omega_{t-1}, y_t)\Big)\Big] \\ &= \Big(1 - E_{t-1}(y_t)\Big)\Big(\psi_0 + \delta_0 h_{0t}\Big) + \Big(E_{t-1}(y_t)\Big)\Big(\psi_1 + \delta_1 h_{1t}\Big) \\ &= (1-p_t)\Big(\psi_0 + \delta_0 h_{0t}\Big) + p_t\Big(\psi_1 + \delta_1 h_{1t}\Big), \end{aligned} \tag{19}$$

where the third equation follows from the fact that $E_{t-1}(\epsilon_t|\Omega_{t-1}, y_t) = 0$. This result shows that the conditional expectation of r_t is dependent on the conditional probability p_t and its complement probability $1 - p_t$. This conditional expectation is also the fitted value of r_t implied by the QR-GARCH-M model when the information set is Ω_{t-1}.

In forecasting, the conditional expectation of $\boldsymbol{z}_t$,

$$\hat{\boldsymbol{z}}_t = E_{t-1}(\boldsymbol{z}_t) = \Big(E_{t-1}(y_t) \quad E_{t-1}(r_t)\Big)'. \tag{20}$$

can also be interpreted as the mean-square sense optimal one-period forecast where $E_{t-1}(y_t)$ and $E_{t-1}(r_t)$ are given in (5) and (19). In Section 4.3, we concentrate on one-period forecasts for the U.S. stock market return. In other applications also multiperiod forecasts may be of interest. However, computation of multiperiod forecasts is more complicated than that of one-period forecasts (cf. multiperiod forecasting in nonlinear models, e.g., Franses and van Dijk, 2000, 118–121).

The regime switching dynamics of the QR-GARCH-M model also indicates that the conditional expectation of the conditional variance of r_t, derived in Appendix, is different from the simple GARCH-M model. That is,

$$\mathrm{Var}_{t-1}(r_t) = (1-p_t)h_{0t} + p_t h_{1t} + p_t(1-p_t)\Big((\psi_0 + \delta_0 h_{0t}) + (\psi_1 + \delta_1 h_{1t})\Big)^2, \tag{21}$$

where the regime switching volatility process in the two regimes of y_t, h_{0t} and h_{1t}, is given in (9).

3.5 Comparison to Related Models

To the best of our knowledge, no regime switching model of this type has been proposed in the previous literature. However, several closely related models have been considered. Dueker (2005) has proposed a Qual VAR model where binary and continuous dependent variables, such as (3), are modeled endogenously within the same vector autoregressive model. The main difference is that on the Qual VAR model a binary variable is replaced by a continuous latent variable y_t^*. This latent variable determines the values of the binary variable y_t which in our model is observable. For instance, $y_t^* > 0$ ($y_t^* \leq 0$) determines the outcome $y_t = 1$ ($y_t = 0$).

In contrast to the VAR modeling of the latent y_t^* and continuous variables in the Qual VAR model, in this study, the joint density of $\boldsymbol{z}_t$ is constructed in a proposed regime switching context, where the binary variable defines the regime. Further, the main implication of use of the autoregressive model (7) is that it facilitates the method of maximum likelihood in estimation (Section 3.2) and one-period forecasts can be calculated with explicit formulae (Section 3.4) without using Bayesian methods.

As already mentioned, the QR-GARCH-M model shares some characteristics with regime switching models, such as Markov switching GARCH models (see, e.g., Hamilton and Susmel, 1994; Hamilton and Lin, 1996). However, in the QR-GARCH-M model the regime, such as the state of the economy, is observed, whereas in Markov switching models this is not the case. Consequently, compared with RS-GARCH models (see, e.g., Bauwens *et al.*, 2006; Lange and Rahbek, 2009, 871–887), the QR-GARCH-M model has the advantage that it is not "path dependent". In many RS-GARCH models, path dependence occurs because the conditional variance is dependent on the entire history of past unobserved regimes. Thus, one needs to integrate over all possible 2^T past regime paths when computing the value of the likelihood function which is clearly computationally infeasible.

In previous literature, Bauwens *et al.* (2006) have proposed Bayesian methods, whereas Gray (1996), Lanne and Saikkonen (2003) and Haas, Mittnik and Paolella (2004), among others, have suggested alternative methods and new regime switching models to circumvent the path dependence problem. The essential difference between these above-mentioned models and the QR-GARCH-M model is that now the regime is observed as the value of a binary time series.

4 Empirical Results

4.1 Data and Descriptive Analysis

In this section, we consider an application of the QR-GARCH-M model to the risk-return tradeoff in the U.S. excess stock returns. The monthly data set consists of the period from January 1960 to March 2009. The first 12 observations are used as initial values in estimation. It is assumed that the recent U.S. recession period that began after the business cycle peak in December 2007 is still going on at the end of the sample period in March 2009. This assumption is based on the evidence of various U.S. economic indicators.

The monthly U.S. excess stock return series (r_t) is constructed as the difference between the monthly CRSP value-weighted return on all NYSE, AMEX, and NASDAQ stocks and the return on the risk-free one-month Treasury Bill rate. Business cycle recession and expansion periods (y_t) are obtained from the NBER business cycle chronology. The NBER defines the recession as "a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales." [4]

In estimating the conditional probability of recession and expansion we employ financial predictive variables which have been found the most reliable leading indicators for the state of the economy in the previous literature. Another reason why we restrict ourselves to financial predictors is that they are available on a continuous basis without informational delays and revisions. Therefore, the information set Ω_{t-1} in our model consists of the information available at period $t-1$ in real time.

Much of the previous research on predictive variables lends support to the term spread between the long-term and the short-term interest rate being the main recession predictor, but the stock market return and the foreign term spread have also

[4] We use the CRSP stock return data and the one-month Treasury Bill rate from Ibbotson Associates (see details at Kenneth French's data library http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). Details on the NBER recession periods, i.e. the chronology of business cycle peaks and troughs, see http://www.nber.org/cycles/cyclesmain.html.

been found to have some additional predictive power (see, e.g., Estrella and Mishkin, 1998; Bernard and Gerlach, 1998; Rudebusch and Williams, 2009; Nyberg, 2010a, and the references therein). In addition, the results of Ang and Piazzesi (2006) and Wright (2006) suggest that the short term interest rate, such as the three-month Treasury Bill rate or the Federal funds rate, may have some additional predictive power. In contrast, King, Levin and Perli (2007), and the references therein, provide evidence in favor of the default spread between the corporate bonds as a recession predictor.[5]

Descriptive statistics of the excess stock market returns are presented in Table 1. One of the contributions of this study is the extension of the sample period with the recent U.S. business cycle expansion period from December 2001 to December 2007 and the beginning of the subsequent recession. Hence, although there have been only few recession periods in last decades, the sample period appears to be reasonably long to be used in the QR-GARCH-M model because there are now 89 recession months ($y_t = 1$) in the sample period which is 15.1% of the whole sample ($T = 591$).

The mean of excess stock market returns is positive, but the returns also exhibit clear variation over the business cycle. As expected, during the recession the returns have mainly been negative. The standard deviation is also higher in the recession regime when compared with the expansion regime, indicating that volatility is also time-varying. Excessive kurtosis and negative skewness lead to the result that the normal distribution does not describe returns adequately. The values of the Jarque-Bera normality test confirm this finding. However, in the recession regime, the skewness and kurtosis are close to zero and three, respectively, as implied by the normal distribution. Thus, the null hypothesis of normality is not rejected. It should be noted, however, that the limited number of observations in the recession regime may have a salient effect on this result.

4.2 Estimation Results

In this section, the main interest is in the estimated coefficients related to the risk-return tradeoff. However, in the QR-GARCH-M model, a model for the U.S. business

[5] Interest rate data are extracted from http://www.federalreserve.gov/releases/h15/data.htm, http://www.bundesbank.de/statistik/statistik.en.php and http://stats.oecd.org/index.aspx.

cycle recession and expansion periods should first be specified. For that purpose, we consider the autoregressive probit model (7). To complete the specification of the model, the predictive variables included in the vector $\boldsymbol{x}_{t-1}$ must be selected. As emphasized in Section 4.1, we restrict ourselves to financial predictors. In model selection, we include predictors one by one in the vector $\boldsymbol{x}_{t-1}$ as long as the lowest value of the Schwarz information criterion (BIC) (Schwarz, 1978) is found. In other words, the model selection procedure is stopped if the estimated value of BIC increases when any additional predictor is included in the previous model. The whole sample period from January 1961 to March 2009 is used in estimation.

In accordance with the findings of Nyberg (2010a), the U.S. term spread (SP_t^{US}), the German term spread (SP_t^{GE}), used as a "representative" foreign term spread in the model reflecting the state of the economy in euro area, and the lagged U.S. stock market return turn out to be the best predictors. This three variable combination yields the lowest value of BIC. Therefore, neither the three-month short-term interest rate, its first difference, nor the default spread provides statistically significant additional predictive power when considered as a fourth predictor in the model (cf. Ang and Piazzesi, 2006; Wright, 2006; King *et al.*, 2007).

Throughout the paper, we employ these three financial predictors in the autoregressive probit model (7) for the business cycle indicator y_t. The best lags of the predictors in terms of their predictive power based on the BIC are mentioned in Table 2. We also report the estimated parameter coefficients and some commonly used statistical goodness-of-fit measures for binary time series. In the case of predictive variables, all estimated coefficients are negative and statistically significant. Thus, low values of the term spreads and stock returns predict a high probability of recession. Overall, the value of the pseudo-R^2 measure of Estrella (1998) provides evidence that the model is successful in predicting the state of the U.S. business cycle very accurately. For example, if the 50% threshold value is employed to construct recession and expansion signals, the percentage of correct predictions, denoted by $CR_{50\%}$, is very high ($CR_{50\%}$=0.967). The p-value of the sign predictability test of Pesaran and Timmermann (1992) is 1e-05 showing that recession and expansion periods are well predictable in sample.

Figure 2 depicts the in-sample recession probability p_t which is also the estimated mixing probability between the two GARCH-M regimes in the QR-GARCH-M model. As the statistical goodness-of-fit measures show, the recession probability matches very well with the U.S. recession and expansion periods. The recession probability is high during the recession periods and very close to zero during the expansions. This is also the case with the beginning of the recent recession where the recession probability increased at the same time as the recession started after the business cycle peak in December 2007.

Next we turn our interest to the estimation results of the GARCH-M specifications for the U.S. excess stock returns. At first, it is worth reminding that the GARCH-M part of the model does not affect the autoregressive probit model for y_t. Thus, the estimation results of the probit model presented in Table 2, and also the mixing proportion between the regimes, are the same for all QR-GARCH-M specifications.

In Table 3, the first two models (Model 1 and Model 2) are GARCH-M models (1), where the state of the U.S. business cycle is not taken into account. The last three models (Models 3–5) are different specifications of the QR-GARCH-M model. In all models, the GARCH(1,1) model for the conditional variance is employed. In Model 1, we observe a positive risk-return tradeoff, but the estimated risk aversion coefficient δ is statistically insignificant. On the other hand, if the intercept term is excluded from the mean equation, as the ICAPM indicates, the risk aversion coefficient is indeed positive and statistically significant at the 5% level (Model 2). This result is in line with the findings of Lanne and Saikkonen (2006).

The QR-GARCH-M models indeed suggest substantial business cycle variation in the excess stock returns. The likelihood ratio (LR) test of the GARCH-M model (Model 1) against the unrestricted QR-GARCH-M model (Model 3) is statistically significant at all traditional significance levels. Although the unrestricted QR-GARCH-M model outperforms the simple GARCH-M model, there are several statistically insignificant coefficient estimates. As the objective is to select as parsimonious a model as possible, we consider a restricted model with the restrictions $\beta_0 = \beta_1$ and $\alpha_0 = \alpha_1$. Thus, in the restricted model (Model 4) the GARCH and ARCH parameters are the same in both regimes, but the intercept terms can be different. The LR test indicates that the above-mentioned restrictions hold (p-value 0.651). However, the intercept term in the recession regime (ω_1) is significantly higher than in the expansion regime (ω_0) indicating that the level in the conditional variance is higher in the recession regime. Hamilton and Lin (1996) and Chauvet and Potter (2001), among others, have obtained similar results.

In the restricted model (Model 4) the intercept in the mean equation of the recession regime (ψ_1) is statistically significant, but the intercept ψ_0 in the expansion regime is not. Based on the findings of Lanne and Saikkonen (2006), in Model 5, we exclude the latter intercept. Once again the LR test indicates that this restriction holds in the model with the restrictions $\alpha_0 = \alpha_1$ and $\beta_0 = \beta_1$ imposed. In Model 5, the estimated coefficients are all statistically significant and the value of BIC is minimized among the considered models. Therefore, this model seems adequate. Residual diagnostics confirm that there is no significant autocorrelation left in the residuals, but some remaining conditional heteroskedasticity is still found.[6]

Perhaps the most interesting finding in Model 5 is that the estimates of the risk aversion coefficients δ_0 and δ_1 are positive and statistically significant (p-values are 1.098e-05 and 0.012, respectively). Thus, a higher conditional variance tends to increase expected stock return in both regimes. However, it is worth pointing out that the estimated value of the intercept ψ_1 is negative. It appears necessary to include an intercept in the recession regime. If it is excluded, the estimated risk aversion coefficient δ_1 in fact becomes negative and statistically insignificant (results not reported). This is in accordance with the findings of Kim and Lee (2008). They found a positive risk-return relationship only in the expansion period, whereas in recession the risk aversion coefficient is negative and insignificant.

All in all, in Model 5 the positive estimates of δ_0 and δ_1 are consistent with the conditional ICAPM. Furthermore, the fact that the estimate of δ_1 is greater than that δ_0 leads to the conclusion that the risk aversion, or equivalently the required risk premium, is significantly higher in recession. Estimated risk aversion coefficients

[6] We also considered some extensions of the GARCH(1,1) model for the conditional variance. The values of BIC obtained with the GARCH(1,2) and GARCH(2,1) models were higher than in the GARCH(1,1) model. Overall, these extensions of the GARCH(1,1) model essentially lead the same conclusions concerning the risk-return tradeoff as obtained with Model 5.

reported in the previous literature (see, e.g, Lanne and Saikkonen, 2006; Bali and Engle, 2008) have typically been between 0.04 and 0.06, and hence quite close to the corresponding estimate obtained for the expansion regime. In contrast, the risk aversion coefficient (0.13) in the recession regime in Model 5 is considerable higher than in the previous studies.

The estimated in-sample predictions for the excess stock market return and the conditional variance (see (21)) are depicted in Figure 3. As in Section 3.4, the information set is Ω_{t-1} indicating that the stock return is dependent on the estimated recession probability p_t. We see that the conditional variance is typically relatively high during the recession periods. Interestingly in the upper panel, there are some months where the fitted value of the excess stock return has been negative. Those months are related to the recession periods, and especially to the beginnings of the recessions, showing very low investment opportunities. At those periods, it seems that even higher conditional variance (i.e. higher risk) does not guarantee the positiveness of the excess return. This is in line with the findings of Guo and Whitelaw (2006). They argue that the inclusion of the hedge component in the conditional ICAPM may lead to the result that the expected stock can be negative although this is intuitively implausible. Perez-Quiros and Timmermann (2001) also find similar evidence in their Markov switching model that the expected stock returns are negative from the late expansion to early recession stage of the business cycle.

The results are also in accordance with the findings of Bauwens *et al.* (2006) who, among others, have suggested that neglected regime switches may lead to the excessively persistent GARCH models. In Model 5, the sum of GARCH and ARCH parameters is 0.868. In Model 2, where the regime switches are not taken into account, the sum is considerably higher (0.948) indicating that the QR-GARCH-M model indeed implies less persistent conditional variance.

4.3 Out-of-Sample Forecasting Performance

The estimation results in Section 4.2 suggest that there is indeed statistically significant business cycle specific variation in the conditional mean and variance of the U.S. excess stock returns. Although the main interest lies in the risk-return tradeoff it is also interesting to explore out-of-sample forecasts of the QR-GARCH-M model. This can also be seen as a robustness check against potential overfitting.

In our limited forecasting experiment we compute forecasts for two out-of-sample periods. The first one consists of observations from January 1989 to March 2009, whereas the second period begins in January 1996. The second sample period includes one recession period more compared to the first one. The parameters are estimated using an expansive window approach.[7] We restrict ourselves to one-period forecasts ($h = 1$) for the excess stock market return.

Table 4 reports the root mean square forecasting error (RMSE) and the mean absolute forecast error (MAE) measures of the QR-GARCH-M model relative to the benchmark GARCH-M model. In addition, we also consider a potential qualitative difference between forecasts when the sign of the loss-differential series is examined. In the QR-GARCH-M model, we impose the restrictions implied by Model 5 in Table 3 (i.e. $\beta_1 = \beta_0$, $\alpha_1 = \alpha_0$ and $\psi_0 = 0$). In the GARCH-M model the intercept is excluded (i.e. we use Model 2).

It turns out that the conditional variance is very high related to the recent recession in the U.S. at the end of the out-of-sample in November 2008. This means that especially the QR-GARCH-M model predicts a very high expected excess stock return (over 16%). As the realized return was -8.54% this aberrant observation has a huge impact on the out-of-sample forecasting results. Therefore, in Table 4 we report results with and without this observation.[8]

The results show that excluding the above-mentioned observation, the QR-GARCH-M model yields only a bit better out-of-sample forecasts compared with the simple GARCH-M model when RMSE and MAE are used to measure forecasting accuracy. However, when comparing the residual series of the two models, the forecast error is often smaller in the QR-GARCH-M model. The p-values of the sign test of Diebold and Mariano (1995) are 0.062 (sample 1989 M1–2009 M3) and 0.131 (sample 1996

[7] We execute our out-of-sample forecasting in the same way as Kim and Lee (2008). In particular, they employed an expansive estimation window which is the only feasible selection also in this study. Due to the limited number of recession periods the use of a rolling estimation window is complicated.

[8] In the QR-GARCH-M model the contribution of this single observation to the overall sum of mean square forecast error is as much as about 15%.

M1–2009 M3) showing that qualitatively the QR-GARCH-M model produces slightly superior forecasts. Thus, the in-sample findings of the importance of regime switches based on the business cycle regimes in the risk-return tradeoff are also confirmed in our out-of-sample forecast experiment.

5 Conclusions

We study the risk-return tradeoff in the U.S. stock market by means of a new QR-GARCH-M model. In the model the binary dependent U.S. business cycle indicator is modeled simultaneously with the continuous dependent U.S. excess stock market return with a regime switching GARCH-M model. The QR-GARCH-M model has several advantages related to maximum likelihood estimation and forecast computation compared with closely related models, such as previously suggested regime switching GARCH models.

In the previous literature, findings on the sign of the risk-return tradeoff have been ambiguous. Our empirical results show that there is evidence for a positive relationship between the conditional mean and the conditional variance of returns irrespective of the state of the business cycle. Recently, Lanne and Saikkonen (2006) failed to find a positive risk-return tradeoff in the simple GARCH-M model with an intercept in the conditional mean equation. However, when allowing for regime switching, it is necessary to include an intercept term in the mean equation for the recession regime to find a positive relation. This is consistent with the idea of the conditional ICAPM because the regime switching structure of the model, based on the state of the business cycle, can be interpreted as describing time-varying investment opportunities implied by the conditional ICAPM. The results also show that the strength of the risk aversion appears to be significantly higher in the recession regime compared with the expansion regime. In addition, in accordance with previous studies, the conditional variance turns out to be higher in recession periods.

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Appendix: Conditional Variance in the QR-GARCH-M Model

This appendix derives the conditional variance of r_t in the QR-GARCH-M model ((8) and (11)). That is,

$$
\begin{aligned}
\mathrm{Var}_{t-1}(r_t) &= E_{t-1}\Big[r_t - E_{t-1}(r_t)\Big]^2 \\
&= E_{t-1}\Big[(1-y_t)\Big(\psi_0 + \delta_0 h_{0t} + h_{0t}^{1/2}\epsilon_t\Big) + y_t\Big(\psi_1 + \delta_1 h_{1t} + h_{1t}^{1/2}\epsilon_t\Big) \\
&\quad -(1-p_t)\Big(\psi_0 + \delta_0 h_{0t}\Big) - p_t\Big(\psi_1 + \delta_1 h_{1t}\Big)\Big]^2 \\
&= E_{t-1}\Big[(p_t - y_t)\Big(\psi_0 + \delta_0 h_{0t}\Big) + (y_t - p_t)\Big(\psi_1 + \delta_1 h_{1t}\Big) \\
&\quad +(1-y_t)h_{0t}^{1/2}\epsilon_t + y_t h_{1t}^{1/2}\epsilon_t\Big]^2 \\
&= E_{t-1}\Big[(p_t - y_t)^2\Big(\psi_0 + \delta_0 h_{0t}\Big)^2 + (y_t - p_t)^2\Big(\psi_1 + \delta_1 h_{1t}\Big)^2 \\
&\quad +(1-y_t)^2 h_{0t}\epsilon_t^2 + y_t^2 h_{1t}\epsilon_t^2 + 2(p_t - y_t)\Big(\psi_0 + \delta_0 h_{0t}\Big)(y_t - p_t)\Big(\psi_1 + \delta_1 h_{1t}\Big) \\
&\quad +2(p_t - y_t)\Big(\psi_0 + \delta_0 h_{0t}\Big)(1-y_t)h_{0t}^{1/2}\epsilon_t + 2(p_t - y_t)\Big(\psi_0 + \delta_0 h_{0t}\Big)y_t h_{1t}^{1/2}\epsilon_t \\
&\quad +2(y_t - p_t)\Big(\psi_1 + \delta_1 h_{1t}\Big)\Big(1-y_t)h_{0t}^{1/2}\epsilon_t + 2(y_t - p_t)\Big(\psi_1 + \delta_1 h_{1t}\Big)y_t h_{1t}^{1/2}\epsilon_t \\
&\quad +2(1-y_t)h_{0t}^{1/2}\epsilon_t^2 y_t h_{1t}^{1/2}\Big].
\end{aligned}
$$

According to the properties of Bernoulli distribution, $E_{t-1}(y_t) = p_t, (0 \le p_t \le 1)$ (see (5)), and

$$E_{t-1}(y_t^2) = p_t.$$

Furthermore, as assumed in Section 3.2, the error term ε_t is independent of y_t. Therefore, we obtain

$$
\begin{aligned}
\mathrm{Var}_{t-1}(r_t) &= (-p_t^2 + p_t)\Big(\psi_0 + \delta_0 h_{0t}\Big)^2 + (-p_t^2 + p_t)\Big(\psi_1 + \delta_1 h_{1t}\Big)^2 + (1-p_t)h_{0t} + p_t h_{1t} \\
&\quad +2(p_t - p_t^2)\Big(\psi_0 + \delta_0 h_{0t}\Big)\Big(\psi_1 + \delta_1 h_{1t}\Big) + 2(p_t - p_t)h_{0t}^{1/2}h_{1t}^{1/2} \\
&= p_t(1-p_t)\Big(\psi_0 + \delta_0 h_{0t}\Big)^2 + p_t(1-p_t)\Big(\psi_1 + \delta_1 h_{1t}\Big)^2 + (1-p_t)h_{0t} + p_t h_{1t} \\
&\quad +2p_t(1-p_t)\Big(\psi_0 + \delta_0 h_{0t}\Big)\Big(\psi_1 + \delta_1 h_{1t}\Big) \\
&= (1-p_t)h_{0t} + p_t h_{1t} + p_t(1-p_t)\Big((\psi_0 + \delta_0 h_{0t}) + (\psi_1 + \delta_1 h_{1t})\Big)^2.
\end{aligned}
$$

Table 1: Descriptive statistics.

	whole sample	expansion $y_t = 0$	recession $y_t = 1$
Mean	0.366	0.548	-0.660
St. Deviation	4.465	4.012	6.356
Skewness	-0.577	-0.701	-0.039
Kurtosis	5.056	5.832	2.881
Observations	591	502	89
Jarque-Bera	136.872	208.770	0.074
p-value	0.000	0.000	0.964

Notes: Descriptive statistics for monthly U.S excess stock returns. The Jarque-Bera test tests the normality of excess stock returns.

Table 2: Estimation results of the autoregressive probit model (7) for the business cycle indicator in the QR-GARCH-M model.

π_t	QR-GARCH-M		
w	0.034		
	(0.023)		
π_{t-1}	0.880		
	(0.011)		
r_{t-1}	-0.113		
	(0.016)		
SP^{US}_{t-6}	-0.138		
	(0.025)		
SP^{GE}_{t-3}	-0.087		
	(0.018)		
log-likelihood	-64.132	$CR_{50\%}$	0.967
pseudo-R^2	0.650	PT	20.687
BIC	92.473	(p-value)	(0.000)

Notes: The sample period is 1961 M1–2009 M3. In table, SP^{US} is the U.S. term spread and SP^{GE} is the German term spread. Robust standard errors (18) are given in parentheses. The pseudo-R^2 measure (Estrella, 1998) is the counterpart to the coefficient of determination used in models with continuous dependent variables. The BIC is the Schwarz information criterion and $CR_{50\%}$ the percentage of correct signal predictions when the 0.50 threshold is applied for probability forecasts. PT is the test statistic of the the market timing test of Pesaran and Timmermann (1992).

Table 3: Estimation results of QR-GARCH-M models for the excess stock returns.

	GARCH-M Model 1	GARCH-M Model 2	QR-GARCH-M Model 3	QR-GARCH-M Model 4	QR-GARCH-M Model 5
ψ_0	0.366		0.203	0.133	
	(0.343)		(0.447)	(0.486)	
δ_0	0.018	0.035	0.036	0.040	0.048
	(0.018)	(0.009)	(0.028)	(0.030)	(0.011)
ω_0	1.100	1.213	1.526	1.880	1.987
	(0.522)	(0.546)	(0.804)	(0.907)	(0.869)
β_0	0.821	0.826	0.794	0.777	0.773
	(0.041)	(0.041)	(0.057)	(0.061)	(0.061)
α_0	0.136	0.122	0.107	0.098	0.095
	(0.038)	(0.032)	(0.037)	(0.032)	(0.030)
ψ_1			-8.743	-5.805	-5.916
			(5.911)	(2.071)	(2.083)
δ_1			0.209	0.127	0.131
			(0.163)	(0.052)	(0.052)
ω_1			12.286	7.259	7.450
			(8.418)	(3.046)	(3.058)
β_1			0.637	β_0	β_0
			(0.232)		
α_1			0.072	α_0	α_0
			(0.056)		
ν	7.212	7.292	7.748	7.753	7.757
	(1.917)	(1.948)	(2.100)	(2.081)	(2.082)
log-likelihood	-1715.24	-1715.77	-1703.38	-1703.81	-1703.84
BIC	1734.32	1731.67	1738.36	1732.43	1729.28

Notes: The sample period is 1961 M1–2009 M3 and the number of observations is 578. Robust standard errors (18) are given in parentheses. The reported value of the log-likelihood function is for the whole QR-GARCH-M model including also the autoregressive probit model for the business cycle indicator y_t (see Table 2). The BIC is the Schwarz information criterion. In Models 2 and 5, the intercept ψ_0 is excluded from the model, whereas in Models 4 and 5, the GARCH and ARCH parameters are restricted to the same in both regimes ($\beta_0 = \beta_1$ and $\alpha_0 = \alpha_1$).

Table 4: Out-of-sample performance of the QR-GARCH-M model for excess stock returns.

	Sample	RMSE	MAE	sign
	1989 M1-2009 M3	1.102	1.017	0.560
	1996 M1-2009 M3	1.119	1.021	0.560
excl. Nov 2008	1989 M1-2009 M3	0.999	1.000	0.562
excl. Nov 2008	1996 M1-2009 M3	0.990	0.998	0.563

Notes: Table reports the ratio of the forecast error criteria of the QR-GARCH-M model relative to the benchmark GARCH-M model. The employed QR-GARCH-M model is Model 5 presented in Table 3. RMSE denotes the root mean square error and MAE the mean absolute forecasting error. "Sign" states the percentage of months when the forecast error has been smaller in the QR-GARCH-M model compared with the GARCH-M model. In both models, the parameters are estimated by the expansive window of observations. In the last two cases November 2008 is excluded from the forecast evaluation sample.

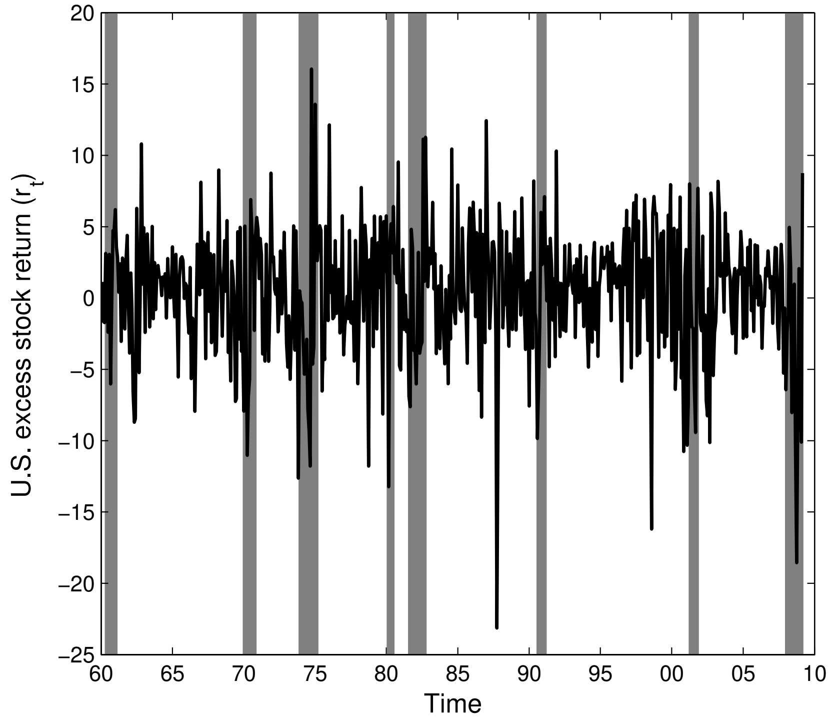

Figure 1: Excess stock returns r_t on the CRSP index and the values of the U.S. business cycle indicator y_t for the sample period from February 1960 to March 2009. The shaded areas are the recession periods ($y_t = 1$).

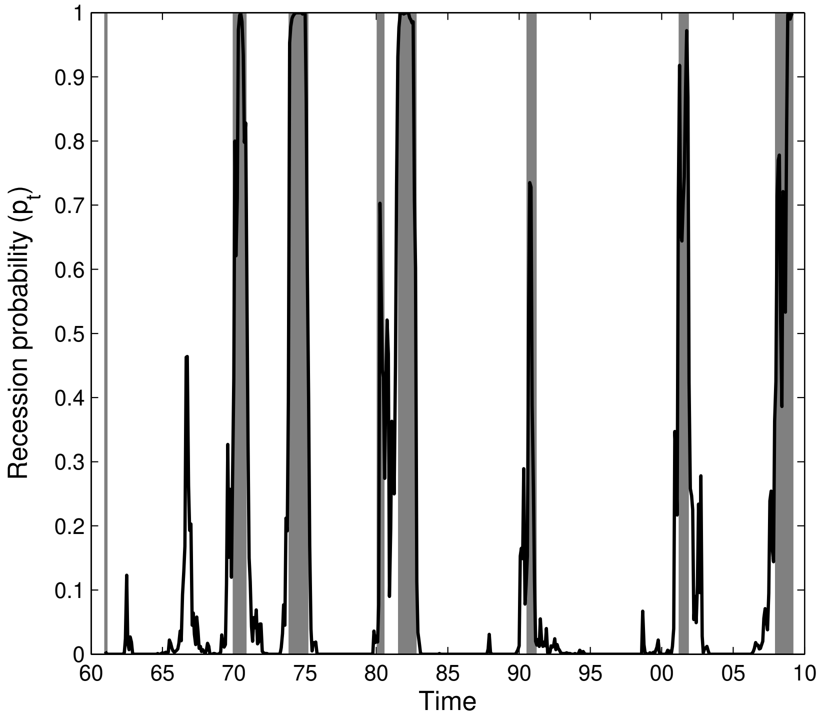

Figure 2: Recession probability p_t implied by the autoregressive probit model (7) presented in Table 2. The shaded areas are the recession periods ($y_t = 1$).

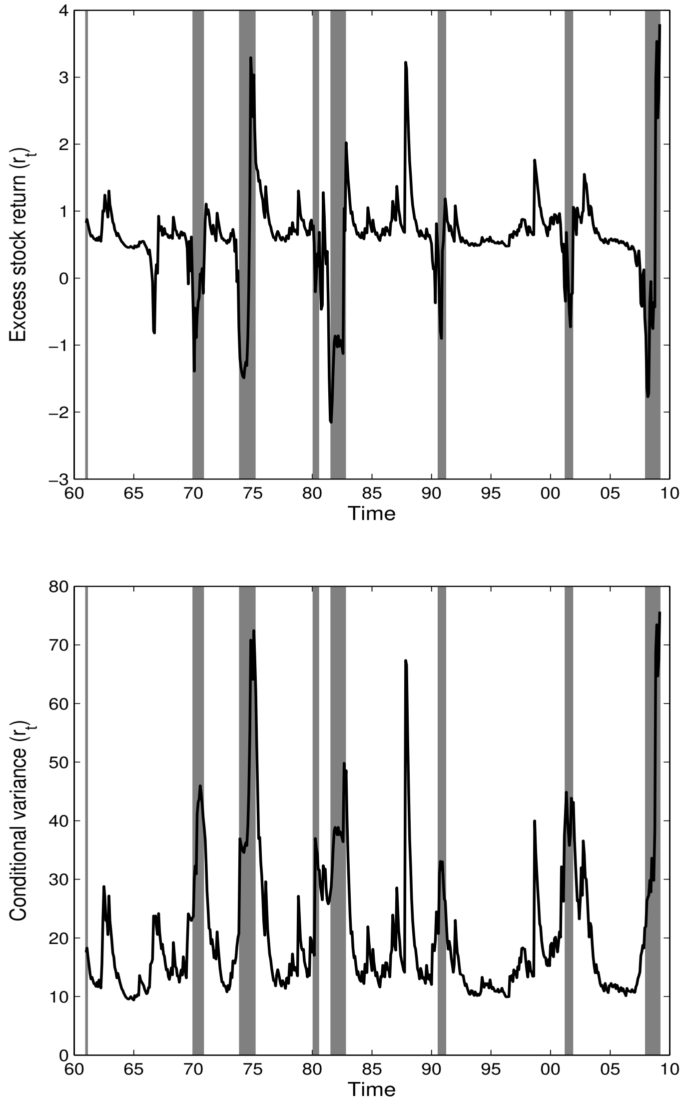

Figure 3: Fitted values for the U.S. excess return (upper panel) and its conditional variance (see (21)) (lower panel) from Model 5 in Table 3. The shaded areas are the recession periods ($y_t = 1$).