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A DIFFERENTIAL GAME APPROACH IN THE CASE OF A POLLUTING OLIGOPOLY

By

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Abstract
In this paper we propose an oligopolistic market model of pollution, where demand is not linear and firms are revenue maximizers. Additionally we assume that the rate of purification is very small tending to zero and that each firm accumulates a pollution share depending for example on firm's size. The game ends up with Markov strategies employed by all firms. Our findings show that under conditions it is possible a marginal decrease on the total pollution stock to increase firms' discounted revenues. A reallocation caused by a uniform decrease in all firms pollution, reorders the marginal change of the pollution stocks in reverse of the original order of the allowed pollution.

Keywords: Non linear strategies; Markov equilibrium; allowed pollution stock.

JEL classification codes: C72, C73, Q58

1. Introduction

In this study we present a dynamic model of polluting oligopolists, where firms compete $\dot{\alpha}$ la Cournot. Former models of Cournot oligopolistic markets are static and with or without linear demand functions. The existing literature on such models of polluting oligopolists can be distinguished in two general categories, depending on the resulting strategies employed by the rivals. The first originates in Tsutsui and Mino (1990) who established non-linear rivals strategies in a linear quadratic setting and the second is the broad category that results in linear such strategies into the same dynamic setting (see Dockner and Long, 1993; Carlier, 2008). However all of the existing models focus on a linear demand function resulting in a linear quadratic differential game.

Here, we consider a more general formulation of the demand function, which covers the above demand functions and the linearity of demand is a special case. The assumption about demand results in a non quadratic differential game. The usual assumptions in this class of pollution games is of quadratic utility functions and linear constraints depending both on the output of all firms and on the rate of the natural purification too. Specifically, we relax the last purification assumption, which means that the damage caused by pollution is permanent or the rate of purification is very small approaching zero, a fact that is verified for every kind of pollutants having very long life time.

To that end we tight the assumption that pollution is accumulated by all firms in the industry. We assume that each firm accumulates its own pollution stock that is allowed by the regulator and the above variable size stock is dependent for example on the firm's size. This assumption in our opinion is more flexible than the usual accumulated pollution assumption and results in an easier manipulation by the regulator (government or organization) of the total industrial pollution.

The proposed model relies on a very simple idea. Once a representative oligopolistic firm is endowed with its own allowed pollution stock and Markovian strategies together with the value function of the quantity produced, as a function of the pollution stock, is computed then it is easy for the regulator to manage the pollution allowance in such a way as to achieve lower total pollution together with higher revenues without taxes levying. Although it is expected that the more the firm's revenues by higher production the more the firm's own pollution, it is possible as further analysis reveals to have lower private pollution levels together with higher private revenues. In the rest of paper we explore the conditions under which the latter will be true. Applying a more general than linear demand function in which curvature plays the crucial role the conditions of optimality with respect to lower pollution are achieved.

The structure of the paper is as follows. Section 2 reviews the existing relative literature. Section 3 sets up the basic model of polluting oligopolists, while section 4 presents the strategies of the Markov Perfect Nash Equilibrium (MPNE) and the value function of the oligopolists discounted revenues. Section 5 proposes policies that may be followed by a social planner in order to reduce industry's pollution. The last section concludes the paper.

2. Literature review

A dynamic game is considered as a case of extensive game (Fudenberg and Tirole, 1993). Following the economic theory and in the case of the classical oligopoly game, the player who sets the lowest price gains the entire market (Cayseele and Furth, 1996). In the dynamic duopoly case one player gains what the other looses (Sorger, 1995; Howroyd and Russell, 1984).

The Cournot duopoly model is probably the first case of bounded rationality in the economics literature, which defines the Cournot-Nash strategy in the space of quantities (Szidarovszky and Yen, 1995). However it relies on the strict assumption that firms are able to observe both the quantity produced and sold by their rivals. There are attempts in modeling the lack of exact knowledge by duopolists (Kirman, 1975). Gates et al. (1982) consider also linear demand functions for differentiated products assuming a learning process.

Naimzada and Sbragia (2005) using the gradient dynamics adjustment process adjust the firms' production in the direction indicated by their (correct) estimate of the marginal profit. Leonard and Nishimura (1999) employ an arbitrary non linear demand without full information. Firms in a Cournot model do not observe their rival's actions, making mistaken beliefs. The above assumptions destroy the stability of equilibrium and create cycles, so the dynamics of the Cournot model are also affected.

A different approach, given by Bylka et al. (2000), studies the situation where oligopolistic firms compete with a global demand constraint. The evolution of firms' market demand is determined by all firms' price decisions. Their interests focus into analysis of some simple classes of strategies and to find the best responses to them. To that end, Huck et al. (2002) report results of the Cournot best reply process. In a Cournot oligopoly with four firms, linear demand and linear cost functions, the best reply process explodes. They also investigate the power of several learning dynamics to explain the unpredicted stability.

The game theory aspects that have been explored rely on the existence of open-loop and Markov Perfect Nash Equilibrium (MPNE) (Fershtman and Muller, 1984; Amir, 1989; Sundaram, 1989; Dockner and Sorger, 1996) as well as the equilibrium stability paths in the case of complex dynamic behavior (Dockner et al., 1996; Mitra and Roy, 2007; Zhang, 2007). On the other hand, Fudenberg and Tirole (1983) and Reynolds (1991) study Nash equilibria in the case when players use closed-loop (Markovian) strategies. Fudenberg and Tirole examine Markov strategies when players maximize the time average payoff. Reynolds assumes a symmetric linear-quadratic differential game but his game is a standard one in continuous-time games solved using dynamic programming methods.

Applications of the Markov concept can be found on resource extraction (Dutta and Sundaran, 1988; Levhari and Mirman, 1980; Sundaran, 1989), on dynamic monopoly or oligopoly (Benadou 1989; Eaton and Engers, 1990; Harris, 1988; Kirman and Sobel, 1974; Maskin and Tirole, 1987, 1988a,b), on bequest equilibria (Bernheim and Ray, 1989; Harris, 1985) and on research and development (Harris and Vickers 1987).

Another model of pollution found in literature that follows the original Tsutsui and Mino (1990) paper with respect to extracted strategies is by Fujiwara and Matsueda (2007). In their note they report the property of a nonlinear feedback Nash strategy equilibrium in the linear quadratic dynamic game. In their dynamic model of pollution, with a linear demand function, no state variables enter the objective functionals of the players, which are two polluting firms. But with this deviation, the lack of the state variables into payoffs, one would expect static and dynamic equilibrium coincidence. However, they demonstrate that the above conjecture is not true if a nonlinear feedback strategy is employed. Their results indicate broader applicability of the novel finding of Tsutsui and Mino (1990), i.e. certain feedback strategy equilibrium approximates the monopoly outcome, provided that the discount rate is sufficiently small.

Our model differs ultimately from Fujiwara and Matsueda's note. One basic difference is the demand function, which is a nonlinear one. Another difference is the value function of the firm's discounted revenues that is computed and its implications on firm's output and pollution stock. The only similarity to that model it seems to be the fact that no state variable enters into the players' objective function. To that end the basic difference is not only the statement of the problem approach but the main result as well.

In the field of pollution regulation Haucap and Kirstein (2003) compare the incentive effects of pollution taxes versus pollution permits for a budget oriented government. Pollution permits are analyzed as durable goods, while pollution tax is seen as being equivalent to leasing out pollution permits. Implicitly they ask what policy instruments different types of government would prefer and, given the government incentives, how environmental policy should be designated from a welfare perspective. They start from the idea that pollution taxes can be seen as a leasing solution to preserve a monopoly power of a revenue maximizing government.

In their two period model they analyze the efficiency characteristics of durable pollution permits versus taxes and identify circumstances under which either policy instrument is preferable from an efficient point of view, given policymaker's incentives. One major conclusion extracted from the paper is the fact that environmental policy based on durable pollution permits can yet be welfare superior to a pollution tax. This is caused by the credibility problem that weakens the government ability to commit to the monopoly quantity in the long run.

In the same field of pollutants regulation Wirl (1996) explores the hypothesis that the politicians have, in contrast to the normative literature, an interest in tax revenues. Wirl conjectures that the Leviathan motive will raise taxes and this will mitigate the tragedy of commons, despite the ignorance of international spillovers by national government. The political message extracted form Wirl's paper, in the topic of global warming, is a warning to "Greens" that appealing to the Leviathan motives of governments may ease the introduction of carbon taxes but may ultimately harm the environment.

Here in our proposed model we make use of the dynamic modeling because is the more general modern perspective that extracts several results depending on the informational structure employed by the game. For this reason we study the stronger with respect to time consistency version, that is the closed loop or Markovian type of equilibrium (while the weaker is open loop), and extract the associated conclusions. We could infer that the choice between using open or closed loop strategies is a matter of analytical feasibility and tractability but not only economic justification. At the same time and in our case we use differential games but we maximize discounted revenues.

3. The basic model

Let us assume that there are N firms in an oligopoly market. Firm i accumulates a stock of pollution $P_i(t)$ at time t, with $P_i(0) = P_i^0$. Let $P(t)$ denote the sum of all pollutants at time t, that is

$$P(t) = \sum_{i=1}^{N} P_i(t)$$

We define $P_{-i}(t) = P(t) - P_i(t)$. We assume that the rate of change of firm's i pollution stock is

$$\dot{P}_i(t) = q_i(t)$$

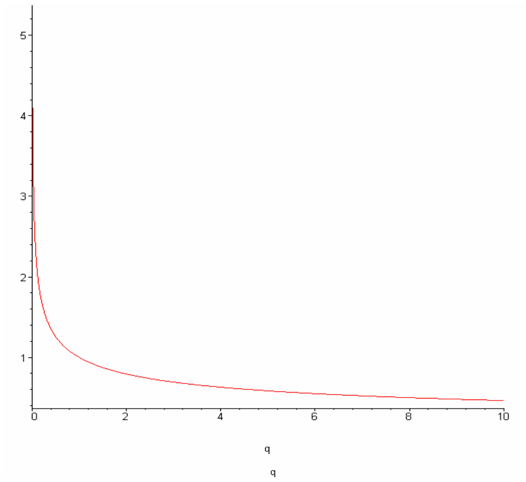
where $q_i(t)$ is firm's i output at time t. The inverse demand function is given by

$$D\left(q(t)\right)=\left(q(t)\right)^{-\left(\frac{1}{a}\right)}$$

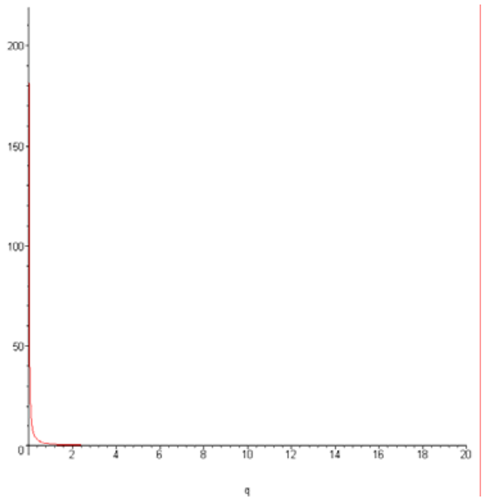
where $\alpha \in (0,+\infty)$ and $q(t)=\sum_{i=1}^{N} q_i(t)$ is the total quantity.

The negative parameter $-\left(\frac{1}{a}\right)$ determines, in absolute value, the elasticity of demand, i.e. the inverse demand function is elastic if $a \in (0,1)$, inelastic if $a \in (1,+\infty)$ and takes the hyperbolic shape if $a=1$, but is always convex as figure 1 shows.

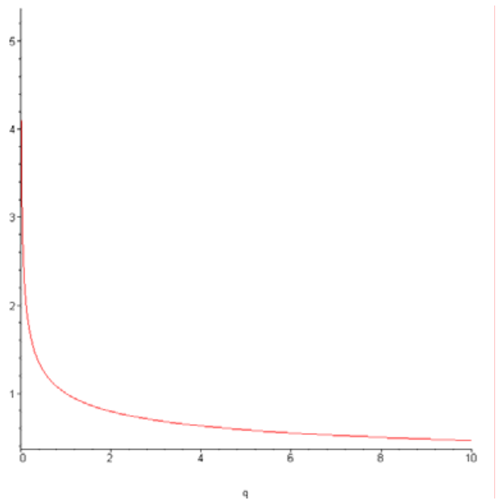
Figure 1: Industry's demand



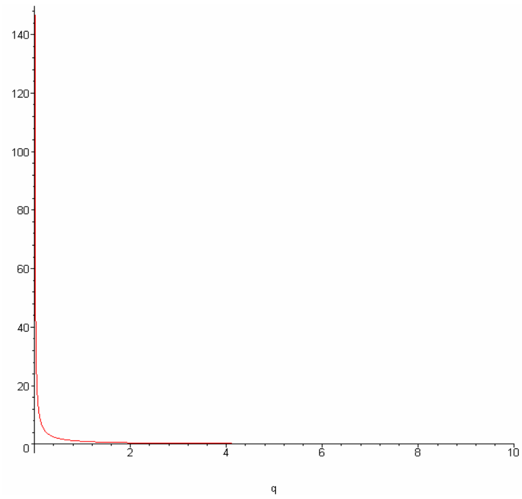
(a) (α=10)



(b) (α=0.8)



(c) (α=3)



(d) (α=1)

Here in order to form the dynamic problem we neglect the production costs, so firms in industry are rather revenues maximizers, an approach that is not usual in practice, since no state variable enters into the objective functional, but still compatible with the differential game literature. Having these assumptions the dynamic can be presented as follows. Firm's i revenues are given by the expression:

$$R_i\left(q_i, q_{-i}\right) = q_i\left(q_i + q_{-i}\right)^{-\left(\frac{1}{a}\right)}$$

where $$q_{-i} = q - q_i$$

The objective function of firm i is to maximize the present value of the stream of cash flow subject to the system dynamics, that is the problem

$$\max \int_0^\infty q_i\left(q_i + q_{-i}\right)^{-\left(\frac{1}{a}\right)} e^{-rt} dt \qquad (1)$$

subject to $$\dot{P}_i(t) = q_i(t) \qquad (1a)$$

with $$P_i(0) = P_i^0$$

The choice variable of firm i is its quantity q_i, while the state variable is its accumulated pollution stock P_i.

We seek to find a strategy and the value function of the dynamic problem under the Closed Loop[11] or Markovian Nash informational structure equilibrium which is by definition the concept of equilibrium in which the choice of player's i current action is conditioned on current time t and on state vector too.

Under the closed–loop informational structure and stationarity of the game the player's i strategy space is this of mappings

$$\phi_i : \mathbb{R}_+^n \to \mathbb{R}$$

[1] For more details about the informational structures of the dynamics games, see Olsder and Basar (1998).

which associates to a vector of pollution stock $(P_1, P_2, ..., P_N) \in \mathbb{R}_+^n$ the quantity $\phi_i(P_1, P_2, ..., P_N)$ to produce. Each player i of the game has to choose a quantity $q_i(t) \in \mathbb{R}$ of good to produce. The price of that good is then set according to

$$D(q_1, q_2, ..., q_N) = \left(\sum_{i=1}^{N} q_i \right)^{-\left(\frac{1}{a}\right)}$$

The payment (total revenues) of the firm i is the given by

$$U_i : (\phi_1, \phi_2, ..., \phi_N) \to \int_0^\infty D(\phi_1(P), \phi_2(P), ..., \phi_i(P), ..., \phi_N(P)) e^{-\rho t} dt$$

where $(P_k)_{k=1,...,N}$ evolve according to the differential equation determined by $(1a)$. An equilibrium should then be defined as a set of strategies for which no player has a profitable deviation.

Imposing this assumption on informational structure of the game, clearly the history of the game is important and is reflected in the current value of the state vector. Consequently, player's i optimal time paths take into account at any point of time the control variables of the other players. This type of equilibrium affects the state variables, requiring a revision of the player's i controls at any time instant. Here we apply the Hamilton – Jacobi – Bellman (HJB) equation in order to prove that the conjectured strategy we propose is a Markovian strategy and consequently a strong time consistent one. In contrast to the open loop informational structure the closed loop is a strongly time consistent one, but the open loop is not. Here the time consistent property is in the sense of sub–game perfectness (for more details see Dockner et al. 2000).

4. Markov Perfect Nash Equilibrium (MPNE)

We denote by ϕ_i the strategy that specifies firm's i production rate as a function of time t and the vector of pollution stocks accumulated at the same time. This is the strategy

$$q_i(t) = \phi_i(P(t))$$

Each firm takes competitors strategies as given and determines its optimal strategy that solves problem (1) with constraint $(1a)$.

Proposition 1.

A MPNE exists, where the equilibrium strategy of firm i has the property that its output level depends only on its pollution stock. That is

$$q_i = arP_i \qquad i = 1, ..., N$$

The discounted sum of firm's i revenues $V_i(P)$, when the total pollution is P, are given by

$$V_i(P) = (ar)^{-\left(\frac{1}{a}\right)} P_i \left(\sum_{k=1}^{N} P_k\right)^{-\left(\frac{1}{a}\right)} \tag{2}$$

Proof (See appendix)

A special case

Consider for a moment that elasticity of demand equals to one, $a = 1$. As it is simply clear in this case the market demand function collapses to a hyperbolic shape, this being a special case of a more general class of models based on isoelastic demand curves. It is well known from the literature[2] in such a case the maximum problem of a firm choosing the output level is indeterminate if marginal cost is zero, since the

[2] For an exposition of a differential oligopoly model where firms face implicit menu costs of adjusting output over time due to sticky market price, see Lambertini (2007).

revenues generated by a hyperbolic demand are constant, thus economically unacceptable. But even in this special case our model under closed loop informational structure yields economically acceptable strategies and value function as well. More precise setting demand elasticity to one, $a=1$, the model solution yields the following results for strategies and value function respectively:

$$q_i = rP_i \qquad i=1,...,N \tag{3}$$

$$V_i\left(P\right)=\left(r\right)^{(-1)} P_i\left(\sum_{k=1}^{N} P_k\right)^{(-1)} \tag{4}$$

The latter reasoning leads us to conclude the following corollary.

Corollary 1

The above proposed model of pollution even in the case of hyperbolic demand, so for constant revenues, yields economically acceptable strategies and value functions given by $\left(3\right),\left(4\right)$ *respectively.*

5. Policies in the allowed pollution stock

We consider now the impact of a small change in the allowed pollution stock imposed by an authority into the firms' value function. For this purpose we investigate the total differentiation of the value function

$$V_i\left(P\right)=\left(ar\right)^{-\left(\frac{1}{a}\right)} P_i\left(\sum_{k=1}^{N} P_k\right)^{-\left(\frac{1}{a}\right)}$$

with respect to pollution, that is

$$dV_i = \frac{\partial V_i}{\partial P_i} dP_i + \sum_{j\neq i, j=1}^{N} \frac{\partial V_i}{\partial P_j} dP_j \tag{5}$$

In order to have a unified result into the previous found value function of each firm we record the following proposition.

Proposition 2.

A marginal increase in the total pollution stock, affects incrementally the discounted firm's i revenues, if the inequality $\frac{1}{a}\frac{P_i}{P} > \frac{dP_i}{dP}$ holds, otherwise an increase in the total pollution stock reduces the discounted sum of firm's i revenues.

Proof (See Appendix)

The total derivative of the value function after manipulations (see in the appendix) is given by the expression

$$dV_i = \left(arP\right)^{-\left(\frac{1}{a}\right)} dP\left[\frac{P_i}{aP} - \frac{dP_i}{dP}\right] \qquad (6)$$

Since the term $\left(arP\right)^{-\left(\frac{1}{a}\right)}$ of (6) always measures the aggregate demand, as $q = \sum_{i=1}^{N} q_i = \left(ar\sum_{i=1}^{N} P_i\right)^{-\left(\frac{1}{a}\right)} = \left(arP\right)^{-\left(\frac{1}{a}\right)}$ and we have set $\sum_{i=1}^{N} P_i = P$, the rest of term (6) $dP\left[\frac{P_i}{aP} - \frac{dP_i}{dP}\right]$ measures the amount multiplied with the total demand, giving the total marginal change on the discounted revenues.

Furthermore, we assume that the sign of (6) is negative and results in a decrement of the discounted revenues, that is, $\frac{1}{a}\frac{P_i}{P} < \frac{dP_i}{dP}$ and firms are ranked by an increasing order of the allowed pollution stock $P_1 < P_2 < \ldots P_i < P_{i+1} < \ldots < P_N$, so the initial pollution shares are $\frac{P_i}{P} < \frac{P_{i+1}}{P}$. We have

$$\frac{dP}{a}\frac{P_i}{P} < dP_i \qquad\qquad \frac{dP}{a}\frac{P_{i+1}}{P} < dP_{i+1}$$

Subtracting the LHS and RHS of the two relations we have

$$\frac{dP}{aP}\left(P_i - P_{i+1}\right) < dP_i - dP_{i+1} \qquad (7)$$

The LHS of (7) is a negative number. So we have $dP_i > dP_{i+1}$ $(\forall i)$. Following (7) we conclude the following corollary.

Corollary 2

In the case of a decrement in the discounted revenues caused by a marginal increment of the total allowed pollution stock, dP*, the order of marginal increments of individual firms,* dP_i*, is ranked by the reverse order rather than the originally allowed set of pollution stocks. That is, if* $P_1 < P_2 < ... < P_N$ *the result in the above marginal decrease is* $dP_1 > dP_2 > ... > dP_N$

The impact of an absolute increase to the pollution stocks $dX_i = \varepsilon$ $(\forall i)$ can be expressed as follows.

Corollary 3

A uniform absolute increase in all pollution stocks by $dP_i = \varepsilon > 0$ *reduces firm's* i *discounted revenues if and only if* $\frac{P_i}{P} < \frac{a}{N}$.

Proof

The result is easily obtained since $dP = \sum_{i=1}^{N} dP_i = \varepsilon N \Rightarrow \frac{dP_i}{dP} = \frac{1}{N}$ and $\frac{P_i}{aP} < \frac{dP_i}{dP}$.

Next we consider a new allocation of the allowed pollution stocks. With P_i^O we denote firm's i old allowed pollution and with P_i^N the reallocated (new) allowed pollution stock. Moreover we assume that the new allowed pollution is less than the original, $P_i^O > P_i^N \quad (i = 1,..,N)$.

The next proposition joints the two pollution stocks assuming the last given order.

Proposition 3.

The discounted revenues of each firm increases while the total pollution stock falls, caused by a new allocation, if and only if $\sigma_i > (1-\sigma)^{\left(\frac{1}{a}\right)} + 1$, *where*

$$\sigma = 1 - \frac{\left(\sum_{k=1}^{N} P_k^N\right)}{\left(\sum_{k=1}^{N} P_k^O\right)} \textit{ and } \sigma_i = 1 - \frac{P_i^N}{P_i^O}.$$

Proof (See Appendix)

Remark

The results of Proposition 3 may be used as follows. Suppose that an authority decides to decrease the total pollution by an amount ΔP ($\sigma = \frac{\Delta P}{\sum_{i=1}^{N} P_i}$), so in order to have each firm higher revenues, its allowed pollution stock must be reduced by the amount $P_i^O - P_i^N = \Delta P_i = \left((1-\sigma)^{\left(\frac{1}{a}\right)} + 1\right) P_i^O$.

In the same way we consider a uniform decrease η to all firms' pollution. Then

$$\sigma = 1 - \frac{N\eta + \sum_{i=1}^{N} P_i^O}{\sum_{i=1}^{N} P_i^O} = \frac{N\eta}{\sum_{i=1}^{N} P_i^O}$$

and the raised revenues requirement is $\sigma > 1 - (\sigma_i - 1)^{-\left(\frac{1}{a}\right)}$

and finally $$\frac{N\eta}{\sum_{i=1}^{N} P_i^O} > 1 - (\sigma_i - 1)^{-\left(\frac{1}{a}\right)}$$

where $P^O = \sum_{i=1}^{N} P_i^O$ is the initial allocation of the pollution and $\sigma_i = 1 - \frac{P_i^N}{P_i^O}$ the percentage change on firm's i allowed pollution stock.

6. Concluding remarks

In this paper we set up a very simple model of polluting oligopolists where the demand is not linear and the resulting game is not a linear quadratic one. We also make the assumptions that each firm is allowed to pollute to a variable size depending on the criterion that is given by an authority. The results, in our opinion, are useful for an authority to make distributed pollution policies on the industry in total as well as partially on a firm.

One conclusion that could be drawn as a result of the above model is that a new technology that reduces the total amount of the accumulated pollution stock is not necessarily welcomed by all firms in the industry. If for example any authority decides to improve the technology that is used by firms previous analysis shows that the bigger, with respect to the pollution stock, firm does not always benefit from this decision.

Specifically, our results on a strong time consistent (Markov) equilibrium with conjectured value function and strategies are surprising. Although without exposing the solutions of the problem in full generality, as Tsutsui and Mino (1990) face their linear quadratic differential game in a duopoly with sticky prices, a strong time consistent solution is obtained using the conjectured method.

Moreover testing the above strategies and value function obtained we are able to conclude many policy results. As the pollution is accumulated by product in an industry we expect for each firm the higher the production process is the more the pollution stock will be. However, the findings of the model are slightly different. It is possible a marginal decrease on the total pollution stock to increase the firms' discounted revenues, provided that the original allowed pollution share multiplied by the elasticity of demand is greater than the marginal change share.

Additionally, a reallocation caused by a uniform decrease into all firms pollution, reorders the marginal change of the pollution stocks in reverse to the original order of the allowed pollutions and again the reallocation is possible to raise the discounted revenues of each firm.

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Appendix
Proofs of Propositions

Proof of Proposition 1

First we check that if firm's j strategy is $q_j = arP_j$, then firms' i best response will be $q_i = arP_i$. The Hamilton-Jacobi-Bellman (hereafter HJB) equation for firm's i maximization problem is the following

$$rV_i = q_i \left(q_i + arP_{-i}\right)^{-\left(\frac{1}{a}\right)} + \frac{\partial V_i}{\partial P_i} q_i + \sum_{j \neq i, j=1}^{N} \frac{\partial V_i}{\partial P_j} \left(arP_j\right)$$

Maximization of the RHS of the HJB equation with respect to q_i gives

$$\left(q_i + arP_{-i}\right)^{-\left(\frac{1}{a}\right)} - \frac{q_i \left(q_i + arP_{-i}\right)^{-\left(\frac{1}{a}\right)}}{a\left(q_i + arP_{-i}\right)} + \frac{\partial V_i}{\partial P_i} = 0$$

or equivalently

$$\frac{\partial V_i}{\partial P_i} = \left(q_i + arP_{-i}\right)^{-\left(\frac{1}{a}\right)} \left[\frac{q_i}{a\left(q_i + arP_{-i}\right)} - 1\right] \qquad (A.1)$$

Where P_{-i} represents the sum of all pollution stocks except firm's i pollution stock, that is $P_{-i} = P - P_i$ and $P = \sum_{j=1}^{N} P_j$

Now we make use of the nonlinear conjectured value function

$$V_i = \left(ar\right)^{-\left(\frac{1}{a}\right)} P_i \left(\sum_{j=1}^{N} P_j\right)^{-\left(\frac{1}{a}\right)}$$

Differentiation of the value function with respect to P_i yields

$$\frac{\partial V_i}{\partial P_i} = -\left(arP\right)^{-\left(\frac{1}{a}\right)} + \frac{\left(arP\right)^{-\left(\frac{1}{a}\right)} P_i}{aP} = \left(arP\right)^{-\left(\frac{1}{a}\right)} \left(\frac{P_i}{aP} - 1\right) \qquad (A.2)$$

with $P = \sum_{j=1}^{N} P_j$ the same as above.

Equating the terms with the same power of $(A.1)$ and $(A.2)$ we have the resulting equations

$$\frac{P_i}{aP}-1=\frac{q_i}{a\left(q_i+arP_{-i}\right)}-1 \qquad (A.3)$$

and
$$\left(arP\right)^{\left(-\frac{1}{a}\right)}=\left(q_1+arP_{-i}\right)^{\left(-\frac{1}{a}\right)} \qquad (A.4)$$

Both equations $(A.3)$ and $(A.4)$ have the same solution

$$q_i=arP_i.$$

Now we prove that substituting the above strategies into the RHS of the HJB function we have equality with the LHS of the same equation. The partial derivative of the value function V_i with respect to P_j is

$$\frac{\partial V_i}{\partial P_j}=\frac{\left(ar\right)^{-\left(\frac{1}{a}\right)}P_iP^{-\left(\frac{1}{a}\right)}}{aP} \qquad (A.5)$$

so the RHS of the HJB is

$$RHS(HJB)=arP_i\left(arP_i+arP_{-i}\right)^{-\left(\frac{1}{a}\right)}+arP_i\left(arP\right)^{-\left(\frac{1}{a}\right)}\left(\frac{arP_i}{a\left(arP_i+arP_{-i}\right)}-1\right)+$$

$$+\sum_{j\neq i,j=1}^{N}\frac{\left(ar\right)^{-\left(\frac{1}{a}\right)}P_iP^{-\left(\frac{1}{a}\right)}}{aP}arP_j=$$

$$=arP_i\left(arP\right)^{\left(-\frac{1}{a}\right)}-arP_i\left(arP\right)^{-\left(\frac{1}{a}\right)}+arP_i\left(arP\right)^{-\left(\frac{1}{a}\right)}.\frac{P_i}{P}-arP_i\left(arP\right)^{-\left(\frac{1}{a}\right)}\frac{P_{-i}}{P}=$$

$$=rP_i\left(arP\right)^{-\left(\frac{1}{a}\right)}\frac{P_i+P_{-i}}{P}=r\left(arP\right)^{-\left(\frac{1}{a}\right)}P_i=rV_i\left(P\right)=LHS(HJB)$$

Where as above we have set $\qquad P=\sum_{j=1}^{N}P_j$

Proof of Proposition 2

The total derivative of the value function is

$$dV_i = \frac{\partial V_i}{\partial P_i} dP_i + \sum_{j \neq i, j=1}^{N} \frac{\partial V_i}{\partial P_j} dP_j \qquad (A.6)$$

Substituting the partial derivatives $(A.2)$ and $(A.5)$ previously found, into $(A.6)$ the derivative of the value function takes the form:

$$dV_i = (arP)^{-\left(\frac{1}{a}\right)} \left(\frac{P_i}{aP} - 1 \right) dP_i + \sum_{j \neq i, j=1}^{N} \frac{(ar)^{-\left(\frac{1}{a}\right)} P_i P^{-\left(\frac{1}{a}\right)}}{aP} dP_j$$

Putting the term $(arP)^{-\left(\frac{1}{a}\right)} \frac{P_i}{aP} dP_i$ inside the sum, the above expression simplifies to

$$dV_i = (arP)^{-\left(\frac{1}{a}\right)} (-1) dP_i + \sum_{j=1}^{N} \frac{(ar)^{-\left(\frac{1}{a}\right)} P_i P^{-\left(\frac{1}{a}\right)}}{aP} dP_j$$

Multiplying and divide the first term the RHS of the latter by $\sum_{j=1}^{N} \frac{(ar)^{-\left(\frac{1}{a}\right)} P_i P^{-\left(\frac{1}{a}\right)}}{aP} dP_j$

we have $$dV_i = (arP)^{-\left(\frac{1}{a}\right)} \left(\sum_{j=1}^{N} dP_j \right) \left(\frac{P_i}{aP} - \frac{dP_i}{\sum_{j=1}^{N} dP_j} \right)$$

Setting $dP \equiv \left(\sum_{j=1}^{N} dP_j \right)$ the latter simplifies to

$$dV_i = (arP)^{-\left(\frac{1}{a}\right)} dP \left[\frac{P_i}{aP} - \frac{dP_i}{dP} \right] \qquad (A.7)$$

The meaning of $(A.7)$ is, as we expect that a change in the allowed pollution stocks results in the same sign change on firm's i discounted revenues depending on the sign of the term inside the brackets. That is, if the sign of the bracketed term is positive an increase in the total pollution stock dP increases the discounted revenues of firm's i, as the term outside brackets reveals and vice versa.

Now consider the term of $(A.7)$ $\left(\frac{P_i}{aP}-\frac{dP_i}{dP}\right)$, which shows how the change on firm's i revenues responds to a marginal change in the pollution stock. The term under consideration has positive sign which means that

$$\frac{1}{a}\frac{P_i}{P}>\frac{dP_i}{dP} \qquad (A.8)$$

i.e. the original allowed pollution share multiplied by the elasticity is greater than the marginal change share.

Proof of Proposition 3

From solution of the original value function we have the two value functions of the discounted revenues

$$V_i\left(P^O\right)=\left(ar\right)^{-\left(\frac{1}{a}\right)}P_i^O\left(\sum_{k=1}^{N}P_k^O\right)^{-\left(\frac{1}{a}\right)} \qquad (A.9)$$

$$V_i\left(P^N\right)=\left(ar\right)^{-\left(\frac{1}{a}\right)}P_i^N\left(\sum_{k=1}^{N}P_k^N\right)^{-\left(\frac{1}{a}\right)} \qquad (A.10)$$

Subtracting $(A.9)$ from $(A.10)$ to have incremental revenues, the positive change in firm's i revenues due to reallocation is

$$\Delta V_i=V_i\left(P^N\right)-V_i\left(P^O\right)=\left(ar\right)^{-\left(\frac{1}{a}\right)}P_i^N\left(\sum_{k=1}^{N}P_k^N\right)^{-\left(\frac{1}{a}\right)}-\left(ar\right)^{-\left(\frac{1}{a}\right)}P_i^O\left(\sum_{k=1}^{N}P_k^O\right)^{-\left(\frac{1}{a}\right)}$$

or $$\Delta V_i=V_i\left(P^N\right)-V_i\left(P^O\right)=\left(ar\right)^{-\left(\frac{1}{a}\right)}P_i^N\left(\sum_{k=1}^{N}P_k^O\right)^{-\left(\frac{1}{a}\right)}\left[\frac{\left(\sum_{k=1}^{N}P_k^N\right)^{-\left(\frac{1}{a}\right)}}{\left(\sum_{k=1}^{N}P_k^O\right)^{-\left(\frac{1}{a}\right)}}-\frac{P_i^O}{P_i^N}\right]$$

The latter expression simplifies denoting by $\sigma = 1 - \frac{\left(\sum_{k=1}^{N} P_k^N\right)}{\left(\sum_{k=1}^{N} P_k^O\right)}$ the percentage decrement into the total pollution and with $\sigma_i = 1 - \frac{P_i^N}{P_i^O}$ the percentage change into firm's i pollution. In order to have an increment into firms' i discounted revenues it suffices to hold the condition

$$(1-\sigma)^{-\left(\frac{1}{a}\right)} > \frac{1}{\sigma_i - 1}$$