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# Information Acquisition and Innovation under Competitive Pressure\*

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## Abstract

This paper studies information acquisition under competitive pressure and proposes a model to examine the relationship between product market competition and the level of innovative activity in an industry. Recent empirical papers point to an inverted-U shape relationship between competition and innovation. Our paper offers theoretical support to these results while employing a more accurate definition of innovation than the previous literature; more precisely, we isolate innovation from riskless technological progress. The firms in our model learn of an invention and decide on whether and when to innovate. In making this decision, firms face a trade-off between seeking a first-mover advantage and waiting to acquire more information. By recognizing that a firm can intensify its innovative activity on two dimensions, a temporal and a quantitative one, we show that firms solve this trade-off precisely so as to generate the inverted-U shape relationship. When the competition in the pre innovation market is sufficiently high, the level of competition in the post innovation market is endogenous. We investigate the welfare effects of innovation under competitive pressure and find conditions that determine the socially optimal level of competition. We study the effects that the degree of technological spread in the industry has on innovation and highlight the roles that strategic uncertainty and the discreteness of the information acquisition process play in this context.

*JEL Classification:* D40, L10

*Keywords:* Innovation, Competitive Pressure, Information Acquisition.

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# 1 Introduction

This paper studies information acquisition under competitive pressure and employs the resulting model to investigate the relationship between the degree of competition in an industry and the level of innovative activity. The clear policy implications of the nature of this relationship generated a large body of literature investigating it. Starting with the seminal work of Schumpeter(1943), the objective of these studies has been to determine whether there is an optimal market structure that results in the highest rate of technological advance. In particular, this literature tried to reconcile the intuitive appeal of the Schumpeter's assertion that only large firms possessing a significant amount of monopoly power have the resources and incentives to engage in risky innovative activity, with a substantial amount of empirical literature that suggested the opposite. Although results vary, more recent empirical papers, such as Aghion, Bloom, Blundell, Griffith and Howitt (2005) (henceforth, ABBGH(2005)) suggest *an inverted-U shape relationship* between product market competition and innovation.<sup>1</sup> Thus, according to these studies, for low levels of competition an increase in competition induces more innovation, while for higher values of competition, as competition increases, firms become less innovative.

Our paper adopts a microeconomic approach and studies the innovation process at firm level by following a new project through its stages of development. The firms in our dynamic model become sequentially aware of an invention and decide on whether and when to undertake a costly investment in innovation. In taking this decision, firms face a trade-off between seeking a first-mover advantage and waiting to acquire more information. There are three novel contributions that our study brings to the literature on innovation. First, we uncover the trade-off between information acquisition and competitive pressure as sufficient to generate the empirically observed inverted-U shape relationship. Second, we identify two dimensions on which a firm can intensify the level of its innovative activity, a risk dimension and a quantitative dimension, and reveal the effect of this breakdown in explaining that relationship. Finally, our model offers theoretical support to the newest empirical findings while employing a more accurate definition of innovation than the one used in ABBGH(2005), which is the only other paper to present a theoretical model that obtains the inverted-U shape relationship.<sup>2</sup> More precisely, the results in ABBGH(2005) hinge on including in the definition of innovation the technological advancements made at no cost by laggard firms who copy the technology of the leader.<sup>3</sup> In contrast, the definition in our model is consistent with

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<sup>1</sup>Scherer (1967) is the first empirical paper to uncover this shape. See also Scott (1984) or Levin et al. (1985).

<sup>2</sup>The vast majority of the theoretical literature on this topic suggested a monotone relationship. For instance, Caballero and Jaffe (1993) or Martin (1993) support the Schumpeterian hypothesis while Schmidt (1997) or Aghion, Harris and Vickers (1997) predict a positive relationship between competition and innovation. Boone (2000) finds conditions under which more competitive pressure induces either more or less innovation, while Boone (2001) presents a model that can generate non-monotone relationships of any nature. See also the discussion in Section 2 of this paper of the Kamien and Schwartz (1976) model.

<sup>3</sup>If the definition of innovation in ABBGH(2005) does not include these zero cost technological advancements, their

the standard interpretation of innovation as being something new, different and usually better than what existed before.<sup>4</sup> We therefore isolate the innovative activity from riskless technological progress. Finally, we also study the effects that the degree of technological spread in the industry has on innovation and highlight the roles that strategic uncertainty and the discreteness of the information acquisition play in this context.<sup>5</sup>

The typical stages in the development of a new commercial product or process are presented in the following figure:<sup>6</sup>

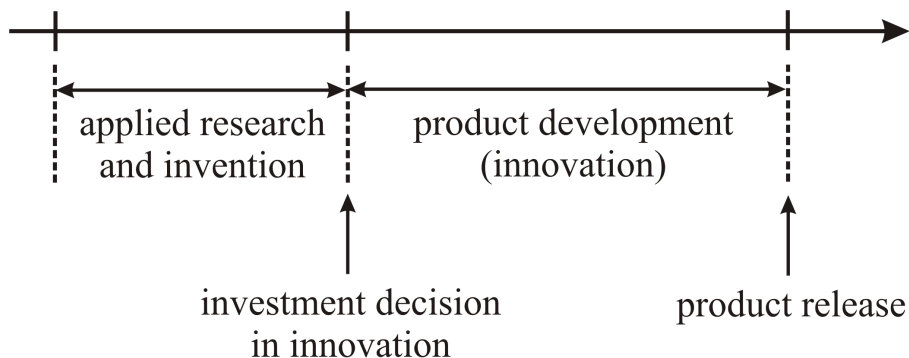


Figure 1

Applied research is aimed at gaining knowledge that will address a specific problem or meet a specific need within the scope of that particular entity; successful applied research results in an invention or the discovery of an idea that should work. In the invention stage, the idea also passes through its first tests. This is the research part of the phrase "research and development". The product development, also called the innovation stage, is the first commercial application of an invention; it requires refinement of the invention and the developing of a marketable product. Large R&D labs spend most of their resources on innovation.<sup>7</sup> The ideas generated through applied

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main monotonicity result does not hold anymore unless the hazard rate of these events is insignificant. However, for small values of this hazard rate, it is straightforward to see that their model predicts that the industry structure will be such that the Schumpeterian effect will always dominate.

<sup>4</sup>Schumpeter(1934) defines economic innovation as the introduction of a new good, the introduction of a new method of production, the opening of a new market, the use of a new input of production or the implementation of a new organizational structure.

<sup>5</sup>Also, our model can be seen as a study of *product* innovation in which new products are introduced in the market. This differs from most of the current theoretical literature on innovation, including ABBGH(2005), which focuses on the study of technological advancements that consist of *process* innovations in which existing products are produced at a lower average cost.

<sup>6</sup>See Mansfield (1968b) for an excellent in depth analysis of the innovation decision making at firm level.

<sup>7</sup>For instance, of the \$208.3 billion spent on industrial R&D in the United States in 2004, \$155.1 billion (or 74.5 percent) were spent for development. Source: National Science Foundation, Division of Science Resources Statistics. 2008. *Research and Development in Industry: 2004*.

research go through a screening process and a small share of them end up being implemented in marketable products.<sup>8</sup>

The model in our paper has a set of firms who, sequentially, become informed about an invention that could render future gains to its investors, provided that it is a success from technological and business standpoints. Once a firm learns of the invention, it has the option of investing in the project at any time. Initially, the firm's knowledge about the feasibility of the project is scarce, so investment is relatively risky.<sup>9</sup> As time passes, the firm acquires additional information, and is able to better assess its chances of success.<sup>10</sup> The additional information may eventually lead the firm to decide not to invest in the project. This may make waiting beneficial because it can potentially help avoid the financial losses associated with the development of an unsuccessful product. On the other hand, in our model, earlier investors end up releasing the product earlier, and thus enjoy a natural first-mover advantage.<sup>11</sup>

These two features of the model induce a trade-off in the firm's problem between investing early to enjoy the first-mover advantage, and waiting to acquire new information and reduce the risk of investment. Mansfield(1968b, p. 105) underlies this trade-off in the firm's decision making process. Thus, as he states, on the one hand, "there are often considerable advantages in waiting, since improvements occur in the new product and process and more information becomes available regarding its performance and market". On the other, "there are disadvantages... in waiting, perhaps the most important being that a competitor may beat the firm to the punch...". He concludes: "if the expected returns... justify the risks and if the disadvantages of waiting outweigh the advantages, the firm should innovate. Otherwise it should wait. Pioneering is a risky business; whether it pays off is often a matter of timing". Now clearly, the presence of this trade-off suggests

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<sup>8</sup>To illustrate this breakdown of the development process, Kotler and Armstrong (2005) quote a management consultant as saying: 'For every 1000 ideas, only 100 will have enough commercial promise to merit a small-scale experiment, only 10 of those will warrant substantial financial commitment, and of those only a couple will turn out to be unqualified successes'.

<sup>9</sup>Mansfield et al. (1977, p. 9) found that the probability that an R&D project would result in an economically successful product or process was only about 0.12; the average probability of technical completion for a project was estimated to be 0.57.

<sup>10</sup>This information can be technological, in the form of test results, or knowledge about the technological trend for the complementary products. For instance, the potential developers of a new hybrid car may have had an incentive to wait, so that more efficient electric batteries would be produced. Second, this new information could also be commercial in the form of marketing research. For instance, the same hybrid car manufacturer may have waited to study whether and how many consumers would be willing to compromise and accept the relatively weaker performance of this new product. Third, the information may come in the form of knowledge about the overall economic environment. For instance, hybrid cars were only moderately, if at all, successful until just a few years ago, but they are in relatively high demand now.

<sup>11</sup>For instance, while there are plenty of hybrid models that have been launched recently, the earlier investors, Toyota and Honda have a clear technological and commercial advantage in that market. As Porter (1990) states: "early movers gain advantages such as being first to reap economies of scale, reducing costs through cumulative learning, establishing brand names and customer relationships without direct competition, getting their pick of best sources of raw materials and other inputs... The innovation itself may be copied but the other competitive advantages often remain". See Lieberman and Montgomery (1988) for a comprehensive review of the theoretical and empirical literature analyzing the first mover advantage in innovation.

that, generically, firms will neither invest immediately in all inventions nor wait until all uncertainty is removed. However, it is not immediately clear how firms adjust their innovative activity in response to a change in competition. Moreover, it is not a priori obvious whether this adjustment is monotonic as most other theoretical papers concluded or non-monotonic as the newest empirical evidence suggests. Studying how firms make these adjustments is the main objective of the paper.

A more innovative industry is defined to be one in which firms allocate a larger budget to the innovative activity.<sup>12</sup> There are two channels for a firm to increase its innovative expenditures. First, the firm can invest earlier in any given project, thus undertaking riskier projects.<sup>13</sup> Given a constant flow of ideas, this leads to more inventions reaching the innovation stage where the substantial financial commitment to the project is made. Second, the firm may decide to invest in an increasing fraction of the projects that attain a certain probability of success. An increase in this fraction leads to an increase in the level of innovative activity. We will show that for low levels of competition, firms invest in all projects that attain a likelihood of success higher than a certain threshold. For these low values, as competition increases, the threshold decreases and thus firms undertake riskier projects and become more innovative. On the other hand, for high levels of competition, firms react to an increase in competition by investing in a decreasing fraction of the projects that reach a given threshold, thus being less innovative. This suggests that when the pre innovation level of competition is high, the competition in the post innovation markets is *endogenous*. More precisely, as competition increases, the fraction of firms that undertake any specific project decreases, lowering the post innovation level of competition. From a policy perspective, this finding implies that the positive welfare effects of increasing competition have only a limited scope. Finally, we show that for high values of competition, if the technological spread in the industry increases when competition increases, firms also respond by investing in safer projects; this further decreases the level of the innovative activity.

The key driving force in our model is the effect of an increase in product market competition on the marginal cost of waiting for more information. For low levels of competition, firms expect positive profits from innovation and invest in all projects that are sufficiently safe. They decide on the optimal moment of investment by comparing the marginal benefit and the marginal cost of waiting for more information. When competition increases, the marginal cost of waiting increases exceeding the marginal benefit earlier and thus inducing firms to invest earlier. On the other

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<sup>12</sup>AGBBH (2005) employ patent count data as a primary measure of innovation, but as a robustness check, they also use R&D expenditures as an alternative measure. The same inverted-U shape relationship emerges.

<sup>13</sup>From a policy perspective, taking a riskier decision is, on the one hand, decreasing total welfare, because of the expenses incurred on projects that ultimately prove unsuccessful. On the other hand, it also implies that a successful product will be released earlier; this improves welfare by delivering the corresponding benefits earlier and by generating other further inventions based on that new product. Basically, firms in any industry almost always develop a new product that would deliver positive economic profits almost surely, provided that the required financial means are available. But if the firm or industry has waited a long period of time before making the investment, it cannot necessarily be considered innovative. For instance, any major car manufacturer would invest in the development of a hybrid car now, but only a few of them were willing to take that risk 20 years ago.

hand, for the higher values of product market competition, each firm's expected profit from the innovation approaches the competitive outcome and becomes virtually zero. When competition further increases, to continue to break even, firms need to be less innovative. They do this by investing in a decreasing fraction of projects. While investing later would ensure non negative expected profits, the resulting strategy profile would not be an equilibrium. This is because the marginal cost curve would continue to shift up and therefore the trade-off between marginal cost and marginal benefit of waiting would continue to be solved at earlier times.

The dynamic setting of the model allows studying the case when the increase in competition alters the technological spread in the industry, defined as the length of time it takes for all firms to make the technological breakthrough. In our model, firms are not informed of the exact moment when other firms learned of the same invention.<sup>14</sup> The absence of this piece of information and the fact that the firm's payoff depends on the investment decisions of the other firms in the industry introduces *strategic uncertainty* in the firms' decision problems. An *increase* in technological spread then leads each firm to assign a higher probability to the event that the innovation has already started in the industry at any given moment. This induces more pessimistic beliefs about the number of firms who will invest before the next piece of information arrives and thus increases the marginal cost of waiting. As argued above, the upward shift of the marginal cost curve induces firms to invest earlier for low levels of competition. For high values of competition, the effect of the belief updating is of second order, and is compensated by the first order effect of the decrease in the fraction of projects that are undertaken. On net, the marginal cost decreases, which induces firms to invest later, thus further reducing innovation. Thus, for higher values of competition, when the technological spread increases, firms invest in a decreasing fraction of projects *and* wait longer.

In addition to the key comparative static result with respect to the value of product market competition discussed above, the model offers other predictions of interest. First, when the innovation costs increase, firms react by investing later for all values of competition. Second, an increase in the speed of learning induces firms to invest in safer projects. Third, the innovation maximizing level of competition is essentially independent of the cost of innovation. Fourth, the model is successful at supporting additional empirical regularities that ABBGH(2005) observed. Thus, we show that a lower level of technological dispersion in an industry results in an inverted-U shape with a higher peak attained for a lower level of competition. Finally, we also investigate the welfare effects of innovation under competitive pressure. A social planner that aims at designing the market structure most conducive of innovation has to take into account the effects of an increase in competition on the post innovation social welfare, on the firms' risk taking behavior, on the timing of innovations and on the degree of redundancy in parallel innovations. We find conditions that determine the level of competition that optimizes these welfare effects of innovation and argue

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<sup>14</sup>The sequential awareness assumption from our model is similar to the one used in Abreu and Brunnermeier (1993) in a model of financial bubbles and crashes.

that, generically, this level is different than the one that maximizes the industry-wide innovative activity.

A more comprehensive review of the literature is presented in section 2. The model is presented in section 3, while the analytical results, their discussion and a numerical example are presented in section 4. We also discuss in section 4 the welfare effects of innovation and the case of continuous information acquisition. The conclusion is in section 5. Most of the proofs are relegated to the appendix.

## 2 Review of the literature

Schumpeter (1943) considered innovation to be the main determinant of technological progress and an engine of economic growth and development. Discussing the role of market structure in enhancing innovation, he distinguished between static and dynamic efficiency by arguing: "a competitive market may be perfectly suitable vehicle for static resource allocation, but the large firm operating on a concentrated market is the most powerful engine of progress and ... long-run expansion of total output" (Cohen and Levin, 1989, p. 1060). Motivated by Schumpeter's conjecture, many empirical studies have investigated the role that the firm's size and the level of product market competition play in influencing innovative activity. However, the results have been inconclusive.<sup>15</sup> The first of Schumpeter's claims to be extensively tested is that the possession of some *ex ante* market power is required for firms to have the means and incentives to engage in significant innovation activity. While studies by Scherer (1984), Pavitt (1987), Blundell, Griffith and Van Reenen (1999) found a positive linear relationship between firms' size and the intensity of the R&D activity, Bound et al. (1984) found evidence contrary to this claim and Cohen et al. (1987) found no conclusive effect.

Supporters of the Schumpeterian view argued that larger firms have better access to capital, are less risk averse due to diversification and enjoy economies of scale (the returns from innovation are higher when the firm has a large volume of sales over which to spread the fixed costs of R&D) and economies of scope (large firms can benefit from positive spillovers between various research programs) from innovation. On the other hand, the opponents of the theory argued that as the size of the firm increases, the efficiency in R&D is undermined by the loss of managerial control, while the incentives of individual scientists and engineers become attenuated. Second, Schumpeter argued that innovation should increase with *ex post* market power because less competition increases the rewards that are associated with successful innovations. This argument is the basis of the current patent laws, which provide the expectation of ex-post market power as an incentive to innovate. Studies such as Fellner (1951), Arrow (1962), Bozeman and Link (1983) have supported

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<sup>15</sup>A detailed literature review of the earlier empirical literature can be found in Cohen and Levin (1989).



Schumpeter's hypothesis, whereas Porter (1990), Comanor and Scherer (1995), Geroski (1995), Baily and Gersbach (1995) and Nickell (1996) rejected it. On the other hand, more recent studies, that allowed for flexible *non linear* relationships, such as Scherer (1967), Scott (1984), Levin et al. (1985) and ABBGH (2005) found evidence of the inverted-U relationship between R&D intensity and market concentration.<sup>16</sup>

The theoretical literature on this topic is also vast and the results have been mixed. Earlier papers, such as Loury (1979), Grossmann and Helpman (1991), Aghion and Howitt (1992), Caballero and Jaffe (1993), Martin (1993) sought to confirm the Schumpeterian hypothesis.<sup>17</sup> On the other hand, inspired by the seminal work of Hart (1983), more recent theoretical papers focusing on managerial incentives, such as Schmidt (1997), Aghion, Dewatripont and Rey (1997) or Aghion, Dewatripont and Rey (1999), proposed models in support of the positive correlation between competition and innovation observed in some of the empirical papers.<sup>18</sup> The main drawback of this strand of literature is that the profit maximization assumption at the managerial level is replaced with a less convincing assumption of minimizing innovation costs, subject to the constraint that the firm does not go bankrupt.<sup>19</sup> Other theoretical papers in support of this positive relationship are Aghion, Harris and Vickers (1995), whose approach is close to the one from ABBGH (2005) and Aghion and Howitt (1996) who endogenize the rate at which firms switch from old technologies to new and show that an increase in the substitutability between the old and new product lines will induce firms to adopt the new technologies faster. Boone (2000) obtained conditions under which more competitive pressure induce either more or less innovation to individual firms depending on their efficiency level. Finally, Kamien and Schwartz (1976) and ABBGH (2005) are the first theoretical models to obtain the inverted-U shape relationship.

ABBGH (2005) argue that the escaping the competition effect is stronger in neck-and-neck industries,<sup>20</sup> while the Schumpeterian effect is stronger in less neck-and-neck industries. The inverted-U shape curve emerges because the fraction of neck-and-neck industries in the economy changes in response to a change in competition. Kamien and Schwartz (1976) present a model of innovation under rivalry in which firms decide on the optimal moment to innovate while facing the following trade-off. On the one hand, spending more time on developing a product induces an decreased and *convex* cost of innovation. On the other, it increases the risk that some rival firm would innovate first. As in our model, firms adjust their innovation behavior in a manner consistent with the inverted-U shape relationship between competition and innovation. However, there are a number of issues with the model in this paper. First, the measure of rivalry that Kamien and Schwartz use,

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<sup>16</sup>When ABBGH(2005) imposed a linear relationship in their regression analysis, the results were consistent with the ones from the earlier empirical literature, which had yielded a positive slope. This suggests a possible explanation for the previous spurious conclusions.

<sup>17</sup>See Cohen and Levin (1989) for a detailed overview of this literature.

<sup>18</sup>See Aghion and Howitt (1998) and Boone (2000) for excellent reviews of this literature.

<sup>19</sup>These innovation costs are seen to increase the manager's effort in adapting to the new technology.

<sup>20</sup>Neck-and-neck industries are defined as industries in which firms are at technological par.

the expected time of innovation by the other firm, leaves aside other interesting cases - for instance, the case in which an increase in competition is associated with a decrease in post innovation profits. Second, and more importantly, the model ignores potential strategic considerations. While the firm under consideration changes its behavior by investing earlier or later as a response to the rival's expected time of innovation, the rival does not do so.

Our paper is also related to the very broad literature on timing of irreversible actions under uncertainty. Jensen (1982) presents a model of information acquisition in which the incentive to innovate earlier is provided by the discounting of future revenues rather than the competitive pressure. Chamley and Gale (2005) study a model of endogenous information acquisition in which firms learn about the profitability of a common value investment from the actions of the other players, while Decamps and Mariotti (2004) allow in addition for a private value component of the investment and for exogenous information. Caplin and Leahy (1993) develop a model in which investors learn of the profitability of new industries from the success of the earlier entrants. Finally, the strategic experimentation literature (see Bolton and Harris (1999), Cripps, Keller and Rady (2005)) studies the problem of costly information acquisition about the common value of a project when each agent can learn from the experimentation of the other players. In a different direction, our paper shares the first-mover advantage feature with the patent race literature (see for instance the seminal paper by Reinganum (1983)). What distinguishes the current model from this literature is mainly the source of uncertainty. In the patent race literature, the uncertainty was generated by the fact that the technological advancements were the outcome of a random process, or by the fact that the finish line was random. In contrast, in our model, the uncertainty stems from the fact that the firm does not know whether the project is successful or not; in other words, it does not know the state of the world.

## 3 The Model

### 3.1 The Technology

There is a continuum of identical and risk neutral firms who, sequentially, learn of an invention at moments denoted by  $t_i$  for firm  $i$ . A mass  $a$  of firms becomes aware at each instant  $t$ , with  $t \in [t_0, t_0 + \eta\delta]$ ,  $\eta > 1$  and  $\delta > 0$ .<sup>21</sup> The moment  $t_0$  is not known by any of the firms, but it has a prior distribution, which is common knowledge among the firms in the industry. This prior distribution is uniform on the real line.<sup>22</sup> After the firm learns of the new idea, it may invest in its

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<sup>21</sup>As standard in the literature, the continuum hypothesis employed here can be interpreted simply as the distribution of the unknown locations on the timeline of a finite number of firms.

<sup>22</sup>We use this nonstandard distribution to avoid boundary effects. As an alternative to using it we may discard the assumption of the existence of a common prior. Thus, instead of having the posterior beliefs of the firms about  $t_0$  at the initial moment when they become aware of the invention be derived from a common prior about  $t_0$ , we may

development at any time with a one-time sunk fixed cost  $c$  of innovating.<sup>23</sup> As we argue later,  $a$  can be interpreted as a counterpart of the Lerner index, that is, an inverse measure of the ability of the firms in an industry to collude. On the other hand,  $\eta\delta$ , the length of the so called awareness window, constitutes a measure of the technological spread in the industry.

The timeline corresponding to the case in which the firm  $i$  waits  $\tau$  time units before investing, is presented in the Figure 2.

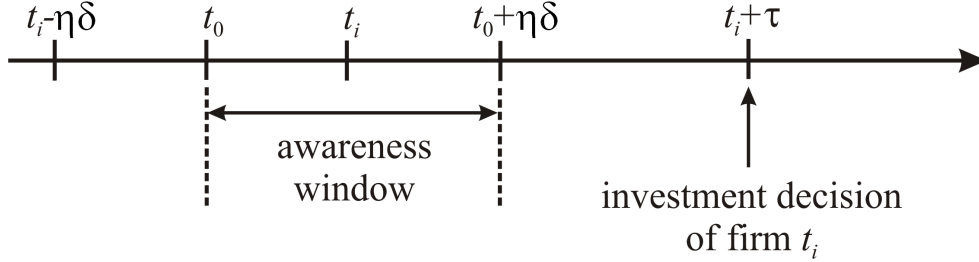


Figure 2

The information acquisition is modelled as follows.<sup>24</sup> Before moment  $t_i$ , the firm's R&D department engages in applied research aimed at gaining knowledge with the purpose of using that knowledge for commercial purposes.<sup>25</sup> At moment  $t_i$ , when firm  $i$  becomes aware of an invention, it has a belief  $p_0$  about the chance of the investment project being ultimately successful.<sup>26</sup> Then, after every  $\delta$  units of time, the firm performs a test against some of the potential technological or commercial problems that the project might encounter. In the state of the world in which the

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consider directly that these beliefs are actually the firm's prior on  $t_0$  at that moment.

<sup>23</sup>It is straightforward to see that the cost  $c$  can be interpreted in the rest of the model as the *expected* present value of all future expenditures on the development of this new product, without changing the qualitative results. Thus, the specification of a *one time* cost is inessential. Also, to simplify the analysis, we assumed that the research phase is costless; this assumption can be easily dropped, but all the salient results remain the same.

<sup>24</sup>Since the necessary tests should be identical for all firms, it is natural to have all firms go through the same information acquisition process. This can be slightly relaxed to have each firm  $i$  *believe* that the rest of the firms go through the same information acquisition process as the process that firm  $i$  goes through.

<sup>25</sup>Note here that, before becoming aware of the invention, the firm is completely unaware of the model as a whole. In other words, only at the exact instant when the firm learns about the new idea, does the firm also learn about the model as presented here. This is intuitive in that it makes sense for a firm to not hold beliefs about the characteristics of an object of which it is unaware. Moreover, this unawareness assumption can be discarded if we interpret the notion of "becoming aware" as *discovering the product*. For instance, all firms in an industry might be looking for a cure for some disease and have the same common prior about the model as described above. However, only those that discover a potential cure for that disease contemplate an investment decision. This interpretation allows for a common prior *of the model*, while still preserving the sequential awareness assumption.

<sup>26</sup>Mansfield (1968a, ch.3) presents in detail the process by which R&D proposals and budgets were generated and evaluated within the central research laboratory of a major electronic, electrical equipment and appliance manufacturer. The three dimensions on which projects were evaluated were: (1) probabilities of commercial and technical success; (2) additional profit generated; (3) cost of innovation. Each proposed project was assigned precise estimations for each of these variables at different levels of management. This anecdotal evidence underlies the importance that measurable risk of investment plays in the innovative decision-making process at firm level.

project is successful, the tests are always passed. If the project is unsuccessful, some tests may still be passed. Slightly more formally, if the project is successful, the signal received is always 'Pass'. If the project is unsuccessful, the signal may be either 'Pass' or 'Fail'. The *unconditional* probability of a 'Pass' signal to be received at moment  $t_i + t$ , where  $t \in \delta\mathbb{Z}_+ \equiv \{\delta, 2\delta, 3\delta, \dots, t_M - \delta\}$ , is  $e^{-\mu\delta} < 1$ . At the time  $t_M \equiv \min\{t \in \delta\mathbb{Z}_+ : t \geq -\delta - \mu - \ln p_0\}$ , which is the first moment when the posterior probability that the project is successful is no lower than  $e^{-\mu\delta}$ , the firm receives a "Pass" signal if and only if the project is successful. Thus, at  $t_M + \delta$  if all tests have been passed, the firm knows for sure that the project is successful. In Appendix A4, we present the specific sequence of conditional probabilities that generate this signal structure in a manner consistent with Bayesian updating.

There are three salient features in our model. The first is that firms are not informed of the exact moment when other firms became aware of the same invention. This assumption has three merits. First, it captures the real world uncertainty that firms face. Second, it effectively induces a smooth marginal cost of waiting and thus a smooth payoff function essential for equilibrium existence. Finally, it helps obtaining the inverted-U shape when the increase in competition is associated with an increase in the technological spread in the industry. The second feature is that of a *discrete information acquisition*. Besides being in line with the reality of our particular application, this discreteness helps deliver the main results of the paper in together with the strategic uncertainty assumption. The last salient feature of the model is a *single crossing* property between the marginal cost of waiting and the marginal benefit of waiting as functions of time *in equilibrium*. The marginal cost of waiting for more information is naturally increasing over time since, in expectation, the marginal loss from a late release is higher when the product is closer to being a success than when the product is just in an early stage of testing. On the other hand, the specific signal structure defined above is chosen precisely because it implies a marginal benefit of waiting that is constant in time, which immediately ensures the single crossing property. The constant marginal benefit and the rest of the functional forms make the analysis of our discrete dynamic model tractable, and allows for a closed form solution and thus for potential further applications. However, by following the intuitive arguments offered for the main results of the paper, it will be clear that these results hold for more general specifications that imply the single crossing property.

### 3.2 The Payoffs

At moment  $t_i + t$ , if firm  $i$  invests in a project which will ultimately turn out to be successful, its post innovation profits are given by:<sup>27</sup>

$$\pi(t, t_i, t_0) = 1 - m(t|t_i, t_0) - \frac{1}{2}n(t|t_i, t_0) \quad (1)$$

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<sup>27</sup>Note that  $t$  does not represent a calendar time, as  $t_0$  or  $t_i$  represent, but it is the length of time passed since the firm became aware of the innovation. The conditioning on  $t_0$  is required because this determines the measure of firms who became aware of the innovation before  $t + t_i$ .

where  $m(t|t_i, t_0)$  is the measure of firms that innovate before firm  $i$  and  $n(t|t_i, t_0)$  is the measure of firms that innovate at the same time as firm  $i$ . To isolate the effect of the competitive pressure in inducing firms to invest earlier, we assume no intertemporal discounting. The functional form in (1) can be seen as a reduced form of a model in which firms that invest in innovation earlier have a higher chance of releasing the product earlier and thus of enjoying the first mover advantage. Alternatively, one may think of the functional form as a reduced form for a patent race model in which firms that engage earlier have a higher chance of winning.<sup>28</sup> The motivation for the particular effect of  $n(t|t_i, t_0)$  on  $\pi(t, t_i, t_0)$  is a natural rationing rule in which, if a mass of firms innovate at the same time, each of these firms is considered to have a median rank in the group. The functional form in (1) ensures that the *total* amount of profits available from a successful innovation across the industry does *not* depend on the particular distribution of the moments when firms in the industry innovate. We state this fact formally in the following Remark. For an arbitrary distribution of innovation times in the industry, denote by  $G(t)$  the measure of firms who has invested by time  $t$ .

**Remark 1** *The total amount of profits earned in the industry is independent of the distribution  $G(\cdot)$ .*

*Proof.* See Appendix A1.

We will consider throughout the paper that  $c$  is high enough so that if all firms released the product before firm  $i$ , then, even if the project succeeds, firm  $i$  makes negative profits, i.e.

$$(1 - a\eta\delta) - c < 0 \tag{2}$$

This condition eliminates the uninteresting case in which firms wait so long that they eventually invest in innovation under almost certainty. In real world, laggard firms frequently choose not to invest in innovation, and instead they either purchase the license for the new product, wait for the patent to expire, or copy the new technology through reverse engineering if it is not protected.

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<sup>28</sup>One could make the post innovation profits depend also on the measure of firms that invest *after* firm  $i$ . In that case a sufficient statistic for the firm  $i$ 's profits would be the pair  $(\Gamma, \lambda_i)$ , where  $\Gamma$  is the total measure of firms that stay in the post innovation market and  $\lambda_i \in [0, \Gamma]$  is the rank of firm  $i$ . In this case, in a second stage of the game that would follow all investment decisions and full information revelation, the laggard firms that would experience negative post innovation profits would exit the market. The only off equilibrium path actions would be for some firms incurring negative profits to stay in the market and for some firms making positive profits to exit it. Standard backward induction arguments reveal these possible deviations to be inconsequential for the first stage. Thus, in the first stage of the game, which is the model we are analyzing in this paper, all firms would know  $\Gamma$ . To avoid uninteresting complications, we specify the post innovation profits only as a function of the rank  $\lambda_i$  as in (1).

### 3.3 The Measure of Competition

Competition has been modeled in the literature in several ways. Boone (2008) shows that the salient feature common to all theoretical parametrizations of competition is that an increase in competition always raises the *relative* profit shares of the more advanced firms and reduces the profits of the least advanced firm active in the industry. We show in Appendix A2 that in our model these conditions are satisfied when the increase in competition is parametrized by an increase in  $a$ , an increase in  $\eta$ , or an increase in  $a\eta$ .<sup>29</sup> A higher value of  $a$  means that the product market competition increases while the length of the awareness window  $\eta\delta$  remains fixed. We define the *technological spread* in an industry to be the length of time it takes for all firms to learn of the innovation. Thus, an increase in  $a$  is associated with an increase in competition that does *not* change the technological spread in the industry.

On the other hand, an increase in  $\eta$  parametrizes an increase in competition that also increases the technological spread. For instance, if the number of firms in the industry increases, one would expect that it takes longer for all of them to discover a solution to a certain problem. In fact, using the total factor productivity as a proxy for the technological level of a firm and the price-cost margin to measure competition, ABBGH(2005) show empirically that the average technological gap in an industry increases with competition.<sup>30</sup> In our paper, we will focus the discussion on the two polar cases when the competition is measured by  $a$  or  $\eta$ . To simplify exposition, with a slight abuse of notation, we denote by  $x$  the parameter that measures the value of competition, and we will specify precisely which case we consider only when the distinction is meaningful.

As a side point here, note that while in the above the parameter  $a$  was interpreted as the mass of firms in the industry that learn of the invention at any particular time, it can also be interpreted simply as the inverse measure of the degree to which the firms in the industry are able to collude.<sup>31</sup> Therefore, our model can also be used, for instance, to describe a duopoly, in which an increase in  $a$  is associated with a decrease in the ability of the two firms to collude. Since each candidate parameter for measuring competition in our model can be interpreted in a variety of ways, depending on how competition is measured in a particular application, we remain agnostic

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<sup>29</sup>To understand this, assume that there are  $m$  firms in the industry and that starting at  $t_0$  firms become aware of the invention at constant rate  $a$ . Then, if firms become aware independently of each other, the resulting arrival process is distributed *Poisson*( $a$ ), while, conditional on  $t_0$ , the time  $T^m$  when the  $m^{\text{th}}$  firm becomes aware is distributed *Gamma*( $m, a$ ). In particular, the expected time for the  $m^{\text{th}}$  firm to learn of the invention is  $E(T^m) = \frac{m}{a}$ . In other words, the total number of firms in the industry can be written as  $m = aE(T^m)$ . Thus, an increase in the number of firms can be parametrized either by an increase in  $a$  or an increase in  $E(T^m)$  or an increase in  $aE(T^m)$ . For tractability reasons, in our model, conditional on  $t_0$ , the time  $T^m$  is deterministic and takes value  $\eta$ .

<sup>30</sup>Clearly that, by parametrizing an increase in competition with an increase in  $a\eta$  coupled with a decrease in  $\eta$ , one could study the effects on innovation of an increase in competition that decreases the technological spread.

<sup>31</sup>To see this, note first that the expected measure of firms who became aware of the invention before any given firm is  $a\frac{\eta\delta}{2}$ . Therefore, in a symmetric equilibrium, the expected profit of any firm, given that the project ends up being successful, is  $1 - a\frac{\eta\delta}{2} - c$ . This is also the average profit of the firms across the industry. So an increase in  $a$  lowers the average profits in the industry.

with respect to exactly what these parameters mean precisely. This preserves the highest level of generality for our model.

## 4 Results

### 4.1 The Equilibrium

The main result of the paper describes the equilibrium of our model. This equilibrium is completely characterized by the time firms wait before investing,  $\tau(x)$ , and by the probability with which firms pursue the project,  $\alpha(x)$ . As common for other models with mixed-strategy equilibria,  $\alpha(x)$  can also be interpreted as the fraction of projects into which the firm invests, and we will sometimes refer to it as such in the rest of the paper. Proposition 2 and its corollaries describe the salient qualitative features of the equilibrium of our model. The proof of these results as well as more precise statements, with the exact conditions determining  $\tau(x)$ ,  $\alpha(x)$  and the cutoff  $\bar{x}$ , can be found in Appendix B.

**Proposition 2** *In equilibrium, there exists a threshold  $\bar{x}$  such that:*

- (i) *For  $x < \bar{x}$ ,  $\alpha(x) = 1$  and firms expect strictly positive profits from innovation.*
- (ii) *For  $x > \bar{x}$ ,  $\alpha(x) < 1$  and firms expect zero profits from innovation.*

**Corollary 3** *When  $x := a$ , for  $a > \bar{a}$ , all firms wait the same amount of time  $\tau(\bar{a})$  before mixing between investing and not investing. When  $x := \eta$ , there exists a sequence  $\eta_0 = \bar{\eta} < \eta_1 < \eta_2 < \dots$  such that:*

- (i) *for  $\eta \in [\eta_{2k}, \eta_{2k+1}]$  firms wait  $\tau(\eta)$  before mixing between investing and not investing.*
- (ii) *for  $\eta \in [\eta_{2k+1}, \eta_{2k+2}]$ , firms mix among investing after  $\tau(\eta)$ , investing after  $\tau(\eta) + \delta$ , and not investing at all.*

In Appendix B4 we find conditions under which firms invest as soon as they learn of the invention and conditions under which they invest only in perfectly safe projects. As expected, firms invest immediately when  $p_0$  is sufficiently high, and wait until they remove all uncertainty if the level of competition is sufficiently low.<sup>32</sup> The next two results discuss comparative statics. First, denote by  $p_t$  the belief of firm  $i$  in the success of the project at  $t_i + t$ .

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<sup>32</sup>For certain values of the parameters, there exist equilibria as described by Proposition 2 in which  $\tau(\theta) > \eta\delta$ . Thus, in this type of equilibrium there is a moment when there are more than enough firms in the industry aware of the invention to invest and render the profits of the remaining firms negative without any of them actually investing yet. Moreover, in an equilibrium in which  $\tau(\theta) > 2\eta\delta$ , there is a moment when all firms know that the remaining

**Corollary 4** (i)  $\tau(x)$  is increasing in  $c$  for all levels of competition.

(ii)  $p_{\tau(x)}$  is increasing in  $\mu$  for all levels of competition.

The proof of this corollary follows immediately from the precise characterization of the equilibrium in Proposition 2. The first statement of the corollary suggests that when the innovation costs are higher, firms wait more before innovating. Put differently, the higher the profits that the innovations promise in case of success, the more risky the projects undertaken. Second,  $\mu$  measures the speed of learning. Thus, the corollary states that, all else being equal, when firms learn faster about the profitability of new products, they end up investing in safer projects. The effect on the equilibrium value of waiting time is ambiguous because while an increase in  $\mu$  increases the equilibrium value of the belief in the ultimate success of the project, it also increases the speed of learning and thus, that belief level may be attained earlier. To pin down the sign of that effect precisely, one needs to know the values of the rest of the parameters of the model.

The comparative static of interest is the one with respect to the measure of competition. The result is presented in the next corollary.

**Corollary 5** For  $x < \bar{x}$ ,  $\tau(x)$  is decreasing. For  $x > \bar{x}$ ,  $\alpha(x)$  is decreasing, while  $\tau(x)$  is constant when  $x := a$ , and increasing when  $x := \eta$ .

We present first the intuition for this Corollary for the case when the increase in competition is parametrized by an increase in  $a$ . First, for low levels of competition, the post innovation profits from a successful project are significant, and thus firms expect strictly positive profits from innovation. Therefore, the optimality condition that drives the firm's response is the one that solves the trade-off between the marginal cost (henceforth, denoted  $MC$ ) and marginal benefit ( $MB$ ) of waiting for an additional informative signal. The  $MC$  of waiting for firm  $i$  is the expected decrease in post innovation rents due to the expected loss in first mover advantage.<sup>33</sup> On the other hand, the  $MB$  is the additional information provided by the signal; in monetary terms, the  $MB$  can be measured as the expected forgone costs on an *unsuccessful* project. As explained in Section 3.1, the  $MC$  and  $MB$  curves satisfy a single crossing property.

An increase in  $a$  shifts the  $MC$  curve upwards, while the  $MB$  curve is unaffected. Therefore, for small values of  $a$ , as  $a$  increases, firms respond by waiting less in equilibrium. In the literature, this is called the "escaping the competition effect". Above a certain level of competition,  $\bar{a}$ , there is

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firms know about the invention and still nobody invests. These types of equilibria are sustainable because, while all firms may know of the invention and know that everyone else knows and so on up to any finite level, the invention is not common knowledge among the firms until the product is actually released in the market for the first time.

<sup>33</sup>We use the terms "marginal cost" and "marginal benefit" in a loose sense, without giving a precise formal definition. We employ them only to provide intuition regarding the trade-off that the firms face in deciding on whether to wait for one more piece of information or to invest immediately.



no symmetric pure strategy equilibrium. If all firms that became aware of the new product before some firm  $i$ , have already invested at the first moment when the  $MC$  of waiting exceeds the  $MB$  for firm  $i$ , then the expected measure of firms who would release the product before firm  $i$  would be too high for firm  $i$  to expect non-negative profits from investing in the project. Conversely, if firms were to just respond to the increase in competition by investing later, then each firm would have an incentive to deviate and invest earlier. Thus, no symmetric pure strategy equilibrium, in which all firms invest in the project is sustainable.

Instead, for  $a > \bar{a}$  firms mix between innovating and not innovating, thus effectively investing in only a fraction of the projects. Therefore, they respond to increased competition by being less innovative. In line with the Schumpeterian argument, the explanation is that in highly competitive industries, the potential revenues from a successful new product are divided among many firms and thus each firm's expected profit from the innovation is virtually zero. When the technological spread does *not* change, firms reduce their level of innovative activity by investing with a decreasing probability  $\alpha(a) \in (0, 1)$ . This endogenizes the level of competition in the post innovation markets and allows firms to expect nonnegative profits. Note that if  $\alpha(a)$  did not decrease, but instead firms would continue to invest later, the  $MC$  curve would continue to shift up as  $a$  increases. Thus, the trade-off between the  $MC$  and  $MB$  of waiting, which determines the moment when the expected profits from innovation are at their highest level, would continue to be solved earlier. But these maximum profits would be negative. Thus,  $\alpha(a)$  needs indeed to decrease when  $a$  increases.

To understand the intuition for Corollary 5 in the case when the increase in competition also increases the technological spread in the industry, we first provide intuition for why the  $MC$  of waiting increases when  $\eta$  increases, and discuss the elasticity of the  $MC$  curve with respect to  $\eta$ . We do these for the simplest case in which all firms wait for  $\tau$  periods before investing and restrict attention to the case when  $t < \tau$ . Thus, note first that at  $t_i$  firm  $i$ 's posterior of  $t_0$ ,  $F_0(\cdot)$  is uniform on  $[t_i - \eta\delta, t_i]$ . Second, conditional on any  $t_0$ , firm  $i$  will know that, according to the equilibrium strategies of the other firms, the measure of firms who have already invested at any moment  $t_i + t$  is

$$m(t|t_i, t_0, \tau, x) \equiv a \min(\eta\delta, \max(t_i + t - \tau - t_0, 0)) \quad (3)$$

Note that innovation has already started at moment  $t_i + t$ , that is,  $m(t|t_i, t_0, \tau, x) > 0$  if and only if  $t_0 \in [t_i - \eta\delta, t_i - (\tau - t)]$ . Then, the expected measure of firms who have invested at  $t_i + t$  is

$$\lambda(t|t_i, \tau, x) \equiv E_{t_0} [m(t|t_i, t_0, \tau, x)] = \int_{t_i - \eta\delta}^{t_i} m(t|t_i, t_0, \tau, x) dF_0(t_0) \quad (4)$$

It follows that  $\frac{1}{a} \frac{\partial}{\partial t} \lambda(t|t_i, \tau, x) = \int_{t_i - \eta\delta}^{t_i - (\tau - t)} dF_0(t_0)$ ; this is precisely the measure of the set of values of  $t_0$  for which innovation has already started in the industry at moment  $t_i + t$ . Thus, *waiting for additional  $\Delta t$  time units increases the expected measure of firms who have invested*

by  $a\Delta t$  multiplied by the probability that innovation has started in the industry. Straightforward calculations show that  $\frac{\partial^2}{\partial\eta\partial t}\lambda(t|t_i, \tau, \eta) > 0$ . It follows that the *instant MC* of waiting at  $t_i + t$ ,  $MC(t, \eta) = p_t \frac{\partial}{\partial t}\lambda(t|t_i, \tau, \eta)$  is increasing in  $\eta$ . This is because an increase in technological spread induces more pessimistic beliefs about the moment when firms started learning of the innovation. To understand this, note that from the view point of firm  $i$ , the earliest moment that innovation could have started is  $t_i + \tau - \eta\delta$ , and second that  $\tau$  and  $t$  are fixed. Therefore, when  $\eta$  increases, firm  $i$  assigns a higher probability at moment  $t_i + t$  to the event that innovation has already started.<sup>34</sup> This makes waiting more costly. Finally,  $\frac{\partial \ln MC(t, \eta)}{\partial \ln \eta} \leq 1$  for  $t \in [\tau - \frac{\eta\delta}{2}, \tau]$ , so the *MC* curve is inelastic with respect to  $\eta$ , for  $t$  close to  $\tau$ . Intuitively, when  $t$  is close to  $\tau$ , firm  $i$  already assigns a high probability that innovation has started, so an increase in the technological spread does not alter the beliefs significantly. For later use, note also that the *MC* curve becomes almost perfectly inelastic at  $\tau - \delta$  as  $\delta \rightarrow 0$ . A somewhat similar argument shows that when  $t \geq \tau$ ,  $MC(t, \eta)$  is again increasing in  $\eta$  and inelastic with respect to  $\eta$  for  $t \in [\tau, \tau + \frac{\eta\delta}{2}]$ . Since for  $t \geq \tau$ , the event that some firms have started investing has probability one, the main difference is that an increase in  $\eta$  increases the measure of the set of values of  $t_0$  for which *all* firms have already invested.

Now, for  $\eta < \bar{\eta}$ , as argued above, when  $\eta$  increases the *MC* increases and firms invest earlier. Above  $\bar{\eta}$ , where the expected profits become zero, if firms were to just wait longer as  $\eta$  increases, without a corresponding decrease in  $\alpha(\eta)$ , the *MC* curve would continue to shift up. This would lead firms to solve the trade-off earlier and incur negative profits. Thus  $\alpha(\eta)$  must decrease. More precisely,  $\alpha(\eta)$  must decrease so that the expected profits in equilibrium,  $p_{\tau(\eta)}(1 - \frac{1}{2}a\alpha(\eta)\eta\delta)$  stay at zero, as necessary for the firms to be willing to randomize. As shown above, in equilibrium, the *MC* curve is inelastic with respect to  $\eta$  around  $\tau$ . On the other hand, the *MC* curve is unit elastic with respect to  $\alpha$ .<sup>35</sup> This is because the decrease in  $\alpha(\eta)$  decreases the density of firms who invest at any particular moment; therefore the total measure of firms who invest in any time period  $\delta$  decreases by exactly the same fraction that  $\alpha(\eta)$  decreases. Since the magnitudes of the percentage changes in  $\alpha(\eta)$  and  $\eta$  must be equal to keep expected profits at zero, the effect of the decrease in  $\alpha(\eta)$  dominates around  $\tau$  and thus lowers the marginal cost. On the other hand, since the *MC* is increasing in time, the only relevant section of the *MC* curve for determining the new equilibrium waiting time is the middle segment around  $\tau$ . Since the curve shifts down on this part, firms end up investing later.

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<sup>34</sup>The probability as of moment  $t_i + t$  that the innovation has started is  $\Pr(t_0 \in [t_i - \eta\delta, t_i - (\tau - t)]) = \frac{\eta\delta - (\tau - t)}{\eta\delta}$ . This closed form solution is due to the simplicity of the posterior  $F_0(\cdot)$ , but since  $\tau - t$  is fixed, the intuition is clearly valid for all sufficiently well behaved families of distributions  $\{F_0^\eta(\cdot) : \eta > 0\}$ , parametrized by  $\eta$ , the finite length of the support of  $F_0^\eta(\cdot)$ .

<sup>35</sup>As shown in the proof of Proposition 2, the *MC* of waiting for one more signal is  $p_t\alpha(x)a\delta(1 - \frac{1}{2\eta})$ . The unit elasticity is an artifact of the linearity of profits in (1). A more general sufficient condition for this argument to go through is the the *MC* is elastic with respect to  $\alpha a$ . This is equivalent to the *MC* being convex in  $\alpha a$  and with  $\pi$  in (1) being concave and decreasing in  $m(t|t_i, t_0) + \frac{1}{2}n(t|t_i, t_0)$ . A concave specification of  $\pi$  allows for a less steep fall in profits for the earliest innovators. This is consistent with the presence of some further uncertainty regarding the time of release which smooths the expected payoffs and thus weakens the first mover advantage in innovation.

Note the distinct channels through which the two parameters  $a$  and  $\eta$  increase the  $MC$  of waiting. An increase in  $a$  increases the expected measure of firms who invest in the time it takes to acquire a new signal or increase the potential loss in post innovation profits from being beaten to the punch by another firm.<sup>36</sup> On the other hand, an increase in  $\eta$  alters the *beliefs* that firms have regarding the event that innovation has already started in the industry. This underlies the role that uncertainty plays in delivering the results of the model. Absent uncertainty, the  $MC$  does not increase when  $\eta$  increases; the  $MC$  of waiting would be either  $a$  or 0 depending on whether innovation has started or not in the industry. This would imply, for instance, that firms do not respond by investing earlier for low values of competition.<sup>37</sup>

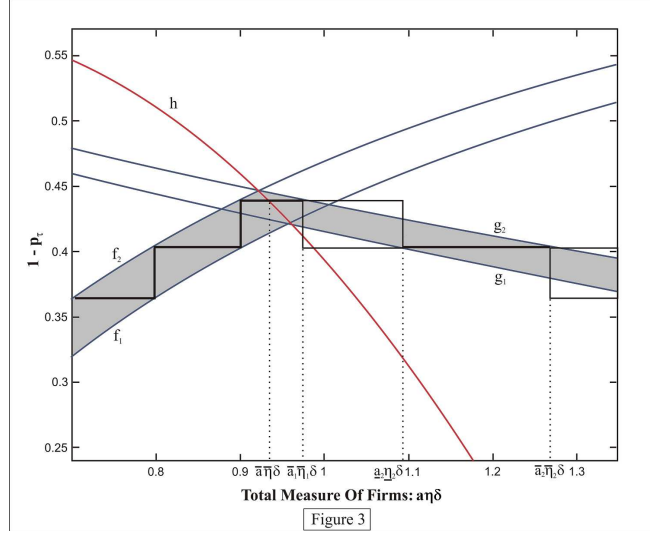
## 4.2 A Numerical Example

In this section, we illustrate the above theoretical results with a numerical example. We calibrated the model with the following values of the parameters:  $c = 0.3$ ,  $\mu = 0.4$ ,  $\delta = 0.2$ . To also describe the timing of innovation for the higher values of competition, we allowed for the technological spread increases with competition and considered  $\eta = 100a$ . The following figure presents the results of Proposition 2 by plotting the range of possible values for the equilibrium value of  $1 - p_\tau$  against the total measure of competition  $a\eta\delta$ , as the level of competition increases. Note that  $1 - p_\tau$  measures the risk that firms undertake, and thus is a measure of the innovation intensity in the industry. The step function from Figure 3 depicts the equilibrium values of  $1 - p_\tau$ .

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<sup>36</sup>To understand this, assume a simple setup in which, if the project is successful, the first firm investing in the product has a payoff of 1 and all the others have a payoff of  $x$ . Then, conditional on the product being ultimately successful and on no other firm having already invested, the expected loss in post innovation rents from waiting for one more piece of information is  $1 - x$  multiplied by the probability  $p$  that some other firm invests in the product in that period. Clearly, if an increase in competition is associated with an increase in the number of firms in the industry, then  $p$  should increase when the number of firms in the industry increases. On the other hand, if the increase in competition affects the post innovation profits, the following Boone (2008), the increase in competition should decrease  $x$ . In both cases, the conditional  $MC$ , which is  $p(1 - x)$  increases.

<sup>37</sup>As a side point, note that firms invest *earlier* for high values of competition if the increase in competition is associated with a lower level of technological spread in the industry. To see this, consider an increase in  $a$  coupled with a decrease in  $\eta$  that increases the value of  $a\eta$ . To keep  $\alpha(a, \eta)a\eta$  constant as required by the zero profit condition,  $\alpha(a, \eta)$  must increase by the same fraction that  $\eta$  decreases. Since  $MC(\cdot, \eta)$  is more elastic with respect to  $\alpha(a, \eta)$  than with  $\eta$ , this would make  $MC(\cdot, \eta)$  shift up and thus it would induce firms to continue investing earlier even after they make zero profits. In this case the additional risk undertaken would be compensated by a lower level of competition in the post innovation market.



For any given value of competition, firms stop waiting for additional information if two conditions are satisfied. First, their belief in the feasibility of the project should be high enough that they expect non-negative profits from that investment. When all firms invest in the project, for any amount of time  $t$  spent on acquiring additional information, this condition is satisfied whenever  $1 - p_t$  is below the curve  $h$  in Figure 3. Second, the  $MC$  of waiting for one more piece of information should exceed the  $MB$  of waiting for that information. In Figure 3, this is the case whenever  $1 - p_t$  is below the curve  $f_2$ . Since the  $MC$  is increasing in time, it is sufficient to impose this condition at  $\tau$ , so  $1 - p_\tau$  should fall below the curve  $f_2$ .

On the other hand, firms need to postpone investing in innovation for  $\tau$  periods, which can only happen if their belief in the feasibility of the project is low enough. More precisely, if  $1 - p_t$  were below  $f_2$  and below  $h$ , for some  $t < \tau$ , firms would deviate from the equilibrium strategy and invest earlier. Since the  $MC$  is increasing in time, a sufficient condition for the equilibrium to be sustainable is that firms have an incentive to wait at  $\tau - \delta$ . In Figure 3 this is the case whenever  $1 - p_{\tau - \delta}$  is above  $f_2$ . Now,  $f_1$  is defined such that whenever  $1 - p_{\tau - \delta}$  is above  $f_2$ ,  $1 - p_\tau$  is above the curve  $f_1$ . Therefore, in order for *all* firms to wait for no more or less than  $\tau$  time units,  $1 - p_\tau$  should fall in the area between the curves  $h$ ,  $f_1$  and  $f_2$ . Therefore, for low levels of competition the bounds that drive the equilibrium value of  $1 - p_\tau$  are  $f_1$  and  $f_2$ , which as Corollary 5 also shows are both increasing in competition. Thus, for small values of the competitive fringe, firms react to an increase in competition by being more innovative.

The shaded area between the curves  $g_1$ ,  $g_2$  and  $h$  represents the range of possible equilibrium values of  $1 - p_\tau$  for the higher levels of competition. Curves  $g_1$  and  $g_2$  are the counterparts of  $f_1$  and  $f_2$  subject to the constraint that the expected profits are zero. Notice that there exist certain values of  $a\eta\delta$  in between  $\bar{a}_1\bar{\eta}_1\delta$  and  $\underline{a}_2\underline{\eta}_2\delta$  or above  $\bar{a}_2\bar{\eta}_2\delta$ , where the shaded area between the curves

$g_1$  and  $g_2$  does not contain any of the possible values of  $1 - p_\tau$ .<sup>38</sup> The motivation for this fact is rather involved and its presentation is deferred to the Appendix. As stated in Corollary 3, for those values of competition, firms mix among three options and thus there exist two different possible values of the equilibrium belief. It is possible that when competition increases above the highest level that allows for a pure strategy equilibrium, firms mix among three options rather than two as in Figure 3. Graphically, this occurs if the curve  $h$  does not intersect at the peak a horizontal hashed line representing a value of  $1 - p_\tau$ , but instead intersects one of the vertical hashed lines. Figure 3 confirms the fact that for these higher values of competition, firms react in equilibrium by being less innovative when the competition in the industry increases.

### 4.3 The Innovation Maximizing Level of Competition

To provide some additional testable implications of our model, we examine the behavior of the peak of the inverted-U shape curve. More precisely, we first investigate the effect of a change in the cost of innovation on the values of competition that maximize the innovation in the industry and on the corresponding equilibrium risk of innovation. Second, we argue that our model supports theoretically two additional empirical facts uncovered by ABBGH(2005). These facts describe the behavior of peak of the curve in response to a change in the average technological gap in the industry. For uniformity of exposition, we define the set of innovation maximizing levels of competition to be the set of values of competition that induce the pure strategy equilibrium with the shortest waiting time. An inspection of Figure 3 reveals that due to the discrete information acquisition process, there exists a range of values of  $x$  that maximize innovation. In particular, in Figure 3, these values are in between the levels of competition corresponding to the intersections of the highest horizontal line  $1 - p_\tau$  with the curves  $f_2$  and  $h$ . However, as mentioned at the end of section 4.2, if the curve  $h$  intersects a vertical hashed line at the peak, the upper bound of this set is instead determined by the intersection of  $1 - p_\tau$  with the curve  $f_1$ .

To understand the conditions that determine this set of values, note first that as stated in section 4.2, the  $MC$  of waiting at  $\tau$  is higher than the  $MB$  of waiting at  $\tau$  when  $1 - p_\tau$  is below the curve  $f_2$ , and is equal precisely when the two curves intersect. So the lower bound of the interval, denoted by  $\bar{x}_0$  is determined by the condition that the  $MC$  and the  $MB$  of waiting at  $t_i + \tau$  are equal at the lowest possible equilibrium value of  $p_\tau$ . Denoting by  $S$  the event that the project is successful and by  $F$  the event that the signal received is "Fail", the  $MB$  of waiting is:  $c \cdot \Pr(F|\bar{S}) \cdot \Pr(\bar{S})$ . Since  $\Pr(F|S) = 0$ , it follows that the  $MB$  of waiting is  $c \cdot \Pr(F)$ . On the other hand, assuming that all other firms invest after waiting for  $\tau$  time units, the  $MC$  of waiting at moment  $t_i + t$  for firm  $i$  is the (expected) profit at moment  $t_i + t$ ,  $p_t [1 - \lambda(t|t_i, \tau, x)]$ , minus the expected value as of moment  $t_i + t$  of the (expected) profits at moment  $t_i + t + \delta$ . This second value

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<sup>38</sup>The set of possible levels for  $1 - p_\tau$  can be inferred in the figure from the values that the step function takes below  $\bar{a}\eta\delta$ .

is  $p_{t+\delta} [1 - \lambda(t + \delta|t_i, \tau, x)] \cdot \Pr(\bar{F}) + 0 \cdot \Pr(F)$ . Since  $p_t = p_{t+\delta} \Pr(\bar{F})$ , it follows then immediately that the *MC* at  $t_i + t$  equals  $p_t [\lambda(t + \delta|t_i, \tau, x) - \lambda(t|t_i, \tau, x)]$ . We have then that  $\bar{x}_0$  is determined by:

$$p_{\tau(\bar{x}_0)} [\lambda(\tau(\bar{x}_0) + \delta|t_i, \tau(\bar{x}_0), \bar{x}_0) - \lambda(\tau(\bar{x}_0)|t_i, \tau(\bar{x}_0), \bar{x}_0)] = c \Pr(F) \quad (5)$$

The upper bound of the interval, denoted by  $\bar{x}$ , is determined by one of the following two conditions. If the bound is at the intersection of  $h$  and  $1 - p_\tau$ , then the condition is that firms expect precisely zero profits from innovation for that value of  $p_\tau$  while they *all* invest. Using the notation introduced in (4), we can write this condition as<sup>39</sup>

$$p_{\tau(\bar{x}_1)} [1 - \lambda(\tau(\bar{x}_1)|t_i, \tau(\bar{x}_1), \bar{x}_1)] - c = 0 \quad (6)$$

On the other hand, if the upper bound is instead at the intersection of  $f_1$  and  $1 - p_\tau$ , the condition is that the *MC* and the *MB* of waiting at  $t_i + \tau - \delta$  are equal at the lowest possible equilibrium value of  $p_\tau$ . Using the same reasoning as above, it can be argued that the corresponding equation is:

$$p_{\tau(\bar{x}_2)-\delta} [\lambda(\tau(\bar{x}_2)|t_i, \tau(\bar{x}_2), \bar{x}_2) - \lambda(\tau(\bar{x}_2) - \delta|t_i, \tau(\bar{x}_2), \bar{x}_2)] = c \Pr(F) \quad (7)$$

For any value of the parameter of interest, equations (5), (6) and (7) determine the two bounds  $\bar{x}_0(\cdot)$  and  $\bar{x}(\cdot) = \min\{\bar{x}_1(\cdot), \bar{x}_2(\cdot)\}$ .

**Comparative statics with respect to the cost of innovation.** The following proposition describes the comparative statics of the set  $[\bar{x}_0(c), \bar{x}(c)]$  with respect to  $c$ :

**Proposition 6** *There exist four values of competition  $\bar{x}_L < \bar{x}_M < \bar{x}_H$  such that for all values of  $c$ , we have  $\bar{x}_0(c) \in [\bar{x}_L, \bar{x}_M]$  and  $\bar{x}(c) \in [\bar{x}_M, \bar{x}_H]$ . Moreover, there exists a sequence  $c_0 < c_1 < c_2 < \dots$  such that:*

(i) *when  $c = c_k$  for some  $k \geq 0$ ,  $\bar{x}_0(c) = \bar{x}_M = \bar{x}(c)$ ;*

(ii) *as  $c$  increases on  $(c_{k-1}, c_k]$ ,  $\bar{x}_0(c)$  increases continuously from  $\bar{x}_L$  to  $\bar{x}_M$ , while  $\bar{x}(c)$  first increases continuously from  $\bar{x}_M$  to  $\bar{x}_H$  and then decreases continuously from  $\bar{x}_H$  to  $\bar{x}_M$ ;*

(iii)  *$\tau(\bar{x}) = k\delta$  for all  $\bar{x}_0(c) \leq \bar{x} \leq \bar{x}(c)$  and  $c \in (c_{k-1}, c_k]$ .*

*Proof.* See Appendix C1.

The upper bound increases when driven by  $f_1$  and decreases when driven by  $h$ . To understand the result, note that since  $\tau(\bar{x}_0(c)) = \tau(\bar{x}(c))$ , by substituting  $p_{\tau(\bar{x}_0(c))}$  from (6) into (5), it follows that the *MC* and *MB* curves *at the peak of the inverted-U curve* are proportional to the cost of innovation  $c$ . Thus the adjustments of the *MC* and *MB* curves have the same amplitude at

<sup>39</sup>Since  $\lambda(\tau(x)|t_i, \tau(x), a, \eta) = \frac{1}{2}a\eta\delta$ , equation (6) provides a simple functional form for  $p_{\tau(\bar{x})}$ .

the peak. Therefore, the set of values of competition that make the values of the  $MC$  and  $MB$  approximately equal at the peak is the roughly the same for all values of  $c$ . This is precisely what Proposition 6 suggests. In the limit, as  $\delta \rightarrow 0$ , the interval collapses to a single point and the maximizing level of competition is the same for all values of  $c$ .<sup>40</sup> Therefore, a change in the cost of innovation does *not* have a meaningful effect on the level of competition that maximizes innovation. Finally, (iii) states that as the cost of innovation increases, the minimum possible equilibrium waiting time increases. This is natural since at the peak of the inverted-U curve, firms make almost zero expected profits so when the cost of innovation is higher, firms need to invest in safer projects to ensure non negative profits.

**Comparative statics with respect to the average technological gap.** ABBGH(2005) also study the relationship between the properties of the peak of the inverted-U shape curve and the degree of what ABBGH(2005) call the "neck-and-neckness" of an industry. The measure of the degree of neck-and-neckness in an industry that ABBGH(2005) use is the inverse of the average technological gap between the firms in an industry and the technological leader of that industry.<sup>41</sup> Using as a proxy for the technological gap between two firms the time between the moments when the firms make a certain technological breakthrough, that is, between the moments when the firms learn of the new invention, the average technological gap in our model is  $\frac{1}{\eta\delta} \int_0^{\eta\delta} x dx = \frac{\eta\delta}{2}$ . So  $\eta$  is also a measure of the average technological gap. In the empirical part of the paper, ABBGH(2005) show in Figure III that for the subsample of industries with a *higher* degree of neck-and-neckness, the inverted-U curve has a *higher peak* and attains this peak at a *lower* level of competition than the curve corresponding to the entire sample of industries. However, while the theoretical model in ABBGH(2005) does support the first of these two results, it does *not* support the second one. We will show next that our model replicates both of these two additional empirical regularities if our measure of technological gap can be considered as a proxy for the measure that ABBGH(2005) use.

In order to study the comparative statics with respect to the average technological gap, we will vary  $s \equiv \eta\delta$ , while holding the total level of competition as measured by  $a\eta\delta$  constant. This means that when  $s$  increases,  $a$  will decrease precisely so as to keep  $\beta \equiv a\eta\delta$  constant. Thus, in the following,  $\beta$  will parametrize the level of competition. Holding  $\beta$  constant ensures that the change in the technological gap does not induce a change in competition, so that the desired effect is precisely identified. In line with the notation introduced above,  $[\bar{\beta}_0(s), \bar{\beta}(s)]$  will denote the set of values of competition that maximize innovation.

**Proposition 7** *There exists a sequence  $s_0 < s_1 < s_2 < \dots$  such that:*

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<sup>40</sup>Note that this result, as well as the result of Proposition 7 below, does not hinge on a constant value of the  $MB$  of waiting.

<sup>41</sup>ABBGH(2005) use the total factor productivity of a firm as a measure of the firm's technological level.

- (i) as  $s$  increases on  $(s_{k-1}, s_k]$ ,  $\bar{\beta}_0(s)$  and  $\bar{\beta}(s)$  increase continuously in  $s$ ;
- (ii) when  $s = s_k$  for some  $k \geq 0$ ,  $\bar{\beta}_0(s) = \bar{\beta}(s)$ ;
- (iii)  $\bar{\beta}_0(s_{k-1}) < \bar{\beta}_0(s_k)$ ,  $\bar{\beta}(s_{k-1}) < \bar{\beta}(s_k)$ ;
- (iv)  $\tau(\bar{\beta}) = \tau_k$  for all  $\bar{\beta}_0(s) \leq \bar{\beta} \leq \bar{\beta}(s)$  and  $s \in (s_{k-1}, s_k]$  with  $\tau_{k+1} = \tau_k + \delta$ .

*Proof.* See Appendix C2.

Thus, as  $s$  increases, the set of values of competition that maximize innovation essentially moves to the right. On the other hand, the minimum possible pure strategy equilibrium waiting time weakly increases. Conversely, when  $s$  decreases, that is, when the average technological gap decreases or the degree of neck-and-neckness increases, the peak of the inverted-U curve will move up and to the left. This is precisely what ABBGH(2005) uncovered empirically. Intuitively, when the average technological gap decreases, each firm  $i$  expects that the moments when the rest of the firms learned of the same invention are closer to the moment when firm  $i$  learned. In other words, it increases the density of firms in the awareness window. This increases the *MC* of waiting for more information at any moment, and therefore induces firms to invest earlier for any value of competition. Moreover, since at the peak of the inverted-U curve firms make zero profits, the lower equilibrium belief about the ultimate success of the investment that results from investing earlier must correspond to a value of competition which is also lower.

#### 4.4 Welfare Analysis

A frequent critique of the standard models of welfare analysis is that they are rooted in static economic analysis aiming at minimizing the deadweight loss from monopoly while ignoring the dynamic effects of improvements in productivity or of the introducing of new products. A social planner that aims at designing the market structure that generates the welfare maximizing level of innovative activity has to account for a number of effects that a change in product market competition induces. First, in line with the standard model of welfare analysis, an increase in competition lowers the deadweight loss in the post innovation market. Second, as shown in our paper, the increase in competition changes the time firms wait before innovating, thus potentially generating the benefits from innovation earlier. Third, it induces an adjustment in the firms' risk taking behavior, thus determining the amount of resources spent on unsuccessful innovations. Finally, it affects the number of firms that engage into a specific line of research, thus potentially inducing a change in the amount of resources exhausted on parallel innovations. A fully fledged model of the social planner's problem would have to capture the welfare considerations in the post innovation market. In particular, it should specify the type of innovation under consideration and it should make a number of assumptions about the welfare benefits, in terms of consumer and



producer surpluses, that the innovation creates. This analysis is beyond the scope of our paper. However, by employing a reduced form model of the post innovation market, one can define the social planner's problem so that it captures all the other effects that the market structure has on innovation.

We denote by  $w(x)$  the ex post social welfare from a successful innovation as a function of the ex post level of competition. According to the standard models of social welfare, it follows that generically  $w'(x) \geq 0$ , for  $x \leq x_c$  where  $x_c$  corresponds to a perfectly competitive market. For instance, if the innovation results in a new good,  $x_c$  is the level of competition at which the marginal benefit for society of one more unit - the market price - equals average cost of producing all units. Thus, at  $x_c$ , firms make zero profits in the post innovation market. Note that since at  $\bar{x}$ , where  $\bar{x}$  is the threshold given by Proposition 2, firms make zero *expected* profits taking into account the risk and cost of innovation, it must be that  $\bar{x} \leq x_c$ . We assume that the social planner has a discount factor  $\gamma$ . Finally, to capture the effect of parallel innovations, we assume that the number of firms in the pre innovation market is fixed at  $N$ , that they all contemplate the idea of innovating, and that one firm is enough to develop the new product or process as long as the invention is feasible. Then, given the equilibrium values of  $\tau(x)$  and  $\alpha(x)$  derived in the previous analysis, the social planner's problem is to choose  $x$  to maximize:<sup>42</sup>

$$v(x) = e^{-(\gamma+\mu)\tau(x)} \{p_{\tau(x)}w(x\alpha(x)) [1 - (1 - \alpha(x))^N] - cN\alpha(x)\} \quad (8)$$

For  $x < \bar{x}$ , we showed that  $\alpha(x) = 1$ , so the social planner's value function reduces to  $v(x) = p_0w(x)e^{-\gamma\tau(x)} - cNe^{-(\gamma+\mu)\tau(x)}$ . Abstracting away from discreteness issues, we can write:

$$v'(x) = p_0w'(x)e^{-\gamma\tau(x)} - \gamma\tau'(x)p_0w(x)e^{-\gamma\tau(x)} + (\gamma + \mu)\tau'(x)cNe^{-(\gamma+\mu)\tau(x)} \quad (9)$$

Thus, an increase in competition increases the social welfare by increasing the post innovation welfare ( $p_0w'(x)e^{-\gamma\tau(x)}$ ) and by generating the innovation benefits earlier ( $-\gamma\tau'(x)p_0w(x)e^{-\gamma\tau(x)}$ ), and it lowers it through expenses incurred on unsuccessful projects ( $(\gamma + \mu)\tau'(x)cNe^{-(\gamma+\mu)\tau(x)}$ ). Certain conditions would determine an optimal level of competition lower than  $\bar{x}$ , the level that maximizes the innovative activity in the industry. First, this may happen when the industry-wide innovation costs are high relative to the expected benefits as measured by  $w(x)$ . This may happen when either the firm level innovation cost  $c$  or the number of firms in the industry  $N$  is high. Also a lower level of competition is beneficial when the innovation in the industry under consideration is characterized by an inherent high degree of technological or commercial risk, that is when  $p_0$  is small. On the other hand, more competition is better when the social marginal benefits in the post innovation market, as measured by  $w'(x)$  are sufficiently high, or when the speed of learning

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<sup>42</sup>Note that the probability of innovation is  $e^{-\mu\tau(x)}$ , that is the probability that the first  $\tau(x)$  signals are positive.

$\mu$  is high - in this last case the society benefits from the additional time spent on acquiring new information.

On the other hand, when  $x > \bar{x}$ , we have  $\alpha'(x) < 0$  and  $\tau'(x) \geq 0$ . On the other hand, as argued in section 4.1, for  $x > \bar{x}$ , we have  $\alpha(x)x = \bar{x}$ . Thus,

$$v'(x) = -\gamma\tau'(x)p_0w(\bar{x})e^{-\gamma\tau(x)} [1 - (1 - \alpha(x))^N] + (\gamma + \mu)\alpha(x)\tau'(x)cNe^{-(\gamma+\mu)\tau(x)} + (10) \\ + N\alpha'(x)p_0w(\bar{x})e^{-\gamma\tau(x)}(1 - \alpha(x))^{N-1} - \alpha'(x)cNe^{-(\gamma+\mu)\tau(x)}$$

First, when  $x = \eta$ , as we showed,  $\tau'(\eta) > 0$  so the increase in competition induces a later  $(-\gamma\tau'(x)p_0w(\bar{x})e^{-\gamma\tau(x)} [1 - (1 - \alpha(x))^N])$ , but safer  $((\gamma + \mu)\alpha(x)\tau'(x)cNe^{-(\gamma+\mu)\tau(x)})$  innovation. Second, the increase in competition decreases the probability that each firm would pursue that particular line of research. On the one hand, this has the negative effect that it lowers the chance that the innovation would be completed in the industry  $(N\alpha'(x)p_0w(\bar{x})e^{-\gamma\tau(x)}(1 - \alpha(x))^{N-1})$ ; on the other, it reduces the redundancies in innovation  $(-\alpha'(x)cNe^{-(\gamma+\mu)\tau(x)})$ . For values of  $x$  close enough to  $\bar{x}$ ,  $\tau'(x) \simeq 0$  because  $\tau$  achieves its minimum at  $\bar{x}$ , while  $(1 - \alpha(x))^{N-1} \simeq 0$  because  $\alpha(x) \simeq 1$ . Therefore, just above  $\bar{x}$  the only non-negligible effect of an increase in ex ante competition is the decrease in the redundancy effect. Therefore, when the optimal level of competition is not lower than  $\bar{x}$ , it will always be higher. However, note that even when  $x = a$ , so that the delay in innovation does not occur, it is never socially optimal to induce a too high level of product market competition. To see this, note that  $p_{\tau(a)}w(a\alpha(a)) \geq p_{\tau(a)}\theta(a\alpha(a)) = c$ , where the first inequality comes from the fact that the firms' profits are included in the social welfare, while the second from the zero profit condition. Since  $\tau'(a) = 0$ , it follows that  $v'(a_c) < 0$  and thus the optimal value of competition is lower than  $a_c$ . Intuitively, if the post innovation rents were too small, firms would end up investing in few projects so many otherwise successful lines of innovation would not be pursued.

#### 4.5 Alternative Equilibria

As in many dynamic discrete time settings (see Oberfield and Trachter (2010) for other examples) alternative symmetric equilibria might emerge in our model. In particular, in our case firms could mix among investing at different moments. Lemma 19 in Appendix C3 presents necessary conditions for such an equilibrium. In the set-up of our model, these conditions are very restrictive for symmetric equilibria other than the one from Proposition 2, and as illustrated numerically in the appendix, generically, these conditions are not satisfied. This is because the single crossing property between the  $MC$  and  $MB$  of waiting induces a concavity of the expected profit from innovation in the waiting time which makes it impossible for firms to be indifferent among more than two waiting times. Moreover, the two moments need to be  $\delta$  time units apart; otherwise firms would deviate and invest in between the two moments. But these are precisely the conditions that define

the equilibrium of Proposition 2.

## 4.6 The Case of Continuous Information Acquisition

To underscore the relevance of the discreteness of the information acquisition process specification from our model, we also present the main comparative statics with respect to the value of product market competition under the assumption that the information acquisition is continuous. While this is not always the case (see Oberfield and Trachter (2010) for counterexamples), for this model it is straightforward to show that assuming continuous information acquisition is equivalent to taking the limit  $\delta \rightarrow 0$ , while the length of the awareness window  $\eta\delta$  stays constant.

**Proposition 8** *When firms update their beliefs continuously, in a symmetric equilibrium there exist two thresholds  $\bar{a}, \bar{\eta}$  such that:*

(i)  $\tau(a)$  is decreasing in  $a$  for  $a < \bar{a}$  and a constant function of  $a$  for  $a > \bar{a}$ ;  $\alpha(a)$  is 1 for  $a < \bar{a}$  and is decreasing in  $a$  for  $a > \bar{a}$ .

(ii)  $\tau(\eta)$  is constant in  $\eta$  for  $\eta < \bar{\eta}$  and increasing in  $\eta$  for  $\eta > \bar{\eta}$ ;  $\alpha(\eta)$  is 1 for  $\eta < \bar{\eta}$  and is decreasing in  $\eta$  for  $\eta > \bar{\eta}$ .

*Proof.* See Appendix C4.

Therefore, while the inverted-U shape curve emerges again when competition is measured by  $a$ , it does not fully do so when the increase in competition is associated with an increase in the technological spread in the industry. This is because the value of the  $MC$  around the equilibrium waiting time does not increase when the length of the awareness window increases. To understand why, recall from the discussion motivating the results in Corollary 5 that the  $MC$  curve is very inelastic in the neighborhood around  $\tau$ . In particular, as  $\delta$  approaches 0 since according to the equilibrium strategies of the other firms, innovation has almost surely started at  $\tau - \delta$ , no measurably meaningful information arrives between  $\tau - \delta$  and  $\tau$ . Thus, an increase in  $\eta$  does not change the value of the  $MC$  at  $\tau - \delta$ . A similar argument explains why the  $MC$  at  $\tau$  does not change. Since  $MC$  is increasing in time, these are the only two values of the  $MC$  relevant for pinning down the equilibrium strategies. Therefore, firms do not change their waiting time for the lower levels of competition when  $\eta$  increases.<sup>43</sup> For higher values of  $\eta$ , as  $\eta$  increases,  $\alpha$  must decrease to satisfy the zero profit condition. This decreases the  $MC$  and induces firms to invest later.<sup>44</sup> The results

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<sup>43</sup>Note that this result does not hinge on a constant marginal benefit of waiting in time.

<sup>44</sup>Note that when the increase in competition is associated with an increase in  $a$ , the effect on the  $MC$  is of the first order and thus firms do wait less for the lower levels of competition. For the higher values, the effects of the increase in  $a$  and decrease in  $\alpha$  perfectly compensate each other and the equilibrium waiting time stays constant.

in Proposition 8 underscore the importance of the discrete information acquisition process in this setting.

## 5 Conclusion

The issue of innovation is complex and has many facets, some of which have been studied extensively in the industrial organization literature over the past half a century. Our model uncovers two of the main driving forces influencing the level of innovative activity in an industry. These two forces have not only the merit that they are sufficient to generate the empirically documented inverted-U shape relationship between competition and innovation, but they also offer reason to believe that they are indeed some of the major forces that influence a firm's innovation decisions. In order to isolate the effect of the trade-off that we focus on, we abstract away from other factors that may play a role in the firms' decision making process. Clearly, enriching the model to include some of these additional forces would improve the predictive power of the model.

The main policy implication of the results in our paper is that the way a policy maker should stimulate the innovative activity in an industry is not by always decreasing the level of product market competition, as Schumpeter suggested, or by always increasing it, as other economists who looked for a linear relationship concluded. Instead, my paper argues that a more thorough empirical analysis should be performed in order to find the right way to use the tool of the market structure design in promoting innovation for each industry under consideration.

A reduced form version of the model in this paper would have the marginal cost of waiting and the marginal benefit of waiting curves satisfying two conditions. First, they would exhibit the single crossing property. Second, the marginal cost curve would shift up in response to an increase in competition, while the marginal benefit curve would stay fixed. Then, an increase in competition would decrease the time at which the two curves intersect and thus explain the increase in innovation for the small values of competition. When this equilibrium waiting time is sufficiently low, firms would expect zero profits from innovation and thus a further increase in competition would require firms to become less innovative. The need for the fully fledged model in this paper stems mainly from three considerations. First, the reduced form model does not explain the link between the level of competition, which is a parameter with immediate empirical interpretation, and the marginal cost of waiting, whose interpretation is difficult in the absence of a well defined model. This is even more problematic when the increase in competition is associated with an increase in the technological spread rather than an increase in the technological density. Second, the reduced model would not immediately suggest the way in which firms can become less innovative for higher values of competition. Simply stating that they would invest later is unsatisfactory since the marginal cost of waiting would continue to increase and thus the trade-off would be solved earlier rather than later. The model in this paper allows distinguishing between

decreases in innovation that lead to a delay in innovation and decreases in innovation that lead to a decrease in the number of projects undertaken. Finally, the model predicts additional testable regularities that a reduced form model would not uncover.

## Appendix

### Appendix A1. Proof of Remark 1

Since  $G$  is a cumulative distribution function, it is right continuous and therefore the set of points of discontinuity is countable. Denote this set by  $S_d = \{s_1, s_2, s_3, \dots, s_{|S_d|}\}$ , where  $|S_d|$  can be  $\infty$ , and let  $s_0 \equiv t_0$ . Also, for any  $s \in S_d$  denote by  $G(s_-) \equiv \lim_{t \rightarrow s_-} G(s)$ , and for any  $t \in [t_0, \infty) \setminus S_d$  denote by  $g(t)$  the probability distribution function associated with  $G$ . Then, the total amount of profits earned from the innovation in the industry is:

$$\begin{aligned} \Pi &= \sum_{s \in S_d} \left[ 1 - G(s_-) - \frac{G(s) - G(s_-)}{2} - c \right] [G(s) - G(s_-)] + \int_{t \in [t_0, \infty) \setminus S_d} [1 - G(t) - c] g(t) dt \\ &= a\eta\delta(1 - c) - \sum_{i=1, \dots, |S_d|} \left[ \int_{s_{i-1}}^{s_i} G(t)g(t)dt + \frac{G^2(s) - G^2(s_-)}{2} \right] \end{aligned} \quad (12)$$

where we used the fact that  $G(\infty) = a\eta\delta$ . Integrating by parts  $\int_{s_{i-1}}^{s_i} G(t)g(t)dt$ , we obtain:  
 $\int_{s_{i-1}}^{s_i} G(t)g(t)dt = \frac{G^2(s_i) - G^2(s_{i-1})}{2}$ . Therefore, as claimed  $\Pi = a\eta\delta(1 - c) - \frac{(a\eta\delta)^2}{2}$ . ■

### Appendix A2.

The fact that the absolute profits of all but the most advanced firm in the industry decrease with competition is immediate. On the other hand, by Remark 1, the total profits in the industry are  $a\eta\delta(1 - c) - \frac{(a\eta\delta)^2}{2}$ , so the relative profit shares of the firm with rank  $\nu$  is:  $r(\nu) = \frac{1 - \nu - c}{a\eta\delta(1 - c) - \frac{(a\eta\delta)^2}{2}}$ . Taking derivatives with respect to  $x$ , where  $x$  is either  $a$  or  $\eta$  or  $a\eta$ , we obtain that:

$$\frac{\partial}{\partial x} r(\nu) = - \frac{(1 - \nu - c)(1 - a\eta\delta - c)}{\left[ a\eta\delta(1 - c) - \frac{(a\eta\delta)^2}{2} \right]^2} \frac{\partial (a\eta\delta)}{\partial x} \quad (13)$$

By (2),  $1 - a\eta\delta - c < 0$ , so  $\frac{\partial}{\partial x} r(\nu) > 0 \iff \nu < 1 - c$ . Therefore, as claimed, the relative profit shares of the most advanced firms increase with an increase in  $x$ . ■

### Appendix A3.

Since at  $t_i$ , from the perspective of firm  $i$ ,  $t_0$  is distributed uniformly on  $[t_i - \eta\delta, t_i]$ , the expected profit of firm  $i$  at  $t_i + t$  from investing in the innovation is  $p_t [1 - \lambda(t|t_i)] - c$ , where

$$\lambda(t|t_i) \equiv \int_{t_i - \eta\delta}^{t_i} \left[ m(t|t_i, t_0) + \frac{n(t|t_i, t_0)}{2} \right] \frac{1}{\eta\delta} dt_0 \quad (14)$$

the expected measure of firms that invested before and simultaneously to firm  $i$ . We dropped the conditionals on  $t_i$ ,  $a$  and  $\eta$  which are self evident from the context.

The following results describe the expected measure  $\lambda(t|t_i)$  for the various strategy profiles of interest.

**Lemma 9** *Consider a strategy profile under which firms mix between investing at moments  $\{\tau + \theta_1, \dots, \tau + \theta_n\}$  with probabilities  $\{\alpha_1, \dots, \alpha_n\}$ , where  $n \geq 1$ ,  $\theta_1 = 0$ , and the sequence  $\{\theta_k\} \subset \delta\mathbb{Z}_+$  is strictly increasing. Denote by  $\theta_0 \equiv \max(0, \tau + \theta_1 - \eta\delta) - \tau$ . Then, the expected measure of firms who have already invested before firm  $t_i$  at moment  $t_i + t$  is the following.*

$$\lambda(t|t_i) = \begin{cases} 0, & \text{for } t \in [0, \tau + \theta_0] \\ \sum_{k=1}^m a\alpha_k \left\{ \frac{\eta\delta}{2} + \min(t - \tau - \theta_k, \eta\delta) - \frac{1}{2\eta\delta} [\min(t - \tau - \theta_k, \eta\delta)]^2 \right\} + \\ \quad + \sum_{k=m+1}^n a\alpha_k \left[ \frac{(t - \tau - \theta_k + \eta\delta)^2}{2\eta\delta} \right], & \text{for } t \in [\tau + \theta_m, \tau + \theta_{m+1}], m \in \{0, \dots, n\} \\ a\eta\delta, & \text{for } t \geq \tau + \theta_n + \eta\delta \end{cases} \quad (15)$$

*Proof.* We may assume without loss of generality that the randomization over  $\{\tau + \theta_1, \dots, \tau + \theta_n\}$  is made at the beginning of the game so that at each instant  $t \in [t_0, t_0 + \eta\delta]$ , the mass  $a$  of firms who becomes aware at that moment can be distributed among a set of  $n$  groups, where group  $k \in \{1, \dots, n\}$  will contain the firms that will invest at moment  $\tau + \theta_k$ . Thus, at each instant  $t \in [t_0, t_0 + \eta\delta]$  there is a mass  $a\alpha_k$  of firms who become aware of the product and invest with probability 1 after exactly  $\tau + \theta_k$  time units. Denote by  $\lambda_k(t|t_i)$  the expected measure of firms out of group  $k$  that invested before  $t_i + t$ , from the perspective of firm  $t_i$ . Then, we will have:

$$\lambda(t|t_i) = \sum_{k=1}^n \lambda_k(t|t_i) \quad (16)$$

Firstly, note that due to the sequential awareness assumption and the fact that all firms wait the same number of time units,  $n(t|t_i, t_0) = 0$  for all  $t$ . Secondly, according to the strategy profile that we are considering, for a fixed value of  $t_0$  and for any  $t \geq 0$ , the last firm out of group  $k$  who has invested is the one that became aware at  $t_i + t - \tau - \theta_k$ , provided that  $t_i + t - \tau - \theta_k \in [t_0, t_0 + \eta\delta]$ . Denote by:

$$\varphi(t|t_i, t_0, \bar{\tau}) \equiv t_i + t - \bar{\tau} - t_0. \quad (17)$$

Consider now an arbitrary group  $k \in \{1, \dots, n\}$ . Note firstly that for  $t_i + t - \tau - \theta_k < t_0$ , no firm of the group has invested yet while for  $t_i + t - \tau - \theta_k \geq t_0 + \eta\delta$  all firms from the group have already invested. Secondly, for the remaining case the measure of firms out of the group who have invested at  $t_i + t$  is  $a\alpha_k\varphi(t|t_i, t_0, \tau)$ . In conclusion, the measure of firms who have already invested at  $t_i + t$  for a fixed value of  $t_0$  is:

$$m_k(t|t_i, t_0) = a\alpha_k \min(\eta\delta, \max(\varphi(t_i, t, \tau, t_0), 0)) \quad (18)$$

If  $\tau + \theta_1 > \eta\delta$ , take some  $t \in [0, \tau + \theta_1 - \eta\delta)$ , and note that the earliest moment when any firm could have invested is  $t_i - \eta\delta + \tau + \theta_1$ . This corresponds to the case when the first firm became aware exactly at  $t_i - \eta\delta$ . But for  $t \in [0, \tau + \theta_1 - \eta\delta)$  we have  $t_i - \eta\delta + \tau + \theta_1 > t_i + t$ , so at  $t_i + t$  the expected measure of firms who have invested before firm  $i$  is 0. Therefore,  $\lambda(t|t_i) = 0$  for  $t \in [0, \max(0, \tau + \theta_1 - \eta\delta)]$ .

Take some moment  $t \in [\tau + \theta_m, \tau + \theta_{m+1}]$ , for some  $m \in \{0, 1, \dots, n\}$ . Consider a group  $k$ , with  $k \geq m + 1$ , so that  $t \leq \tau + \theta_k$ . Note that  $\varphi(t|t_i, t_0, \tau + \theta_k) \leq \eta\delta \Leftrightarrow t_0 \geq t_i - \eta\delta + (t - \tau - \theta_k)$ . But  $t_0 \geq t_i - \eta\delta$  and  $0 \geq t - \tau - \theta_k$  together imply  $t_0 \geq t_i - \eta\delta + (t - \tau - \theta_k)$ , so  $\varphi(t|t_i, t_0, \tau + \theta_k) \leq \eta\delta$  for any possible value of  $t_0$ , as long as  $t \leq \tau + \theta_k$ . On the other hand,  $\varphi(t|t_i, t_0, \tau + \theta_k) \geq 0 \Leftrightarrow t_0 < t_i + t - \tau - \theta_k$ . Therefore, for  $t \in [\tau + \theta_m, \tau + \theta_{m+1})$  and  $k \geq m + 1$  we have:

$$\lambda_k(t|t_i) = \int_{t_i - \eta\delta}^{t_i + t - \tau - \theta_k} a\alpha_k [(t_i + t - \tau - \theta_k) - t_0] \frac{1}{\eta\delta} dt_0 = a\alpha_k \left[ \frac{(t - \tau - \theta_k + \eta\delta)^2}{2\eta\delta} \right] \quad (19)$$

Consider now a group  $k$  with  $k \leq m$  so that  $t > \tau + \theta_k$ . For any  $t_0$  we have  $\varphi(t|t_i, t_0, \tau + \theta_k) \geq 0$  because  $t_i + t - \tau - \theta_k - t_0 \geq 0 \Leftrightarrow t_0 \leq t_i + t - \tau - \theta_k$ , which is true because  $t \geq \tau + \theta_k$  and  $t_0 \leq t_i$ . On the other hand,  $\varphi(t|t_i, t_0, \tau + \theta_k) > \eta\delta$  when  $t_i + t - \tau - \theta_k > t_0 + \eta\delta \Leftrightarrow t_0 < t_i + t - \tau - \theta_k - \eta\delta$  so over this range of  $t_0$  we have that the measure out of group  $k$  is  $a\alpha_k\eta\delta$ . Thus, for  $t \in [\tau + \theta_m, \min(\tau + \theta_{m+1}, \tau + \theta_k + \eta\delta))$  and  $k \leq m$  we have:

$$\begin{aligned} \lambda_k(t|t_i) &= \int_{t_i - \eta\delta}^{t_i - \eta\delta + t - \tau - \theta_k} (a\alpha_k\eta\delta) \frac{1}{\eta\delta} dt_0 + \int_{t_i - \eta\delta + t - \tau - \theta_k}^{t_i} a\alpha_k [(t_i + t - \tau - \theta_k) - t_0] \frac{1}{\eta\delta} dt_0 = \\ &= a\alpha_k \left[ \frac{\eta\delta}{2} + (t - \tau - \theta_k) - \frac{(t - \tau - \theta_k)^2}{2\eta\delta} \right]. \end{aligned} \quad (20)$$

For  $t \geq \tau + \theta_k + \eta\delta$ , we have  $\lambda_k(t|t_i) = a\alpha_k\eta\delta$ . It is easy to see that these can be written concisely as we do in the text of the Lemma in (15).

Then, (16), (19) and (20) will give the measure as in the text of the Lemma for all  $t \leq \tau + \theta_n + \eta\delta$ . After  $t_i + \tau + \theta_n + \eta\delta$ , the measure is  $a\eta\delta$  for sure. ■

**Corollary 10**  $\lambda(t|t_i)$  is continuous, strictly increasing on  $[\max(0, \tau + \theta_1 - \eta\delta), \tau + \eta\delta]$  and differ-

entiable with a continuous derivative.

*Proof.* From (19) and (20), it is clear that  $\lambda_k(t|t_i)$  is continuous and differentiable with a continuous derivative on  $[0, \tau + \eta\delta] \setminus \bigcup_{k=0}^r \{ \tau + \theta_k, \tau + \theta_k + \eta\delta \}$ . For  $k \geq 1$ , we have:

$$\begin{aligned} \lim_{t \searrow \tau + \theta_k} \lambda_k(t|t_i) &= \lim_{t \nearrow \tau + \theta_k} \lambda_k(t|t_i) = a\alpha_k \frac{\eta\delta}{2} \text{ and } \lim_{t \searrow \tau + \theta_k} \frac{\partial}{\partial t} \lambda_k(t|t_i) = \lim_{t \nearrow \tau + \theta_k} \frac{\partial}{\partial t} \lambda_k(t|t_i) = a \\ \lim_{t \searrow \tau + \theta_k + \eta\delta} \lambda_k(t|t_i) &= \lim_{t \nearrow \tau + \theta_k + \eta\delta} \lambda_k(t|t_i) = a\alpha_k \eta\delta \text{ and } \lim_{t \searrow \tau + \theta_k + \eta\delta} \frac{\partial}{\partial t} \lambda_k(t|t_i) = \lim_{t \nearrow \tau + \theta_k + \eta\delta} \frac{\partial}{\partial t} \lambda_k(t|t_i) = 0 \end{aligned}$$

On the other hand,

$$\lim_{t \searrow \max(0, \tau + \theta_1 - \eta\delta)} \lambda_k(t|t_i) = \lim_{t \nearrow \max(0, \tau + \theta_1 - \eta\delta)} \lambda_k(t|t_i) = 0 \text{ and} \quad (21)$$

$$\lim_{t \searrow \max(0, \tau + \theta_1 - \eta\delta)} \frac{\partial}{\partial t} \lambda_k(t|t_i) = \lim_{t \nearrow \max(0, \tau + \theta_1 - \eta\delta)} \frac{\partial}{\partial t} \lambda_k(t|t_i) = 0 \quad (22)$$

Therefore,  $\lambda(t|t_i)$  is continuous with a continuous derivative on  $[0, \tau + \eta\delta]$  as a finite sum of functions with these properties.  $\lambda(t|t_i)$  is strictly increasing on  $[\max(0, \tau + \theta_1 - \eta\delta), \tau + \eta\delta]$  because  $\lambda_1(t|t_i)$  is strictly increasing on this interval, while  $\lambda_k(t|t_i)$  with  $k \geq 2$  are weakly increasing. ■

**Corollary 11** *Consider a strategy profile under which all firms wait exactly  $\tau$  time units before investing and then invest with probability 1. Then, the expected measure of firms who have invested before firm  $i$  at moment  $t_i + t$  is the following.*

$$\lambda(t|t_i) = \begin{cases} 0, & \text{for } t \in [0, \max(0, \tau - \eta\delta)] \\ a \left[ \frac{(t - \tau + \eta\delta)^2}{2\eta\delta} \right], & \text{for } t \in [\max(0, \tau - \eta\delta), \tau] \\ a \left[ \frac{\eta\delta}{2} + (t - \tau) - \frac{(t - \tau)^2}{2\eta\delta} \right], & \text{for } t \in [\tau, \tau + \eta\delta] \\ a\eta\delta, & \text{for } t \geq \tau + \eta\delta \end{cases} \quad (23)$$

*Proof.* This follows immediately from Lemma 9 by taking  $n = 1$  and  $\alpha_1 = 1$ .

**Corollary 12** *Consider a strategy profile under which each firm  $t_i$  mixes between investing and not investing with probability  $\alpha \in (0, 1)$  at moment  $t_i + \tau$  and invests with zero cumulative probability in the rest of the time. Then, the expected measure of firms who have already invested before firm  $t_i$  at moment  $t_i + t$  is  $\alpha\lambda(t|t_i)$ , where  $\lambda(t|t_i)$  is given by Corollary 11.*

*Proof.* Because in the mixed strategy equilibrium each firm invests after  $\tau$  time units with probability  $\alpha$  as opposed to probability 1 in the Corollary 11, it is as if the distribution of firms would be uniform with density  $\alpha a$  instead of  $a$  over a timespan of length  $\eta\delta$  and each firm would invest



after  $\tau$  time units with probability 1. By replacing  $a$  with  $\alpha a$  as the density in the Corollary 11, we obtain the measure  $\alpha\lambda(t|t_i)$ . ■

#### Appendix A4.

We will show that if *conditional* on an unsuccessful project, the probability of receiving a 'Pass' signal at  $t_i + t$  is:

$$r_t = \max\left(\frac{e^{-\mu t} - p_0}{e^{-\mu(t-\delta)} - p_0}, 0\right) \quad (24)$$

then the resulting *unconditional* probability of receiving a 'Pass' signal is precisely the one defined in Section 3.1. Thus, denoting by  $F_t$  the event of receiving a 'Fail' signal at  $t_i + t$  and by  $S$  the event that the project is successful, we have:

$$\Pr(\overline{F}_t) = \Pr(\overline{F}_t|\overline{S})(1 - p_{t-\delta}) + \Pr(\overline{F}_t|S)p_{t-\delta} = r_{t-\delta}(1 - p_{t-\delta}) + p_{t-\delta} \quad (25)$$

Given the signal structure introduced in Section 3.1, it is straightforward to see that the resulting law of motion for the posterior belief that the project is successful is the following:

- If  $p_0 \leq p_t < 1$ , then

$$p_{t+\delta} = \begin{cases} \min(1, p_t e^{\mu\delta}), & \text{with probability } \max(e^{-\mu\delta}, p_t) \\ 0, & \text{with probability } 1 - \max(e^{-\mu\delta}, p_t) \end{cases} \quad (26)$$

with  $\mu > 0$  and  $t \in \delta\mathbb{Z}_+$ .

- If  $p_t \in \{0, 1\}$ , for some  $t$ , then  $p_{t'} = p_t$  for all  $t' \geq t$ . Clearly, for a fixed  $p_0$ ,  $p_t$  can take values only in the set  $P(p_0, \mu, \delta) \equiv \{0, p_0, p_0 e^{\mu\delta}, p_0 e^{2\mu\delta}, p_0 e^{3\mu\delta}, \dots, 1\}$ .

Now, if  $t < t_M$ , (25) becomes  $\Pr(\overline{F}_t) = \frac{e^{-\mu t} - p_0}{e^{-\mu(t-\delta)} - p_0} (1 - p_0 e^{\mu(t-\delta)}) + p_0 e^{\mu(t-\delta)} = e^{-\mu\delta}$ . On the other hand, at  $t = t_M + \delta$ , we have  $e^{-\mu(t_M+\delta)} \leq p_0$  so  $\Pr(\overline{F}_{t_M+\delta}) = 0 \cdot (1 - p_{t_M}) + 1 \cdot p_{t_M} = p_{t_M}$ . This completes the argument.

#### Appendix B. Proof of Proposition 2.

We will prove Proposition 2 in three steps. Firstly, in Proposition 13, we will describe the pure strategy equilibria for the low levels of competition. Then, in Proposition 16 we will argue that there is a set of maximal values of competition above which the equilibria in Proposition 13 do not exist. Finally, in Proposition 17 we will describe the equilibria for the higher values of competition.

We start the proof by arguing that we can always restrict attention to action spaces that consist of investment decisions of firm  $i$  at times  $t_i + t$ , with  $t \in [\max(0, \tau - \eta\delta), \min(\tau + \eta\delta, t_M + \delta)] \cap \delta\mathbb{Z}_+$ .

Firstly, as we will show in Appendix A3, under any strategy profile of interest,  $\lambda(t|t_i)$  is strictly increasing in  $t$  on the interval  $[\max(0, \tau - \eta\delta), \tau + \eta\delta]$ , where  $\tau$  is the minimum amount of time that any of the competing firms waits before investing according to that strategy profile. Thus, in this interval, whenever a firm's optimal strategy is not to invest as soon as it learns some new information, then the strategy should prescribe the firm to wait at least  $\delta$  more units of time, until it receives the next piece of information. Also, we will show that if  $\tau > \eta\delta$ , then  $\lambda(t|t_i) = 0$  on  $[0, \tau - \eta\delta]$  so the firm does not have any incentive to invest before  $t_i + \tau - \eta\delta$  because, up to that moment, the expected profit from investing is strictly increasing. This is because the firm can gather information without risking that other firms invest it. Finally, the firm does not have any incentive to wait after  $t_M + \delta$  because there is no additional information left to acquire. On a separate note, we mention that given the sequential awareness assumption and the fact that, apart from this assumption, all firms are identical, in the symmetric strategy profiles that we will consider, no two firms will invest at the same time. Therefore,  $n(t|t_i, t_0) = 0$  for all  $t$ .

Denote by

$$MC_{\tau, \alpha}(t) = p_t [\lambda(t + \delta|t_i, \tau, \alpha) - \lambda(t|t_i, \tau, \alpha)] \quad (27)$$

to be the  $MC$  of waiting at  $t$  if all firms invest at  $\tau$  with probability  $\alpha$  and by

$$MC_{\tau, \alpha_\tau, \tau + \delta, \alpha_{\tau + \delta}}(t) = p_t [\lambda(t + \delta|t_i, \tau, \alpha_\tau, \tau + \delta, \alpha_{\tau + \delta}) - \lambda(t|t_i, \tau, \alpha_\tau, \tau + \delta, \alpha_{\tau + \delta})] \quad (28)$$

the  $MC$  of waiting at  $t$  if all the other firms mix among investing at one of the two moments  $\tau, \tau + \delta \in \delta\mathbb{Z}_+$  and not investing at all, with probabilities  $\alpha_\tau, \alpha_{\tau + \delta}$  and  $1 - \alpha_\tau + \alpha_{\tau + \delta}$  respectively. The marginal benefit of waiting is defined as  $MB = c(1 - e^{-\mu\delta})$ . We will show that the interpretation of the functional forms in the right hand side of these equations is indeed the one suggested by the name we associate them. We ignore for the time being the constraint that  $\tau \leq t_M + \delta$  assuming that it does not bind and then in Appendix B4 find conditions under which the constraint binds. Also, in Appendix B4, we find conditions under which firms invest as soon as they learn of the invention.

### Appendix B1. Proposition 13.

**Proposition 13** *In a symmetric strategy equilibrium, a necessary and sufficient set of conditions for the firms to wait for  $\tau \in \delta\mathbb{Z}_+$  before mixing between investing and not investing with probabilities  $\alpha$  and  $1 - \alpha$ , respectively, with  $\alpha \in [0, 1]$  is that:*

$$(1) \quad MC_{\tau, \alpha}(\tau - \delta) \leq MB \leq MC_{\tau, \alpha}(\tau) \quad (29)$$

$$(2) \quad p_\tau \left( 1 - \alpha a \frac{\eta\delta}{2} \right) - c \geq 0, \text{ with equality when } \alpha < 1 \quad (30)$$

$$(3) \quad 1 - \alpha a \eta\delta \leq c, \text{ when } \alpha < 1. \quad (31)$$

*Proof.* Assume all other firms invest after  $\tau$  time periods and denote by

$$\Psi(t) \equiv p_0(1 - \lambda(t|t_i)) - ce^{-\mu t}, \text{ for } t \geq 0. \quad (32)$$

Since the probability of not receiving a 'Fail' signal in the first  $t$  periods, with  $t \in \delta\mathbb{Z}_+$  and  $t < t_M$  is  $e^{-\mu t}$ , it follows that  $\Psi(t) = e^{-\mu t} [p_t(1 - \lambda(t|t_i)) - c]$  is the expected profits of firm  $i$  as of moment  $t_i$  from waiting  $t < t_M$  periods before investing. Thus, to prove the result it is enough to show that  $\Psi(\cdot)$  is maximized at  $t = \tau$  in the set  $[\max(0, \tau - \eta\delta), \min(\tau + \eta\delta, t_M + \delta)] \cap \delta\mathbb{Z}_+$ . From (23), when  $t < \tau$ , we have  $\Psi''(t) = -p_0\lambda''(t|t_i) - \mu^2 ce^{-\mu t} = -p_0 \frac{a\alpha}{\eta\delta} - \mu^2 ce^{-\mu t} < 0$ . On the other hand, for  $\tau < t < t_M$ , we have  $\Psi'''(t) = \mu^3 ce^{-\mu t} > 0$ .

Now firstly, the condition  $p_\tau \left(1 - \alpha a \frac{\eta\delta}{2}\right) \geq 0$ , ensures that  $\Psi(\tau) \geq 0$ . Secondly, the condition  $MC_{\tau,\alpha}(\tau - \delta) \leq MB$  is equivalent to  $\Psi(\tau) \geq \Psi(\tau - \delta)$ . Therefore, since  $\Psi$  is concave for  $t \leq \tau$  and it is increasing at  $\tau - \delta$ , it must be that it is increasing for all  $t \leq \tau - \delta$  and thus  $\Psi(t) \leq \Psi(\tau)$  for  $t \leq \tau$ . On the other hand,  $MB \leq MC_{\tau,\alpha}(\tau)$  is equivalent to  $\Psi(\tau) \geq \Psi(\tau + \delta)$ . Since  $\Psi'''(t) > 0$ , it follows that once  $\Psi$  is convex, it will be convex for all higher values. Since  $\Psi(\tau) \geq \Psi(\tau + \delta)$ ,  $\Psi$  is decreasing at  $\tau$ . But,  $\Psi$  can start increasing only after it becomes convex so after it starts increasing, it will increase forever. Since (31), for the case  $\alpha < 1$ , and (2) for the case  $\alpha = 1$ , ensure that  $\Psi(\tau + \eta\delta) < 0$ , it means that  $\Psi(t) < 0$  for  $t \leq \tau + \eta\delta$  when  $\tau + \eta\delta \leq t_M$ . Therefore, as desired,  $\Psi(\tau) \geq \Psi(t)$  for all  $0 \leq t \leq \tau + \eta\delta$  when  $\tau + \eta\delta \leq t_M$ . Since there is no new information arrival after moment  $t_M$ , it is straightforward to see that  $\Psi(\tau) \geq \Psi(t)$  for all  $0 \leq t \leq t_M$  when  $\tau + \eta\delta \leq t_M$ . Finally, note that when  $\alpha < 1$ , (30) is necessary to be satisfied with equality to have the firms willing to mix, while (31) is necessary because otherwise the firms could deviate and invest after they remove all uncertainty. ■

**Corollary 14** *In a symmetric pure strategy equilibrium, a necessary and sufficient condition for firms to wait for  $\tau \in \delta\mathbb{Z}_+$  before investing is that:*

$$\max \left( \frac{c}{1 - a \frac{\eta\delta}{2}}, \frac{2\eta}{2\eta - 1} \frac{c(1 - e^{-\mu\delta})}{a\delta} \right) \leq p_\tau \leq \min \left( \frac{2\eta}{2\eta - 1} \frac{c(e^{\mu\delta} - 1)}{a\delta}, 1 \right). \quad (33)$$

*Proof.* This follows immediately from Proposition 13 and Corollary 11. ■

**Proposition 15** *In a symmetric equilibrium in which all firms mix among investing at one of the two moments  $\tau, \tau + \delta \in \delta\mathbb{Z}_+$  and not investing at all, with probabilities  $\alpha_\tau \in (0, 1)$ ,  $\alpha_{\tau+\delta} \in (0, 1)$*

and  $1 - \alpha_\tau + \alpha_{\tau+\delta}$  respectively, a set of necessary and sufficient conditions is:

$$p_\tau \left[ 1 - a\alpha_\tau \frac{\eta\delta}{2} - a\alpha_{\tau+\delta} \frac{(\eta-1)^2\delta}{2\eta} \right] = c \quad (34)$$

$$p_{\tau+\delta} \left[ 1 - a\alpha_\tau \left( \frac{\eta\delta}{2} + \delta - \frac{\delta}{2\eta} \right) - a\alpha_{\tau+\delta} \frac{\eta\delta}{2} \right] = c \quad (35)$$

$$1 - a(\alpha_\tau + \alpha_{\tau+\delta})\eta\delta \leq c \quad (36)$$

*Proof.* Assume all other firms mix among investing at one of the two moments  $\tau, \tau + \delta \in \delta\mathbb{Z}_+$  and not investing at all, with probabilities  $\alpha_\tau \in (0, 1)$ ,  $\alpha_{\tau+\delta} \in (0, 1)$  and  $1 - \alpha_\tau + \alpha_{\tau+\delta}$  respectively. Then, as in (32), denote by  $\Psi(t) \equiv p_0\Pi(t|t_i) - ce^{-\mu t}$  and we will show that  $\Psi''(t) < 0$  for  $t < \tau$  and  $\Psi'''(t) > 0$  for  $t > \tau + \delta$  so that the argument from the proof of Proposition 13 will go through in this case as well with a slight modification.

Employing (15), we have for  $t < \tau$  that

$$\Psi''(t) = \begin{cases} -p_0a\frac{1}{\eta\delta} - \mu^2ce^{-\mu t}, & \text{for } t < \tau \\ p_0a\alpha_\tau\frac{1}{\eta\delta} - p_0a\alpha_{\tau+\delta}\frac{1}{\eta\delta} - \mu^2ce^{-\mu t}, & \text{for } \tau < t < \tau + \delta \end{cases} \quad (37)$$

On the other hand,  $\Psi'''(t) = \mu^3ce^{-\mu t}$  for all  $t$ . Note that  $\Psi''(t) < 0$  for  $t < \tau$  and  $\Psi'''(t) > 0$  for  $t > \tau + \delta$ . Moreover, it can be shown that  $\lim_{t \nearrow \tau+\delta} \Psi''(t) = p_0a\alpha_\tau\frac{1}{\eta\delta} - p_0a\alpha_{\tau+\delta}\frac{1}{\eta\delta} - \mu^2ce^{-\mu(\tau+\delta)} < \lim_{t \searrow \tau+\delta} \Psi''(t) = p_0a(\alpha_\tau + \alpha_{\tau+\delta})\frac{1}{\eta\delta} - \mu^2ce^{-\mu(\tau+\delta)}$ . Finally, the condition from (34) and (35) in the text of the Proposition imposes that  $\Psi(\tau) = \Psi(\tau + \delta) = 0$ .

We will argue now that  $\Psi(\tau) \geq \Psi(t)$  for all  $t \in \delta\mathbb{Z}_+$ . Consider two cases. *Case 1:*  $\alpha_\tau \leq \alpha_{\tau+\delta}$ . In this case, from (37) it follows that  $\Psi''(t) < 0$  for  $t \in (\tau, \tau + \delta)$ . Therefore the function  $\Psi$  is concave for  $t < \tau + \delta$ . It is clear then that this and  $\Psi(\tau) = \Psi(\tau + \delta)$  imply that  $\Psi'(\tau) > 0$  and  $\Psi'(\tau + \delta) < 0$ .<sup>45</sup> Since  $\Psi'''(t) > 0$  for  $t > \tau + \delta$ , and (36) implies  $\Psi(\min(\tau + \eta\delta + \delta, t_M)) < 0$ , an argument similar to the one from the proof of Proposition 13 shows the result. *Case 2:*  $\alpha_\tau > \alpha_{\tau+\delta}$ . In this case,  $\Psi'''(t) > 0$  for  $t \geq \tau$  together with the facts that  $\lim_{t \nearrow \tau+\delta} \Psi''(t) < \lim_{t \searrow \tau+\delta} \Psi''(t)$  implies that once  $\Psi$  is convex, it will be convex for all higher values. If  $\Psi$  were increasing at  $\tau + \delta$ , then it should already be convex there and thus it would be increasing for all values above  $\tau + \delta$ . But this contradicts the fact that  $\Psi(\tau) > 0 > \Psi(\min(\tau + \eta\delta + \delta, t_M))$ . Therefore,  $\Psi'(\tau + \delta) < 0$ . Also,  $\Psi'(\tau) > 0$  because otherwise, in order for  $\Psi$  to be decreasing at  $\tau + \delta$ , it should have increased somewhere between  $\tau$  and  $\tau + \delta$ , which would imply that  $\Psi$  was convex at that point and therefore convex and increasing from that point to  $\tau + \delta$ . But this would contradict the fact that  $\Psi$  should be decreasing at  $\tau + \delta$ . The rest of the argument goes as in the previous case. ■

<sup>45</sup>Note that Corollary 10 ensures that the function  $\Psi$  is differentiable at both  $\tau$  and  $\tau + \delta$ .

## Appendix B2. Proposition 16.

**Proposition 16** For any  $P(p_0, \mu, \delta)$ , denote by  $\bar{x}$ , a maximal value of competition for which there exists a waiting time  $\Upsilon \in \delta\mathbb{Z}_+$  such that

$$\frac{2\eta}{2\eta-1} \frac{c(1-e^{-\mu\delta})}{a\delta} \leq p_\Upsilon = \frac{c}{1-a\frac{\eta\delta}{2}} \leq \min\left(\frac{2\eta}{2\eta-1} \frac{c(e^{\mu\delta}-1)}{a\delta}, 1\right) \quad (38)$$

Then, for any  $x > \bar{x}$ , there is no equilibrium with firms expecting strictly positive profits.

*Proof.* Let  $(\Upsilon, \bar{x})$  be a pair with the properties from the text of the Claim. Then this pair must satisfy:

$$p_\Upsilon e^{\mu\delta} > \frac{2\eta}{2\eta-1} \frac{c(e^{\mu\delta}-1)}{a\delta} \quad (39)$$

because otherwise there would exist a higher value of competition  $\bar{x}'$  such that  $(\Upsilon + \delta, \bar{x}')$  would satisfy the properties from the text of the Claim. Now, when  $x$  increases above  $\bar{x}$ , firms would make strictly negative profits if they were *all* to continue to invest at  $\Upsilon$ . Therefore, they would need to start investing at  $\Upsilon + \delta$  or later. But, (39) is in that case inconsistent with the necessary conditions for a pure strategy equilibrium as defined by (33). ■

## Appendix B3. Proposition 17.

**Proposition 17** Let  $\bar{x}$  be any value of competition satisfying (39). Then, for all values of  $x \geq \bar{x}$  there exists a symmetric mixed strategy equilibrium. In this equilibrium, for any value of  $p_0$ , one and only one of the following two cases is true:

1. there exists  $\tau \in \delta\mathbb{Z}_+$  and  $\alpha \in (0, 1)$  such that all firms wait for a period  $\tau \in \delta\mathbb{Z}_+$  and then in period  $\tau$  mix between investing and not investing with probability  $\alpha$ .  $\alpha$  satisfies the condition:

$$p_\tau \left(1 - \alpha a \frac{\eta\delta}{2}\right) - c = 0 \quad (40)$$

2. there exists  $\tau \in \delta\mathbb{Z}_+$  such that the equilibrium prescribes that all firms mix among investing at one of the two moments  $\tau, \tau + \delta \in \delta\mathbb{Z}_+$  and not investing at all, with probabilities  $\alpha_\tau \in (0, 1)$ ,  $\alpha_{\tau+\delta} \in (0, 1)$  and  $1 - \alpha_\tau + \alpha_{\tau+\delta}$  respectively, satisfying:

$$p_\tau \left[1 - a\alpha_\tau \frac{\eta\delta}{2} - a\alpha_{\tau+\delta} \frac{(\eta-1)^2 \delta}{2\eta}\right] - c = 0 \quad (41)$$

$$p_{\tau+\delta} \left[1 - a\alpha_\tau \left(\frac{\eta\delta}{2} + \delta - \frac{\delta}{2\eta}\right) - a\alpha_{\tau+\delta} \frac{\eta\delta}{2}\right] - c = 0 \quad (42)$$

The resulting equilibrium profile of  $p_\tau$  and thus of  $\tau$  is weakly increasing in the degree of competition when  $x = \eta$  and constant when  $x = a$ .

*Proof.* Denote by  $\Upsilon$ , the equilibrium waiting time at  $\bar{x}$  and assuming for the time being that  $\Upsilon - \delta$  is not sustainable any type of equilibrium, denote by  $\underline{\theta}_1$  be a minimal value of competition for which  $\Upsilon$  is an equilibrium waiting time in pure strategies. Note that at  $\underline{\theta}_1$ , the firms just switched from investing after  $\Upsilon + \delta$  time units to investing after  $\Upsilon$  time units, so they were just indifferent between investing at  $\Upsilon$  and investing at  $\Upsilon + \delta$ . Alternatively put,  $MC_{\Upsilon+\delta,1}(\Upsilon) = MC_{\Upsilon,1}(\Upsilon) = MB$ . Now, in between  $\underline{\theta}_1$  and  $\bar{x}$ ,  $MC_{\Upsilon+\delta,1}(\Upsilon)$  increases exceeding  $MB$  thus making firms deviate and invest at  $\Upsilon$  if all other firms were to invest at  $\Upsilon + \delta$ .  $MC_{\Upsilon,1}(\Upsilon)$  also increases above  $MB$  thus making the firms want to invest at  $\Upsilon$  in an equilibrium in which all firms invest at  $\Upsilon$ . At  $\bar{x}$  though, the nonnegative profit constraint binds. Above  $\bar{x}$ , if the all firms were to continue to invest at  $\Upsilon$ , they would make negative profits. To avoid this, firms can either switch immediately to investing at  $\Upsilon + \delta$  or can invest in fewer projects. But switching to investing at  $\Upsilon + \delta$  immediately is not a feasible equilibrium strategy because  $MC_{\Upsilon+\delta,1}(\Upsilon)$  would still be higher than  $MB$  so provided that the rest of the firms invest at  $\Upsilon + \delta$ , any firm would be better off deviating and investing at  $\Upsilon$ . Therefore, the firms need to start investing in fewer projects which would have the effect of reducing the total measure of firms in the market and thus allow for the non negative profits condition to continue to be satisfied.

Now, by Corollary 12, we have that in an equilibrium in which all firms mix with probability  $\alpha$ , the expected measure of firms who already invested at  $t + t_i$  is  $\alpha\lambda(t|t_i)$ , with  $\lambda(t|t_i)$  as in Corollary 11. Note that since the firm mixes in period  $\tau$  between investing at that moment and never investing, its expected profits from investing in period  $\tau$ ,  $p_\tau[1 - \alpha\lambda(\tau|t_i)] - c = p_\tau[1 - a\alpha\frac{\eta^\delta}{2}] - c$  should be zero, which is the condition in (40).

**Claim 18** Let  $\alpha(x)$  be defined implicitly by the equation  $p_\tau[1 - a\alpha\frac{\eta^\delta}{2}] - c = 0$ . Then: (i)  $MC_{\tau,\alpha(a)}(\tau)$  and  $MC_{\tau,\alpha(a)}(\tau - \delta)$  are constant in  $a$ ; (ii)  $MC_{\tau,\alpha(\eta)}(\tau)$  and  $MC_{\tau,\alpha(\eta)}(\tau - \delta)$  are decreasing in  $\eta$ .

*Proof.*  $MC_{\tau,\alpha}(\tau) = p_\tau a \alpha \delta \left(1 - \frac{1}{2\eta}\right)$ , so since  $\alpha = \frac{2}{a\eta^\delta} \left(1 - \frac{c}{p_\tau}\right)$ , from the zero profit condition, we will have  $MC_{\tau,\frac{2}{a\eta^\delta} \left(1 - \frac{c}{p_\tau}\right)}(\tau) = p_\tau \frac{2}{\eta} \left(1 - \frac{c}{p_\tau}\right) \left(1 - \frac{1}{2\eta}\right)$ , which is constant in  $a$  and decreasing in  $\eta$ . Clearly,  $MC_{\tau,\alpha}(\tau - \delta) = p_{\tau-\delta} a \alpha \delta \left(1 - \frac{1}{2\eta}\right)$  satisfies the same properties. ■

Thus, when  $x = a$ , the conditions of Proposition 13 are satisfied for  $\tau = \Upsilon$  and  $\alpha(a) = \frac{2}{a\eta^\delta} \left(1 - \frac{c}{p_\Upsilon}\right)$ .

On the other hand,  $MC_{\Upsilon,\alpha(\eta)}(\Upsilon - \delta)$  and  $MC_{\Upsilon,\alpha(\eta)}(\Upsilon - \delta)$  decrease when  $\eta$  increases above  $\bar{\eta}$ . Since  $MC_{\Upsilon,\alpha(\bar{\eta})}(\Upsilon - \delta) < MB < MC_{\Upsilon,\alpha(\bar{\eta})}(\Upsilon)$ , these inequalities will be also satisfied for a range of values of  $\eta$  above  $\bar{\eta}$ . Moreover, from the zero profit conditions at  $\bar{\eta}$  and  $\eta$ , we have that  $a\alpha\eta = \bar{a}\bar{\eta}$ , so

since  $1 - a\eta\delta|_{x=\bar{x}} < c$ , we will also have  $1 - a\alpha\eta\delta < c$ . Therefore, by Proposition 13, in equilibrium firms will invest at  $\Upsilon$  with probability  $\alpha$  such that  $p_{\Upsilon}[1 - a\alpha\frac{\eta\delta}{2}] - c = 0$ . Now, denote by  $\bar{\eta}_1$ , the value of  $\eta$  for which  $MC_{\Upsilon,\alpha(\eta)}(\Upsilon) = MB$  and note that as  $\eta$  increases above  $\bar{\eta}_1$ , the equilibrium in which all firms invest at  $\Upsilon$  is no longer sustainable. This is because  $MC_{\Upsilon,\alpha(\eta)}(\Upsilon) < MB$  so the firms would deviate and invest at  $\Upsilon + \delta$ .

However, note that immediately above  $\bar{\eta}_1$ , a profile of strategies in which *all* firms would mix at  $\Upsilon + \delta$  does not constitute an equilibrium. To see this, denote by  $\alpha'$  the mixing probability at  $\alpha(\bar{\eta}_1)$  such that  $p_{\Upsilon}[1 - a\alpha'\frac{\bar{\eta}_1\delta}{2}] - c = 0$ . Note that  $MC_{\Upsilon,\alpha(\bar{\eta}_1)}(\Upsilon) = p_{\Upsilon}a\alpha(\bar{\eta}_1)\delta\left(1 - \frac{1}{2\bar{\eta}_1}\right) = MB$ . Now, if immediately above  $\bar{\eta}_1$ , firms were to mix at  $\Upsilon + \delta$ , in order for the zero profit condition to be satisfied, since the firms invest in safer projects,  $\alpha$  should have an immediate upward jump to  $\alpha'' > \alpha(\bar{\eta}_1)$ , where  $\alpha''$  satisfies  $p_{\Upsilon+\delta}[1 - a\alpha''\frac{\bar{\eta}_1\delta}{2}] - c = 0$ . But then,  $MC_{\Upsilon+\delta,\alpha''}(\Upsilon) = p_{\Upsilon}a\alpha''\delta\left(1 - \frac{1}{2\bar{\eta}_1}\right) > MC_{\Upsilon,\alpha(\bar{\eta}_1)}(\Upsilon) = MB$ . So if all the other firms invest at  $\Upsilon + \delta$ , any firms would have an incentive to deviate and invest at  $\Upsilon$ .

Therefore, for a range of values of  $\eta$  above  $\bar{\eta}_1$ , firms would mix among investing at  $\Upsilon$ , investing at  $\Upsilon + \delta$  and not investing at all. The corresponding mixing probabilities  $\alpha_{\Upsilon}$  and  $\alpha_{\Upsilon+\delta}$  will be given by the zero profit conditions as in (41) and (42). Note that

$$\alpha_{\Upsilon}(\eta) = \frac{1}{a\delta} \frac{2\eta}{2\eta - 1} \left(1 - \frac{c}{p_{\Upsilon+\delta}}\right) - \frac{1}{a\delta} \frac{2\eta^3}{(2\eta - 1)^2} \frac{c(1 - e^{-\mu\delta})}{p_{\Upsilon}} \quad (43)$$

$$\alpha_{\Upsilon+\delta}(\eta) = \frac{1}{a\delta} \frac{2\eta^3}{(2\eta - 1)^2} \frac{c(e^{\mu\delta} - 1)}{p_{\Upsilon+\delta}} - \frac{1}{a\delta} \frac{2\eta}{2\eta - 1} \left(1 - \frac{c}{p_{\Upsilon}}\right) \quad (44)$$

so  $a[\alpha_{\Upsilon}(\eta) + \alpha_{\Upsilon+\delta}(\eta)]\eta\delta = \frac{2\eta^2}{2\eta - 1} \frac{c(1 - e^{-\mu\delta})}{p_{\Upsilon}}$  is increasing in  $\eta$ . Therefore, since  $\alpha_{\Upsilon+\delta}(\bar{\eta}_1) = 0$ , we will have  $1 - a[\alpha_{\Upsilon}(\eta) + \alpha_{\Upsilon+\delta}(\eta)]\eta\delta < 1 - a\alpha(\bar{\eta}_1)\bar{\eta}_1\delta < c$  so by Proposition 15, this will constitute an equilibrium strategy.

As  $\eta$  increases above  $\bar{\eta}_1$ ,  $\alpha_{\Upsilon}(\eta)$  decreases and  $\alpha_{\Upsilon+\delta}(\eta)$  increases. Denote by  $\underline{\eta}_2$  the value of  $\eta$  that satisfies  $\alpha_{\Upsilon}(\eta) = 0$ . Then, as competition increases above  $\bar{\eta}_1$  the equilibrium prescribes that the firms gradually shift the weight of the mixing probabilities from moment  $\Upsilon$  towards moment  $\Upsilon + \delta$ , until at  $\underline{\eta}_2$ , firms no longer invest at  $\Upsilon$ , and mix only between investing and not investing at  $\Upsilon + \delta$  with probability  $\alpha_{\Upsilon+\delta}(\underline{\eta}_2)$ . Note that above  $\underline{\eta}_2$ , the equilibrium in which firms mix at  $\Upsilon$  and  $\Upsilon + \delta$  no longer exists. However, it can be shown that at  $\underline{\eta}_2$ , we have  $MC_{\Upsilon+\delta,\alpha_{\Upsilon+\delta}(\underline{\eta}_2)}(\Upsilon) = MC_{\Upsilon,\alpha_{\Upsilon+\delta}(\underline{\eta}_2)}(\Upsilon) = MB$ . Using the same arguments as before, it can be shown that above  $\underline{\eta}_2$ , it is an equilibrium for all firms to mix at  $\Upsilon + \delta$ , with a decreasing probability. As shown in Claim 18,  $MC_{\Upsilon+\delta,\alpha(\eta)}(\Upsilon + \delta)$  and  $MC_{\Upsilon+\delta,\alpha(\eta)}(\Upsilon)$  will be decreasing until  $\bar{\eta}_2$  where  $MC_{\Upsilon+\delta,\alpha(\eta)}(\Upsilon + \delta) = MB$  and the firms will start mixing among investing at  $\Upsilon + \delta$ , investing at  $\Upsilon + 2\delta$  and not investing at all and then the process will repeat.

Finally, the fact that  $\alpha_{\Upsilon}(\eta)$  is decreasing in  $\eta$ , while  $\alpha_{\Upsilon+\delta}(\eta)$  is increasing also shows that there

can be no equilibrium in which firms mix at two moments for those values of competition for which there exists an equilibrium that prescribes firms to mix only at one moment. To see this, note that whenever firms mix at two moments, the mixing probability on the later moment should increase in  $\eta$  in order for the firms to make zero profits at both of those moments. However, we know that at  $\bar{\eta}_1$  firms are making zero profit at both  $\Upsilon$  and  $\Upsilon + \delta$ , while the mixing probability at  $\Upsilon + \delta$  is zero. Therefore, that probability could have not increased below  $\bar{\eta}_1$ .

The argument for the case in which  $\Upsilon - \delta$  is sustainable in a mixed strategy equilibrium is similar to the one above. If  $x := a$  firms start mixing immediately above  $\bar{x}$  between investing at  $\Upsilon - \delta$  and not investing at all, with probabilities strictly lower than 1. If  $x := \eta$ , then immediately above  $\bar{x}$ , firms mix among investing at  $\Upsilon - \delta$ , investing at  $\Upsilon$  and not investing at all. As competition continues to increase, the weight on  $\Upsilon - \delta$  decreases, while the weight on  $\Upsilon$  increases until a point at which firms mix only between investing after  $\Upsilon$  time units and not investing at all. Then the rest of the argument is as above. This completes the proof of Proposition 17. ■

#### Appendix B4.

Assume first that  $-\mu - \ln p_0 \in \delta\mathbb{Z}_+$ , so that  $t_M = -\mu - \ln p_0$  and  $p_{t_M} = e^{\mu\delta} p_{t_M - \delta} = 1$ . Then, condition (29) from the text of Proposition 13 becomes  $MC_{t_M, \alpha}(t_M - \delta) \leq MB$ , because the marginal benefit of waiting at  $t_M$  is zero. In the text of the Corollary 14 this condition becomes  $\frac{2\eta}{2\eta-1} \frac{c(e^{\mu\delta}-1)}{a\delta} \geq 1$ . Therefore, when  $\frac{2\eta}{2\eta-1} \frac{c(e^{\mu\delta}-1)}{a\delta} \geq 1$  and  $1 - a\frac{\eta\delta}{2} - c \geq 0$  are satisfied, firms wait precisely  $t_M$  periods in equilibrium. Since these two conditions are satisfied precisely when the level of competition is small, we conclude as expected that when for low enough values of competition, firms wait until they remove all uncertainty.

On the other hand, if  $\bar{x}$  that satisfies  $1 - a\frac{\eta\delta}{2} - c \Big|_{x=\bar{x}} = 0$ , also satisfies  $\frac{2\eta}{2\eta-1} \frac{c(e^{\mu\delta}-1)}{a\delta} \Big|_{x=\bar{x}} \geq 1$ , then no pure strategy equilibrium exists for  $x > \bar{x}$ ; this is the counterpart of Proposition 16. Thus, for  $x > \bar{x}$ , as in Proposition 17, firms start mixing. While  $MC_{t_M, \alpha(x)}(t_M)$  is weakly decreasing in  $x$ , where  $\alpha(x)$  is defined as in the text of Claim 18, the fact that the marginal benefit of waiting at  $t_M$  is zero induces firms to invest no later than  $t_M$ . Since  $MC_{t_M, \alpha(x)}(t_M - \delta)$  is also weakly decreasing in  $x$ , firms will never invest earlier than  $t_M$ . Thus, in this case, the equilibrium will specify that all firms wait precisely  $t_M$  periods and invest with probability  $\alpha(x)$ . When  $-\mu - \ln p_0 \notin \delta\mathbb{Z}_+$ , the main difference is that if  $MC_{t_M, \alpha}(t_M)$  is smaller than the marginal benefit of waiting at  $t_M$ , which is  $c(1 - p_{t_M})$  firms will invest at  $t_M + \delta$ , but the rest of the analysis is similar.

Finally, note when  $p_0 \geq \max\left(\frac{c}{1-a\frac{\eta\delta}{2}}, \frac{2\eta}{2\eta-1} \frac{c(1-e^{-\mu\delta})}{a\delta}\right)$ , that is when  $MB \leq MC_{0,1}(0)$  and the non-negative profit condition is satisfied at belief  $p_0$ , then firms invest in development of the new product as soon as they learn of it. Since  $\frac{2\eta}{2\eta-1} \frac{c(1-e^{-\mu\delta})}{a\delta}$  decreases in  $x$ , this condition will continue to be satisfied as long as  $p_0 \geq \frac{c}{1-a\frac{\eta\delta}{2}}$ . For high enough levels of competition, the non negative profit



condition is no longer satisfied and thus firms will start mixing. As in Proposition 17, if  $x = a$  they will mix immediately, while if  $x = \eta$  then they will start mixing later as  $x$  increases. This concludes the proof of Proposition 2. ■

### Appendix C1. Proof of Proposition 6.

Define  $\bar{x}_M$  as the value of competition that satisfies:

$$\frac{1}{1 - a\frac{\eta\delta}{2}} = \frac{2\eta}{2\eta - 1} \frac{(1 - e^{-\mu\delta})}{a\delta} \quad (45)$$

and for  $k \geq 0$ , let

$$c_k \equiv p_{k\delta} \left[ 1 - a\frac{\eta\delta}{2} \right] \Big|_{x=\bar{x}_M}$$

Note then that by Proposition 13, Corollary 14 and (45),  $\tau(\bar{x}_M) = k\delta$  when the cost of innovation is precisely  $c_k$ . From (45) it also follows that at  $\bar{x}_M$  we have  $MC_{k\delta,1}(k\delta) = MB$ , and that when the cost of innovation is  $c_k$ , the only value of competition for which a pure strategy exists is  $\bar{x}_M$ . Now, as  $c$  increases above  $c_k$ , there is *no value of competition* for which  $p_{k\delta}$  can be sustained in a pure strategy equilibrium. To see this, note that if that was possible, it must be that the corresponding level of competition, call it  $\bar{x}$ , is lower than  $\bar{x}_M$  to ensure non-negative profits. But in that case,  $MC_{k\delta,1}(k\delta) = p_{k\delta}a\delta \left( 1 - \frac{1}{2\eta} \right) \Big|_{x=\bar{x}} > MB$  so firms would wait at  $k\delta$ . Therefore, when  $c$  increases above  $c_k$ , the minimum possible equilibrium value of the belief in the success of the project is  $p_{(k+1)\delta}$ .

For a fixed value of  $c$ , the values of competition for which  $p_{(k+1)\delta}$  is a pure strategy equilibrium belief are given by Corollary 14. Since

$$p_{(k+1)\delta} = e^{\mu\delta} p_{k\delta} = \frac{e^{\mu\delta} c_k}{1 - \bar{a}_M \frac{\bar{\eta}_M \delta}{2}} \Big|_{x=\bar{x}_M} \quad (46)$$

the condition from Corollary 14 becomes:

$$\max \left( \frac{c}{1 - a\frac{\eta\delta}{2}}, \frac{2\eta}{2\eta - 1} \frac{c(1 - e^{-\mu\delta})}{a\delta} \right) \leq \frac{e^{\mu\delta} c_k}{1 - a\frac{\eta\delta}{2}} \Big|_{x=\bar{x}_M} \leq \min \left( \frac{2\eta}{2\eta - 1} \frac{c(e^{\mu\delta} - 1)}{a\delta}, 1 \right) \quad (47)$$

As  $c$  increases above  $c_k$ , (47) reduces to  $\frac{2\eta}{2\eta - 1} \frac{c(1 - e^{-\mu\delta})}{a\delta} \leq \frac{e^{\mu\delta} c_k}{1 - a\frac{\eta\delta}{2}} \Big|_{x=\bar{x}_M} \leq \frac{2\eta}{2\eta - 1} \frac{c(e^{\mu\delta} - 1)}{a\delta}$ . Define

therefore  $\bar{x}_0(c)$  and  $\bar{x}(c)$  by the following equations:

$$\frac{c_k e^{\mu\delta}}{1 - a \frac{\eta\delta}{2}} \Big|_{x=\bar{x}_M} = \frac{2\eta}{2\eta - 1} \frac{c(1 - e^{-\mu\delta})}{a\delta} \Big|_{x=\bar{x}_0(c)} \quad (48)$$

$$\frac{c_k e^{\mu\delta}}{1 - a \frac{\eta\delta}{2}} \Big|_{x=\bar{x}_M} = \frac{2\eta}{2\eta - 1} \frac{c(e^{\mu\delta} - 1)}{a\delta} \Big|_{x=\bar{x}(c)} \quad (49)$$

and note then that for a fixed value of  $c$ , the set of values of competition for which  $p_{(k+1)\delta}$  is sustainable in a pure strategy equilibrium is given by  $\bar{x}_0(c) \leq x \leq \bar{x}(c)$ . Define  $\bar{x}_L \equiv \bar{x}_0(c_k)$  and note when the cost of innovation is just above  $c_k$ , the values of competition for which this equation is satisfied are precisely the ones that satisfy  $\bar{x}_L \leq x \leq \bar{x}_M$ . It is straightforward to see that both  $\bar{x}_0(c)$  and  $\bar{x}(c)$  is increasing. However, as  $c$  increases, since  $\bar{x}(c)$  increases, at some point  $c'_k$  we will have  $\frac{2\eta}{2\eta-1} \frac{(e^{\mu\delta}-1)}{a\delta} \Big|_{x=\bar{x}(c'_k)} = \frac{1}{1-a\frac{\eta\delta}{2}} \Big|_{x=\bar{x}(c'_k)}$ . Define  $\bar{x}_H \equiv \bar{x}(c'_k)$  and note that in fact it does not depend on  $k$ . When  $c$  increases above  $c'_k$ , the non negative profit condition will bind for some values of competition and the upper bound  $\bar{x}(c)$  will be defined by the equation:

$$\frac{c_k e^{\mu\delta}}{1 - a \frac{\eta\delta}{2}} \Big|_{x=\bar{x}_M} = \frac{1}{1 - a \frac{\eta\delta}{2}} \Big|_{x=\bar{x}(c)} \quad (50)$$

Clearly,  $\bar{x}(c)$  is decreasing in  $c$ . It is straightforward to see that the two bounds,  $\bar{x}_0(c)$  and  $\bar{x}(c)$ , will be equal precisely when (45) is satisfied. Define  $c_{k+1}$  to be the smallest value higher than  $c_k$  for which  $\bar{x}_0(c_{k+1}) = \bar{x}(c_{k+1})$ . This completes the proof of the Proposition. ■

## Appendix C2. Proof of Proposition 7.

The proof of this result is similar to the one of Proposition 6 and thus it will be presented in less detail. Let  $\tau_0$  be the minimum waiting time sustainable in an equilibrium for all possible values of  $\beta$  and  $s$ . It follows then that that for a fixed value of  $s$ ,  $\tau_0$  is sustainable for  $\beta \in (\bar{\beta}_0(s), \min\{\bar{\beta}_1(s), \bar{\beta}_2(s)\}]$ , where  $\bar{\beta}_0(s)$ ,  $\bar{\beta}_1(s)$  and  $\bar{\beta}_2(s)$  are the implicit solutions to the equations:

$$p_{\tau_0} = \frac{2s^2}{2s - \delta} \frac{c(1 - e^{-\mu\delta})}{\bar{\beta}_0} \quad (51)$$

$$p_{\tau_0} = \frac{2s^2}{2s - \delta} \frac{c(e^{\mu\delta} - 1)}{\bar{\beta}_1} \quad (52)$$

$$p_{\tau_0} = \frac{c}{1 - \frac{\bar{\beta}_2}{2}} \quad (53)$$

It is straightforward to see that as  $s$  increases,  $\bar{\beta}_0(s)$  and  $\bar{\beta}_1(s)$  increase, while  $\bar{\beta}_2(s)$  is constant so  $\bar{\beta}(s) \equiv \min\{\bar{\beta}_1(s), \bar{\beta}_2(s)\}$  is weakly increasing. Denote by  $s_0$  the minimum value of  $s$  for which  $\tau_0$  is

sustainable and by  $s_1$  the value of  $s$  such that  $\bar{\beta}_0(s_1) = \bar{\beta}(s_1)$ . When  $s$  increases above  $s_1$ ,  $\tau_0$  is no longer sustainable in a pure strategy equilibrium and the new minimum pure strategy equilibrium waiting time is  $\tau_0 + \delta$ . Clearly,  $\bar{\beta}_0(s)$  as given by (51) will have a downward jump to the value that satisfies the equation:

$$p_{\tau_0+\delta} = \frac{2s^2}{2s-\delta} \frac{c(1-e^{-\mu\delta})}{\bar{\beta}} e^{\mu\delta} \quad (54)$$

The rest of the values of  $s_k$  can be computed iteratively in a similar manner. Finally, to see that  $\bar{\beta}_0(s_{k-1}) < \bar{\beta}_0(s_k)$ , note that they are defined by equations of the type (51) and (54), which are essentially identical. However,  $\bar{\beta}_0(s_{k-1})$  satisfies that equation for a lower value of  $s$  and thus is smaller than  $\bar{\beta}_0(s_k)$ . Since  $\bar{\beta}_0(s_k) = \bar{\beta}(s_k)$  we also have  $\bar{\beta}(s_{k-1}) < \bar{\beta}(s_k)$ .<sup>46</sup> ■

### Appendix C3.

**Lemma 19** *If the firms mix with strictly positive probabilities between investing at arbitrary moments  $\{\tau + \theta_1, \dots, \tau + \theta_n\}$ , with  $\theta_1 = 0$ , and  $\tau + \theta_k \in \delta\mathbb{Z}_+$ , for all  $k$ , then this set of moments needs to satisfy one of the following two sets of conditions: (i)  $\theta_2 - \theta_1 \geq \eta\delta$ ,  $\theta_3 - \theta_2 = \delta$ ,  $\theta_4 - \theta_3 \geq (\eta - 1)\delta$ ,  $\theta_5 - \theta_4 = \delta$ ,  $\theta_6 - \theta_5 \geq (\eta - 2)\delta$ , etc., (ii)  $\theta_2 - \theta_1 = \delta$ ,  $\theta_3 - \theta_2 \geq (\eta - 1)\delta$ ,  $\theta_4 - \theta_3 = \delta$ ,  $\theta_5 - \theta_4 \geq (\eta - 2)\delta$ ,  $\theta_6 - \theta_5 = \delta$ , etc.*

*Proof.* Assume that there is an equilibrium in which firms mix between investing at moments  $\{\tau + \theta_1, \dots, \tau + \theta_n\}$  with probabilities  $\{\alpha_1, \dots, \alpha_n\}$ , where  $n \geq 2$ ,  $\theta_1 = 0$ , the sequence  $\{\theta_k\}$  is strictly increasing,  $\alpha_k > 0$  for all  $k$  and  $\sum_{k=1}^n \alpha_k = 1$ . Then, as argued before, since  $\lambda(t|t_i)$  is strictly increasing, we may restrict attention to equilibria corresponding to  $\{\theta_k\} \subset \delta\mathbb{Z}_+$ .

Take some  $k \in \{1, \dots, n-1\}$  arbitrarily, and denote by:

$$h(t) \equiv e^{-\mu(t-\tau-\theta_k)} \{p_t [1 - \lambda(t|t_i)] - c\} - \{p_{\tau+\theta_k} [1 - \lambda(\tau + \theta_k|t_i)] - c\}, \text{ for } t \in [0, \tau + \eta\delta] \quad (55)$$

which can be rewritten, using the fact that  $e^{-\mu(t-\tau-\theta_k)} p_t = p_{\tau+\theta_k}$ , as:

$$h(t) = p_{\tau+\theta_k} [\lambda(\tau + \theta_k|t_i) - \lambda(t|t_i)] + c [1 - e^{-\mu(t-\tau-\theta_k)}] \quad (56)$$

Note that given its definition in (55),  $h(t)$  represents the deviation profit from investing at  $\tau + \theta_k$  to investing at some other point  $t$ . Thus, in order for this to be an equilibrium, we would firstly need to have  $h(t) \leq 0$  for all  $t \in \delta\mathbb{Z}_+$ .<sup>47</sup> Secondly, the fact that the firm is willing to mix between

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<sup>46</sup>Note however that  $\lim_{s \rightarrow s_k^+} \bar{\beta}_0(s) < \bar{\beta}_0(s_k)$  and thus the lower bound does have a downward jump at  $s_k$ . On the other hand,  $\lim_{s \rightarrow s_k^+} \bar{\beta}(s) = \bar{\beta}(s_k)$ . However, the set  $[\bar{\beta}_0(s), \bar{\beta}(s)]$  essentially moves to the right as  $s$  increases.

<sup>47</sup>Note that even if  $h(t) > 0$  on for some  $t \notin \delta\mathbb{Z}_+$  a profitable deviation to that  $t$  is not possible. This is because  $h(t)$  represents the deviation surplus only for  $t \in \delta\mathbb{Z}_+$ . In between elements of  $\delta\mathbb{Z}_+$ , the deviation surplus is no higher

investing at moments  $\tau + \theta_k$  and  $\tau + \theta_l$  implies that  $h(\tau + \theta_l) = 0$ , for all  $l \in \{1, \dots, n\}$ . Now, using Lemma 9, we have that for each  $m \in \{1, \dots, n-1\}$  and for all  $t \in [\tau + \theta_m, \tau + \theta_{m+1}]$ :

$$h(t) = p_{\tau+\theta_k} \left\{ \begin{aligned} & - \sum_{j=1}^m \left( a\alpha_j \left[ \frac{\eta\delta}{2} + \min(t - \tau - \theta_j, \eta\delta) - \frac{[\min(t - \tau - \theta_j, \eta\delta)]^2}{2\eta\delta} \right] \right) - \\ & - \sum_{j=m+1}^n \left( a\alpha_j \left[ \frac{(t - \tau - \theta_j + \eta\delta)^2}{2\eta\delta} \right] \right) + \lambda(\tau + \theta_k | t_i) \end{aligned} \right\} + c \left[ 1 - e^{-\mu(t - \tau - \theta_k)} \right] \quad (57)$$

Firstly, since by Corollary 10,  $\lambda'(t|t_i)$  is continuous, it follows immediately that  $h'(t)$  is also continuous. By considering for each term in the sum that gives  $h(t)$ , the cases when  $t \leq \tau + \theta_j + \eta\delta$  and then  $t > \tau + \theta_j + \eta\delta$ , it is straightforward to see that we can write concisely:

$$h'(t) = p_{\tau+\theta_k} \left\{ \begin{aligned} & - \sum_{j=1}^m a\alpha_j \left[ \max \left( 1 - \frac{(t - \tau - \theta_j)}{\eta\delta}, 0 \right) \right] + \\ & - \sum_{j=m+1}^n a\alpha_j \left[ \frac{(t - \tau - \theta_j + \eta\delta)}{\eta\delta} \right] \end{aligned} \right\} + \mu c e^{-\mu(t - \tau - \theta_k)} \quad (58)$$

For  $t \in (\tau + \theta_m, \tau + \theta_{m+1}) \setminus \bigcup_{j=0}^n \{\tau + \theta_j + \eta\delta\}$ , we also have:

$$h''(t) = p_{\tau+\theta_k} \left\{ - \sum_{j=1}^m a\alpha_j \left[ -\frac{1}{\eta\delta} \mathbf{1}_{\{t \leq \tau + \theta_j + \eta\delta\}} \right] - \sum_{j=m+1}^n a\alpha_j \left[ \frac{1}{\eta\delta} \right] \right\} - \mu^2 c e^{-\mu(t - \tau - \theta_k)} \quad (59)$$

$$h'''(t) = \mu^3 c e^{-\mu(t - \tau - \theta_k)} \quad (60)$$

Take  $k = 1$ , and assume  $\theta_{k+1} - \theta_k < \eta\delta$ , in which case  $\mathbf{1}_{\{t \leq \tau + \theta_j + \eta\delta\}} = 1$ , for all  $t \in (\tau + \theta_k, \tau + \theta_{k+1})$  and  $j \in \{1, \dots, n\}$ . Note that  $\lim_{t \nearrow \tau + \theta_m} h''(t) < \lim_{t \searrow \tau + \theta_m} h''(t)$  for  $m \in \{k, k+1\}$  and  $h'''(t) > 0$  imply that  $h''(t)$  is strictly increasing on  $[0, \tau + \theta_{k+1}]$ . Therefore, there exists  $\bar{t} \in [0, \tau + \theta_{k+1}]$ , such that  $h''(t) < 0$  for  $t \in [0, \bar{t})$ , and  $h''(t) > 0$  for  $t \in (\bar{t}, \tau + \theta_{k+1}]$ . On the other hand, since  $h$  is continuous on  $[\tau + \theta_k, \tau + \theta_{k+1}]$ , and since  $h(\tau + \theta_k) = h(\tau + \theta_{k+1}) = 0$ , we have either  $h'(\tau + \theta_k) < 0 < h'(\tau + \theta_{k+1})$  or  $h'(\tau + \theta_k) > 0 > h'(\tau + \theta_{k+1})$ .

*Case 1:*  $h'(\tau + \theta_k) < 0 < h'(\tau + \theta_{k+1})$ . This means that  $h'(\cdot)$  must be already increasing at  $\tau + \theta_{k+1}$ , so  $\bar{t} < \tau + \theta_{k+1}$ . Thus,  $h''(t) > 0$  for all  $t > \tau + \theta_{k+1}$  and since  $h'(\tau + \theta_{k+1}) > 0$  and  $h'(\cdot)$  is continuous, it must be that  $h'(t) > 0$  for all  $t \in [\tau + \theta_{k+1}, \tau + \eta\delta]$ . In particular,  $h(\tau + \theta_{k+1} + \delta) > 0$ , so this case cannot be part of an equilibrium. Therefore, it must be that  $\theta_{k+1} - \theta_k \geq \eta\delta$ .

*Case 2:*  $h'(\tau + \theta_k) > 0 > h'(\tau + \theta_{k+1})$ . We will show that in this case, it must be that  $\theta_{k+1} - \theta_k = \delta$ . To see this, assume by contradiction that  $\theta_{k+1} - \theta_k \geq 2\delta$ , and note that in that case, we must have  $h(\tau + \theta_k + \delta) < 0$ , because otherwise the firm would deviate and invest at  $\tau + \theta_k + \delta$ . Since  $h$  is continuous and  $h'(\tau + \theta_{k+1}) < 0$ , there must exist some  $t' \in (\tau + \theta_k + \delta, \tau + \theta_{k+1})$  such that  $h'(t') > 0$ , because  $h$  must be strictly positive just below  $\tau + \theta_{k+1}$ . Similarly, there must exist some  $t'' \in (\tau + \theta_k, \tau + \theta_k + \delta)$  such that  $h'(t'') < 0$ . But since  $h'(t'') < 0 < h'(t')$ , it means that

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than the than the deviation surplus to the highest element of  $\delta\mathbb{Z}_+$  that is smaller than  $t$ . This is due to the discrete updating of the belief.

$h'$  is already increasing at  $t'$ , so  $h'(\tau + \theta_{k+1}) > h(t') > 0$ . This contradicts the initial assumption. Therefore, if  $\theta_{k+1} - \theta_k < \eta\delta$ , it must be that  $\theta_{k+1} - \theta_k = \delta$ .

Therefore, the analysis of the two cases reveals that in order to have an equilibrium, we must either have  $h'(\tau + \theta_1) < 0$  and  $\theta_2 - \theta_1 \geq \eta\delta$ , or  $h'(\tau + \theta_1) > 0$  and  $\theta_2 - \theta_1 = \delta$ .

If  $h'(\tau + \theta_1) < 0$ , then note that since  $\theta_2 - \theta_1 \geq \eta\delta$ , then it must be that  $1_{\{t \leq \tau + \theta_1 + \eta\delta\}} = 0$ , for all  $t \in (\tau + \theta_2, \tau + \theta_3)$  and  $1_{\{t \leq \tau + \theta_j + \eta\delta\}} = 1$ , for all  $t \in (\tau + \theta_2, \tau + \theta_3)$  and  $j \in \{2, \dots, n\}$ . Therefore,  $h''$  is again increasing on  $(\tau + \theta_2, \tau + \theta_3)$ . So repeating the argument in Case 2 above, we conclude that it must be that  $\theta_3 - \theta_2 = \delta$ , and that  $h'(\tau + \theta_3) < 0$ . Assume now that  $\theta_4 - \theta_3 < (\eta - 1)\delta$ , and note that in this case, we have  $1_{\{t \leq \tau + \Delta_1 + \eta\delta\}} = 0$  for all  $t \in (\tau + \theta_3, \tau + \theta_4)$  and  $1_{\{t \leq \tau + \theta_j + \eta\delta\}} = 1$ , for all  $t \in (\tau + \theta_3, \tau + \theta_4)$  and  $j \in \{2, \dots, n\}$ . Therefore, repeating the argument that we used for the Case 1 above, we conclude that this cannot be part an equilibrium. Thus, it must be that  $\theta_4 - \theta_3 \geq (\eta - 1)\delta$  and  $h'(\tau + \theta_3) > 0$ . Then, using the same type of arguments, it follows that  $\theta_5 - \theta_4 = \delta$ ,  $\theta_6 - \theta_5 \geq (\eta - 2)\delta$  and so on.

If  $h'(\tau + \theta_1) > 0$ , then  $h'(\tau + \theta_2) < 0$  and it can be showed as we did above for Case 1, that unless  $\theta_3 - \theta_2 > (\eta - 1)\delta$ , this cannot be part of an equilibrium. It follows then iteratively that it must be that  $\theta_4 - \theta_3 = \delta$ ,  $\theta_5 - \theta_4 \geq (\eta - 2)\delta$ ,  $\theta_6 - \theta_5 = \delta$  and so on. ■

Therefore, in order to have an equilibrium in which firms mix between investing at various moments in time, the function  $h$  should have an extremely specific shape, in which it takes values of 0 at points in between which the distances alternate between very large and very small values. Generically, such a shape is impossible to be attained. For instance, consider the following virtually randomly chosen values of the parameters:  $\mu = 0.4$ ,  $\delta = 0.2$ ,  $\eta = 5$  and assume that there is an equilibrium in which firms mix between innovating after three different waiting times. By Lemma 19, the three waiting times can either be (i)  $\tau$ ,  $\tau + \theta \geq \tau + \eta\delta$  and  $\tau + \theta + \delta$  or (ii)  $\tau$ ,  $\tau + \delta$  and  $\tau + \theta \geq \tau + \eta\delta$ , for some  $\tau > 0$ . To simplify the illustration, assume that  $\theta = \eta\delta$ .<sup>48</sup>

For case (ii), consider first the case when the three mixing probabilities add up to 1. Then, we have a system of 3 equations in 3 unknowns  $(\alpha_1, \alpha_2, \alpha_3)$  given by  $h(\tau + \delta) = 0$ ,  $h(\tau + \eta\delta) = 0$  and  $\alpha_1 + \alpha_2 + \alpha_3 = 1$  which gives us  $\alpha_1, \alpha_2, \alpha_3$  as a function of  $x \equiv \frac{ap\tau}{c}$ . In order for this to be an equilibrium, it is also necessary that  $h(\tau - \delta) < 0$ , which can be calculated that holds only if  $x < 0.7805$ . But when this condition is satisfied, it can be shown that  $\alpha_2 < 0$ , which is impossible. Second, consider the case when  $\alpha_1 + \alpha_2 + \alpha_3 < 1$ . In this case firms should expect zero profits so  $\alpha_1, \alpha_2, \alpha_3$  are given by  $h(\tau + \delta) = 0$ ,  $h(\tau + \eta\delta) = 0$ ,  $p_\tau[1 - \lambda(\tau|t_i)] - c = 0$ . Solving this system for  $\beta_i \equiv a\alpha_i$ ,  $i = 1, 2, 3$  as a function of  $y \equiv \frac{p\tau}{c}$ , we obtain that  $x > 1.08$  from  $\beta_3 > 0$ , which then can be shown that implies  $\beta_1 < 0$ . For case (i), if  $\alpha_1 + \alpha_2 + \alpha_3 = 1$ , the other two equations are  $h(\tau + \eta\delta) = 0$ ,  $h(\tau + \eta\delta + \delta) = 0$ . Solving the system for  $\alpha_1, \alpha_2, \alpha_3$  as a function of  $x \equiv \frac{ap\tau}{c}$ , we obtain that  $x > 2.80$  from  $\alpha_1 > 0$ , which then can be shown that implies  $\alpha_2 < 0$ . Finally, if

<sup>48</sup>It is actually intuitive that when  $\theta$  is higher, the set of necessary conditions that  $h$  needs to satisfy is stricter.

$\alpha_1 + \alpha_2 + \alpha_3 < 1$ , the three equations are:  $h(\tau + \eta\delta) = 0$ ,  $h(\tau + \eta\delta + \delta) = 0$ ,  $p_\tau[1 - \lambda(\tau|t_i)] - c = 0$ . Solving this system for  $\beta_i \equiv a\alpha_i$ ,  $i = 1, 2, 3$  as a function of  $y \equiv \frac{p_\tau}{c}$ , we obtain immediately that  $\beta_2 < 0$ , which is impossible.

#### Appendix C4. Proof of Proposition 8.

As mentioned in the main text, we will find the equilibrium under the continuous information acquisition process by taking the limit  $\delta \rightarrow 0$  and keeping the length of the awareness window  $\eta\delta$  constant. This can be accomplished by denoting  $M \equiv \eta\delta$ , substituting  $\frac{M}{\delta}$  for  $\eta$  and taking the limit  $\delta \rightarrow 0$  in the results obtained from the analysis of the discrete case. It can be shown that the necessary condition (29) becomes now  $\frac{2M}{2M-\delta} \frac{c(1-e^{-\mu\delta})}{a\alpha\delta} \leq p_\tau \leq \frac{2M}{2M-\delta} \frac{c(e^{\mu\delta}-1)}{a\alpha\delta}$  so taking the limit  $\delta \rightarrow 0$  we obtain  $p_\tau = \frac{c\mu}{a\alpha}$ . On the other hand, (30) becomes  $p_\tau \geq \frac{c}{1-\alpha a \frac{M}{2}}$ . Therefore, a pure strategy equilibrium in which  $\alpha = 1$  exists when  $\frac{\mu}{a} (1 - \alpha a \frac{M}{2}) \geq 1$ . Clearly, this condition is satisfied for values of  $x$  that are small enough. Finally, note that the equilibrium value of  $p_\tau$  is decreasing in  $a$  but does not depend on  $M$ .

Now, using an argument similar to the one from Appendix B2, it can be shown that for values of  $x$  higher than  $\bar{x}$  where  $\frac{\mu}{a} (1 - \alpha a \frac{M}{2}) \Big|_{x=\bar{x}} = 1$  a pure strategy equilibrium does not exist. Instead, employing again Proposition 13, we obtain that a necessary and sufficient set of conditions for the firms to mix at moment  $\tau$  between investing and not investing with probability  $\alpha < 1$  is that  $p_\tau = \frac{c\mu}{a\alpha}$ ,  $p_\tau = \frac{c}{1-\alpha a \frac{M}{2}}$  and  $1 - \alpha a M \leq c$ . As in Appendix B3 it can be shown that the last condition is satisfied whenever the first two conditions are. The mixing probability  $\alpha$  satisfies thus:  $\frac{c\mu}{a\alpha} = \frac{c}{1-\alpha a \frac{M}{2}}$ , which can be rewritten as:  $a\alpha \left(1 + \frac{\mu M}{2}\right) = \mu$ . Therefore, if  $a$  increases,  $\alpha$  must decrease at the same rate to keep  $a\alpha$  constant; since  $p_\tau = \frac{c\mu}{a\alpha}$ , the equilibrium value of  $p_\tau$  also remains constant. On the other hand, if  $M$  increases,  $\alpha$  decreases and  $p_\tau = \frac{c\mu}{a\alpha}$  increases. This completes the proof. ■

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