The U.S. Excess Money Growth and Inflation Relation in the Long-Run: A Nonlinear Analysis

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Abstract

This paper specifies, estimates, and evaluates the relation between inflation rate and excess money growth, defined as the difference between money supply growth and real GDP growth, using a smooth transition regression model and U.S. data. The results indicate that the relation is a nonlinear one as supported by the linearity tests. Although deterministic extrapolation exercises indicate that both the linear and nonlinear models are stable, the nonlinear model is favored by several misspecification tests. Deterministic extrapolation exercises also indicate that an increase in excess money supply has positive effect on the long-run inflation rate but the effect is not one-to-one even in high-inflation regime.

1. Introduction

The debate whether money growth and inflation rate are related seems to be an interesting one. There are many studies in the literature concerning the relation. While some find that the relation is significant, others claim that it is not.

Let's first look at some of the studies that find no or little evidence regarding the relation. Bachmeier, Leelahanon, & Li (2007) reject a linear forecasting model for inflation based on several specification tests. Using a fully nonparametric model and a threshold regression model, they evaluate out-of-sample inflation forecasts. Their main results are that nonlinear models are more successful at forecasting inflation than linear models and that including money growth in the model does not yield significant improvements. They conclude that money growth is not a good inflation indicator as long as linear models are concerned. However, money growth is a better forecaster for inflation in nonlinear models. De Gregorio (2004), analyzing several low inflation countries with very rapid growth of money, finds that money growth does not necessarily cause inflation. Milas (2007), using a Markov switching regression model and UK data, finds that the relation is not stable over time and that inflationary pressures created by money growth depends on the condition that money growth exceeds 10% threshold level. Even in that case, Milas finds that the inflationary effects are negligible. Nikolic (2000) claims that the effect of changes in money growth on inflation is much weaker as the Russian economy is more stabilized. Roffia and Zaghini (2007), studying the short-run impact of strong monetary growth on inflation for 15 industrialized economies, report mixed results. In approximately half of the cases they investigate, they find a positive relation between inflation and money growth.

On the other hand, there are several studies finding a relation between money growth and inflation. Assenmacher-Wesche, Gerlach, & Sekine (2007), using a band spectrum regression, find that inflation is correlated with money growth and that money growth unidirectionally Granger-causes to inflation. Christensen (2001) provides empirical evidence that money growth and inflation are related one-to-one in the long-run for the US. The paper attributes the short-run deviations from this relation to global real supply shocks. Crowder (1998), using a co-integration analysis, claims that the relation between money growth and inflation is statistically significant in the long-run. In an interesting study, De Grauwe and Polan (2005) find that the long-run relation between money growth and inflation is strong and positive for high-inflation economies whereas it is weak for low-inflation economies. Dwyer, Jr. (2001), using US data, finds that money growth is better in forecasting inflation than any other variables besides past inflation. Dwyer, Jr. and Hafer (1999), without employing an econometric model, also find a significant relation for many countries over long and short periods. Hossain (2005), using annual data for the period 1954-2002, finds that a short-run bi-directional causality between money supply growth and inflation for Indonesia. Kaufmann (2007) employs a vector error correction model and quarterly data for the period 1980-2006 for the Euro area and finds that monetary growth and inflation are nonstationary but co-integrated. Kugler and Kaufmann (2005) also find a robust co-integration between money growth and inflation for the Euro area and conclude that deviations of the real money growth from its long-run average are a good indicator for inflation. Neumann and Greiber (2004), analyzing data from the Euro area, "attributes an impact on inflation not to actual money growth but to its core component", which is "defined as the long-lasting, low-frequency component of nominal money growth in excess of real money demand". As a result, they conclude that money growth and inflation have a clear and stable relation. Shelley & Wallace (2005), using band-pass filter, find a strong, positive correlation between money growth and inflation for the US. Siklos (1991) analyzes the Hungarian hyperinflation and finds that money growth and inflation contain a common trend.

The standard quantity theory of money postulates that the following equation[1] holds for each specific period of time

$$MV = PQ$$

where M is the money supply, V is the velocity or the number of times each money unit is spent, P is the price level, and Q is the quantity of goods and services sold. The equation can be written in percentages as follows

$$\%P = \%M - \%Q + \%V$$

where $\%P$ is inflation, $\%M$ is money growth, $\%Q$ is output growth, and $\%V$ is percentage change in velocity. This equation is called "basic inflation identity". At this point, we make a big assumption and state that $\%V$ has zero or negligible effect on inflation. Then we obtain the following approximate equation

$$\%P \cong \%M - \%Q$$

This equation is the main motivation of the paper and one of the points of departure of the paper from the literature. In other words, we use excess money growth defined as money supply growth minus real output growth[2], $\%M - \%Q$, rather than simply money growth, $\%M$, in explaining the inflation in addition to its own past values. The idea is that the portion of money supply growth exceeding real GDP growth and past values of inflation are the main determinants of inflation.

Another point of departure of the paper from the literature is that the approximate relation between excess money growth and inflation in the equation above can be investigated using a nonlinear time series model, namely, the smooth transition regression (STR) model instead of a linear one. While the relation has been subjected to mostly linear models, to our best knowledge, it has never been analyzed using the STR model. One advantage of the STR model is that one can identify high- and low-inflation regimes in the economy and investigate long-run inflation under each regime.

[1] The equation is called "monetary exchange equation".
[2] This is also known as "unproductive debt expansion". See http://en.wikipedia.org/wiki/Money_supply.

The organization of the paper is as follows. Section 2 describes the econometric model, namely, the smooth transition regression (STR) model. Section 3 combines the economic problem and the econometric model. Section 4 discusses the data. Section 5 evaluates the results. Section 6 concludes.

2. Smooth Transition Regression (STR) Models[3]

2.1. The General Model

Smooth transition regression models are parametric nonlinear time series regression models. The general formulation of the models is as follows:

$$y_t = \underset{1\times(m+1)}{\boldsymbol{\varphi}'}\underset{(m+1)\times 1}{\mathbf{z}_t} + \underset{1\times(m+1)}{\boldsymbol{\theta}'}\underset{(m+1)\times 1}{\mathbf{z}_t} G(\bullet) + u_t = \left(\underset{(m+1)\times 1}{\boldsymbol{\varphi}} + \underset{(m+1)\times 1}{\boldsymbol{\theta}} G(\bullet)\right)' \underset{(m+1)\times 1}{\mathbf{z}_t} + u_t$$

where $\underset{1\times(m+1)}{\boldsymbol{\varphi}'}\underset{(m+1)\times 1}{\mathbf{z}_t}$ is the linear part, $\underset{1\times(m+1)}{\boldsymbol{\theta}'}\underset{(m+1)\times 1}{\mathbf{z}_t} G(\bullet)$ the nonlinear part, $u_t \sim iid\left(0,\sigma^2\right)$ a sequence of independent, identically distributed random errors, and $t = 1,\ldots,T$. The model variables $\mathbf{z}_t$ may include lags of the endogenous variable, lags of exogenous variables, and deterministic variables. The model parameters are $\boldsymbol{\varphi}$, which is the parameter vector of the linear part, and $\boldsymbol{\theta}$, which is the parameter vector of the nonlinear part. The equation $y_t = \left(\underset{(m+1)\times 1}{\boldsymbol{\varphi}} + \underset{(m+1)\times 1}{\boldsymbol{\theta}} G(\bullet)\right)' \underset{(m+1)\times 1}{\mathbf{z}_t} + u_t$ indicates that the models can be interpreted as linear models with stochastic time-varying coefficients $\boldsymbol{\varphi} + \boldsymbol{\theta} G(\bullet)$ since $G(\bullet)$ depends on time.

The most important feature of the STR models is that they allow regime switches. This is achieved through the transition function $G(\bullet)$. There are basically two types of transition functions in the literature: logistic transition function (LTF) and exponential transition function (ETF). The specific form of the logistic transition function depends on the number of regimes it allows for, which is usually 2 or 3 in practice. In this paper, we focus only on the logistic transition function that allows two regimes.

2.2. Logistic Transition Function (LTF)

The general logistic transition function takes the following form:

[3] This section is based on Terasvirta (1997), Terasvirta (2004), and Kratzig (2005).

$G\left(s_t;\gamma,\underset{K\times 1}{\mathbf{c}}\right)=\left\{1+\exp\left\{-\gamma\prod_{k=1}^{K}\left(s_t-c_k\right)\right\}\right\}^{-1}$. s_t is the continuous transition variable which can be part of $\mathbf{z}_t$ or any other variable, e.g. time trend $s_t = t$. γ is the slope parameter (slope of the transition function) where $\gamma > 0$. $\underset{K\times 1}{\mathbf{c}} = \left[c_1 \quad c_2 \quad \cdots \quad c_K\right]'$ is a vector of K location parameters (location of the transition function) where $c_1 \leq \cdots \leq c_K$. s_t and $\mathbf{c}$ together determine the regimes. The general logistic transition function $G\left(s_t;\gamma,\underset{K\times 1}{\mathbf{c}}\right)$ is a bounded function, $0 \leq G\left(s_t;\gamma,\underset{K\times 1}{\mathbf{c}}\right) \leq 1$, and is continuous everywhere in the parameter space for any value of s_t. The slope parameter γ indicates how rapid the transition from zero to unity is as a function of s_t. When $\gamma = 0$, the transition function becomes $G\left(s_t;\gamma,\underset{K\times 1}{\mathbf{c}}\right)=\left\{1+\exp\left\{-\gamma\prod_{k=1}^{K}\left(s_t-c_k\right)\right\}\right\}^{-1} \equiv \frac{1}{2}$. In that case, there is virtually no difference between the linear and the nonlinear models and thus the STR model nests the linear model. The model coefficients become $\boldsymbol{\varphi} + \frac{1}{2}\boldsymbol{\theta}$, which is basically a linear model. In other words, the smaller γ is, the less influential the nonlinear part is[4].

The location parameters determine where the transition occurs. Together with the general logistic transition function, $y_t = \underset{1\times(m+1)}{\boldsymbol{\varphi}'}\underset{(m+1)\times 1}{\mathbf{z}_t} + \underset{1\times(m+1)}{\boldsymbol{\theta}'}\underset{(m+1)\times 1}{\mathbf{z}_t} G\left(s_t;\gamma,\underset{K\times 1}{\mathbf{c}}\right) + u_t$ defines the general logistic STR (LSTRK) model. The most common choices for K are K = 1 (LSTR1) and K = 2 (LSTR2). As stated above, we focus on LSTR1 in this paper.

2.3. Logistic Transition Function with 1 Location Parameter (LTF1)

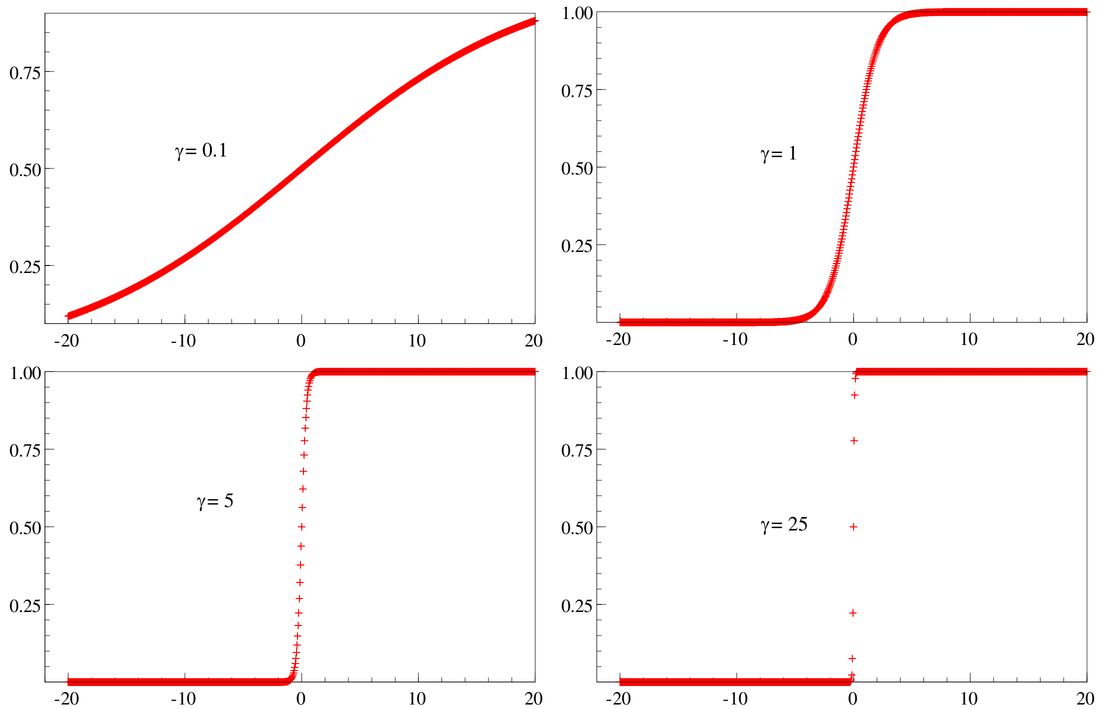
The logistic transition function with 1 location parameter takes the following form: $G\left(s_t;\gamma,c_1\right)=\left\{1+\exp\left\{-\gamma\left(s_t-c_1\right)\right\}\right\}^{-1}$ where $\gamma > 0$. The function is a monotonically increasing function of s_t.

The LTF1 has the following properties:

- $\lim_{\gamma\to 0} G\left(\gamma,c_1,s_t\right)=\lim_{\gamma\to 0}\frac{1}{1+\exp\left\{-\gamma\left(s_t-c_1\right)\right\}}=\frac{1}{2}$ and thus the LSTR1 model nests the linear model in that case.
- $G\left(\gamma,c_1,s_t\right)=\frac{1}{1+\exp\left\{-\gamma\left(s_t-c_1\right)\right\}}=\frac{1}{2}$ if $s_t = c_1$.

[4] As we will see later, the linearity test is basically a test for $\gamma = 0$.

Figure 1: Shapes of the LTF1 for Different Values of γ When $c_1 = 0$



- $\lim_{\gamma \to \infty} G(\gamma, c_1, s_t) = \lim_{\gamma \to \infty} \frac{1}{1+\exp\{-\gamma(s_t - c_1)\}} = 0 \quad \text{if} \quad s_t - c_1 < 0$
- $\lim_{\gamma \to \infty} G(\gamma, c_1, s_t) = \lim_{\gamma \to \infty} \frac{1}{1+\exp\{-\gamma(s_t - c_1)\}} = 1 \quad \text{if} \quad s_t - c_1 > 0$

Figure 1 shows the LTF1 for various γ values. In each subfigure, $c_1 = 0$ and $-20 \le s_t \le 20$. The values of γ are $\gamma = 0.1$, $\gamma = 1$, $\gamma = 5$, $\gamma = 25$. As can be seen from the figure, for very small values of γ, the logistic transition function is very smooth. As the value of γ increases, the transition function becomes a less smooth curve for the given interval.

2.4. Logistic Smooth Transition Regression with 1 Location Parameter (LSTR1) Model

The LSTR1 model is jointly defined by the following equations:

$$y_t = \underset{1\times(m+1)}{\boldsymbol{\varphi}'} \underset{(m+1)\times 1}{\mathbf{z}_t} + \underset{1\times(m+1)}{\boldsymbol{\theta}'} \underset{(m+1)\times 1}{\mathbf{z}_t} G(\bullet) + u_t = \left(\underset{(m+1)\times 1}{\boldsymbol{\varphi}} + \underset{(m+1)\times 1}{\boldsymbol{\theta}} G(\bullet) \right)' \underset{(m+1)\times 1}{\mathbf{z}_t} + u_t \text{ and}$$

$G\left(s_t;\gamma,c_1\right)=\left\{1+\exp\left\{-\gamma\left(s_t-c_1\right)\right\}\right\}^{-1}$. The model parameters, $\boldsymbol{\varphi}+\boldsymbol{\theta}G\left(s_t;\gamma,c_1\right)$, change monotonically as a function of s_t from $\boldsymbol{\varphi}$ ($G(\bullet)=0$) to $\boldsymbol{\varphi}+\boldsymbol{\theta}$ ($G(\bullet)=1$). In other words, the parameters are bounded between $\boldsymbol{\varphi}$ and $\boldsymbol{\varphi}+\boldsymbol{\theta}$. As the value of $G(\bullet)$ changes from zero to one, the values of the parameters/coefficients change from $\boldsymbol{\varphi}$ to $\boldsymbol{\varphi}+\boldsymbol{\theta}$.

When $\gamma\to\infty$, the LSTR1 model approaches the threshold regression model with two regimes that have equal variances[5]. In this special case, $s_t=c$ is the switch-point between the regimes $y_t=\boldsymbol{\varphi}'\mathbf{z}_t+u_t$ and $y_t=\left(\boldsymbol{\varphi}+\boldsymbol{\theta}\right)'\mathbf{z}_t+u_t$. If $\mathbf{z}_t=\begin{bmatrix}1 & y_{t-1} & \cdots & y_{t-p}\end{bmatrix}'$ and $s_t=y_{t-d}$, the limiting model is a two-regime self-exciting threshold autoregressive (SETAR) model.

The usefulness of the LSTR1 model comes from its capability of characterizing asymmetric behavior. For instance, if s_t measures the phase of the business cycle, then the LSTR1 model can describe dynamic processes which exhibit different behaviors under recession and expansion and the transition from one extreme to the other is less or more smooth. One can use the LSTR1 model if the economy behaves differently in expansion from how it does in contraction.

An important special case is $s_t=t$. Then the model becomes $y_t=\left(\boldsymbol{\varphi}+\boldsymbol{\theta}G\left(t;\gamma,c\right)\right)'\mathbf{z}_t+u_t$. This can be interpreted as a linear model whose parameters change over time as a function of time. If we also let $\gamma\to\infty$, the model contains as a special case the presence of a single structural break.

When the model is purely autoregressive and $s_t=y_{t-d}$ or $s_t=\Delta y_{t-d}$, $d>0$, STR model becomes STAR (smooth transition autoregressive) model. The literature is full of studies employing these models. Among others, STAR models have been applied by Arango & Gonzalez (1998) to analyze Columbian inflation, Bardsen, Hurn, & McHugh (2004) to examine the Australian unemployment rate, Baum, Barkoulas, & Caglayan (1999) to analyze international long-run purchasing power parity, Bruinshoofd & Candelon (2004) to study monetary policy in Europe, Gregoriou & Kontonikas (2005) to analyze inflation deviations from the target for several OECD countries, Guerra (2001) to analyze several European exchange rates, Michael, Nobay, & Peel (1997) to examine the relation between transaction costs and real exchange rates, Persson & Terasvirta (2003) to examine the net barter terms of trade for several industrial countries, Terasvirta (1995) to model US GNP, and Terasvirta & Anderson (1992) to characterize business cycles in several countries.

A somewhat more complicated and less employed STR model is the smooth transition autoregressive distributed lag (STARDL) model in which lags of the endogenous variable

[5] See Hansen (1996), Hansen (1997), and Hansen (1999)

and current value and lags of an exogenous variable appear as regressors and the transition variable is a lag of the endogenous or exogenous variable or is another variable. STARDL models have been applied by Aslanidis, Osborn, & Sensier (2002) to analyze UK stock returns, Reyes, Osborn, & Sensier (2002) to study real exchange rate and business cycles in Latin America, and Sensier, and Osborn, & Ocal (2002) to model UK interest rates. This paper also uses the STARDL model.

2.5. The Modeling Cycle

The modeling cycle of a smooth transition regression model consists of the following three stages:

1. **Specification:** Specification includes setting up a linear model that forms a starting point for the analysis, testing for linearity, choosing a transition variable, and deciding whether logistic (LSTR) or exponential (ESTR) smooth transition regression should be used. In case of LSTR, it is also determined whether K = 1 (LSTR1) or K = 2 (LSTR2).
2. **Estimation:** Estimation includes finding appropriate starting values and estimating the model parameters.
3. **Evaluation:** Evaluation includes checking the model graphically and testing for misspecification, e.g. error autocorrelation, parameter non-constancy, remaining nonlinearity, ARCH, and non-normality. Non-normality also checks for outliers in the model.

We will cover these steps in detail in the next three subsections.

2.6. Model Specification

Model specification is a two-phase process: 1) setting up a linear model and 2) testing for linearity.

2.6.1. Selecting a Linear Model

In the first phase of the model specification, the researcher sets up a linear model, e.g. autoregressive (AR) model or autoregressive distributed lag (ARDL) model that forms a starting point for the analysis. The linear model may contain lagged endogenous, current and lagged exogenous and deterministic variables, e.g. constant term, seasonal dummies, and other dummies. The lags can be determined with Akaike information criterion (AIC), Bayesian information criterion (BIC), or sequential tests. Whole lags or partial lags may be obtained. The linear model is subjected to various specification tests: residual autocorrelation, test for ARCH effects in the residuals, structural break in the model parameters, heteroskedasticity tests, stability tests, and normality tests.

2.6.2. Testing for Linearity

Once an appropriate linear model is obtained, the second phase is to test for STR type nonlinearity. The test also helps to determine the transition variable and whether LSTR1 or LSTR2 (or ESTR) should be used.

If s_t is an element of $\mathbf{z}_t$ where $\mathbf{z}_t = \begin{bmatrix} 1 & \underset{m\times 1}{\tilde{\mathbf{z}}'_t} \end{bmatrix}'$, the auxiliary regression for the test[6] is

$$y_t = \underset{1\times(m+1)}{\boldsymbol{\beta}'_0} \underset{(m+1)\times 1}{\mathbf{z}_t} + \sum_{j=1}^{3} \underset{1\times m}{\boldsymbol{\beta}'_j} \underset{m\times 1}{\tilde{\mathbf{z}}_t} s_t^j + u_t$$

The null hypothesis of linearity is

$$H_0 : \underset{m\times 1}{\boldsymbol{\beta}_1} = \underset{m\times 1}{\boldsymbol{\beta}_2} = \underset{m\times 1}{\boldsymbol{\beta}_3} = \underset{m\times 1}{\mathbf{0}},$$

that is, the model is linear. If s_t is **not** an element of $\mathbf{z}_t$, then the auxiliary regression is

$$y_t = \underset{1\times(m+1)}{\boldsymbol{\beta}'_0} \underset{(m+1)\times 1}{\mathbf{z}_t} + \sum_{j=1}^{3} \underset{1\times(m+1)}{\boldsymbol{\beta}'_j} \underset{(m+1)\times 1}{\mathbf{z}_t} s_t^j + u_t$$

In that case, the null hypothesis of linearity is

$$H_0 : \underset{(m+1)\times 1}{\boldsymbol{\beta}_1} = \underset{(m+1)\times 1}{\boldsymbol{\beta}_2} = \underset{(m+1)\times 1}{\boldsymbol{\beta}_3} = \underset{(m+1)\times 1}{\mathbf{0}}$$

This is the usual F test. The F-statistic has an approximate F distribution with $3m$ and $T-4m-1$ degrees of freedom under the null hypothesis. If we reject the null hypothesis, we can conclude that the model is nonlinear. The test results can also help choose the transition variable if it is not dictated by economic theory. One can select all potential transition variables and run the test for each one of them[7]. The variable with the strongest test rejection (the smallest p-value) is selected as the transition variable.

If linearity is rejected, one has to choose between LSTR1, LSTR2, and ESTR. The following test sequence is employed for this task

1. $H_{04} : \boldsymbol{\beta}_3 = \mathbf{0}$ (F4)
2. $H_{03} : \boldsymbol{\beta}_2 = \mathbf{0} \,|\, \boldsymbol{\beta}_3 = \mathbf{0}$ (F3)
3. $H_{02} : \boldsymbol{\beta}_1 = \mathbf{0} \,|\, \boldsymbol{\beta}_2 = \boldsymbol{\beta}_3 = \mathbf{0}$ (F2)

If test 2 gives the lowest p-value, then we choose LSTR2 or ESTR. Otherwise, we choose LSTR1. If the test does not provide a clear-cut choice, it is reasonable to fit both LSTR1 and LSTR2 (or ESTR) and decide a model on the evaluation stage by looking at the information criteria or the RSS or forecasting performance.

[6] The first part in the auxiliary regression after the equality sign (and before the plus sign) corresponds to the linear part of the STR equation. The second part corresponds to the nonlinear part.
[7] This is what we do in the section Results.

2.7. Model Estimation

The model is estimated by the method of nonlinear least squares. Estimation involves finding appropriate starting values for the optimization algorithm and estimating the model.

It is important to find good starting values before estimating the model. The unknown parameters are $\mathbf{c}$, γ, $\boldsymbol{\varphi}$, and $\boldsymbol{\theta}$. Notice that the model becomes linear in the parameters when $\mathbf{c}$ and γ in the transition function are fixed. Thus a search grid over $\mathbf{c}$ and γ can be constructed to find the appropriate starting values for $\boldsymbol{\varphi}$ and $\boldsymbol{\theta}$. The linear estimation of the remaining parameters is done conditionally on (γ, c_1) for LSTR1 and (γ, c_1, c_2) for LSTR2 and the sum of squared residuals (RSS) are stored for every combination of $(\mathbf{c}, \gamma)$. The values of $(\mathbf{c}, \gamma)$ that gives the minimum RSS are the starting values[8]. For LSTR2, a further restriction is $c_1 \le c_2$.

Note that γ is not a scale-free parameter. This is important when constructing the grid. In order to make γ scale-free, the exponent of the transition function is divided by $\hat{\sigma}_s^K$, the K[th] power of the sample standard deviation of the transition variable s_t

$$G\left(s_t;\gamma,\underset{K\times 1}{\mathbf{c}}\right)=\frac{1}{1+\exp\left\{-\left(\dfrac{\gamma}{\hat{\sigma}_s^K}\right)\prod_{k=1}^{K}\left(s_t-c_k\right)\right\}} \qquad \gamma>0$$

This makes the slope parameter scale-free, which facilitates the construction of an effective grid.

2.8. Model Evaluation

Just as in the linear model, evaluation of the nonlinear model includes graphical checks as well as various tests for misspecification, such as error autocorrelation, ARCH, parameter non-constancy, remaining nonlinearity, and non-normality. These tests, except for the test for remaining nonlinearity, are generalizations of those for the linear models.

2.8.1. Graphical Analysis

A visual evaluation of the model can be performed by checking various plots drawn against time. Graphical analysis can also help detect problems in the residuals and illustrate the estimation results. Among the plots that can be helpful are estimated residuals $\tilde{u}_t = y_t - \tilde{\boldsymbol{\varphi}}'\mathbf{z}_t - \tilde{\boldsymbol{\theta}}'\mathbf{z}_t G(s_t;\tilde{\gamma},\tilde{\mathbf{c}})$, estimated transition function $G(s_t;\tilde{\gamma},\tilde{\mathbf{c}})$, fitted series (linear $\tilde{\boldsymbol{\varphi}}'\mathbf{z}_t$ and nonlinear $\tilde{\boldsymbol{\theta}}'\mathbf{z}_t G(s_t;\tilde{\gamma},\tilde{\mathbf{c}})$ parts), original series, and transition

[8] If the starting values are in the boundaries of the parameter values, this may mean a problem.

variable. The estimated transition function $G\left(s_t;\tilde{\gamma},\tilde{\mathbf{c}}\right)$ can also be plotted against the transition variable s_t.

2.8.2. Test for Serial Correlation in the Residuals

The test is implemented by regressing the estimated residuals $\tilde{u}_t$ from the STR model on the lagged residuals $\tilde{u}_{t-1},\ldots,\tilde{u}_{t-q}$ and the partial derivatives of the log-likelihood function with respect to the parameters of the model evaluated at the maximizing values. The null hypothesis is that there is no autocorrelation in the residuals.

The test statistic is $F_{LM}=\dfrac{\left(SSR_0-SSR_1\right)/q}{SSR_1/\left(T-n-q\right)}\overset{appr.}{\sim}F\left(q,T-n-q\right)$ where SSR_0 is the sum of squared residuals of the STR model, SSR_1 the sum of squared residuals from the auxiliary regression, and n is the number of parameters in the model.

2.8.3. Test for ARCH Effects in the Residuals

This is a Lagrange Multiplier (LM) test for neglected conditional heteroskedasticity or, briefly, for ARCH in the residuals. Hence it is a diagnostic test. The test is based on fitting an ARCH(q) model to the estimation residuals $\hat{u}_t^2=\beta_0+\beta_1\hat{u}_{t-1}^2+\cdots+\beta_q\hat{u}_{t-q}^2+\varepsilon_t$ where $\hat{u}_t$ is the residuals of the model. The null hypothesis is $H_0:\beta_1=\cdots=\beta_q=0$ (no conditional heteroskedasticity). The alternative hypothesis is $H_1:\beta_1\neq 0$ or $\cdots$ or $\beta_q\neq 0$. The LM test statistic is $ARCH_{LM}\left(q\right)=TR^2$ where R^2 is the coefficient of determination and T is the number of observations. The test statistic has an asymptotic $\chi^2\left(q\right)$ distribution under the null hypothesis. Large values of the test statistic, and hence small p-values[9], indicate that the null hypothesis is false and there may be ARCH in the residuals. There is also an F-version of the test.

2.8.4. Test for No Additive (Remaining) Nonlinearity

This is a test for remaining nonlinearity after the STR model is fitted. The assumption is that the type of the remaining nonlinearity is again of the STR type. The alternative model is

$$y_t=\boldsymbol{\varphi}'\mathbf{z}_t+\boldsymbol{\theta}'\mathbf{z}_tG\left(s_{1t};\gamma_1,\underset{K\times 1}{\mathbf{c}_1}\right)+\boldsymbol{\psi}'\mathbf{z}_tH\left(s_{2t};\gamma_2,\underset{K\times 1}{\mathbf{c}_2}\right)+u_t$$

[9] A p-value represents the probability of getting a test value greater than the observed one if the null hypothesis is true.

where $H(\bullet)$ is another transition function and $u_t \sim iid(0,\sigma^2)$. The auxiliary model used to test this alternative is

$$y_t = \underset{1\times(m+1)}{\boldsymbol{\beta}'_0} \underset{(m+1)\times 1}{\mathbf{z}_t} + \underset{1\times(m+1)}{\boldsymbol{\theta}'} \underset{(m+1)\times 1}{\mathbf{z}_t} G(s_{1t};\gamma_1,\mathbf{c}_1) + \sum_{j=1}^{3} \underset{1\times m}{\boldsymbol{\beta}'_j} \underset{m\times 1}{\tilde{\mathbf{z}}_t} s_{2t}^j + u_t^*$$

The test is carried out by regressing $\tilde{u}_t$ on $\left(\tilde{\mathbf{z}}'_t s_{2t}, \tilde{\mathbf{z}}'_t s_{2t}^2, \tilde{\mathbf{z}}'_t s_{2t}^3\right)'$ and the partial derivatives of the log-likelihood function with respect to the parameters of the model. The null hypothesis of no remaining nonlinearity is $\boldsymbol{\beta}_1 = \boldsymbol{\beta}_2 = \boldsymbol{\beta}_3 = \mathbf{0}$. s_{2t} can be s_{1t} or any variable in $\mathbf{z}_t$. The test is a typical F-test.

2.8.5. Test for Parameter Constancy

The null hypothesis for the parameter constancy test is that the parameters are constant over time. The alternative hypothesis is that the parameters change smoothly and continuously. The auxiliary regression for the test is

$$y_t = \boldsymbol{\beta}'_0 \mathbf{z}_t + \sum_{j=1}^{3} \boldsymbol{\beta}'_j \mathbf{z}_t \tau^j + \sum_{j=1}^{3} \boldsymbol{\beta}'_{j+3} \mathbf{z}_t \tau^j G(s_t;\gamma,\mathbf{c}) + u_t^*$$

where $\tau = \frac{t}{T}$. The test is an F-test for $\boldsymbol{\beta}_j = \mathbf{0}$ for $j = 1,\ldots,6$. Note that the parameters γ and $\mathbf{c}$ are assumed to be constant.

2.8.6. Test for Normality

The Jarque-Bera test for normality is the same as in the linear case. The test is a diagnostic test for non-normality based on the 3rd (skewness) and 4th (kurtosis) moments of a distribution. Let u_t be the true error terms which have a standard deviation σ_u. Then $u_t^s = \frac{u_t}{\sigma_u}$ is the standardized true model errors. The null hypothesis is

$$H_0 : E\left(u_t^s\right)^3 = 0 \text{ and } E\left(u_t^s\right)^4 = 3$$

The alternative hypothesis is $H_1 : E\left(u_t^s\right)^3 \neq 0 \text{ or } E\left(u_t^s\right)^4 \neq 3$. Thus the tests checks whether the 3rd and 4th moments of the standardized residuals are consistent with a standard normal distribution. The test statistic is

$$LJB = \frac{T}{6}\left[T^{-1}\sum_{t=1}^{T}\left(\hat{u}_t^s\right)^3\right]^2 + \frac{T}{24}\left[T^{-1}\sum_{t=1}^{T}\left(\hat{u}_t^s\right)^4 - 3\right]^2$$

where $\hat{u}_t^s$ is the standardized estimated residuals, $T^{-1}\sum_{t=1}^{T}\left(\hat{u}_t^s\right)^3$ and $T^{-1}\sum_{t=1}^{T}\left(\hat{u}_t^s\right)^4$ are measures for the skewness and the kurtosis of the distribution, respectively. Under the null hypothesis, the test statistic has an asymptotic $\chi^2(2)$ distribution. Large values of the test statistics, hence small p-values, indicate non-normality. If the null hypothesis is rejected, the normal distribution is rejected. If the null hypothesis is not rejected, it does not necessarily means that the distribution is normal. It means that the underlying distribution has the same first four moments as the normal distribution. In practice, the first four moments are of particular interest and deviations from the normal distribution beyond that point may not be too important. If the null hypothesis is rejected, this is interpreted as a model defect. Non-normal residuals can also be a consequence of neglected nonlinearities, e.g. threshold, smooth transition, etc. or ARCH effects. Modeling these features may result in a more satisfactory model with normal residuals.

3. The Econometric Model

In this section, we propose the following model to analyze the relation between inflation and money growth:

$$\begin{aligned} y_t = &\left(\alpha_0 + \alpha_1 y_{t-1} + \cdots + \alpha_p y_{t-p} + \beta_0 + \beta_1 x_{t-1} + \cdots + \beta_q x_{t-q}\right) + \\ &\left(\delta_0 + \delta_1 y_{t-1} + \cdots + \delta_p y_{t-p} + \phi_0 + \phi_1 x_{t-1} + \cdots + \phi_q x_{t-q}\right) G(\bullet) + u_t \end{aligned}$$

where y_t is the inflation rate, x_t the excess money growth, and $G(\bullet)$ the logistic transition function with K = 1.

If inflation turns out to have a unit root, which is likely, then the model takes the following form:

$$\begin{aligned} \Delta y_t = &\left(\alpha_0 + \alpha_1 \Delta y_{t-1} + \cdots + \alpha_p \Delta y_{t-p} + \beta_0 + \beta_1 x_{t-1} + \cdots + \beta_q x_{t-q}\right) + \\ &\left(\delta_0 + \delta_1 \Delta y_{t-1} + \cdots + \delta_p \Delta y_{t-p} + \phi_0 + \phi_1 x_{t-1} + \cdots + \phi_q x_{t-q}\right) G(\bullet) + u_t \end{aligned}$$

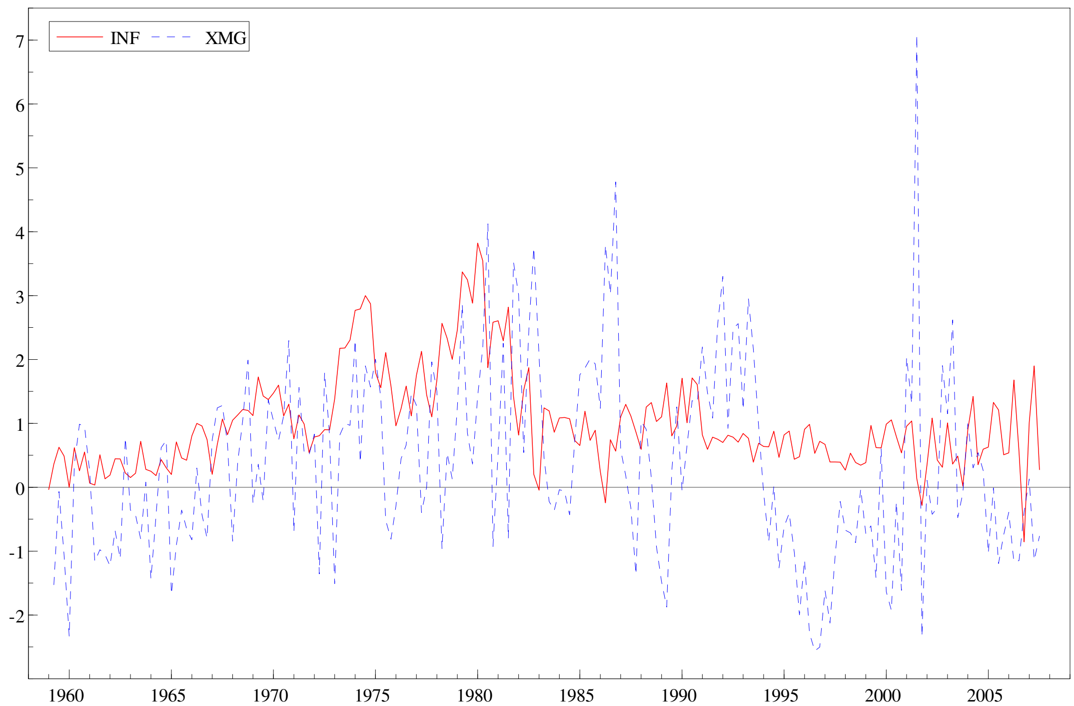
in which case, the acceleration of the inflation is modeled.

4. Data[10]

We collect quarterly data for the US, which include consumer price index (CPI), gross domestic product (GDP), GDP deflator (DEF), and M1 money supply (M1) from the International Financial Statistics (IFS) database. The sample period is 1959Q2-2007Q3, which makes 194 observations. M1 is seasonally adjusted. We divide GDP by DEF to obtain real GDP (RGDP). Then we transform CPI, M1, and RGDP into inflation (INF), money growth (MG), and real GDP growth (RGDPG), respectively, by first taking natural logs and then taking the first differences. Finally, we subtract RGDPG from MG

[10] EViews and PcGive software packages were used to obtain results in this section.

Figure 2: US Inflation and Excess Money Growth (1959Q2-2007Q3)



to obtain excess money growth (XMG). Figure 2 shows INF and XMG together. As can be seen, the excess money growth fluctuates more than the inflation. The inflation is relatively smooth. The large hike in the 3rd quarter of 2001 is probably the effect of the war in Iraq[11].

Table 1 shows some summary statistics for the two series. The average inflation is about 1% per quarter whereas the average excess money growth is about 0.4%. As can be seen from the standard deviation figures, XMG fluctuates almost twice as INF. Both series are left-skewed, which means small values are more often observed than large values, but INF is more skewed. The high kurtosis values for both series show that their peak is greater than a normal distribution. As a result, the Jarque-Bera test rejects the null hypothesis of normality for both series.

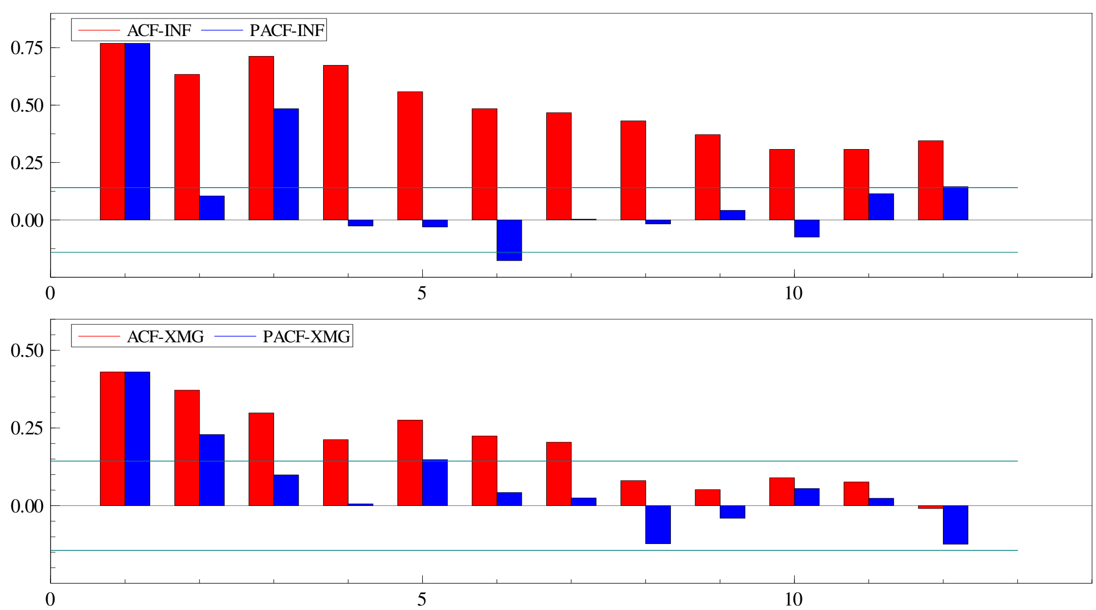
[11] Looking at the figure, one can see that the excess money growth and the inflation seem to move together until around 1990, after which the relation seems to have fallen apart. In order to investigate this further, we run two separate regressions, one using the data until 1990 and the other after 1990. The regressions show that the relation still holds after 1990.

Table 1: Summary Statistics

	INF	XMG
Obs	202.0000	194.0000
Mean	0.9979	0.3616
Median	0.8327	0.2585
Maximum	3.8217	7.0869
Minimum	-0.8560	-2.5620
Std. Dev.	0.7608	1.4819
Skewness	1.1942	0.7797
Kurtosis	4.7092	4.5545
Jarque-Bera	72.6008	39.1924
Probability	0.0000	0.0000

Figure 3 shows the autocorrelation (ACF) and the partial autocorrelation (PACF) functions. The ACF for INF dies out slowly but that for XMG relatively quickly. The PACF for INF indicates an at least AR(6) process whereas that for XMG an AR(2).

Figure 3: ACF and PACF of the Series



Next we run unit root tests to determine if the series is stationary. To obtain robust results, we use the augmented Dickey-Fuller (ADF), the Phillips-Perron (PP), and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests. Table 2 shows the results. All the auxiliary regressions contain an intercept but no time trend. For the ADF tests, the degree of augmentation is selected by a search over 12 lags based on the Akaike information criterion. For the PP and KPSS tests, the spectral estimation method is Barlett kernel and the automatic bandwidth selection is Newey-West. The tests conclude that there is no unit root in both series.

Table 2: Unit Root Tests

Series	Augmentation	Test Statistic	Significance Level %1	%5	%10
		ADF Test			
INF	5	-2.9374	-3.4637	-2.8761	-2.5746
XMG	4	-3.6293	-3.4650	-2.8767	-2.5749
		Phillips-Perron Test			
INF	9	-5.2792	-3.4629	-2.8758	-2.5744
XMG	6	-9.3288	-3.4643	-2.8764	-2.5747
		KPSS Test			
INF	10	0.3064	0.7390	0.4630	0.3400
XMG	9	0.2394	0.7390	0.4630	0.3400

5. Results

This section presents the estimation results and interprets them. The subsections follow the modeling cycle mentioned before. We also conduct a deterministic extrapolation exercise to understand the implications of the linear and nonlinear models for the long-run inflation rates.

5.1. Specification

The specification stage starts with setting up a linear model. The basic issue is to determine which lags of each variable to include in the model. For that purpose, we follow a general-to-specific strategy[12]. The basic idea is to start with a reasonably general model and then to make it parsimonious by eliminating unnecessary lags based on the t-ratios or some information criteria, e.g. Akaike or Schwarz. For that purpose, we include 8 lags for each variable assuming that past values beyond 2 years have no or negligible effect on the current value of the inflation rate. Then we run a comprehensive specification search which estimates every possible subset of the regressors and picks the one that minimizes the Akaike information criterion. The resulting model, which has some lags ignored, is shown in Table 3.

[12] See Hendry (1995).

Table 3: The Linear Model

Model:	Linear Regression			
Estimation Method:	Ordinary Least Squares			
Sample:	1961(2) - 2007(3) (186 Observations)			
Dependent:	INF			
Variable	Coeffs	Std Err	t-Stat	p-Values
Constant	0.1424	0.0555	2.5663	0.0111
INF_1	0.6862	0.0622	11.0321	0.0000
INF_2	-0.2767	0.0790	-3.5049	0.0006
INF_3	0.6284	0.0686	9.1592	0.0000
INF_6	-0.1843	0.0608	-3.0323	0.0028
XMG	-0.0397	0.0219	-1.8128	0.0716
XMG_3	0.0308	0.0222	1.3910	0.1660
XMG_8	0.0421	0.0210	2.0015	0.0469
R-Squared	0.7323	Adj R-Squared	0.7218	
RSS	29.1920	Regression S.E.	0.4050	
Tests	Stat	p-Value	5% Critical Vlue	
Joint Significance	69.5778	0.0000		
F-AR(4)	0.4035	0.8060		
F-AR(8)	0.7138	0.6791		
Jarque-Bera	0.0035	0.9982		
F-ARCH(4)	3.5420	0.0050		
F-ARCH(8)	2.0732	0.0410		
F-Heteroskedasticity	1.6044	0.0821		
RESET y2	3.2445	0.0734		
RESET y3	3.8076	0.0241		
RESET y4	2.5600	0.0566		
Hansen - Variance	0.5208		0.4700	
Hansen - Joint	2.3275		2.3200	

The results suggest that although the inflation rate seems to be a mostly autoregressive process, the excess money growth cannot be eliminated as an explanatory variable. The null hypothesis that each lag, including the zeroth lag, of the excess money growth is equal to zero is rejected by an F-test with a test statistic of 3.2937 and a p-value of 0.0219. However, it seems that the explanatory power of the excess money supply is slight since none of the t-ratios of its coefficients is much greater than 2. The estimated model has moderately high adjusted and unadjusted R-squared values. The joint significance test suggests that the explanatory variables are meaningful as a group. The LM tests for the residual autocorrelation detect no problem up to the eight lag. The Jarque-Bera test does not reject the null hypothesis of normality.

Though the model has the pros mentioned above, it also has some problems. For instance, the residuals show signs of contemporary[13] and autoregressive conditional heteroskedasticity (ARCH). This can be seen from the p-values which are less than 0.10. The RESET tests also indicate model misspecification. Finally, Hansen (1992)'s variance and joint stability tests show that the variance and the coefficients as a group are not stable over time since the test statistics are higher than the 5% critical values. Another problem is related to the estimated coefficients. It seems that the zeroth lag of the excess money growth has the wrong sign, which implies that an increase in the excess money growth decreases the inflation rate, which is counter-intuitive. However, this is not the only study that reports a result like this. For instance, Crowder (1998) also reports "negative short-run relation between base growth and inflation"[14]. A possible explanation is that the coefficient is probably statistically insignificant. The reason is that one cannot expect a significant relation between the current excess money growth and the current inflation rate. In other words, a change in the current excess money growth is not likely to have any effect on the current inflation rate though it is likely to have any effect on the future inflation rates. Besides, it should be remembered that the main reason for specifying a linear model is to create a starting point for the nonlinear model specification. Finally, as will be seen from the deterministic extrapolation exercises at the end of this section, the long-run relation between excess money growth and inflation is still positive for the linear model. Considering all the drawbacks of the linear model, an interesting question to ask at this point is whether a nonlinear model would do better than the linear one.

The next step in the specification phase is to run the linearity test developed by Luukkonen, Saikkonen, & Teräsvirta (1988). Since the linear model indicates that long lags can be significant in modeling inflation with its own lags and the excess money growth, we include a maximum of 8 lags in the linearity tests. The results are shown in Table 4.

As can be seen from the table, the selected transition variable is the second lag of inflation and the suggested model is LSTR1. These results show that the inflation rate can be modeled with a smooth transition regression model containing 2 regimes and the nonlinear dynamic process is governed by the second lag of the inflation itself.

[13] The White test for heteroskedasticity.
[14] Crowder (1998), p. 239.

Table 4: The Linearity Tests

Transition Variable	F	F4	F3	F2	Suggested Model
INF(t-1)	0.0003	0.3294	0.0390	0.0000	LSTR1
INF(t-2)*	0.0002	0.1214	0.1633	0.0000	LSTR1
INF(t-3)	0.0035	0.4223	0.2183	0.0001	LSTR1
INF(t-4)	0.0033	0.2565	0.1436	0.0006	LSTR1
INF(t-5)	0.0401	0.3255	0.4420	0.0048	LSTR1
INF(t-6)	0.0031	0.2100	0.0565	0.0038	LSTR1
INF(t-7)	0.0522	0.9266	0.1852	0.0009	Linear
INF(t-8)	0.0378	0.8150	0.1995	0.0010	LSTR1
XMG(t)	0.0010	0.5072	0.0399	0.0001	LSTR1
XMG(t-1)	0.0043	0.1242	0.0369	0.0266	LSTR1
XMG(t-2)	0.2429	0.7091	0.3160	0.0619	Linear
XMG(t-3)	0.3811	0.7283	0.3757	0.1362	Linear
XMG(t-4)	0.2463	0.6115	0.2449	0.1401	Linear
XMG(t-5)	0.1096	0.1042	0.7378	0.0700	Linear
XMG(t-6)	0.2131	0.5901	0.9015	0.0044	Linear
XMG(t-7)	0.7318	0.7295	0.9302	0.1451	Linear
XMG(t-8)	0.0970	0.2365	0.3266	0.0881	Linear
TREND	0.0042	0.0276	0.0334	0.1911	LSTR1

5.2. Estimation[15]

The model is estimated with nonlinear least squares (NLS). The minimization algorithm is BFGS. The estimated nonlinear model is shown in Table 5. First a few words about how we reached the final model. The estimation started with all of the 8 lags for each variable. Then we eliminated the regressor with the highest p-value or the lowest t-stat in absolute value in order to reduce the number of model parameters (model reduction). We continued in this fashion to eliminate the regressors until we reached the point where it didn’t make too much sense to eliminate one more regressor. This process is similar to the method suggested by Brüggemann & Lütkepohl (2001).

[15] The results in this section were obtained with the JMulTi econometric package.

Table 5: The Nonlinear Model

variable	estimate	s.d.	t-stat	p-value
		Linear Part		
CONST	0.1886	0.0773	2.4418	0.0157
INF(t-1)	0.6275	0.0806	7.7850	0.0000
INF(t-2)	-0.2604	0.1047	-2.4869	0.0139
INF(t-3)	0.3365	0.0858	3.9209	0.0001
INF(t-6)	-0.1883	0.0701	-2.6867	0.0080
INF(t-7)	0.2394	0.0701	3.4146	0.0008
XMG(t-8)	0.0397	0.0191	2.0754	0.0396
		Nonlinear Part		
CONST	0.8327	0.2588	3.2176	0.0016
INF(t-1)	0.2640	0.1347	1.9604	0.0517
INF(t-2)	-0.4916	0.2151	-2.2849	0.0236
INF(t-3)	0.6124	0.1669	3.6686	0.0003
INF(t-7)	-0.4440	0.1222	-3.6326	0.0004
XMG(t)	-0.1906	0.0534	-3.5665	0.0005
XMG(t-1)	0.1553	0.0630	2.4647	0.0148
XMG(t-2)	-0.2296	0.0604	-3.8020	0.0002
XMG(t-4)	-0.1186	0.0643	-1.8449	0.0669
XMG(t-6)	-0.1706	0.0704	-2.4224	0.0165
XMG(t-8)	0.1928	0.0628	3.0696	0.0025
Gamma	13.3477	8.6190	NaN	NaN
C1	1.4805	0.0429	NaN	NaN

The most interesting part of the final model is that although only the 8^{th} lag of the excess money supply is significant in the linear part, almost all of the lags in the nonlinear part are significant. This is a strong indication that the excess money supply affects the inflation nonlinearly.

As can be seen from the table, the inflation is mostly an autoregressive process since its lags are significant in both the linear and the nonlinear parts. However, the linear part is dominated by the lags of the inflation. On the other hand, the lags of the excess money growth are mostly significant in the nonlinear part.

The fit of the model seems to be better compared to the linear model. The unadjusted and adjusted R-squared values are 0.8293 and 0.8302. The estimated value of the gamma is 13.3477, which indicates a moderately smooth transition between regimes. The transition value, c_1 in the logistic transition function, is 1.4805 with a standard error of 0.0429. This means that when the quarterly inflation rate two quarters ago surpasses 1.4805%, the economy switches from one regime to another. The regimes can be interpreted as the

Figure 4: The Transition Function over Time

low- and the high-inflation regimes. In other words, inflation rates below (above) 1.4805% can be regarded as low (high) inflation periods. Figure 4 shows the high-inflation regimes, which are the early 1970s, the late 1970s, the early 1980s, and the early 1990s. As can be seen from the figure, the U.S. experienced mostly low inflation in the sample period. Figure 5 shows the fitted and the actual values.

Figure 5: The Fitted and Actual Values of Inflation

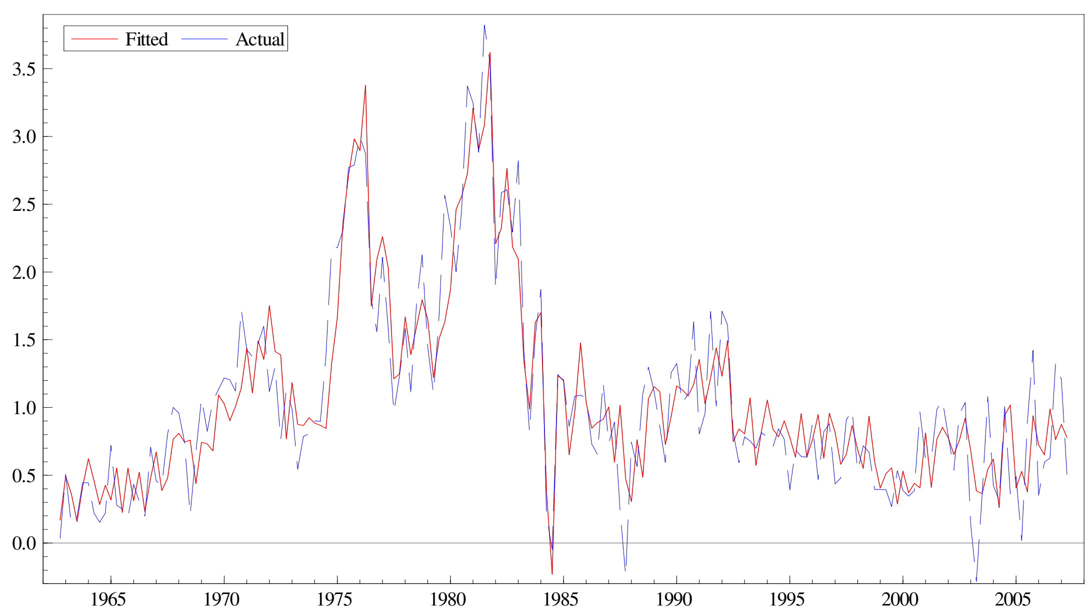


Figure 6: The Linear and Nonlinear Parts of the Fitted Values

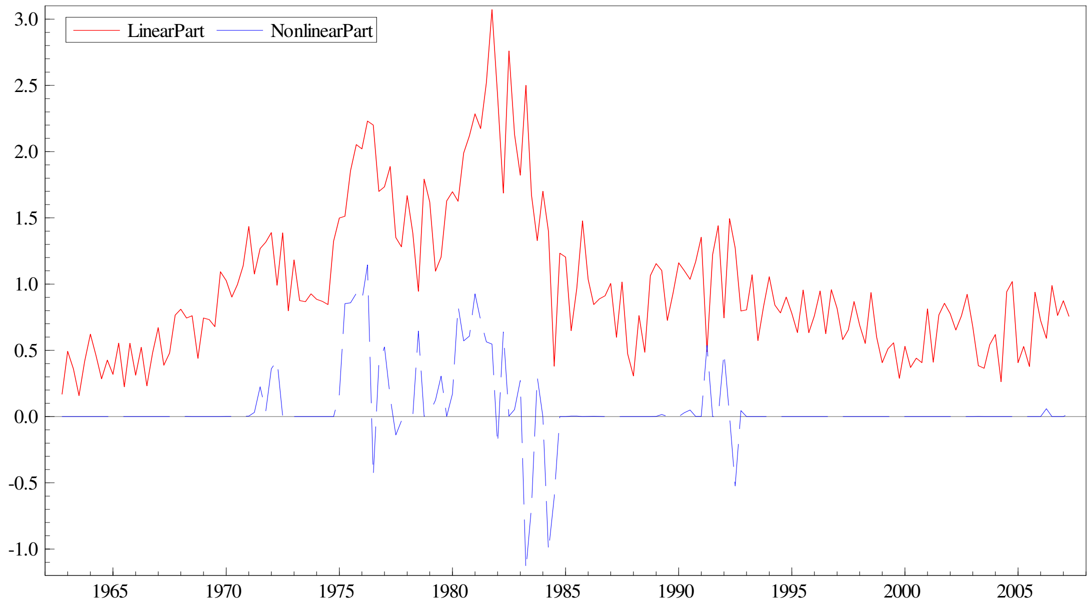
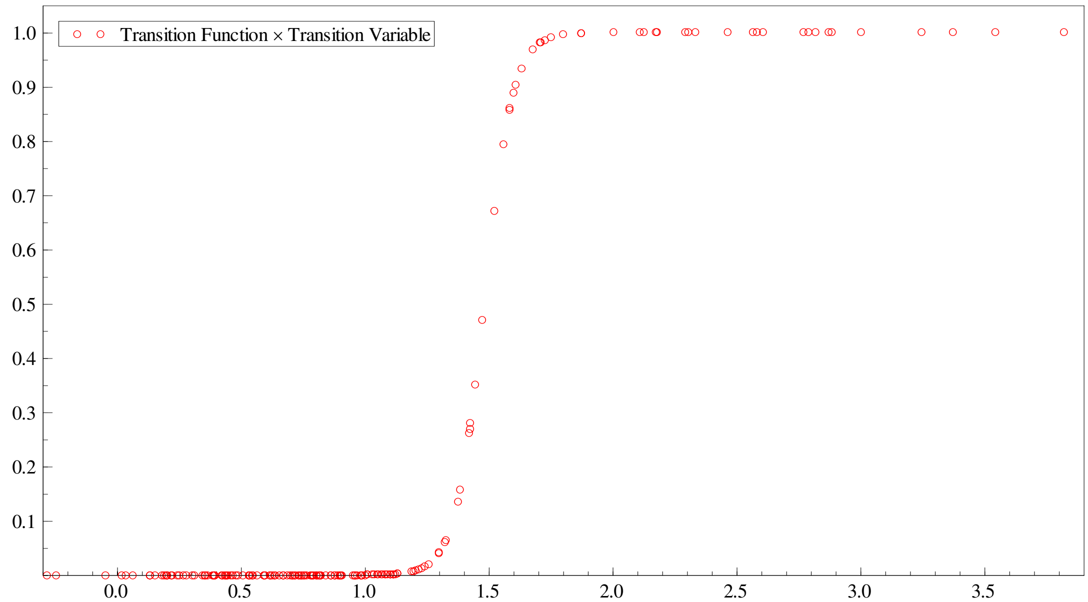


Figure 6 shows the linear and nonlinear parts of the fitted values separately. Since the fitted values for the complete model are obtained from $\hat{y}_t = \hat{\boldsymbol{\varphi}}'\mathbf{z}_t + \hat{\boldsymbol{\theta}}'\mathbf{z}_t G(y_{t-2};\hat{\gamma},\hat{c}_1)$ where y_t is the inflation rate at time t, the fitted values for the linear part are $\hat{\boldsymbol{\varphi}}'\mathbf{z}_t$ and those for the nonlinear part are $\hat{\boldsymbol{\theta}}'\mathbf{z}_t G(y_{t-2};\hat{\gamma},\hat{c}_1)$. Notice that the nonlinear part has fitted values only when the transition function is *non-zero*. It should be remembered that the fitted values for the linear part represent inflation when the economy is in low-inflation regime, in other words, when $G(y_{t-2};\hat{\gamma},\hat{c}_1)=0$. Thus the nonlinear fitted values can be interpreted as adjustments for high-inflation regimes.

Figure 7 shows the transition function versus the transition variable. The vertical axis is the transition function and the horizontal axis is the transition variable. When the transition variable is less (higher) than 1.4805 and thus the transition function is less (higher) than 0.5, the economy is in low-inflation (high-inflation) regime. We should emphasize that the vertical axis should not be interpreted as recession probabilities as in the Markov switching regression model (MSR) developed by Hamilton (1989). Unlike the MSR model, the variable that determines regimes is observable in the STR models. As a result, in the STR models, the regime that the system is in is known given the values of the transition variable, s_t, and the transition value, $\hat{c}_1$.

Figure 7: The Transition Function vs. the Transition Variable



5.3. Evaluation

In this section, we evaluate the linear and nonlinear models. First, we run a couple of misspecification tests for the nonlinear model. Then we conduct a series of deterministic extrapolation experiments with the models to characterize the quantitative properties of the excess money growth and the inflation rate.

5.3.1. Specification Tests

Table 6 shows several misspecification tests for the nonlinear model. The LM test for autocorrelation in the residuals detects no problem up to 8 lags. Similarly, the ARCH tests for the 4^{th} and 8^{th} lags show that, unlike the linear model, the nonlinear model is not affected by autoregressive conditional heteroskedasticity in the residuals. Jarque-Bera test does not reject the null hypothesis of normality. Parameter constancy tests also cannot detect any problem with the model.

Finally, we run the test for no remaining nonlinearity and the results are shown in Table 7. We try every lag of the two model variables as the transition variable. We conclude that there is no remaining nonlinearity in the model since the values in the second column are all greater than or equal to 0.10.

Table 6: Specification Tests for the Nonlinear Model

Test of No Error Autocorrelation				
Lag	F-value	df1	df2	p-value
1	0.0662	1	157	0.7972
2	0.1238	2	155	0.8836
3	0.4412	3	153	0.7239
4	0.3300	4	151	0.8575
5	0.3468	5	149	0.8837
6	0.3885	6	147	0.8855
7	0.3724	7	145	0.9171
8	0.3388	8	143	0.9496
ARCH-LM Test				
Lag	Ch-Sq-Stat	p-value	F-Stat	p-value
4	1.6099	0.8070	0.4062	0.8040
8	3.5406	0.8960	0.4519	0.8879
Jarque-Bera Normality Test				
Test Stat	p-value		Skewness	Kurtosis
1.4922	0.4742		0.2024	3.1902
Parameter Constancy Test				
transition function	F-value	df1	df2	p-value
H1	1.5472	18	139	0.0827
H2	1.2638	36	121	0.1748
H3	1.3852	54	103	0.0789

5.3.2. Deterministic Extrapolation

As further evaluation of the linear and nonlinear models, we conduct a few deterministic extrapolation exercises.

No External Shocks

The basic idea behind the first deterministic extrapolation exercise is explained in the following steps. First, the random term in each model is equated to zero. In other words, external shocks are assumed not to exist. Second, the estimated coefficients are placed in their corresponding equations (deterministic parts of the models). Third, the models are iterated over time. In this iteration, the values of each variable not used in the estimation due to model lags are used as the starting values. Each value of inflation obtained by this iteration is placed back in the equations and the iteration continues. If the iterated values of inflation converges to a constant value, the converged value is the long-run value of inflation. If the iterated values do not converge, the model is labeled as unstable.

Table 7: Test for Remaining Nonlinearity

Test of No Remaining Nonlinearity				
transition variable	F	F4	F3	F2
INF(t-1)	0.1157	0.1261	0.0687	0.4030
INF(t-2)	0.1035	0.0715	0.2016	0.2260
INF(t-3)	0.3921	0.4339	0.1046	0.5389
INF(t-4)	0.7809	0.4685	0.3598	0.6816
INF(t-5)	0.4875	0.6055	0.1407	0.2953
INF(t-6)	0.8066	0.7861	0.1399	0.8873
INF(t-7)	0.5905	0.8919	0.0503	0.6315
INF(t-8)	0.9530	0.8526	0.7113	0.4420
XMG(t)	0.1000	0.3115	0.0458	0.1698
XMG(t-1)	0.3321	0.8438	0.1704	0.0619
XMG(t-2)	0.6036	0.5764	0.8416	0.0723
XMG(t-3)	0.2329	0.3891	0.7810	0.0056
XMG(t-4)	0.4128	0.4812	0.4400	0.1198
XMG(t-5)	0.9041	0.5381	0.8245	0.4509
XMG(t-6)	0.3099	0.1877	0.7142	0.0945
XMG(t-7)	0.5434	0.4052	0.3928	0.2487
XMG(t-8)	0.9837	0.7143	0.8545	0.8495

Figure 8 shows the iterated values of the linear model. As can be seen, the values converges to a stable equilibrium after fluctuating a couple of periods. The long-run equilibrium of the inflation is approximately 1.05%. Interestingly, this is very close to the mean value of the inflation series (0.99%) as can be seen from Table 1.

The nonlinear model draws a different picture, however. We present the results according to the extreme values of the transition function. When $G = 0$, the economy is in the low-inflation regime. The deterministic extrapolation of the nonlinear model for this case is shown in Figure 9. Since the nonlinear part of the model vanishes when G = 0, the linear part and the combined model coincide. That's why Inf and InfLin overlap in the figure. InfNln is zero so it is not drawn. As can be seen from the figure, the long-run inflation for the low-inflation regime is 0.5% per quarter.

Figure 10 shows the results of the deterministic extrapolation when G = 1. When G = 1, the economy is in high-inflation regime. The solid line shows the combined model, that is linear part plus nonlinear part. The dashed line shows the linear part. Finally, the dotted line shows the nonlinear part. During high-inflation regimes, the long-run value of the inflation rate is around 2.5% for the combined model. For the linear part, it is around 2%. For the nonlinear part, it is around 0.5%. The results indicate that the long-run quarterly inflation was in the range 0.5%-2.5% for the sample period and was stable in the US.

Figure 8: Deterministic Extrapolation of Inflation for Linear Model

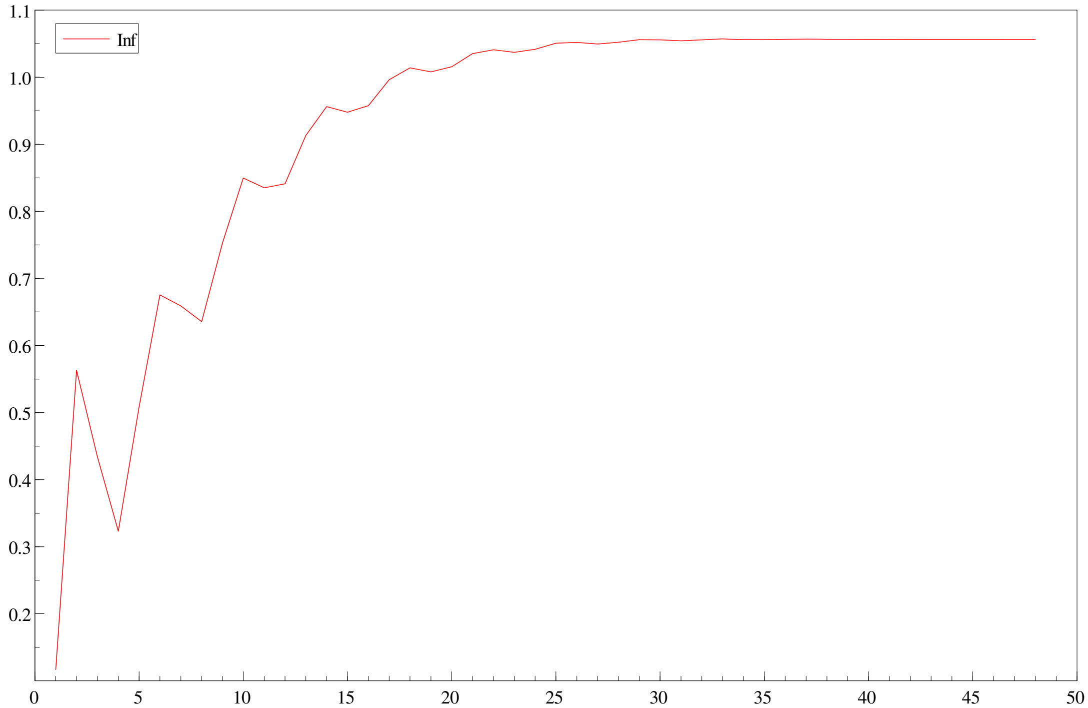


Figure 9: Deterministic Extrapolation of Inflation for Nonlinear Model When G = 0

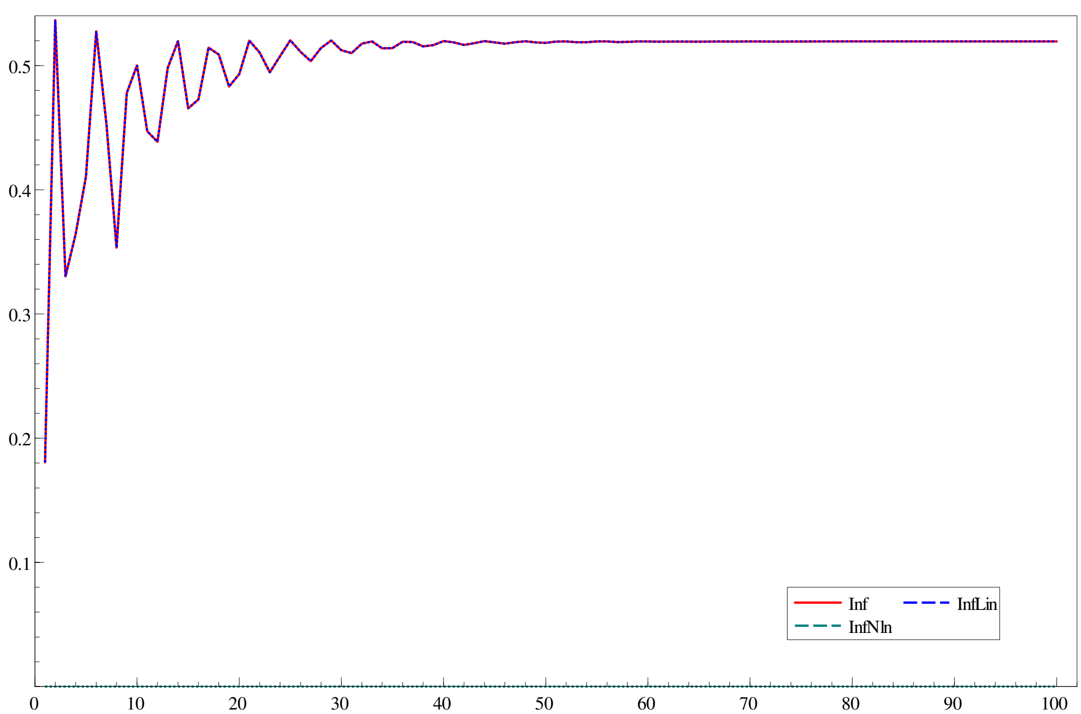
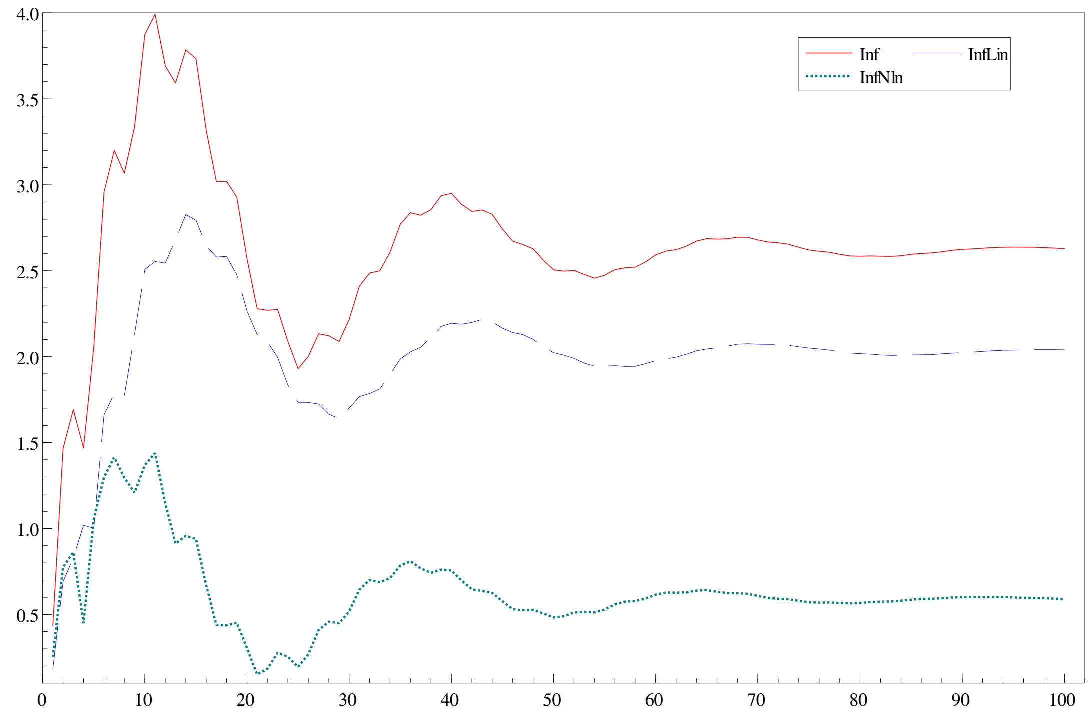


Figure 10: Deterministic Extrapolation of Inflation for Nonlinear Model When G = 1



External Shock to Inflation

The second deterministic extrapolation exercise differs from the first one in that an external shock is applied to the model. This is done by setting the error term to a constant, one to be exact, in the first iteration and to zero in the following iterations. This is an impulse shock, meaning a one-time external shock to the economy. The aim of this exercise is to see its long-run effect on the inflation.

Figure 11 shows the effects of this one-time external shock on the long-run value of the inflation for the linear model. As can be seen from the figure, the external shock is eliminated over time since the inflation rate converges to the same value obtained without the shock. In other words, the long-run value of the inflation is about 1.05% with and without the external shock.

As Figures 12 and 13 show, similar results are obtained for the nonlinear model when G = 0 and G = 1. The effects of the impulse shock disappear over time. The iterated inflation rates corresponding to the linear, nonlinear, and combined parts first fluctuate considerably and then converges to their long-run values obtained without an external shock applied. Once again, the results indicate that the linear and the nonlinear models are stable and the external shocks to the economy are eliminated over time.

Figure 11: Effects of an Impulse External Shock in the Linear Model

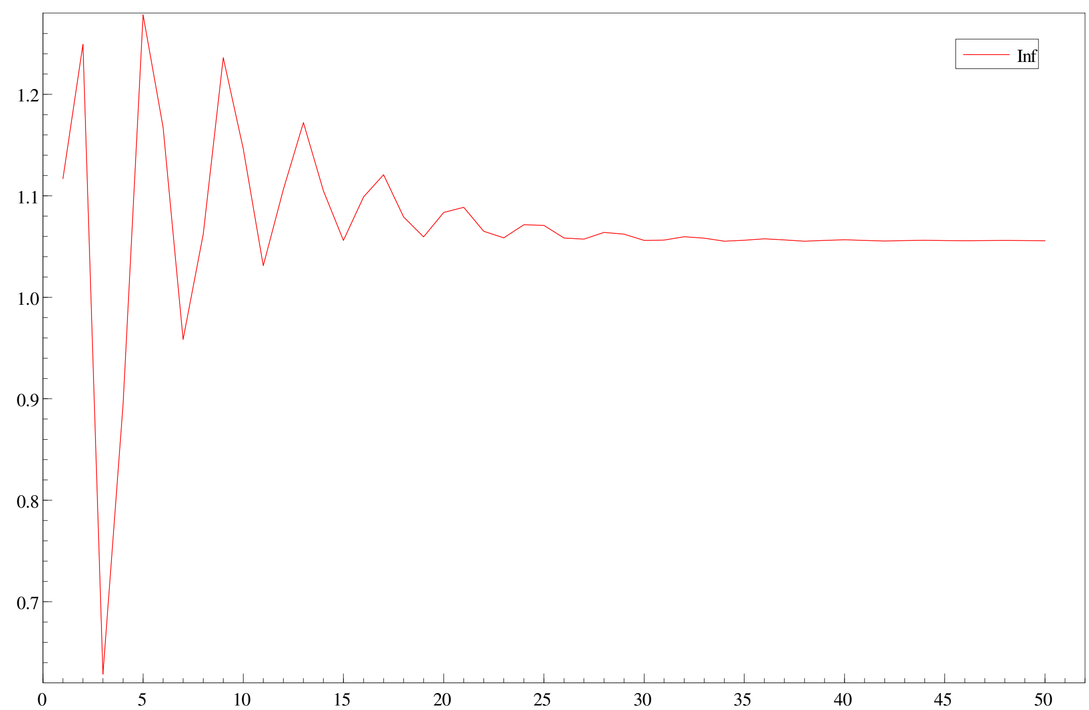


Figure 12: Effects of an Impulse External Shock in the Nonlinear Model When G = 0

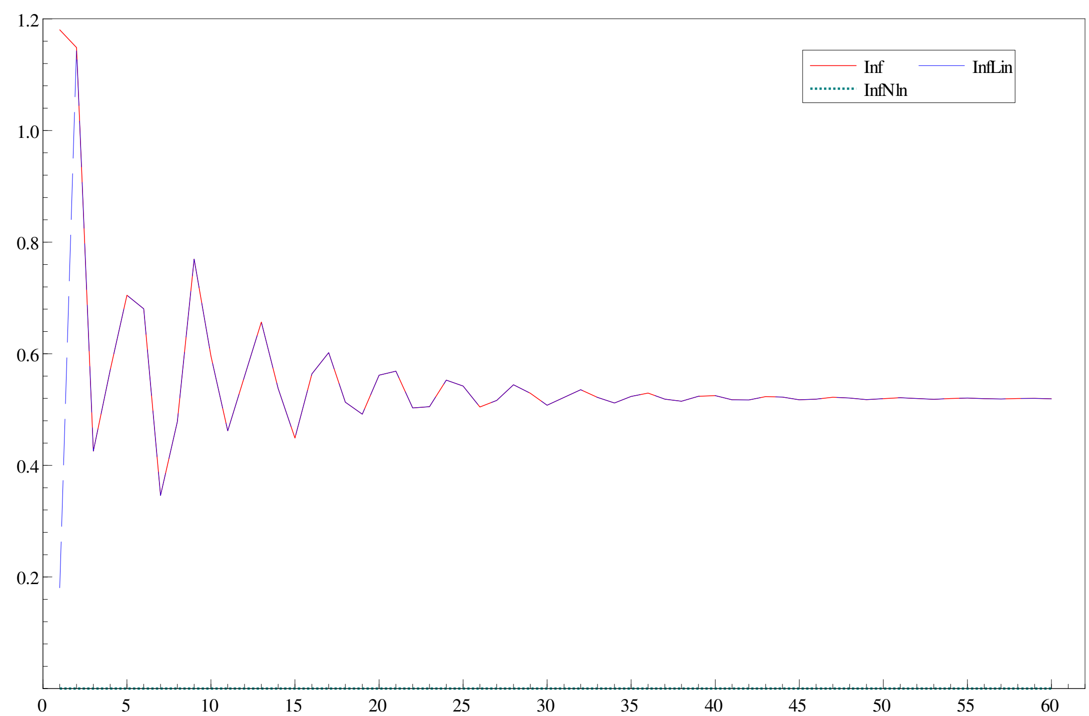
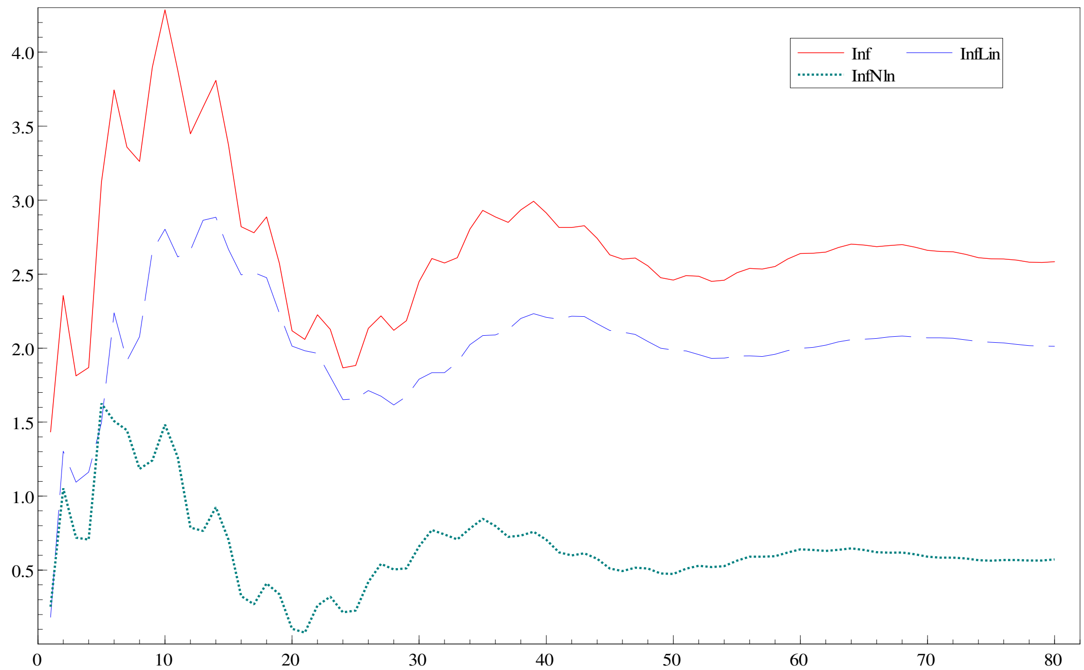


Figure 13: Effects of an Impulse External Shock in the Nonlinear Model When G = 1



External Shock to Excess Money Growth

The third deterministic extrapolation exercise includes an impulse shock to the excess money growth. This exercise is important to understand the effects of a change in the excess money growth on the inflation rate in the long-run. The effect of a 1% increase in the excess money growth in the linear model is shown in Figure 14. As can be seen from the figure, the long-run value of the inflation is 1.34%, which is about 0.3 higher than the long-run inflation in the absence of an increase of 1% in the excess money growth. This result indicates that an increase of 1% in the excess money growth increases the long-run value of the inflation by 0.3%.

Figure 15 shows the effect of an impulse shock of 1% to the excess money growth in the nonlinear model when G = 0 (low-inflation regime). The long-run value of the inflation is about 0.66%. This is approximately 0.16% higher than the inflation rate obtained without the impulse shock. The results indicate that a 1% impulse shock to the excess money growth increases the inflation by 0.16% during low-inflation regime.

Finally, Figure 16 shows the effect of an impulse shock of 1% to the excess money growth in the nonlinear model when G = 1 (high-inflation regime). The long-run value of the inflation is about 3.35%. This is approximately 0.75% higher than the inflation rate obtained without the impulse shock. The results indicate that a 1% impulse shock to the excess money growth increases the long-run inflation by 0.75% during high-inflation regime.

Figure 14: Effects of an Impulse Shock to the Excess Money Growth in the Linear Model

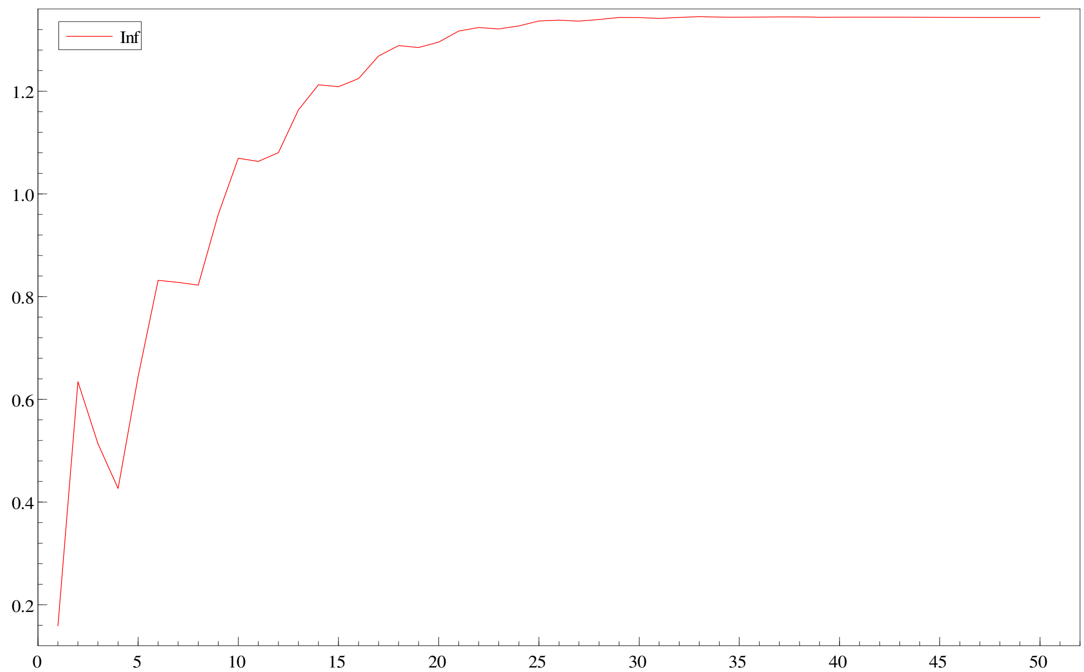


Figure 15: Effects of an Impulse Shock to the Excess Money Growth in the Nonlinear Model When G = 0

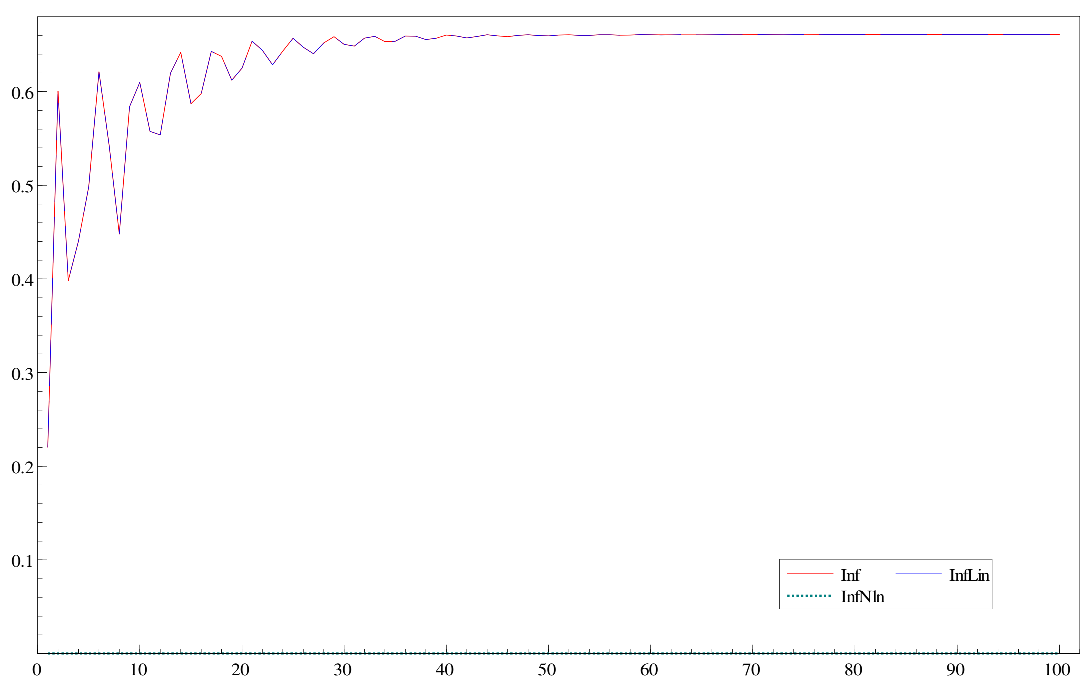
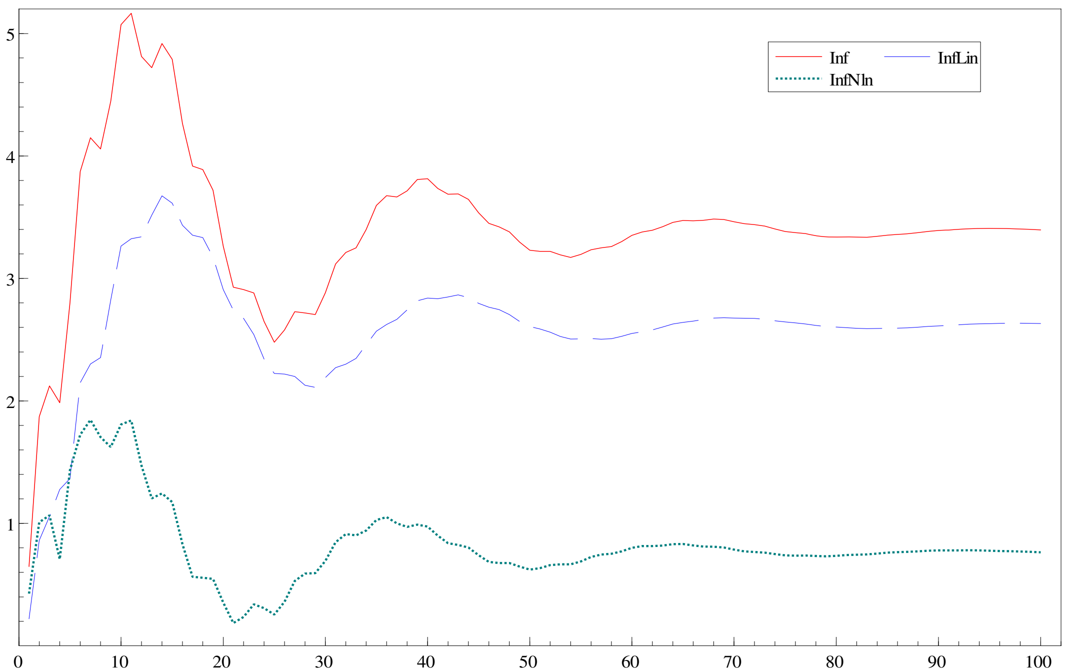


Figure 16: Effects of an Impulse Shock to the Excess Money Growth in the Nonlinear Model When G = 1



6. Conclusion

The basic findings of the paper can be summarized as follows. First of all, the relation between the inflation and the excess money growth in the US seems to be a nonlinear one as indicated by the linearity and specification tests. The nonlinear relation is well captured by the smooth transition regression models. Second, as the first deterministic extrapolation exercise shows, the inflation is stable in both the linear and nonlinear models. Third, an external impulse shock to the inflation has no effect on the long-run inflation rate. This is also supported by both the linear and nonlinear models. Fourth, a 1% impulse shock to the excess money growth has a 0.3% positive effect on the long-run inflation rate in the linear model. On the other hand, the same shock has 0.16% effect in the low-inflation regime and 0.75% effect in the high-inflation regime in the nonlinear model.

It would be useful to compare the results obtained in this paper to those already found in the literature. Christensen (2001) finds a one-to-one long-run relation between money growth and inflation for the US. Our results indicate that the relation is not one-to-one. However, Christensen uses money growth but we use excess money growth. So a direct comparison may not be appropriate. Using impulse response analysis, Crowder (1998) also reports that a shock to money growth effects inflation on a one-to-one basis. A shock to inflation however effects neither money growth nor inflation. This result is in line with our findings. Milas (2007) finds that a 1% increase in annual money growth rate increases annual inflation by only 0.07%. Interestingly, this ratio between money growth and inflation does not hold even under low-inflation regime in our nonlinear model. Kugler and Kaufmann (2005) find that a 5% money growth rate is compatible with a 2% inflation rate. The ratio 2/5 is close to the ratio in our linear model, which is 1.5/5. De Grauwe and Polan (2005) find that the relation between money growth and inflation is not proportional for most of the countries, which is our finding for the US. As a final note, we should emphasize that none of the papers mentioned here cites the quantitative relation between money growth and inflation under different inflation regimes, e.g. high- and low-inflation. This is one of the achievements of this paper.

It should be stressed that more research is required to determine how robust the results in this study are. Using data from other countries or using different money supply and inflation measures seem to be the next steps in that respect. It is also desirable to subject the nonlinear model to more analyses. For instance, it would be interesting to apply Granger causality tests and see the direction of causality between the excess money growth and the inflation. Another interesting analysis would be the comparison of in- and out-of-sample[16] inflation forecasting performances between the linear and nonlinear models. Still another important improvement of the paper might be the use of a multivariate STR analysis where excess money growth and inflation are endogenous variables. Throughout the paper, we assume the effect of a change in velocity on inflation is zero or negligible. A possible improvement of the model may be the addition of a change in velocity to excess money growth and use the resulting series as explanatory variable for inflation. These constitute our research agenda for the future.

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[16] For instance, one can use Corradi & Swanson (2002)'s nonlinear out-of-sample forecasting accuracy tests.

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