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A Remark on Intensive Differential Rent 
and the Labour Theory of Value in Ricardo

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Abstract

One of the foundations of the labour theory of value used by Ricardo in the *Principles* is that rent does not enter into commodity prices. In response to objections raised by Malthus and Say, Ricardo defended this idea by arguing that even where all cultivated land pays rent, the last dose of capital employed on the land does not and no rent is therefore involved in the price of the product of this capital. We will show that this claim, which has been believed true by several generations of economists, is based on a misleading argument and in fact incorrect.

In particular, we will show that the intensive differential rent paid on land of the worst quality under cultivation enters into the agricultural product price and so, even in the most favourable case, commodities are no longer exchanged at a ratio corresponding to the relative quantities of labour they embody.

**Keywords:** differential rent, labour theory of value, methods of production, Ricardo, Sraffa, Smith.

**JEL Classification:** B12, D46.

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1. Introduction

The primary result of Ricardo’s analysis is undoubtedly the presence of an inverse relationship between the wage rate and the rate of profit. With the aid of the labour theory of value, Ricardo succeeded where Adam Smith had failed and pinpointed the link between profits and wages.

Although many of Ricardo’s conclusions have been shown to be valid in general, his arguments are strongly based on the hypothesis that commodities are exchanged with one other at a ratio determined by the quantities of labour they embody.\(^1\) He was therefore obliged first to deny Smith’s claim that prices are not determined by the labour they embody in a developed country and second to refute the objections raised by Say and Malthus.

As is well known, Ricardo’s labour theory of value is essentially grounded on two assertions: i) that the employment of capital in the production of each commodity is proportional to the employment of labour; ii) that rent is not a component part of the prices of commodities. The first was intended as an approximation, in the sense that exceptions were possible and their relevance should be judged case by case. The second is instead a result that Ricardo believed he had established through his theory of differential rent. We shall focus here solely on the latter, as the former has already received a great deal of attention.

If extensive differential rent were the only possible form, then Ricardo’s result would hold, as the price of agricultural products would be determined in this case by the cost of production on the least fertile land under cultivation, and land of this quality pays no extensive differential rent at all. Ricardo therefore appears to have found a reply to Smith’s idea of rent as a component part of commodity prices:

> “that corn which is produced by the greatest quantity of labour is the regulator of the price of corn; and rent does not and cannot enter in the last degree as a component part of its price. Adam Smith, therefore, cannot be correct in supposing that the original rule which regulated the exchangeable value of commodities, namely, the comparative quantity of labour by which they were produced, can be altered by the appropriation of land and payment of rent. Raw material enters into the composition of most commodities, but the value of that raw material, as well as corn, is regulated by the productiveness of the portion of capital last employed on land, and paying no rent; and therefore rent is not a component part of the price of commodities.” (Ricardo 1951-73: vol. 1, p. 46)

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\(^1\) “Ricardo’s basic theorem on distribution—‘a rise of wages ... would invariably lower profits’—is thus strictly dependent on his measure of value” (Stigler 1952, p. 203).
Although differences in the quality of land always involve extensive differential rent, however, it can hardly be maintained that the rents of landowners consist exclusively of this element. Where every piece of land is owned, there is clearly no land that is cultivated without the payment of rent to the owner. This is the objection Say raised against Ricardo’s theory.

In a letter to Say dated 11th January 1820, Ricardo replies to this point as follows:

“You appear to me to have mistaken also an opinion of mine on which you comment in a note of the translation of my book. My argument respecting rent, profit and taxes, is founded on a supposition that there is land in every country which pays no rent, or that there is capital employed on land before in cultivation for which no rent is paid. You answer the first proposition, but take no notice of the second. The admission of either will answer my purpose.” (Ricardo 1951-73: vol. 8, pp. 149, 150)

In the first place, Ricardo expressly states that the presence of rent-free land is a “supposition”, something admittedly not proven. In the second, he does not reject Say’s argument that there is no cultivated land yielding no rent to its owner, which is also summed up in a note\(^2\) to Chapter XXXII of the *Principles*. He confines himself to defending his theory on the basis of intensive differential rent,\(^3\) arguing that even if Say were right to believe that all the cultivated land received rent, the last dose of capital employed on this land would still pay no rent and his theory would thus be valid in any case.

We also find this argument stated in an earlier letter of Ricardo to James Mill, dated December 22nd 1818, in which he reports on a discussion with Malthus:

“He [Malthus] has altered his opinion you know about there being land in every country which pays no rent, and appears like M. Say to think that when that is proved, my doctrine of rent not entering into price is overthrown – they neither of them advert to the other principle which cannot be touched, of capital being employed on land, already in cultivation, which pays no rent.” (Ricardo 1951-73: vol. 7, p. 372)

The ultimate foundation of Ricardo’s idea that rent does not enter into the price of commodities therefore appears to be the theory of intensive differential rent.

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\(^2\) “M. Say, in his notes to the French translation of this work, has endeavoured to show that there is not at any time land in cultivation which does not pay a rent, and having satisfied himself on this point, he concludes that he has overturned all the conclusions which result from that doctrine.” (Ricardo 1951-73: vol. 1, p. 413)

\(^3\) Samuelson (1959, p. 9, footnote 2) proposed a reformulation of the Ricardian theory of rent in terms of differential calculus and marginal productivity. In this case, assuming a continuum of different qualities of land – as in von Thunen theory for example – the extensive differential rent would be compatible with a situation in which almost every cultivated land yields a rent to its proprietor. But clearly this was not the case considered by Ricardo, since he explicitly referred to the intensive rent.
Ricardo ultimately succeeded in defending his idea and convincing several generations of economists\(^4\) that even though all cultivated land receives rent, the last dose of capital employed on land pays no rent and therefore rent does not enter into the price of commodities.

With the aid of Sraffa’s rigorous restatement of differential rent theory\(^5\), we intend to show here that Ricardo’s argument about intensive rent is misleading and the widely accepted conclusion that intensive rent does not enter into the price of agricultural commodities is incorrect.

In particular, after restating Ricardo’s argument, we shall analyse the increase in intensive differential rent from a different point of view and, using the same data as Ricardo’s example, show that it enters into the (relative) price of corn. As a result, when there are intensive rents, commodities cannot be exchanged at prices determined by the embodied quantities of labour.

2. Ricardo’s argument

Ricardo presents his theory of intensive differential rent by means of an example in the second chapter of the *Principles*. He imagines that a capital of £1,000 – which can be regarded as wages anticipated at the beginning of the year – applied on an acre of the most fertile land gives an output of 100 quarters of corn, whereas the same capital applied on an acre of less fertile land would give 80 quarters. He then observes that if the application of a second £1,000 of capital on an acre of the most fertile land gave rise to an increase in product of 85 quarters, this use of capital would be clearly more advantageous, where possible, than the cultivation of less fertile land.

In this case, rent arises even without different qualities of land being cultivated, in that Ricardo views the difference between the 100 quarters obtained by the first dose of capital and the 85 quarters obtained with the second – i.e. 15 quarters, or the value of 15 quarters – as representing the landowner’s rent for an acre. The amount of (gross) profit on a capital of £1,000 is thus the equivalent of 85 quarters of corn for the first investment of capital as well as the second. Ricardo concludes that:

“the capital last employed pays no rent. For the greater productive powers of the first £1000, fifteen quarters is paid for rent, for the employment of the second £1,000 no rent whatever is paid.” (Ricardo 1951-73: vol. 1, p. 72)

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\(^4\) Authoritative examples include Marshall (1893, p. 86) and S. Hollander (1979, p. 203).

\(^5\) We refer in particular to the analysis described by Sraffa (1960: 75, 76 - §§ 87, 88) and put into formal terms and developed in greater depth by various economists, including Montani (1975), Kurz (1978), Guichard (1982) and D’Agata (1983). Cf. also: Kurz and Salvadori (1995, p. 289).
Though capable of showing that more intense cultivation of the most fertile land can be more advantageous than cultivating inferior land, Ricardo’s argument in no way proves that rent does not enter into the price of corn.

The arbitrary division of capital into doses of £1,000 each is in fact misleading and induces the reader to believe that the last dose of capital employed alone gives an output of 85 quarters of corn. In other words, these 85 quarters seems to be the product of £1,000 capital, without any (further) employment of land. This impression is incorrect, however, because corn cannot be produced without land.

The application of a second dose of capital on the most fertile land derives, as Sraffa shows, from the possibility of cultivating land of this quality by two alternative methods. With the first, method $\alpha$, 100 quarters of corn are obtained by employing a capital of £1,000 on an acre of land, which means that one quarter of corn is obtained with a capital of £10 and 0.01 acres of land. With the second, $\beta$, 185 quarters are produced by employing a capital of £2,000 on an acre of land, and so the production of one quarter of corn requires a capital of £10.81 and 0.0054 acres of land. Corn is produced by means of capital and land both with method $\alpha$ and with method $\beta$.

As a result, when the rent rate is equal to the value of 15 quarters of corn, every single unit of capital employed pays a rent. In particular, every £1 of capital employed pays a rent corresponding to the value of 0.015 quarters of corn with method $\alpha$ and 0.0075 quarters with method $\beta$.

With his doses of capital, Ricardo instead seeks to present method $\beta$ as a combination of method $\alpha$ and a third method making it possible produce corn with no land. The argument is misleading, however, since Ricardo then uses this imaginary third method to prove that rent does not enter into the price of corn.

If there were a method of producing corn without land, it would be a mere truism to assert that rent does not enter into the cost of producing corn. No such method exists, however, because the investment of the second dose of capital cannot take place independently of the first. The fact that the second dose of capital is not employed with a third independent method becomes clear when we observe that, according to Ricardo’s reasoning, a change in the quantity of corn obtained from £1,000 with method $\alpha$ – from the first dose of capital – would entail a change in the amount of product obtained from the last dose of capital.\footnote{For example, if capital became more productive with method $\alpha$, so that £1,000 capital produced 110 quarters of corn per acre, the quantity of corn obtained with the second dose of capital would be reduced to 75 quarters because the quantity produced by method $\beta$ (185 quarters per acre), being a well-defined method of production, is independent of any improvement in method $\alpha$. As a result, the second dose of capital cannot be regarded as employed with a third independent method.}
Ricardo’s conclusion thus collapses. Whenever rent is paid for the use of land of a certain quality, every single unit of capital invested on that land pays a rent. As a result, if an intensive differential rent is due for the last piece of land under cultivation, then rent enters into the price of corn. This is proved in the next section by means of a simple example based on the same data used here.

3. A different argument

Let us consider an example with only two commodities: an agricultural product, say “corn”, and an industrial product, say “steel”.

Let us assume that capital in each sector consists exclusively of wages paid in advance for the period of production, which is one year in both sectors. As a result, capital and profits can clearly be seen to be proportional to the employment of labour in each sector. If rents do not enter into the price of commodities, as Ricardo claims, they should therefore be exchanged at a relative price equal to the ratio of the amounts of labour they embody.

An experiment in comparative statics may help to clarify this point. We shall consider two cases. In the first, only one method of corn production is known, namely $\beta$, the more capital and labour-intensive of the two methods in Ricardo’s example. In the second, two methods of corn production – $\alpha$ and $\beta$ – are known and can coexist on land of uniform quality, as in Ricardo. In both cases, the total quantities to be produced are fixed and equal to 740 quarters of corn and 25 tons of steel. The surface area of uniform land available is also fixed and equal to 6 acres.

In the first case, 185 quarters of corn are produced with 20 units of labour per acre of land, while one ton of steel is produced with 2 units of labour and no employment of land. As a result, only 4 of the 6 acres of land are required to obtain 740 quarters of corn and 25 tons of steel. In other words, the availability of land is in excess of production requirements and the differential rent is therefore zero.

If we use $w$ and $r$ respectively to denote the wage rate and the profit rate, the prices of corn and steel, $p_c$ and $p_s$, must therefore be such that:

\[
\begin{align*}
20 w (1 + r) &= 185 p_c \quad (1) \\
2 w (1 + r) &= p_s \quad (2)
\end{align*}
\]

We assume $w = £100$, so that the capital employed on an acre of land with method $\beta$ is £2000, as in Ricardo’s example.
The price of corn in terms of steel is 2/37 and therefore equal to the ratio of the quantities of labour embodied, which are 20/185 (= 4/37) in each quarter of corn and 2 in each ton of steel.

In the second case, method \( \alpha \) is also known and can be used to produce 100 quarters of corn with 10 units of labour per acre of land.

If the rent rate were at the zero level, this method would be cheaper because less labour is employed per quarter of corn. Due to the scarcity of land, however, the use of this method alone would allow the production of only 600 quarters of corn as against the 740 required by the market. Methods \( \alpha \) and \( \beta \) must therefore necessarily coexist and the rent must rise in order to make the unit costs identical within the two methods.\(^8\)

When the rent reaches this level, the 6 acres of land will be entirely cultivated: 1.65 acres (1 acre, 2 roods and 24 perches) with the old method (\( \beta \)) and 4.35 acres (4 acres, 1 rood and 16 perches) with the new one (\( \alpha \)).

In this case, the prices \( p_c \) and \( p_s \) and the rent rate \( \rho \) must be such that:\(^9\):

\[
20 w (1 + r) + \rho = 185 p_c \tag{3}
\]
\[
10 w (1 + r) + \rho = 100 p_c \tag{4}
\]
\[
2 w (1 + r) = p_s \tag{5}
\]

By solving system (3)-(5), we obtain the result that the rent rate \( \rho \) is equal to the value of 15 quarters of corn, as in Ricardo’s example, and the relative price of corn is now 2/34, which means a rise in the price of corn respect the previous case.

Moreover, the quantity of labour embodied in a quarter of corn produced with the least favourable method\(^10\) remains unchanged, so that the relative quantities of labour embodied in corn and steel are 2/37 in this case as well. The price of corn therefore rises with no change in the

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\(^8\) Until 740 quarters of corn are produced, the price of corn is above its cost of production with method \( \alpha \) and agricultural producers therefore obtain extra profits. Since the six acres of land are entirely cultivated, the classical mechanism of capitalistic competition allows landowners to appropriate the extra profits in the form of rents. The rising rents tend to increase the unit cost of production with method \( \alpha \) with respect to the unit cost with method \( \beta \). This process goes on until the unit cost of production for corn is the same with both methods.

\(^9\) As regards the equations determining intensive differential rent simultaneously to prices, we consider the analysis put forward by Sraffa (1960, pp. 75, 76 - §§ 87, 88) and later put into formal terms by various authors, in particular, Montani (1975, pp. 80 – 83), Kurz (1978, pp. 26 – 34) and Kurz and Salvadori (1995, p. 289). Equations (3)-(5) are the same as though found there but written for the simple example addressed. It is also worth observing that although Sraffa and the scholars mentioned above have a correct theory of intensive differential rent, they fail to note that it can be used to refute Ricardo’s conclusions about rent and the labour theory of value (for a discussion of the differences between Sraffa’s and Ricardo’s theory of rent see also Fratini, 2008).

\(^10\) As Ricardo pointed out, “that corn which is produced by the greatest quantity of labour is the regulator of the price of corn”, i.e. in order to determine the relative value of the two commodities in labour embodied we must consider only the most labour intensive method of production of corn, which is, in our case, method \( \beta \).
relative amount of labour embodied.\textsuperscript{11} Rent does enter into the price of corn and causes an increase with respect to the level corresponding to the relative quantities of labour embodied.

4. Conclusion

Contrary to what Ricardo claims and many economists have since believed, if the last piece of land under cultivation receives intensive differential rent, it enters into the price of corn or agricultural products in general.

This has been proved by means of Ricardo’s own example. In particular, even assuming that capital in each sector consists exclusively of wages anticipated for one year, we have shown that in the presence of intensive differential rent, the relative price of commodities cannot be equal to the ratio of the quantity of labour they embody.

The argument Ricardo used in replying to the objections of Malthus and Say is therefore incorrect. They were right to argue that Ricardo’s idea of rent not entering into the price of commodities is based on the presence of cultivated lands which pay no rent, a situation to be found only in very particular cases, such as a newly colonised country. But where all cultivated land pays a rent, which seems to be the general case, rent is an element of the cost and the price of agricultural commodities.

This result also rehabilitates an assertion made by Adam Smith and criticised by Ricardo. In countries where all the land is owned and all cultivated land pays a rent, the “original rule” is in fact altered in that the price of commodities is no longer determined by the relative quantity of labour embodied. In conclusion, therefore:

\textsuperscript{11} It is worth observing that, as Sraffa remarks (1960, 75), the relative price of corn in terms of steel can be determined by a system in which land does not appear. This possibility has been studied in general and formal terms by Guichard (1982).

For the simple case addressed here, we can subtract equation (4) from equation (3) to obtain:

\begin{equation}
10w (1 + r) = 85 pc
\end{equation}

and use equations (5) and (6) in order to determine the relative price of the two commodities and the rate of profit.

This procedure is certainly correct in formal terms and, contrary to what one might think, makes no difference to our argument. The relative price of corn determined by solving equations (5) and (6) must be 2/34 here too and the ratio between the quantities of labour embodied in the two commodities is still 2/37.

Great care must in fact be taken not to infer from equation (6) that 85 quarters of corn embodies 10 unit of labour. This would be true if 85 quarters were the gross product obtained by the employment of 10 workers with the least favourable method, i.e. if equation (6) had exactly the same economic meaning as equation (3). But this is not so. The 85 quarters of corn in equation (6) are not the gross product of 10 workers (which is, on the contrary, 100 quarters with method $\alpha$ and 92.5 with method $\beta$), but a physical amount of gross profits. More precisely, equation (6) tells us only that a capital of 10w employed in agriculture must earn the value of 85 quarters of corn as gross profits regardless of the method used.

There is thus no contradiction between our result and the possibility of eliminating land from the system referred to by Sraffa and Guichard.
“As soon as the land of any country has all become private property, the landlords, like all other men, love to reap where they never sowed, and demand a rent even for its natural produce. The wood of the forest, the grass of the field, and all the natural fruits of the earth, which, when land was in common, cost the labourer only the trouble of gathering them, come, even to him, to have an additional price fixed upon them. He must then pay for the licence to gather them; and must give up to the landlord a portion of what his labour either collects or produces. This portion, or, what comes to the same thing, the price of this portion, constitutes the rent of land, and in the price of the greater part of commodities makes a third component part.” (Smith 1976: vol. 2, p. 67 – I.vi.8)

Appendix. Another example

The present paper, it is now clear, is aimed at showing that intensive differential rent enters into the price of agricultural products. This result, in the main part of the paper, has been reached assuming that capital consists exclusively of wages anticipated at the beginning of the year and proving that, even under this assumption, commodities do not exchange at a ratio determined by the embodied quantities of labour when intensive differential rent is paid for the use of land. Here we will get the same result by a different argument: we will consider a case with a non-basic agricultural commodity and intensive rent – a case which we find, for example, both in Montani (1975) and in Kurz (1978) – in order to show that the latter enters into the price of the former.

In particular, following Montani’s example¹², let us assume there are two commodities: an industrial and basic commodity, named commodity “a”, which is also the numeraire; and an agricultural and non-basic commodity, named commodity “z”.

As for technology, there is a method for the production of “a”, and two different methods for the production of “z”, named “α” and “β”. In symbols:

\[
\begin{align*}
    a_a & \text{ units of “a” } \& \ell_a & \text{ units of labour } \rightarrow 1 \text{ unit of “a”} \\
    a_z^\alpha & \text{ units of “a” } \& \ell_z^\alpha & \text{ units of labour } \& \lambda^\alpha & \text{ units of land } \rightarrow 1 \text{ unit of “z”} \\
    a_z^\beta & \text{ units of “a” } \& \ell_z^\beta & \text{ units of labour } \& \lambda^\beta & \text{ units of land } \rightarrow 1 \text{ unit of “z”}
\end{align*}
\]

with \( \lambda^\alpha > \lambda^\beta \), i.e. method “β” gives a greater output per unit of land.

If both the methods have to be in use in order to fully supply the demand for “z”, then, given a post-factum wage rate \( w \) (or a profit rate \( r \)), the profit rate \( r \) (or the wage rate \( w \)), the rent rate \( \rho \)

¹² A similar case can be found in Kurz (1978), pp. 26-28.
and the price of “z” \( p_z \) in terms of “a” are determine, according to Sraffa’s theory, by solving the system:

\[
\begin{align*}
    a_a (1 + r) + \ell a w &= 1 \quad (A.1) \\
    a^\alpha_z (1 + r) + \ell z^\alpha w + \lambda^\alpha \rho &= p_z \quad (A.2) \\
    a^\beta_z (1 + r) + \ell z^\beta w + \lambda^\beta \rho &= p_z \quad (A.3)
\end{align*}
\]

Therefore, given the wage rate level \( w^* \), and determined the corresponding profit rate \( r^* \) by equation (A.1), we can use equations (A.2) and (A.3) – as both Montani and Kurz did – in order to obtain the “price-rent relation” implied by each of the two methods. More precisely, let us denote by \( c^\alpha \) and \( c^\beta \) the unit cost of production\(^{13} \) of “z” using respectively method \( \alpha \) and \( \beta \), then by equations (A.2) and (A.3) we have:

\[
\begin{align*}
    a^\alpha_z (1 + r^*) + \ell z^\alpha w^* + \lambda^\alpha \rho &= c^\alpha \quad (A.4) \\
    a^\beta_z (1 + r^*) + \ell z^\beta w^* + \lambda^\beta \rho &= c^\beta \quad (A.5)
\end{align*}
\]

Leaving aside the details – for which we refer the reader to Montani’s and Kurz’s mentioned articles – equations (A.4) and (A.5) imply the functions plotted in fig. 1.

Looking at fig. 1, it appears clearly that intensive differential rent enters into the unit cost with both the methods\(^ {14} \), although in different amount, and then into the price of “z” in terms of “a”. The unit costs \( c^\alpha \) and \( c^\beta \) are both increasing functions of the rent rate but being \( \lambda^\alpha > \lambda^\beta \), \( c^\alpha \) grows faster than \( c^\beta \) when the rent rate increases. There is, therefore, a positive rent rate \( \rho \) which allows \( c^\alpha \) to catch up with \( c^\beta \), and, in so doing, allows the two production methods to be simultaneously in use. And this rent rate level makes the price \( p_z \) greater than the costs for wages and gross profits both with method \( \alpha \) – i.e. the length of segment OR in fig. 1 – and with method \( \beta \) – the length of segment OS.

\(^{13}\) It is worth noting that Montani (1975, p. 81) – as well as Kurz (1978, p. 27) – does not include the intensive differential rent into the unit cost of production of commodity “z”. Then, he is considering a notion of cost which excludes entrepreneur’s revenues and is assuming (implicitly) that landowners organize the production; in this case, in fact, rents are viewed as a residuum. But adopting the notion of full-cost, as we are doing here, rents must be included into the cost of production of commodity “z” (even in the case of landowners as entrepreneurs).

\(^{14}\) Our fig. 1 is identical to fig. 4 in Montani (1975, p. 81) and to fig. 3 in Kurz (1978, p. 28), however they seem to use it only in order to show the existence, under certain conditions, of economically meaningful solutions for the equations (A.1), (A.2) and (A.3). Here the same figure is used with a different aim.
In conclusion, the intensive differential rent $\rho$ is entered into the price $p_z$. Here we can easily say that because “z” is a non-basic commodity, and therefore the part of its cost formed by wages and gross profits is unaffected by changes in the rent rate. In this case rents are clearly in addition to this part. Something similar happens in the case of Ricardo treated above, where capital was assumed to consist of anticipated wages only (and therefore no basic commodity exists). In more general case, the linkage between intensive differential rent and the price of agricultural products is decidedly more complex.

References


