Forecasting methods: a comparative analysis

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FORECASTING METHODS: A COMPARATIVE ANALYSIS

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ABSTRACT

Forecasting is an important tool for management, planning and administration in various fields. In this paper forecasting performance of different methods is considered using time series data of Pakistan's export to United States and money supply. It is found that, like other studies of this nature, no single forecasting method provides better forecast for both the series. The techniques considered are ARIMA, Regression Analysis, Vector Autoregression (VAR), Error Correction Model (ECM) and ARCH/GARCH models.

1. INTRODUCTION

Forecasting is a probabilistic estimate or a description of a future value or condition, which includes a mean, range and probability estimate of that range. There is a considerable literature on forecasting in business and economics. Some are aimed at forecasting a particular variable of interest e.g. stock prices, money demand, exchange rates etc. for example Chan and Lee (1997), Kumar (1992), Bhawnani and Kadiyala (1997), Bleaney (1998). Others focus on a comparison of different models or techniques of forecasting, LeSage (1990), Stock and Watson (1996), Fair (1973). This study is of later style. A distinguishing feature of this study is the presentation of forecast confidence intervals besides mean forecasting. The models considered are univariate as well as multivariate. To broaden the scope, the time series considered are quarterly as well as annual. The annual data (1972-1999) for forecasting State Bank's money supply (M2) is used. Forecasting this variable is important for potential investors since money supply affects interest rate and consequently investment. It also has an impact on prices. An estimate of this variable is important for Central Bank itself so that to achieve its target of stabilizing the economy, some alteration in historical pattern of money supply could be made. Secondly quarterly data (1988:1 –1999:4) for forecasting Pakistan's export to U.S., a major trading partner of Pakistan, is considered. The export function involves a variable of economic activity of trading partner (U.S. in this case for which the data are available) instead of the home country. This variable is chosen keeping in view the availability of quarterly data. Since macroeconomic models usually involve a variable of real economic activity such as GDP, or its component. Unfortunately quarterly data for Pakistan on these real variables are not available. The data are collected from various issues of State Bank's monthly bulletin and IMF's International Financial Statistics. The results of almost all studies of this nature indicate that no single model camouflage as the best for out-sample forecasting of different series. This study also provides such an evidence.

Export Function:

\[ E = f (Y_A, R_P, V) \]  

Where E: Pakistan's export to U.S (millions of U.S $), YA: United State's GDP (Billions of U.S.$ seasonally adjusted), RP: Relative prices computed as dividing unit value Index of export to U.S.
consumer price index. V: Exchange rate volatility computed as in Ariz and Shwiff (1998) as an eight quarter moving standard deviation i.e.

\[ V_t = \left[ \sum_{i=0}^{7} (R_{t-i} - \hat{R}_{t-i})^2 / 8 \right]^{1/2} \]  

(1.2)

Where \( R_t \) is the exchange rate (Rs per U.S.$) and \( \hat{R}_t \) is its forecasted value (ARIMA model has been used to forecast it)

Expected signs of the coefficients of the explanatory variables are as follows:

\[ \frac{\partial E}{\partial Y} > 0, \quad \frac{\partial E}{\partial P} < 0, \quad \frac{\partial E}{\partial V} < 0. \]

Money Demand function:

\[ M = f (Y, I) \]  

(1.3)


Expected signs are as follows:

\[ \frac{\partial M}{\partial Y} > 0, \quad \frac{\partial M}{\partial I} < 0. \]

Following this introduction, section 2 describes forecasting models considered in this study. Outsample forecasting is compared in section 3 and section 5 provides conclusion and comments.

2. FORECASTING MODELS

a) ARIMA One of the most popular univariate forecasting model proposed by Box and Jenkins (1970). For a stationary time series \( y_t \), an ARMA(p,q) model is expressed as

\[ y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \ldots + \theta_q \epsilon_{t-q} \]  

(2.1)

where \( \epsilon_t \) is a white noise disturbance term normally and independently distributed with mean 0 and variance \( \sigma^2 \).

This model can be expressed as weighted sum of disturbances \( \epsilon_t \) as

\[ y_t = \epsilon_t + \psi_1 \epsilon_{t-1} + \psi_2 \epsilon_{t-2} + \ldots + \psi_k \epsilon_{t-k} + \ldots \]  

(2.2)

Where \( \psi \) weights are functions of the modal parameters \( \phi \)'s and \( \theta \)'s.

An h-step ahead forecast error variance FEV (h) for \( y \) is given by

\[ \text{FEV} (h) = (1 + \psi_1^2 + \psi_2^2 + \ldots + \psi_h^2) \sigma^2 \]  

(2.3)
A 95% forecast confidence interval for h-step ahead forecast is given by
\[ \tilde{y}_{t+h} \pm 1.96\sqrt{FEV(h)} \] (2.4)

b) **Regression Analysis** A general linear regression model is given by:
\[ y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + \ldots + \beta_k x_{kt} + \epsilon_t \] (2.5)
Which in matrix form is given by
\[ y = X\beta + \epsilon \]
Where the dimensions of \( y \) is \( n \times 1 \), that of \( X \) is \( n \times k \), of \( \beta \) is \( k \times 1 \) and \( \epsilon \) is an \( n \times 1 \) vector of white noise errors with covariance matrix \( \sum = \sigma^2 I \)

Let \( x_0 = [x_{01}, x_{02}, \ldots, x_{0k}]' \) be the vector of values for which forecast is required, then \( \hat{y}_{0|x_0} = x_0' \hat{\beta} \).
Forecast error variance \( FEV(h) \) is given by
\[ FEV(h) = \sigma^2 [1 + x_0' (X'X)^{-1} x_0] \] (2.6)

c) **ARCH/GARCH model** The autoregressive conditional heteroskedasticity (ARCH) model introduced by Engle (1982) is extensively used to model financial time series which are believed to have varying conditional variance or volatility e.g. stock prices, interest rate, inflation etc. In this method, mean and variance are modeled simultaneously. An ARCH model of order \( q \) is expressed as
\[ y_t = x_t' + \epsilon_t \]
Where \( \epsilon_t \sim N(0, \sigma_t^2) \) and
\[ \sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 \epsilon_{t-2}^2 + \ldots + \alpha_q \epsilon_{t-q}^2 \] (2.7)
ARCH model were generalized by Bollerslev (1986) as GARCH (Generalized ARCH), in which the conditional variance depends on past value of itself. The conditional variance in a GARCH (p,q) model is expressed as
\[ \sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 \epsilon_{t-2}^2 + \ldots + \alpha_q \epsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2 + \ldots + \beta_p \sigma_{t-p}^2 \] (2.8)
The most widely used of these models is GARCH (1,1) for which the conditional variance is given by
\[ \sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \] (2.9)
The forecast error variance in GARCH models is given by \( \sigma_t^2 \)

d) **Vector Autoregression (VAR)** VAR introduced by Sim (1980), is used to model interrelated time series simultaneously. This model, unlike structural multi-equation econometric models, relies little on economic theory. Each variable in the model is expressed as function of the lagged values of all the endogenous variables in the system. A VAR of order \( p \) is given by
\[ y_t = A_1 y_{t-1} + A_2 y_{t-2} + \ldots + A_p y_{t-p} + \varepsilon_t \] (2.10)

Where \( \varepsilon_t \sim N(0, \Omega) \) is a vector of white noise errors with covariance matrix \( \Omega \). The errors are assumed to be serially independent in each equation but they may be contemporaneously correlated across equations. \( A_i \) s (i = 1, 2, ..., p) are the matrices of the coefficients to be estimated. \( y_i \) is a vector endogenous variables. For our export model \( y_i = [E, YA, RP, V]' \). A VAR model, unlike structural multi equation modes, does not require the knowledge of explanatory variable for forecast period. An h-step ahead forecast of \( y_i \) is

\[ \hat{y}_{t+h} = A_1 \hat{y}_{t+h-1} + A_2 \hat{y}_{t+h-2} + \ldots + A_p \hat{y}_{t+h-p} \] (2.11)

Where \( \hat{y}_{t+h-1}, \ldots, \hat{y}_{t+1} \) are forecasted using similar scheme. For a VAR (1), we have

\[ \hat{y}_{t+h} = A \hat{y}_{t+h-1} \] and forecast error variance (as given in Clement and Hendry (1993)) is

\[ FEV(h) = \sum_{j=0}^{h-1} A^j \Omega (A^j)' \]. With normality assumption, we can derive simple expression for the forecast intervals for individual time series by considering the diagonal elements of \( FEV(h) \) matrix.

e) **Error Correction Model (ECM)** It is widespread in modern econometric literature that a regression involving non-stationary time series variables may be spurious or misspecified unless the variables are Co-integrated. The time series \( x_1t \) and \( x_2t \), each integrated of order 1, are said to be co-integrated if any linear combination if these e.g. \( z_t = \alpha_1 x_1t + \alpha_2 x_2t \) is stationary. The concept of co-integration in statistics is equivalent to stable long run relationship in economics. The short run disequilibrium can be modeled as an error correction model.

To get the idea lets consider a time series regression

\[ y_t = \alpha + \beta x_t + \varepsilon_t \] (2.12)

Where \( y_t \) is the dependent variable and \( x_t \) is a vector of independent variables. If \( y_t \) and \( x_t \) are in equilibrium, then the error \( y_t - \alpha - \beta x_t \) will equal zero. However in disequilibrium it will be non-zero. This quantity measures the extent of disequilibrium between \( y_t \) and \( x_t \) and hence is called disequilibrium error. In disequilibrium \( y_t \) can be assumed to be related with \( x_t \) and the lagged values of \( y_t \) and \( x_t \), one typical form of which is

\[ y_t = \gamma + \delta_0 x_t + \delta_1 x_{t-1} + \pi y_{t-1} + \varepsilon_t \] (2.13)

Subtracting \( y_{t-1} \) from each side of eq (2.13) above and reparametezing we get

\[ \Delta y_t = \delta_0 \Delta x_t - \mu (y_{t-1} - \alpha - \beta x_{t-1}) + \varepsilon_t \] (2.14)

Where the parameter \( \mu \) depends on parameters in eq (2). The term in bracket is called the error correction term, which incorporates past period's disequilibrium. Eq (2.14) is an example of error correction model. The estimation of ECM involves two steps (Engle and Granger (1987)). First the
long run equilibrium relationship between non-stationary variables is estimated. In our present case this would be the regression \( E = f(\text{YA}, \text{RP}, \text{V}) \) for export function. In the second step ECM model eq (2.14) is estimated using residuals lagged one period from the long run regression. Forecasting of \( y \) can then be performed using the ECM model and the forecast error variance is given as eq (2.6).

3- FORECAST COMPARISON

The comparison of forecasting models is based on Root Mean Square Error (RMS)

\[
RMS = \sqrt{\frac{1}{h} \sum_{i=1}^{h} (y_{t+i} - \hat{y}_{t+i})^2}
\]  

(3.1)

and Theil's Inequality Coefficient (U)

\[
U = \sqrt{\frac{1}{h} \sum_{i=1}^{h} (y_{t+i} - \hat{y}_{t+i})^2} / \sqrt{\frac{1}{h} \sum_{i=1}^{h} y_{t+i}^2 + \frac{1}{h} \sum_{i=1}^{h} \hat{y}_{t+i}^2}
\]  

(3.2)

The former depends on the unit of measurement of the variable to be forecasted and later is unit free lying between 0 and 1 with the forecast getting better as the \( U \) is closer to 0.

These measures are widely used for out-sample forecasting e.g. in Chan and Lee (1997) and Kumar (1992) among others.

Table-I: Forecast Evaluation for Export and Money Supply Series

<table>
<thead>
<tr>
<th>Models</th>
<th>Export</th>
<th>Money Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMS</td>
<td>Theil Coefficient</td>
</tr>
<tr>
<td>Regression</td>
<td>39.975</td>
<td>0.0417</td>
</tr>
<tr>
<td>ARIMA</td>
<td>28.527</td>
<td>0.0304</td>
</tr>
<tr>
<td>ARCH</td>
<td>36.330</td>
<td>0.0381</td>
</tr>
<tr>
<td>VAR</td>
<td>35.421</td>
<td>0.0367</td>
</tr>
<tr>
<td>ECM</td>
<td>96.092</td>
<td>0.0948</td>
</tr>
</tbody>
</table>

Table-I presents RMS and U- measures for the models forecasted. It can be seen that ARIMA model outperforms the others for exports and a GARCH (1,1) model appears to be the best for money supply. These results are consistent with other studies, for example Delurgio(1998) and Bleaney(1998), which indicate that a univariate model usually perform better for monthly or quarterly data and annual data usually are best forecasted by a form of regression model. The ARCH model requires Knowledge of future values of the explanatory variables for mean estimation, however main use of these modes has been for forecasting volatility which, fortunately, does not require forecast of explanatory variables.
The conclusion is further supported by fig. No 1 to 4, which display graphically the realized and forecast from different models and approximately 95% forecast confidence intervals.

Table-II: Augmented Dickey Fuller Test of Unit Root

<table>
<thead>
<tr>
<th>Series</th>
<th>Export Model</th>
<th>Money Supply Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E</td>
<td>YA</td>
</tr>
<tr>
<td>Level</td>
<td>-4.807</td>
<td>-0.747</td>
</tr>
<tr>
<td>1st Difference</td>
<td>-7.46**</td>
<td>-5.413**</td>
</tr>
</tbody>
</table>

(* indicates significant at 5% level  ** indicates significance at 1% level)

It appears that all the series are stationary at the first difference

![Fig. 1 Realized And Forecasted Export](image)

![Fig. 2 Realized And Forecasted Money Supply](image)
Fig. 3: 95% Forecast Intervals From Different Models Of Export

Regression

ARIMA

VAR

ECM

ARCH
Fig. 4: 95% Forecast Intervals From Different Models Of Money Supply
4. CONCLUSIONS

A comparison of forecasting performance of different time series econometric models is considered using data from Pakistan's macroeconomy. Like previous studies no single model appeared the best for both the variables forecasted. The message revealed by the study is that to forecast a particular variable one should not rely on a single forecasting method rather performance of different competing models should be checked, then forecast should be performed from the best models from the comparison.

REFERENCES