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June 2006

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MPRA Paper No. 24019, posted 21 Jul 2010 12:45 UTC

# A Critique of the ‘New Approach’ to the Transformation Problem and a Proposal\*

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## ABSTRACT

This paper shows that the ‘New Approach’ to the ‘problem of transforming values into prices’, first, is subjected to a crucial logical inconsistency and second, is not in a position to deal with the heterogeneous labour case. Thus, the paper proposes an approach, which overcomes these problems and concludes that values of commodities are their actual prices.

## 1. Introduction

In contrast with the *traditional* approach to the so-called ‘transformation problem’, in the context of which the values of commodities are defined as the quantities of labour ‘embodied’, the ‘New Approach or Interpretation’<sup>1</sup> (NA hereafter), which has been developed independently by Duménil (1980) and Foley (1982), does not disregard the lessons of the Sraffian theory and, at the same time, is based on the notion of *abstract social labour*. Thus, according to the NA, the value of ‘labour power’ and surplus product is determined by their *actual* prices and, specifically, is equal to the product of their actual price times a coefficient known as the ‘value of money’.

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\* *Indian Development Review: An International Journal of Development Economics*, Vol. 4, No. 1, June 2006, pp. 23-37 (reprinted in: T. Mariolis and L. Tsoulfidis (eds) (2006), *Distribution, Development and Prices. Critical Perspectives*, New Delhi, Serials Publications).

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In this paper, we shall prove the following three propositions:<sup>2</sup>

**P<sub>1</sub>.** The NA, first, is subjected to a crucial *logical* inconsistency and second, is not in a position to deal with the *heterogeneous* labour case.

**P<sub>2</sub>.** The NA can remove this inconsistency. However, even it is rid of it, then, first, it is economically insignificant and second, it is still unable to deal with the heterogeneous labour case. All this shows that in the context of the NA, the notion of abstract social labour has not been correctly formulated.

**P<sub>3</sub>.** If the notion of abstract social labour is correctly formulated, then it is possible to develop a consistent and general approach to the ‘transformation problem’, which results in the conclusion that values of commodities are their *actual* prices.

Precisely because our proposed approach, first, takes into account the lessons of the Sraffian theory, and, at the same time, is based on the notion of abstract social labour, second, is general enough so as the NA when corrected for its inconsistencies becomes one of its infinite special cases and third, covers the heterogeneous labour case, we *may* call it the ‘Completed New Approach’ (CNA hereafter; an initial sketch of which is in Mariolis (1998)).

The remainder of the paper is organized as follows. Section 2 briefly presents the NA. Section 3 critically evaluates the NA. Section 4 elaborates the CNA. Section 5 determines the relationships between the CNA and the NA. Finally, Section 6 concludes.

## **2. The New Approach**

The NA does not presuppose (it is not necessary for it to presuppose), the existence of linear techniques of single production and prices of production.<sup>3</sup> However, in order to

avoid needlessly burdening the discussion, we shall present the NA under these presuppositions (for a presentation of the NA, see also Glick and Ehbar (1987), Mohun (1994), Saad-Filho (1996), Foley (1997)).

Consider a system of single production that uses the linear, indecomposable and *viable* technique  $[A, a]$ , where  $A$  denotes the  $n \times n$  matrix of input coefficients and  $a$  denotes the  $1 \times n$  vector of inputs of direct *homogeneous* labour, and in which the prices are *production* prices (see, e.g., Kurz and Salvadori, (1995, ch. 4)). Assume that the magnitudes  $p \equiv [p_j]$ , the  $1 \times n$  vector of prices,  $w$ , the money wage rate,  $X$ , the  $n \times 1$  vector of activity levels, are given and economically significant, whilst the composition and level of the real wage rate are unknown *a priori* (for the justification of this choice, see Duménil (1980, ch.7 and pp. 121-5), Foley (1982, p. 43) and Mohun, (1994, pp.400-2)). On the basis of the above, we can calculate the  $1 \times n$  vector  $\omega \equiv [\omega_j]$  ( $> 0$ ) of the quantities of labour ‘embodied’ in the different commodities, the  $1 \times n$  vector  $Y$  of the net output, the total profits,  $P$ ,<sup>4</sup> and the following magnitude  $\lambda_m$ :

$$\lambda_m \equiv (aX / pY) \equiv (\omega Y / pY) \quad (1)$$

which is called ‘value of money’.<sup>5</sup> If, now, the unit value of ‘labour power’,  $V$ , is *defined* as the product of the value of money times the money wage rate, i.e.,

$$V \equiv w\lambda_m \quad (2)$$

then for the total ‘surplus value’,  $S$ , which, is *by definition* equal to  $aX(1 - V)$ , the following will hold:

$$S \equiv aX(1 - w\lambda_m) \quad (3)$$

Consequently, for the total profits, the following holds:

$$S \equiv P\lambda_m \quad (4)$$

Finally, if we rearrange (1) as follows

$$aX (\equiv \omega Y) \equiv pY\lambda_m \quad (1a)$$

then it is clear that (1a) and (4) express the validity of a modified (with respect to the net output) ‘Marxian double equality’: For all  $p$ ,  $w$ ,  $X$ , the value of the net output (the surplus value) is equal to the product of the value of money times the price of the net output (times the total profits).<sup>6</sup>

It is clear that the crucial point of the NA is the definition introduced by (2). Thus, it could possibly be considered that by virtue of (2), the following *premise* is implicitly introduced: The quantity of labour ‘embodied’ in the real wage equals the share of wages in national income. That is, it could be considered that although the real wage rate is unknown *a priori*, the advocates of the NA argue the following: The quantity of labour which has been ‘embodied’ in all those bundles of commodities, which workers can purchase with the money wage rate, equals the share of wages in national income.

*If* this were the case, then the NA would be correct only by way of exception, i.e., when (i) the real wages and the net output have the same composition; or (ii) the real wages and the net output are associated with each other just as the Standard commodity of Sraffa (1960, ch. 6) is associated with the Standard commodities of Miyao (1977); or (iii) the commodity prices are proportional to the quantities of labour ‘embodied’; or (iv)  $w = 0$ ; or (v) by chance, that is, for certain, perhaps, positive values of  $w$ . In reality, however, the NA is *not* founded on the aforementioned premise (see also Mohun (1994, pp. 398-9)) and, to be precise, the advocates of the NA maintain something completely different. Thus, Foley (1982, p. 42) *interprets* (2) as follows: ‘The basic insight of the labor theory of value is its claim that value forms, money, commodities, and so on, are expressions of *abstract*

*social labour*. Thus in any transaction involving value, what is changing hands is control over some part of total abstract social labor time. *The value of labor power, in this perspective, is the fraction of the total abstract social labor time claimed by workers in the form of the wage*. This is an exchange of abstract labor in the form of money for the particular labor power a worker has to offer on the market. If we follow this path, we would *interpret* the value of labor power in general as the money wage multiplied by the value of money...’ [emphasis added], whilst at the same time, at other points he clarifies that: ‘A unit of money [...] can be thought of as a claim to a certain amount of the abstract social labor expended in the economy ... (p. 37). The advantage of interpreting the value of money as the ratio of aggregate labor time to aggregate money value added is that the sum of the value gained and lost by all the producers in exchange will be zero. In other words, this interpretation of the value of money corresponds to the idea that value is created in production but conserved in exchange. (p. 41). [Thus – T.M.] we retain at the global level the relation between money and embodied labor which is central to the idea that money is a form of value and that the substance of value is abstract social labor. (p. 41)’. Finally, Mohun (1994, pp. 404-5) sums up the NA as follows: ‘It is an aggregative theory in which total labour performed defines the value of net output; the value of net output and aggregate value added in money terms defines the value of money; and the value of money and the money wage define the value of labour-power. *A price system is then a method of distributing aggregate labour-time expended across individual commodities*. Hence prices are always [...] representations or forms of value, of abstract labour. [...] Market processes of exchange commensurate commodities, and hence *a posteriori* commensurate the labour-times objectified in their production.

Abstraction from concrete labours occurs as heterogeneous commodities are valued in units of homogeneous money. Value is then labour-time as it appears as money.’ (emphasis added; see also Duménil (1984, pp. 340-3)).

From the above, we may deduce that within the framework of the NA, the following hold:

1. The magnitude  $aX \equiv \omega Y$  is considered as the total quantity of *abstract social* labour that corresponds to a given system  $[A, a, X]$ . Consequently, the one unit of labour that has been ‘embodied’ in the net output,  $Y$ , is *chosen* as the *unit of measurement* of *abstract social* labour.
2. The dimension of the magnitude  $\lambda_m$  is not only: units of ‘embodied’ labour per unit of money, but also: units of abstract social labour per unit of money. For precisely this reason,  $\lambda_m$  is defined ‘as the coefficient which enables a translation between prices into labour-times’ (Mohun (1994, p. 402)), whilst its level expresses how many units of abstract social labour are represented by one unit of money (see also Foley (1982, p. 41)).
3. The magnitude  $V$  defined by (2) has the dimension: units of abstract social labour per unit of labour power expended. It is, therefore, a magnitude, which differs entirely from the unit value of labour power, as this is defined *traditionally*. In addition, the magnitude  $S$  (see (3) and (4)) has the dimension: units of abstract social labour per unit of surplus product. It is, therefore, a magnitude, which differs entirely from the quantity of labour ‘embodied’ in the surplus product, i.e., from the surplus value, as this is defined *traditionally*.
4. Given  $[A, a]$  and  $X$ , the magnitudes  $V$  and  $S$  depend on  $p$  and  $w$ . However, the sum of the surplus value and the value of total labour power is *always* equal to the quantity

of total abstract social labour.

**5.** The value of the net output, of labour power and of the surplus product is measured in units of abstract social labour. When  $[A, a]$  and  $X$  remain constant, and  $p, w$  change (not proportionally), the value of the net output expressed in units of abstract social labour does not change (because it is equal to  $aX$ ), but the value of labour power and of the surplus product change, because to the said commodities there now correspond different proportions of the total abstract social labour of the system ('The amount of labour time expended in a given period is the value of a given bundle of commodities. *A price system expresses the reallocation of these hours of productive labor time to this bundle of commodities.* The total amount of labor time expended in a period being the value of the net output of the period, this quantity of labor must be reallocated to this net output. Thus, Marx's first equalities (sum of value = sum of prices), must be defined for the *net output of the period* and not the gross output as in the old solution.' (Duménil (1984, pp. 341-2); see also Mohun (2000)).

It is precisely these points, which constitute the core of the NA and what remains is to check its consistency and significance.<sup>7</sup>

### **3. Critique of the New Approach**

We have so far shown that the NA is founded on the notion of abstract social labour, which means that in principle it transcends all the well-known critical evaluations that have been made of the *traditional* approach. We shall now show, however, that the NA is neither consistent **(3.1)** nor general **(3.2)**.<sup>8</sup>

**3.1.** Within the framework of the NA, the values of three, and only three, commodities (i.e., the value of the net output, of labour power and of the surplus product) are

defined/expressed in units of abstract social labour, whilst the values of all the other commodities are defined/expressed in units of ‘embodied’ labour.<sup>9</sup> In other words, the NA (and moreover, without explaining why) defines one and the same magnitude, value, in disagreement (in the case of the aforementioned ‘special’ commodities) and, at the same time, in agreement (in the case of any other commodity) with the *traditional* conception. This contradiction is of course not without repercussions, for the question: ‘to what is the value of the surplus product equal?’ accepts, in the context of the NA, two mutually excluding answers: (i) Because the surplus product consists of  $n$  kinds of different commodities, it follows that its value is equal to the aggregate of quantities of labour that have been ‘embodied’ in those commodities. (ii) Because the total abstract social labour is equal to  $aX$  and because the value of total labour power is equal to  $w\lambda_m aX$ , it follows that the value of the surplus product is equal to  $aX(1 - w\lambda_m)$ . So, at this point there is a crucial logical inconsistency in the NA.

**3.2.** The NA identifies total abstract social labour with the quantity  $aX$ . Subsequently, it considers that this quantity, on the one hand *appears* as  $pY$  and, on the other hand, is *redistributed* among the commodity labour power and the surplus product according to the ratio of the price of each of these commodities to the price of the net output of the system (i.e.,  $V \equiv (w/pY)aX$ ,  $S \equiv (P/pY)aX$ ). A ‘rule’ of redistribution, which apparently entails the validity of the following relation:

$$VaX + S \equiv aX \quad (5)$$

So, in accordance with the NA, it must be said that this relation expresses a ‘conservation principle’,<sup>10</sup> which governs the *conversion* of labour from ‘embodied’ to abstract social (a conversion which takes place in the framework of the exchange of commodities mediated by money).<sup>11</sup>

If, however, commodities are produced by means of  $m (> 1)$  types of labour, then, as is well known, the quantities of ‘embodied’ labours are determined by

$$\Omega \equiv L[I - A]^{-1} \quad (6)$$

where  $L$  denotes the  $m \times n$  matrix of labour input coefficients and  $\Omega$  the  $m \times n$  matrix of ‘embodied’ labours.<sup>12</sup> Consequently, the total quantity of direct labours is represented by an  $m \times 1$  vector and, thus, each attempt to find a ‘conservation principle’ (i.e., any connection whatsoever between the multidimensional magnitude  $LX$  and the total quantity of abstract social labour, which corresponds to the given system  $[A, L, X]$  and which is by definition a scalar) proves to be devoid of any economic meaning and content.<sup>13</sup>

We shall therefore show below that in the framework of the NA, the notion of abstract social labour has not, in reality, been correctly formulated.

#### **4. The Completed New Approach**

As is well known, Marx tried to show that in a society of independent producers, in which ‘labour power’ is a commodity (‘capitalist mode of production’), the products of *labour*<sup>14</sup> necessarily appear, and consequently, are exchanged, as products of the various *capitals*. This entails that in the capitalist mode of production, first, the social division/combination of labour is mediated both by exchange<sup>15</sup> and the aim to maximize the individual money rates of profit and second, the commodity prices systematically deviate from the quantities of labour ‘embodied’ (see also Foley (1982, pp. 39-40)).

But when prices deviate from the quantities of labour ‘embodied’, the following occurs: By virtue of the *exchange* defined by the ratio  $(p_g/p_j) (\neq (\omega_g/\omega_j))$ ,  $g$ ,

$j = 1, 2, \dots, n, g \neq j$ , the  $\omega_g$  units of homogeneous labour ‘embodied’ in one unit of commodity  $g$  are *equated with* the  $(p_g/p_j)\omega_j$  units of homogeneous labour ‘embodied’ in  $(p_g/p_j)$  units of commodity  $j$ , and thus constitute equal quantities of a – completely different to their own, and consequently – ‘new’ quality: *abstract social labour*.<sup>16</sup> They constitute, that is,  $B$  (*by convention*, where  $B$  denotes a positive real number) units of abstract social labour.<sup>17</sup> Because, according to Marx,<sup>18</sup> abstract social labour constitutes the substance of value and consequently value is measured in units of abstract social labour, it follows that the unit *value*,  $\omega_g^*$ , of the commodity  $g$  is  $B$  units of abstract social labour and the unit *value*,  $\omega_j^*$ , of the commodity  $j$  is  $(p_j/p_g)B$  units of abstract social labour. Alternatively, we could choose (*arbitrarily*), as a *unit of measurement* of abstract social labour, the  $C$  units of labour that have been ‘embodied’ in a produced commodity  $u$ , where  $u$  denotes an  $n \times 1$  *semi-positive* vector. In this case, which exists *only* when labour is homogeneous, we would have:

$$\omega^* u = \omega u / C \quad (7)$$

$$\omega_j^* = p_j [(\omega u / C) / p u], j = 1, 2, \dots, n \quad (7a)$$

$$V^* = w [(\omega u / C) / p u] \quad (7b)$$

$$S^* = P [(\omega u / C) / p u] \quad (7c)$$

where  $\omega^*$  denotes the  $1 \times n$  vector of unit *values*,  $V^*$  the unit *value* of ‘labour power’, and  $S^*$  the *surplus value*.

Consequently, we conclude that the values of the commodities are always proportional to their actual prices. However, the arbitrary choice of the measure of values and the unit of measurement of abstract social labour is an unnecessary and meaningless act, because this act has ‘already’ been performed within the economic reality *itself*: Exchange is mediated by money and consequently money constitutes the

measure of value, whilst the unit of money constitutes the unit of measurement of abstract social labour. Therefore, values are always equal to the actual prices, that is<sup>19</sup>

$$\omega^* \equiv p \quad (8)$$

$$V^* \equiv w \quad (8a)$$

$$S^* \equiv P \quad (8b)$$

Evidently, it is not money that transforms the products of labour into commensurate magnitudes. Exactly the opposite holds: Exchanged in proportions which express in the main *all* the features of the constituted society, the products of labour become commensurate and can be measured in terms of some homogeneous extensive thing.<sup>20</sup> In this way, the said thing (which is not necessarily a commodity) is transformed into a common measure of the value of commodities or money. Money (the price) constitutes, therefore, the necessary form of existence of ‘embodied’ labour (of the quantity of ‘embodied’ labour), which has ‘first’ been transformed into abstract social labour (into a quantity of abstract social labour). When commodities are exchanged as products of labour, i.e.,  $(p_g/p_j) = (\omega_g/\omega_j)$ ,  $g, j = 1, 2, \dots, n$ , prices express, from a qualitative viewpoint, the corresponding quantities of ‘embodied’ labour in a distorted manner. Although the vectors  $p, \omega$  are collinear, the element  $p_j$  has the dimension: units of money per unit of commodity  $j$ , and the element  $\omega_j$  has the dimension: units of ‘embodied’ labour per unit of commodity  $j$ . In this case, therefore, there is an intrinsic-immanent (‘embodied’ labour) and an extrinsic (money) measure of value. When commodities are not exchanged as products of labour, the prices express, both from a qualitative and a quantitative viewpoint, the corresponding quantities of ‘embodied’ labour in a distorted manner. In this case, therefore, there is one and only one measure of value: money. Precisely for this reason, in the *capitalist*

mode of production the quantities of ‘embodied’ labour cease to have any *valid* significance for the economic subjects: *For example*, each individual capitalist realizes that both that part of the total profits of the system, which he can acquire, and the very existence of a positive, for him, profit, are exclusively dependent (are completely independent) on the money prices of commodities (of labour, i.e., the only factor that, according to Marx, creates the net output and consequently the surplus product).<sup>21, 22</sup> However, it is a situation, which, on the one hand, was created by the subjects themselves, within the framework of which, on the other, their aforesaid realization is not contradicted but rather verified and therefore constitutes a valid condition of their action.<sup>23</sup>

## **5. The relationships between the Completed New Approach and the New Approach**

The CNA is expressed first of all through the relations (7)-(7c). So, if we choose  $u = Y$  and  $C = 1$ , then we will get the NA without the logical inconsistency that we highlighted in Section 3 (3.1). Consequently, it could be said that the NA constitutes a special case (among the infinite cases) of the CNA, because it follows from it by virtue of an appropriate choice of the measure of value and the unit of measurement of abstract social labour.<sup>24</sup> Nevertheless, the following should be noted (in relation to what was said in Sections 3 and 4):

**1.** The choice of the measure of values and the unit of measurement of abstract social labour is an unnecessary and meaningless act, because this act has ‘already’ been performed in economic reality itself: Exchange is mediated by money and consequently money constitutes the measure of value, whilst the unit of money

constitutes the unit of measurement of abstract social labour.

**2.** As we have already seen, the followers of the NA justify their view as follows:

‘The advantage of interpreting the value of money as the ratio of aggregate labor time to aggregate money value added is that the value gained and lost by all the producers in exchange will be zero. In other words, this interpretation of the value of money corresponds to the idea that value is created in production but conserved in exchange’ (Foley (1982, p. 41); see also Duménil (1984, pp. 346-7) and Mohun (1994, pp. 402-3)). However, ‘value’ in terms of ‘embodied’ labour is something *quite different* (as we have showed in Section 4) from value in terms of abstract social labour. Thus, the premise for the existence of any connection whatsoever between these magnitudes (i.e., not only  $\omega^* Y = \omega Y$ , which according to the NA expresses a ‘conservation principle’) has no economic meaning and, consequently, is not logically consistent. Furthermore, it is by no means necessary to introduce the ‘conservation principle’  $\omega^* Y = \omega Y$ , in order to express ‘the idea that value is created in production but conserved in exchange’, because, according to Marx, exchange always presupposes exchange of equivalent things, i.e., exchange of equal values and consequently ‘profit’ and ‘loss’ never arise within the framework of each separate exchange (which, supposedly, cancel each other out when one takes into consideration the entirety of exchanges performed). For one to assert that when one unit of commodity  $g$  is exchanged for  $p_g/p_j$  units of commodity  $j$ , profit and loss arise because  $(p_g/p_j) \neq (\omega_g/\omega_j)$ , has exactly the same economic significance as the assertion that profit and loss arise because the quantities that are exchanged do not have the same weight or the same mass.<sup>25</sup>

**3.** To be precise, the choice of the measure of values is not only an unnecessary and meaningless act, but also leads to ‘perverse’ situations (in the general case). For

example, if we choose  $u = Y$ , then in order for  $\omega^* > 0$  to result, it is necessary and sufficient for  $pY > 0$  to hold (see (7a)). In reality, of course, nothing prevents *a priori* the validity of  $pY \leq 0$ .<sup>26</sup> Furthermore, when this latter relation holds, nothing prevents us from choosing a different  $u$  and converting the indeterminate or negative values of single commodities into positive ones.<sup>27</sup>

4. Even if we for a moment ignore all the preceding issues, the case of heterogeneous labour (see Section 3 (3.2)), in the context of which each syllogism based on the existence of any kind of ‘conservation principle’ collapses, shows conclusively that the NA erroneously sets forth the issue of abstract social labour.

In view of all the above, it therefore follows that in the best case the NA constitutes one of all those infinite special cases of the CNA, which are economically insignificant and unrealistic, and it is clear that all the various problems presented by the NA stem from the fact that it has not correctly formulated the notion that is fundamental to it, namely the notion of abstract social labour. For, in the opposite case, it would be both logically consistent and in a position to deal with the case of the existence of heterogeneous labour.

## 6. Concluding Remarks

In accordance with the Marxian theory, the notion of the capitalist mode of production presupposes the interpretation of profit, the determination of its source. But because profit is the money price of a particular commodity, i.e., of the surplus product, it follows that the Marxian notion of the capitalist mode of production presupposes the *interpretation* of commodity, price and money.

The ‘New Approach or Interpretation’ was different to the *traditional*

approach precisely because it turned its attention to the fundamental issue of abstract social labour, that is, to the analysis of the so-called ‘forms of value’. However, quite paradoxically, it did not manage to put together a coherent and complete formulation of the said issue. Most likely because, at the same time, it displayed an (un)justifiable focus on the validity of ‘Marxian double equality’. As a consequence, it managed to reach, let us put it this way, only the halfway point of the distance it should have covered.

The approach that has been elaborated here has covered (we wish to believe) the ‘remaining distance’, and should not be judged only on the basis of its final conclusion (relations (8) – (8b)), a conclusion which shows, *inter alia*, that each theoretical and empirical effort to find an economically significant, quantitative relationship of any kind between prices and quantities of private and concrete labours is pointless, but primarily on the basis, first, of the logical course followed in order to reach this conclusion, second, the degree to which it sets out and, subsequently, reconstructs in a logically consistent manner the core of Marxian theory and third, its contribution to the attainment of theoretical targets which Marxian theory has set for itself.

#### NOTES

1. We prefer, as being more precise, these terms to the term: ‘New Solution’. In this respect, see Saad-Filho (1996, pp. 116-8) and Foley (1997, p. 45).

2. For brevity's sake, we shall not deal here either with the *traditional* approach (our positions are set out in Mariolis (1998, 2002b, 2003)) or the other more recent approaches / interpretations.
3. Steedman (1992) proved that in the joint production case more than one profit rate can be associated with an exogenously given value of the share of wages in national income. Consequently, the *realistic* case of joint production creates insurmountable problems for the NA (see also the subsequent discussion between Mohun (1993a, b) and Steedman (1993); moreover, see Steedman (2002, pp. 2-4)). It should be clarified that we do not consider an exogenously given wage share as a basic element of the NA *in the sense* that one can present the NA without using this element (see below).

4. As is well known, the following hold:

$$\omega \equiv a[I-A]^{-1}$$

$$Y \equiv [I-A]X$$

$$\omega Y \equiv \alpha X$$

$$P \equiv pY - w\alpha X$$

where  $I$  denotes the  $n \times n$  identity matrix and  $(IX)$  the  $n \times 1$  vector of the gross output.

5. It should be noted that the NA does *not* premise the (semi-) positiveness of the net output. But when prices deviate from the production prices, it demands that  $pY > 0$  (obviously, when prices equal the production prices, and the production technique is indecomposable, the validity of this relation is *always* given), because it

considers that in the opposite case ‘the idea of a ‘transformation problem’ becomes self-contradictory’ (Duménil (1984, p. 347)).

6. If the absolute price level is not given, then we can introduce the normalization equation  $pY = aX$ . Thus, we will get the following:  $S \equiv P$ .
7. We do not consider the criticism of the NA set forth by Roemer (1990, p. 1728), Shaikh and Tonak (1994, p. 179), Sotirchos and Stamatis (1998), Stamatis (1998-99), Febrero Paños (2000) and Mavroudeas (2001), to be ‘fair’, because within the framework of this criticism, the NA is not set out faithfully (see also Mohun (2000)). We believe, however, that it is precisely the issues that we present below (Section 3) which prevent the comprehension, and consequently the faithful impression of the NA.
8. As the reader will ascertain, these two issues are to begin with independent of each other. However, it will subsequently be shown (Section 5) that they are related.
9. ‘[I]t is clear that in general the price of any commodity multiplied by the value of money as defined here will not be equal to the labor value of the commodity. [...] The price of a commodity in this interpretation is the amount of money for which it exchanges on the market, an amount of money which may represent more or less social labor than is embodied in the commodity itself and which determines its labor value’ (Foley (1982, p. 43 and 44)). For an argument that is similar to ours, see Sinha (1997, p. 52).

10. See the excerpts from p. 41 of Foley (1982), which we have already presented here (Section 2), as well as Duménil (1984, pp. 346-7) and Mohun (1994, pp. 402-3).
11. Precisely for this reason, the NA has no choice but to assert that when the price of the net output is not positive, ‘then the idea of a ‘transformation problem’ becomes self-contradictory’ (Duménil (1984, p. 347)).
12. For a critique of each attempt to ‘reduce’ one type of labour to another, see Steedman (1977, ch. 7, 1980, 1985, 1990) and Mariolis (2002a, pp. 208-9).
13. It should be noted that Foley (1997, p. 45) does not deal with this problem directly, but places it on an empirical level: ‘There are also issues of measurement of [...] the living labor expended. In principle it is necessary to adjust total hours worked for the skill levels of workers. A variety of method, some based on relative wage weights, and others on more direct measures of skill, have been proposed to make this adjustment. [...] In the rest of this paper I will assume (contrary to reality) that some agreement has been made on these measurement issues...’.
14. It is clear that if the expression ‘the products of labour’ is to have any meaning, it must be assumed (contrary to what applies in the real world) that labour is homogeneous. This assumption is introduced solely because it facilitates not only

the exposition of the CNA but also the determination of the relationships between the CNA and the NA.

15. Through the exchange of products, exchange that is defined quantitatively by the actual prices and mediated by money, the quantities of corresponding ('embodied' in the various products) *private* and *concrete labours* are also exchanged. Thus, through this process and without it being the aim of the producers or even being realized by them, the products of labour are transformed into commodities, labour and production are socialized, the set of independent producers is formed into a society, and the aforementioned quantities of private and concrete labours are transformed into quantities of social and abstract labour, respectively, and are represented as different, commensurate quantities of one and the same thing: money.
16. Because the exchange of commodities presupposes their production and their production presupposes exchange that took place in an earlier period of time, we are not talking about a 'new' quality from the point of view of time, but about a 'new' quality from the point of view of logic. It is for this reason that the word 'new' appears in inverted commas.
17. If, as we assume here, the quantities of private, concrete labours are homogeneous, then the exchange creates a 'new' measure of equivalence of commodities: abstract social labour, which cancels (as we shall see later) the 'initial' measure of their equivalence: 'embodied' labour. If the quantities of private, concrete labours

are heterogeneous, then there is no ‘initial’ measure of equivalence of commodities. Only as percentages of overall abstract social labour are commodities equivalent.

18. This does not mean that the position maintained here is in agreement with what Marx wrote (and especially with the chapters on the ‘Transformation Problem’ in Volume III of ‘Capital’; see also Steedman, (1977, 1985, 1991) and Mariolis (2000, 2002a, 2005)). We believe, however, that it is in agreement with the core of Marxian theory. Finally, it should be stressed that this position is based on Rubin ([1927] 1978, [1928] 1994) and Goldmann (1959) (despite their various differences and in spite of the existence of certain contradictions and uncompleted analyses in their writings) and that it is very close (as was pointed out to me by Professor D. K. Foley, when he read one of the previous versions of this paper) to the position advocated by Reuten and Williams (1989, Parts 1-2).

19. It goes without saying that this conclusion is independent of the assumption of the existence of homogeneous labour (and/or: single production, one, and only one, linear technique, only reproduced inputs, ‘stationary’ prices, a closed economy, etc.). Thus, in the case where labour is heterogeneous (see (6)), the only thing that varies is the relation (8a), which is written as follows:

$$V_k^* \equiv w_k, k = 1, 2, \dots, m \quad (8aa)$$

where  $V_k^*$  and  $w_k$  denote the unit value and the unit price of type  $k$  ‘labour power’, respectively. Finally, it must be emphasized that Krause (1980, pp. 124-7), who begins with the notion of abstract labour, arrives at the relations (7)-(7c), with  $u =$

$Y$  and  $C = 1$ , but not at this conclusion (which seems paradox to us; see also Krause (1998, p. 8)). Therefore, in order to deal with the heterogeneous labour case, he constructs a ‘reduction’ operator of concrete labours to abstract labour (see Krause (1980, pp. 127-31)). However, the proposed ‘reduction’ is economically unrealistic and not always economically significant, that is, the ‘reduction’ coefficients are not always strictly positive (though it is useful from a formal point of view; see Krause (1981) and Fujimori (1982, pp. 111-3)).

20. Consider the textbook ‘Ricardian’ model of foreign trade, in which two commodities are produced by means of unassisted labours alone (see, e.g., Krugman and Obstfeld (2000, ch. 2)). In equilibrium, the relative price of commodities is determined as a function of the available quantities of ‘labour powers’, the quality of ‘labour powers’ and the consumer preferences in the two ‘countries’-producers (see also Simpson (1975, ch. 14)). So, the *totality* of features of the constituted ‘international’ society is summed up by the ratio of exchange of commodities, and it could not be different. It should be noted, finally, that because it is not out of the question for a certain commodity to be produced by both ‘countries’, it is perfectly possible for the quantities of labour ‘embodied’ in the separate units of that one and the same commodity to be heterogeneous. Therefore, it is not only the various commodities that become commensurate, but, in the general case, also the various units of that one and the same commodity.

21. For other ‘examples’, see Steedman (1977, 1979, Essays 8-11, 1991, 1999a, 2002), Steedman and Metcalfe (1981), Kurz and Salvadori (1995, chs 4-8 and 11), Bidard (1997) and Mariolis (2004) (and take into account Steedman (1999b, 2000) and Steedman and Tomkins (1998)). Moreover, it should be noted that according to the CNA the surplus value  $S^*(\equiv P)$  is positive iff the ‘new value’ produced by the expenditure of one unit of ‘labour power’,  $pY/aX$ , is greater than the value of one unit of ‘labour power’,  $w$ , a condition which neither presupposes nor entails (when prices deviate from the production prices) that the quantity of labour ‘embodied’ in the surplus product is positive (see Mariolis (2000, 2002a, 2005)).
22. What was said in note 17 is therefore correct: The ‘new’ measure of equivalence of commodities cancels the ‘initial’ measure. These measures of equivalence can co-exist only in the mind, a fact that leads certain Marxists to believe in the existence of ‘unequal exchange’, and, therefore, a ‘transfer of value (or surplus value)’.
23. At this point, we could quite clearly go into issues of ‘reification’ and ‘commodity fetishism’. This is, however, not necessary for the purposes of this paper.
24. It is worth noting that Laibman (1998a, b, 2002) arrives at the relations (7)-(7c), with  $u = Y$  and  $C = 1$ . As far as we can judge, however, the process of working out these relations is not based on the formulation of the notion of abstract social labour and is therefore not adequately founded. But even if we do an injustice to

him on this point, we consider that all the other points of our critique of the NA are valid also with respect to Laibman's proposal.

25. See also note 22.

26. As is well known, it is possible for the net output to contain also negative elements. In that case at least one process is reproduced on a lower scale (unless the system disposes stocks; see also Mariolis (2000, 2002a)).

27. It therefore follows that the relevant position of Duménil (see notes 5 and 11) should be considered erroneous.

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