Distribution and Growth in an Economy with Heterogeneous Capital and Excess Capacity

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ABSTRACT  
In a world of heterogeneous capital the aggregate capital-capacity ratio can change in a complicated way as the real wage rate changes and, therefore, nothing useful can be said, a priori, about the relationships between the real wage rate (or the aggregate profit share), the degree of capacity utilization and the rates of profit, capital accumulation and interest.

Key words: Aggregate capital-capacity ratio, capacity utilization, heterogeneous capital, post-Keynesian theory, Sraffian theory  
JEL classifications: D57, E11, E12  

INTRODUCTION  
In modern post-Keynesian theory (in the tradition of Kalecki-Steindl) the interaction of changes in income distribution and effective demand holds centre stage. In this approach, and contrary to the standard growth theories, the redistribution of income ‘has complex, even ambiguous, effect on the level of employment and output’ (Bhaduri and Marglin, 1990, p. 375) or, what amounts to the same thing, the interactions between the real wage rate (or the aggregate profit share) and the rates of capacity utilization, profit and capital accumulation are not necessarily monotonic (ibid., pp. 380-4); furthermore, a ‘bad income distribution’ can be a cause of stagnation (Dutt, 1984).1  
Sraffian theory, on the other hand, begins with the placement of the produced means of production at the centre of the analysis. Thus, one of the key findings is that in a world of heterogeneous capital goods the traditional neoclassical statements about the relationships between the distribution of income, long-period commodity prices and  

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technical conditions of production are not verified and/or make no sense. In this regard
a change in the real wage rate has no longer unambiguous effects on the capital-labour
and capital-capacity ratios.\(^2\)

The objective of this paper is to assess the consequences of the presence of
heterogeneous capital goods in a system, in which (i) the rates of profit and capacity
utilization are uniform; and (ii) the desired rate of capital accumulation is a strictly
increasing function of both the degree of capacity utilization and the aggregate profit
share.\(^3\) It then follows that the aggregate capital-capacity ratio enters into the
determination of the equilibrium between investment and savings and, therefore,
nothing unambiguous can be said, \textit{a priori}, about the directions of change in the rates of
capacity utilization, profit and accumulation when the real wage rate (or, alternatively,
the aggregate profit share) changes.

The remainder of the paper is organized as follows. The next section presents a
basic model for a multi-sector closed economy with excess capacity of capital, which
includes a ‘classical savings function’. In our effort to investigate the role of
heterogeneous capital we employ rather extremely restrictive assumptions. Thus the
analysis is more in the nature of an \textit{exercise}, rather than the formulation of a complete
framework. The following section allows for alternative savings assumptions. The final
section concludes.

\textbf{THE BASIC MODEL}

Consider a closed economy with excess capacity, which produces many basic
commodities (\textit{à la} Sraffa, 1960, pp. 7-8) by linear processes of single production. We
further assume that input coefficients are fixed, homogeneous labour is the only primary
input and capital goods do not depreciate. There are only two classes, workers,
employed in proportion to the level of production, \textit{i.e.}, there is no supplementary or
‘overhead’ labour, and capitalists, and two kinds of income, wages and profits. Wages
are paid at the end of the common production period and there are no savings out of this
income, whilst a given and constant fraction of profits, \(s_p\) \((0 < s_p \leq 1)\), is saved. The
growth of the economy is not constrained by the availability of labour. Both within each
sector and between the sectors there is a uniform degree of capacity utilization,
\(u\) \((0 < u \leq 1)\), which gives the ratio of actual output to potential output, where the latter
is taken to be proportional to the capital stocks in existence. Competitive conditions are
taken to be close to free competition. This allows us to interpret the underutilization of productive capacity as caused essentially by an insufficient effective demand (Kurz, 1995, pp. 96-7; see also Kurz, 1994, Sections 3 and 6). The desired rate of accumulation is a strictly increasing function of both the degree of capacity utilization and the aggregate profit share. Finally, we ignore entirely questions of technological change and questions of government expenditure and taxation.  

On the basis of these assumptions, we may write the following system of relations:

\[ p = pA(r/u) + wa \]  \hspace{1cm} (1)

\[ pb = 1 \]  \hspace{1cm} (2)

\[ x = Ax(g^s / u) + cb \]  \hspace{1cm} (3)

\[ ax \equiv 1 \]  \hspace{1cm} (4)

\[ g^S \equiv S/K = s_p r, \ K \equiv pAx(1/u) \]  \hspace{1cm} (5)

\[ g^l = F(u, h); F(0) \geq 0, F_x = (\partial F / \partial x) > 0, x = u, h \]  \hspace{1cm} (6)

\[ h \equiv 1 - (w/px) = vr/u, \ v \equiv pAx/px \]  \hspace{1cm} (7)

\[ g^l = g^s \]  \hspace{1cm} (8)

\[ (s_p h / v) > F_u \]  \hspace{1cm} (9)

where \( p \) denotes the vector of commodity prices, \( A \) the irreducible matrix of capital coefficients, \( a \) the vector of labour input coefficients, \( r \) the uniform rate of profit, \( w \) the uniform money wage rate, \( b \) a given vector representing the uniform consumption pattern, \( x \) the vector of outputs per unit of labour, \( g^S \) the actual rate of capital accumulation, determined by the amount of savings, \( S \), \( K \) the total savings and value of capital stocks per unit of labour, respectively, \( c \) the index of consumption per unit of labour, \( g^l \) the desired rate of capital accumulation, \( F(\cdot) \) a continuous function, \( h \) the aggregate profit share, and \( v \) the aggregate capital-capacity ratio. Equation (2) fixes the standard of value or \textit{numéraire}. Hence \( w \) also symbolizes the level of the real wage rate. Equation (6) defines an investment function. Equation (8) defines the commodities market equilibrium. Finally, relation (9) gives the short-run Keynesian stability condition for the \( g^l - g^S \) equilibria (\textit{i.e.}, total savings must increase by more than investment demand when \( u \) rises).
It is quite clear that the system has one degree of freedom. From (1) and (2), i.e., the ‘price side’ of the system, we obtain
\[ p = waB(r/u), \quad B(r/u) = [I - A(r/u)]^{-1} \] (10)
and
\[ w = [aB(r/u)b]^{-1} \] (11)
where \( I \) is the identity matrix, and each element in \( B(r/u) \) is homogeneous of degree zero, positive and increases with \( r/u \), tending to infinity as \( r/u \) approaches its maximum feasible value, \( \lambda^{-1} \) (\( \lambda \) denotes the Perron-Frobenius eigenvalue of \( A \)). Thus equation (11) defines a strictly decreasing ‘\( w-(r/u) \) frontier’ for this economy. From equations (3) and (4), i.e., the ‘quantity side’ of the system, we obtain
\[ x = cB(g^S/u)b, \quad B(g^S/u) = [I - A(g^S/u)]^{-1} \] (12)
and
\[ c = [aB(g^S/u)b]^{-1} \] (13)
where each element in \( B(g^S/u) \) is homogeneous of degree zero, positive and increases with \( g^S/u \), tending to infinity as \( g^S/u \) approaches its maximum feasible value, \( \lambda^{-1} \). Thus equation (13) defines a strictly decreasing ‘\( c-(g^S/u) \) frontier’ for this economy. Furthermore, equations (5), (7) and (10)-(13) imply that the aggregate capital-capacity ratio, \( v \), is a complicated expression involving \( r/u \), \( s_p \) and technical conditions, that is,
\[ v = [aB(r/u)AB(s_p,r/u)b][aB(r/u)B(s_p,r/u)b]^{-1} \] (14)

Nevertheless, given that \( h = vr/u \) and that \( h \) is a strictly increasing function of \( r/u \) (see Franke, 1999, pp. 46-9, where \( u = 1 \) holds, by assumption), it follows that (i) \( 0 < vr/u < 1 \) for \( 0 < r/u < \lambda^{-1} \), and \( v = \lambda \) at \( r/u = \lambda^{-1} \); (ii) the elasticity of \( v \) with respect to \( r/u \) is greater than \(-1\); and (iii) the elasticity of \( v \) with respect to \( h \) is less than \( 1 \). Finally, the equality between investment and savings from equations (5)-(8) implies
\[ F(u,h) = s_p hu/v \] (15)
and, recalling (9), the local slope of the ‘\( IS- \) curve’ in \( u \times h \) space is given as
\[ du/dh = [(F_h -(s_p u/v)) + (e_p F(\bullet)/h)][(s_p h/v) - F_u]^{-1} \] (16)
or
\[
\frac{du}{dh} = \left[ F_h - (1-e_r)(s_p u/v) \right] \left[ (s_p h/v) - F_u \right]^{-1}
\]  
(16a)

where \( e_r \) represents the elasticity of \( v \) with respect to \( h \). Thus, equation (15) defines a non monotonic, in the general case, ‘IS – curve’ for this economy.

Given \( w \) from outside the system, (10) and (11) determine a unique solution for \((p, r/u)\). Hence (5), (12) and (13) determine a unique solution for \((x, e)\), (7) determines \((v, h)\), and (15) determines a unique equilibrium value of \( u \). Nevertheless, it is impossible to make any \textit{a priori} prediction concerning the effects of a variation in \( w \) on the equilibrium values of \( u, r \) and \( g \). More specifically, (16) indicates that the movement of the degree of capacity utilization, as a result of a change in the distributive variable, \( i.e., \) the wage rate or the aggregate profit share, can be decomposed into the following two distinct effects: (i) the relative response of investment and savings, represented by the term \([F_h - (s_p u/v)]\); and (ii) the response of the aggregate capital-capacity ratio, represented by the term \((e,F(\ast)/h)\). Thus, when investment responds relatively weakly (strongly) to changes in \( h \), \( i.e., \) \( F_h < (s_p u/v)(F_h > (s_p u/v)) \), \( u \) may rise (fall) due to the fact that \( e_r > (<)0 \). Differentiation of \( r = hu/v \) with respect to \( h \) gives

\[
\frac{dr}{dh} = (1-e_r + e_u)(u/v)
\]
(17)

where \( e_u \) represents the elasticity of the ‘IS - curve’, or, recalling (16a),

\[
\frac{dr}{dh} = \left[ hF_h - uF_u (1-e_r) \right] \left[ 1/v \right] \left[ (s_p h/v) - F_u \right]^{-1}
\]
(17a)

Since \( e_r < 1 \), it follows that \( e_u \geq 0 \) implies \( dr/dh > 0 \). However, neither an elastic, negatively sloped ‘IS – curve’, \( i.e., e_u < -1 \) or, equivalently, \( hF_h - uF_u < -e_u s_p r \), nor \( hF_h < uF_u \), which implies that the elasticity of the desired rate of accumulation with respect to \( h \) is less than its elasticity with respect to \( u \), necessarily imply \( dr/dh < 0 \).5

From (7), (9), \( e_r < 1 \), (16a) and (17a) we may derive the following conclusions:

(i). The model is capable of generating three alternative sets of steady-state equilibria or ‘growth regimes’.6 A ‘regime of overaccumulation’, characterised by \( du/dh < 0 \) and \( dr/dh > 0 \), prevails when

\[
uF_u (1-e_r) < hF_h < (1-e_r)s_p r
\]
(18)

A ‘regime of underconsumption’, characterised by \( du/dh < 0 \) and \( dr/dh < 0 \), prevails when
\[ hF_h < uF_u (1-e_v) \]  
(19)

A ‘Keynesian regime’, characterised by \( du/dh > 0 \) and \( dr/dh > 0 \), prevails when

\[ (1-e_v)s_r r < hF_h \]  
(20)

(ii). Even with a linear investment function, the effects of a redistribution of income (or of a change in \( s_p \)) on the rates of capacity utilization, profit and accumulation are neither known \textit{a priori} nor independent of the initial state of the system. Moreover, nothing rules out the ‘reswitching’ of growth regimes (see Mariolis, 2004, p. 176, for a pertinent numerical example).

(iii). As is well known, the validity of \( e_v = 0 \) cannot, in general, be extended beyond a quasi-one-commodity system, that is, the cases in which \( \mathbf{a} \) or \( \mathbf{b} \) is the Perron-Frobenius eigenvector of \( \mathbf{A} \), and thus \( v = \lambda \) (see, \textit{e.g.}, Marglin, 1984, pp. 240-4). Consequently, it must be said that the great complexity of a multi-sector system is due, in the final analysis, to the fact that the aggregate capital-capacity ratio is not given independent of, and prior to, the determination of prices, distribution and growth.

**SOME EXTENSIONS**

In this section we shall extend the argument to the following cases: (i) there are savings out of wages; (ii) there is a rentier class; and (iii) workers save.

**Savings out of Wages**

Assume that a given and constant fraction of wages, \( s_w \) \((0 < s_w < s_p)\), is saved. Then (5), (9), (16) and (17a) become

\[ S/K = s_p r + s_w (w/K) \]

or

\[ g^s = (s_p - s_w)r + s_w (u/v) \]  
(21)

\[ A > F_u \]  
(22)

\[ du/dh = [F_h - (s_p - s_w)(u/v) + e_v (Au/h)](A - F_u)^{-1} \]  
(23)

\[ dr/dh = [hF_h - uF_u (1-e_v) + (s_w u/v)](1/v)(A - F_u)^{-1} \]  
(24)

where \( A \equiv (s_p - s_w)(h/v) + (s_w/v) \). Differentiation of (21) with respect to \( h \) gives

\[ dg^s / dh = [e_v A - (s_w/v)](u/h) \]  
(25)

where \( e_v = 1 - e_v + e_u \) represents the elasticity of \( r \) with respect to \( h \).
When there are savings out of wages, the relationship between \( g^s / u \) and \( r / u \) depends on the technical conditions, and this implies that \( g^s / u \) and \( r / u \) may be inversely related or \( h \) and \( r / u \) may be inversely related (in that case \( e, <1 \) does not hold).\(^7\) Hence there is a further source of ambiguity in the consequences of redistribution.

**Rentier Class**

Assume that (i) total profits split into income of the capitalists and rentiers’ income, \( i.e., \) interest payments; (ii) rentiers save a given and constant fraction, \( s_r \ (0 < s_r < 1) \), of their income; (iii) the debt-capital ratio is uniform; (iv) the rate of inflation is equal to zero; (v) the desired rate of capital accumulation depends inversely on the interest payments per unit of nominal capital stocks; and (vi) the investment function is linear.\(^8\) Then (5), (6) and (9) become

\[
S / K = s_p [r - (Z / K)i] + s_k (Z / K)i
\]

or

\[
g^s = s_p r - (s_p - s_r) zi
\]

\[
g^i = a_0 + a_1 u + a_2 h - a_3 zi, \quad g^i > 0 \text{ for } r > i
\]

\[
(s_p h / v) > a_i
\]

where \( z \equiv Z / K, \ 0 < z < 1 \), denotes the debt-capital ratio, \( Z \) the nominal stocks of loans per unit of labour, \( i \) the given rate of interest, and \( a_i \) given and positive constants.

The short-run equilibrium is defined as one in which \( z \) is exogenously given. In the first instance consider a quasi-one-commodity economy, \( i.e., \) \( v = \lambda \). Setting \( g^s \) equal to \( g^i \) yields

\[
u = (a_0 + a_2 h + Bzi)[(s_p h / \lambda) - a_1]^{-1}
\]

where \( B = s_p - s_r - a_3 \). Consequently, given \( w \) (or \( h \)) from outside the system, a rise in \( i \) has either positive (iff \( B > 0 \)) or negative effects on \( u \) and \( r \), whilst the effect on the rate of accumulation is positive iff

\[
(s_p h / \lambda a_i) - 1 < B / a_3
\]

Thus it follows that when investment ‘is hardly affected by the interest rate and the propensity to save out of interest income is relatively low, there may arise regimes of
accumulation with positive responses throughout the rates of capacity utilization, accumulation and profit to an increasing interest rate’ (Hein, 1999, p. 15). Furthermore, given \( i \), a ‘Keynesian regime’ prevails iff

\[
C = a_1a_2\lambda + (a_0 + Bz)i < 0
\]  
(31)

whilst a ‘regime of underconsumption’ prevails when \( e_u < -1 \), namely

\[
(s_p h/\lambda a_1) - 1 < (C / \lambda a_2)^{1/2}
\]  
(32)

Nevertheless, in a multi-sector system the aggregate capital-capacity ratio depends on \( u, r \) and \( g^s \), whilst \( g^s \) is related to \( r \) and to \( i \) by (26). Thus nothing useful can be said, \textit{a priori}, about the directions of change in \( u, r \) and the rate of accumulation when \( i \) (or \( h \)) changes.

Finally, the long-run equilibrium is defined as one in which \( z \) remains constant over time. Since the percentage rate of growth of the stocks of loans equals \( s_p i \), it follows that

\[
\dot{z} = (s_p i - g^s)z
\]  
(33)

where \( \dot{z} \) denotes the first derivative of \( z \) with respect to time. Let us first consider a quasi-one-commodity economy. From (27), (29) and (33) we obtain the equation of motion for \( z \):

\[
\dot{z} = D_1 z + D_2 z^2
\]  
(34)

where

\[
D_1 = s_p i - a_0 - a_2h - a_1(a_0 + a_2h)(s_p(h/\lambda) - a_1)^{-1}
\]  
(34a)

and

\[
D_2 = a_1 i - a_1 Bi[(s_p h / \lambda) - a_1]^{-1}
\]  
(34b)

The effects of a variation in \( w \) (or \( i \)) on the equilibrium value \( z = -D_1 / D_2 \), and therefore on the equilibrium values of the rates of capacity utilization, profit and accumulation are vague (see Hein, 2004, pp. 13-20, for a detailed analysis). Nevertheless, in a world of heterogeneous capital, a redistribution of income influences \( D_1 \) and \( D_2 \) both by changing \( r / u \) and by changing the aggregate capital-capacity ratio. Hence there is a further source of ambiguity.
**Workers Save**

Suppose the same economy as before. But now assume that (i) $s_w$ represents the workers’ saving ratio, and thus $Z$, $0 < Z < K$, represents the amount of capital per unit of labour that the workers own indirectly (through loans to the capitalists; see Pasinetti, 1974, chs 5-6); and (ii) there is no rentier class. Then (21) (or (26)), (22) (or (28)) and (29) become

\[
S / K = s_p [r - (Z / K)i] + s_w [(w / K) + (Z / K)i]
\]
or

\[
g^S = (s_p - s_w)(r - zi) + s_w (u / v)
\]

or

\[
A > a_1
\]

\[
u = (a_0 + a_2 h + B_1 zi)(A - a_1)^{-1}
\]

where $B_1 = s_p - s_w - a_3$. Furthermore, since the percentage rate of growth of the stocks of loans equals $s_u[i + (w / Z)]$, it follows that

\[
\dot{z} = (s_u i - g^S) z + s_u(1 - h)(u / v)
\]

(38)

From (27), (37) and (38) we obtain the equation of motion for $z$:

\[
\dot{z} = E_0 + E_1 z + E_2 z^2
\]

(39)

where

\[
E_0 = s_u [(1 - h) / v] (a_0 + a_2 h)(A - a_1)^{-1}
\]

(39a)

\[
E_1 = s_u i - a_0 - a_2 h - [a_1 (a_0 + a_2 h) - s_u (1 - h) B_1 i / v] (A - a_1)^{-1}
\]

(39b)

and

\[
E_2 = a_3 i - a_1 B_1 i (A - a_1)^{-1}
\]

(39c)

Thus it can be concluded that this case combines the main features of the previous two cases.\(^9\)

**CONCLUDING REMARKS**

It has been shown that in a simple model for an economy with heterogeneous capital goods and excess capacity the interaction between distribution and growth is a particularly complex phenomenon. Although the sensitivity of the results to the algebraic expression of the investment function and to the assumptions with respect to savings cannot be disregarded, the principal, and totally independent of the observer, reason for the complexity is that the aggregate capital-capacity ratio cannot be treated as
a datum. And it need hardly be said that this finding casts doubt on the reliability of income redistribution as a macroeconomic policy concerned with removing stagnation.

Taking the robustness of this conclusion as given, future research efforts should, first, examine the possibility of closing the basic model by an endogenous determination of distribution and second, concretize the analysis by considering the existence of ‘overhead’ labour, depreciation, alternative production methods and technological change, differential rates of capacity utilization and profit, pure joint products, and government fiscal activity.

Notes
3. This specification of investment function is due to Marglin and Bhaduri (1990, pp. 160-71). See also Bhaduri and Marglin (1990, pp. 379-80) and Kurz (1990, pp. 218-21). See Lavoie et al. (2004) for a theoretical and empirical investigation of the issue at hand. For two-sector models, with homogeneous capital, a uniform rate of profit, mark-up pricing and, therefore, differential degrees of capacity utilization, see Dutt (1987b, 1990, ch. 6, 1997) and Lavoie and Ramirez-Gaston (1997).
5. This is quite different from the case of a one-commodity model, where $e_v = 0$ and, therefore, $e_u < -1$, $hF_h < uF_u$, and $dr/dh < 0$ are equivalent (see Bhaduri and Marglin, 1990, pp. 382-4).
6. The following terminology is due to Kurz (1990, pp. 222-6), whilst for an alternative terminology see Bhaduri and Marglin (1990, pp. 388-9).
7. See Spaventa (1970, pp. 139-41 and 146) and Marglin (1984, ch. 11); in both analyses $u = 1$ holds, by assumption.
9. It is worth noting that this model is *formally* similar to a model for an open economy with excess capacity of capital, which includes a ‘classical savings function’ (see Mariolis, 2006).

References


