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Abstract

The central question this paper seeks to answer is how monetary policy might affect the equilibrium behaviour of default and sovereign risk premium. The paper is based on “one-interest-rate” model. Public debt becomes risky due to an active fiscal policy, as in Uribe (2006), reflecting the fiscal authority’s limited ability to control primary surplus. The insolvency problem is due to a string of bad luck (negative shocks affecting primary surplus). But in contrast to Uribe’s results, as the sovereign debt cost increases (which result from weak primary surplus), default becomes anticipated and reflected by a rising country risk premium and default probability. The default is defined as reneging on a contractual agreement and so the decision is set by the fiscal authority. However, conflicting objectives between fiscal and monetary authority play an important role in leading fiscal authority to default on its liabilities. The characteristic of the government policy needed to restore the equilibrium after the default is also analyzed.
1 Introduction

Interactions between fiscal and monetary policy in the determination of the price level have been the object of a great debate in monetary theory for years. Sargent and Wallace (1975) argue that if the monetary authorities adopt a policy rule for the interest rate (rather than the money stock) the equilibrium outcome leads to price level indeterminacy. However, the Sargent and Wallace result is not entirely general. McCallum (1981) firstly accounts for the following well-known result in the literature. Monetary policy feedback rules, linking the nominal interest rate to endogenous variables such as the price level, permit to rule out the classical problem of price level indeterminacy advocated by Sargent and Wallace. Following Taylor’s (1993) stimulating article, the so-called 'Taylor rules' have received growing attention in recent years. According to this type of rule, the central bank’s interest rate target is set as an increasing function of the inflation rate and the output gap.\(^1\) In order to rule out multiple equilibria, theoretical studies\(^2\) suggest that the monetary authority has to respond to increases in inflation with a more than one-to-one increase in the nominal interest rate. In term of Leeper (1991), this monetary policy rule is known as 'active'—otherwise, it is called 'passive'.

However, this literature does not account for the fiscal policy behavior. It means that the 'Ricardian equivalence' proposition applies; then, a comprehensive study of the implications of government deficits (and public debt) over the link between interest rate rules and price stability is not possible. All the same, there are important implications to consider within the relation between monetary and fiscal policies. In the recent macroeconomic debate, it is argued that the lack of a sound fiscal policy undermine the objective of price stability.\(^3\)

The seminal contribution of Leeper (1991) made also an important distinction between 'active' and 'passive' fiscal policy. It defines a fiscal policy as 'active' when taxes respond only weakly to public debt levels and 'passive' ones when taxes respond strongly to debt levels.\(^4\) In a standard model the research showed that two combinations, either (i) active

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\(^1\)See, among others, Clarida \textit{et al.}, 1998, 2000, that provide empirical evidence to the view that Taylor-type rules consistently describe the behavior of several central banks.


\(^3\)The Maastricht Treaty and the Stability and Growth Pact in the European Union which set quantitative limits on fiscal deficits and public debt for the Member States is based on this argument.

\(^4\)Later on, Woodford (1995) will identify this type of policies as a non-Ricardian and Ricardian fiscal policy.
monetary and passive fiscal policy or (ii) active fiscal and passive monetary policy, yield determinacy, a unique stationary rational expectations equilibrium. In case (i) the usual monetarist view that inflation depends only on monetary policy is confirmed. However, case (ii) is fiscalist in the sense that fiscal policy, in addition to monetary policy, has an effect on the inflation rate. Leeper (1991) also showed that the steady state is indeterminate, with multiple stationary solutions, when both policies are passive, while the economy is explosive when both policies are active.

Thus, the so-called 'Fiscal Theory of the Price Level' (FTPL), has emerged. This well-known theoretical framework enables to capture the effects of fiscal policy on the dynamic behavior of nominal variables, like price level.

The FTPL asserts that fiscal variables can fully determine the price level independently of monetary variables. More specifically, when fiscal solvency is not ensured for each sequence of the price level, fiscal variables uniquely determine the equilibrium level of nominal variables. This extreme result is the polar opposite of the monetarist statement that the price level and the inflation rate depend primarily on monetary variables. Not surprisingly, the Fiscal Theory approach has triggered criticism and controversy.

Controversy concerns the nature of the intertemporal budget constraint of the government. In different papers Buiter argues that FTPL confuses the roles of budget constraints and equilibrium conditions in models of a market economy. But more interesting, Buiter (2002) criticizes FTPL as a theory of price level determination because it explicitly rules out default. Equilibrium price-level changes each period in response to the (stochastic) fiscal shocks. And with price level changes in each period providing the capital gains and losses on public debt level necessary for equilibrium, default is never necessary. Once the possibility of explicit default is properly allowed for, non-Ricardian regimes become Ricardian regimes and the Fiscal Theory of the Price Level vanishes. Buiter shows that under a non-Ricardian fiscal-monetary programme with an exogenous nominal interest rate rule, the equilibrium conditions are the same as under the Ricardian fiscal-monetary programme without contract fulfilment and with an exogenous nominal interest rate rule.

Uribe (2006) presents a dynamic FTPL model of default in which he allows limited inflation rate flexibility. When a shock is so large that limited inflation rate flexibility cannot provide the necessary capital

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6It has been mainly questioned by Buiter (1999, 2001 and 2002) as well as McCallum (2001) and Niepelt (2004).
gain or loss on government debt, then the government either devalues or revalues its debt. Default is a reduction in debt below its contractual value. This is an interesting application of the FTPL to the problem of default, but it neither exhibits an increasing probability of default nor a positive expected default rate as empirical evidence suggests.

The main objective of this paper is to analyse the price stability and sovereign default risk issue. The model is grounded on a micro-founded equilibrium model with infinitely lived private agents that allow deviations from the Ricardian equivalence. This framework is particularly suitable to study the interactions between monetary and fiscal policy and its effect over both price stability and sovereign risk premium. It is shown that active interest rate rules, overreacting to inflation, are neither necessary nor sufficient to guarantee a unique stable solution for the price level without defaulting. Furthermore, in some cases, even ‘passive’ interest rate rules might drive the economy to an unsustainable path without defaulting. This results suggest that monetary policy matters being able to worsen a given scenario. Then, sovereign default is required to restore fiscal solvency and price stability. But the default rate must be high enough to ensure the economy to reach a stable equilibrium in the post-default dynamics.

The rest of the paper is organized in seven sections. Section II presents the model. Section III describes the three possible scenarios for this economy and section IV, the inflation and default dynamics. Section V explicitly calculates the expected recovery rate and sovereign risk premium. Section VI provides further research showing how detailed specifications of the monetary rule affect the equilibrium dynamics. Finally, in Section VII, the conclusion.

2 The Model

2.1 The Households

Consider a closed economy inhabited by a large number of identical infinitely-lived households. Preferences are described by,

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$  \hspace{1cm} (1)

where $c_t$ denotes household’s consumption level of a perishable good in period $t$, $u(\cdot)$ is the single-period utility function assumed to be increasing, strictly concave and continuously differentiable, $\beta \in (0, 1)$ denotes the subjective discount factor and $E_\tau$ is the mathematical expectation operator conditional on period $\tau$.

Each period, households are assumed to have access to a one-period
nominal government bond, denoted $B_t$. This bond offers, in period $t + 1$, a contractual gross nominal interest rate $R_t$. However, the fiscal authority may default on its debt and in each period it repays a fraction $h_t$ of its liabilities. Therefore, household investment in sovereign bonds in period $t$ is given by $B_t$ whereas the earnings from the last-period investment is expressed as $h_t R_t B_{t-1}$. This expression is called the recovery value of the sovereign debt whereas $h_t \in (0, 1)$ represents the recovery rate.

In our notation, Buiter (1999, 2001 and 2002) do not restrain $h_t$ assuming that both $h_t < 0$ and $h_t > 1$ are possible options. The former assumption $-h_t < 0$ implies that the sovereign can be a net creditor. This seems unrealistic – particularly, in developing countries – and not so relevant in a model focused to analyse scenarios of sovereign debt crisis and default. The last assumption $-h_t > 1$ – also adopted by Uribe (2006), implies that any surplus resources over the contractual value of the outstanding debt are shared out equally among the holders of the contractual government debt. However, this excess of resources should not be interpreted as a government subsidy because in general they are allocated to tax payers; not to bondholders. Buiter names these transfers ‘super-solvency premium’. But even more important, government bonds are fixed-income securities as opposite to any other variable return security, such as stocks. In a more realistic approach we propose constraint $h_t$ as $h_t \in \mathbb{R}$. 

Besides, in each period $t$ households have also the opportunity to invest in a complete set of nominal state-contingent assets. The total investment, in nominal terms, can be expressed as $E_t Q_{t,t+1} D_{t+1}$ where $Q_{t,t+1}$ denotes the stochastic nominal discount factor of an asset with a random nominal payment $D_{t+1}$. The time $t$ revenue from the investment made in the previous period, is denoted as $D_t$.

Finally, households are endowed with a constant and exogenous amount of perishable goods denoted by $y$ and they pay real lump-sum taxes $\tau_t$. Their flow budget constraint can be written as,

$$P_t c_t + B_t + E_t Q_{t,t+1} D_{t+1} \leq P_t (y - \tau_t) + h_t R_{t-1} B_{t-1} + D_t \quad (2)$$

where $P_t$ denotes the price level in period $t$.

Then, the household is subject to an appropriate set of borrowing limits which prevents "Ponzi Games". In the absence of financial market frictions, the borrowing constraint takes the form:

$$h_{t+1} R_t B_t + D_{t+1} \geq -E_{t+1} \sum_{j=t+1}^{\infty} Q_{t+1,j} P_j (y - \tau_j) \quad \forall t + 1 \quad (3)$$

where $Q_{t+1,j} = Q_{t+1,t+2} Q_{t+2,t+3} \cdots Q_{j-1,j}$ and $Q_{t+1,t+1} = 1$. 

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The representative household maximizes its lifetime utility (1) subject to its flow budget constraint (2) and to its borrowing limits (3) by choosing \( \{c_t, B_t, D_{t+1}\}_{t=0}^{\infty} \) taking as given the set of processes \( \{P_t, \tau_t, Q_{t:t+1}, h_t R_{t-1}\}_{t=0}^{\infty} \) and the initial values \( D_0 \) and \( B_{-1} \). In addition to equation (2) holding with equality, the first order conditions are given by,

\[
\begin{align*}
c_t : u_c(c_t) &= \lambda_t \quad (4a) \\
D_{t+1} : Q_{t:t+1} &= \frac{\beta \lambda_{t+1} P_t}{\lambda_t P_{t+1}} \quad (4b) \\
B_t : R_{t}^{-1} &= \beta E_{t+1} \frac{\lambda_{t+1} P_t}{\lambda_t P_{t+1}} \quad (4c)
\end{align*}
\]

where \( \lambda_t \) denotes the Lagrangian multiplier in period \( t \).

Equation (4a) states that the marginal utility of consumption must equal the marginal utility of wealth, \( \lambda_t \), for all time \( t \). Equation (4b) represents the standard pricing equation for each one-period forward nominal contingent asset and equation (4c) represents the pricing equation for the case of the risky sovereign bonds between period \( t \) and \( t+1 \).

The transversality condition for the financial assets is written:

\[
\lim_{T \to \infty} E_t Q_{t:T} [h_T R_{T-1} B_{T-1} + D_T] = 0 \quad (4d)
\]

### 2.2 The Monetary and Fiscal Authorities

The fiscal authority levies lump-sum taxes, \( P_t \tau_t \), which are assumed to follow an exogenous, stochastic process. Recalling that fiscal authority issues nominal bonds, \( B_t \), with a contractual gross nominal interest rate, \( R_t \), but may default on its outstanding debt and repays a fraction \( h_t \) of its liabilities \( R_{t-1} B_{t-1} \), the sequential budget constraint\(^7\) is given by,

\[
B_t = h_t R_{t-1} B_{t-1} - P_t \tau_t \quad (5)
\]

where \( B_{t-1} R_{t-1} \) is given in period \( t \) and the recovery rate satisfies \( h_t \in (0, 1) \).

#### 2.2.1 The Monetary Rule

Following Uribe (2006), we suppose that the monetary policy takes the form of an interest-rate feedback rule whereby the short-term nominal interest rate is set as a function of inflation. But while Uribe uses a simple linear Taylor rule, active in the sense of Leeper (1991), and with an explicit inflation targeting objective. We wish to consider a slightly

\(^7\)For sake of simplicity, in this paper, we ignore money and seigniorage revenues.
different, asymmetric, monetary regime. The central bank behavior can be expressed as,

\[ R_t = \phi(\pi_t) = \begin{cases} \bar{R} & \text{if } \pi_t \leq \hat{\pi} \\ \bar{R} + \alpha(\pi_t - \hat{\pi}) & \text{otherwise} \end{cases} \quad \text{with } \alpha > \beta^{-1} \]

where \( \bar{R} = \beta^{-1}\hat{\pi} \) is the stationary value of the gross nominal interest rate associated to the inflation target \( \hat{\pi} \), and \( \hat{\pi} \) represents an inflation threshold. It will be useful to define: \( \bar{\hat{R}} = \beta^{-1}\hat{\pi} \), about which we make the following assumption:

**Assumption 1:** \( \bar{R} > \bar{\hat{R}} \), or, equivalently: \( \hat{\pi} > \bar{\hat{\pi}} \).

The monetary rule (6) implies that if current inflation increases beyond the inflation threshold \( \hat{\pi} \), the central bank reacts actively\(^8\): \( \alpha > \beta^{-1} \). Otherwise, the central bank pegs\(^9\) the current interest rate to its target \( \bar{R} \) which is associated to an inflation target \( \hat{\pi} \) lower than \( \bar{\hat{\pi}} \). Note that central bank is more concerned about tackling high inflation levels than dealing with scenarios dominated by low inflation and by deflation. In most developing countries, high inflation is a relatively frequent phenomena whereas deflation is quite rare and not so deep. Stylized facts on inflation rates in these countries shape an asymmetric behavior. So it seems to be reasonable to suppose an asymmetric behavior of the central bank. This monetary policy can be called "monitoring policy of current inflation".

In developed countries, much debate has been devoted to the suitability of the Taylor rule in characterizing the behaviour of central banks, especially in abnormal times. Rabanal (2004), for instance, presents evidence that Taylor rule coefficients changed significantly both with time and economic conditions in the United State between 1960 and 2003 using quarterly data. The qualitative interpretation is that the US Federal Reserve places much more weight on inflation stabilization during expansion periods, while it shifts its focus to output stabilization when in recessions. Analogous reasoning applies to the monetary rule (6). In developing countries, Brazil constitutes a successful example of inflation targeting. After being forced to abandon the crawling peg to the US dollar, Brazil adopted an inflation targeting regime in July 1999 which brought annual inflation down to one-digit figures in less than three years.

\(^8\)This condition is identical to that which led Leeper to describe the monetary rule as "active".

\(^9\)Actually, as demonstrated by Uribe (2006), a forward-looking rule of type: \( R_t = \bar{R} + \alpha \left( (E_t\pi_{t+1}^{-1}) - \bar{\pi} \right) \) will lead to the same results as our simpler pegging rule.
2.2.2 The Debt Recovery Rule

Given that the fiscal authority does not control the primary surplus, it is useful to suppose the existence of a rule $H(\cdot)$ which specifies how the fiscal authority chooses the recovery rate $h_t$. We will suppose that such a rule is a (non increasing) function of the nominal interest rate, denoted $R^{nd}_t$, to be determined by the monetary authority in the No-Default case. Then, $R^{nd}_t = \phi(\pi^{nd}_t)$ represents the potential cost of honoring the whole debt in the future. More precisely, the fiscal authority’s behavior is supposed to be defined by:

$$h_t = H(R^{nd}_t) = \begin{cases} 1 & \text{if } R^{nd}_t < \bar{R} \\ \bar{h}(R^{nd}_t) & \text{otherwise} \end{cases}$$  

where the threshold $\bar{R}$ denotes the maximum nominal interest rate that the fiscal authority will accept on its new issued debt without defaulting on its current liability. Finally, $\bar{h}(R^{nd}_t)$ denotes the fraction of the sovereign debt honored by the fiscal authority in case of default as a decreasing function of $R^{nd}_t$, function which will be specified later on.

**Assumption 2:** $\bar{R} > \hat{R}$.

This assumption implies that the fiscal authority is more tolerant – said, *lax* – than its monetary counterpart in terms of equilibrium inflation and interest rates.

However, it is important to point out that the main objective of the central bank is to monitor inflation whereas the fiscal authority only cares about the cost of its debt. Then, note that in order to control current inflation the central bank uses the current interest rate affecting, in this way, the cost of the sovereign debt. Consequently, a conflict of interests between both authorities may arise defining the equilibrium outcome. For instance, an aggressive central bank fighting against inflation may trigger the sovereign default as well as affect its size.

2.3 Market Clearing

At equilibrium, the goods market must clear: $c_t = y$, meaning that the consumption level is constant along time $t$. Thus, from equation (4a) it turns out that the marginal utility of consumption, $\lambda_t$, is also constant. Equation (4b) becomes $Q_{t,t+1} = \beta P_t / P_{t+1}$. Applying conditional expectations operator $E_t$ to the last expression, we obtain $E_t Q_{t,t+1} = \beta E_t (1/\pi_{t+1})$ where $\pi_{t+1} = P_{t+1}/P_t$ is the the gross rate of inflation and $E_t Q_{t,t+1}$ denotes the nominal price of a risk-free portfolio which pays one
unit of currency in all states of the nature. Consequently, the risk-free interest rate can be expressed as,

\[ R^f_t = \beta^{-1} \left[ E_t \frac{1}{\pi_{t+1}} \right]^{-1} \] (8)

Using the constancy of \( \lambda_t \), equation (4c) becomes,

\[ R_t = \beta^{-1} \left[ E_t h_{t+1} / \pi_{t+1} \right]^{-1} \] (9)

Finally, given that all households are assumed to be identical, at equilibrium, there is no borrowing or lending among them, \( i.e. D_t = 0 \) \( \forall t \). Thus all the assets held by private agents are in the form of government debt. Using this result and, again, \( Q_{t,t+1} = \beta P_t / P_{t+1} \), the transversality condition can be rewritten (in real terms):

\[ \lim_{T \to \infty} E_t \beta^{T-t} \left[ \frac{h_T R_{T-1} b_{T-1}}{\pi_T} \right] = 0 \] (10)

where \( b_t = B_t / P_t \).

The sovereign debt dynamics, described by equation (5), can also be written in real terms as:

\[ b_t = h_t R_{t-1} b_{t-1} / \pi_t - \tau_t \] (11)

Therefore, the equilibrium can be defined as follows:

**Definition 1** A rational expectations competitive equilibrium is defined as a set of processes \( \{ \pi_t, b_t, R_t, R^f_t, h_t \}_{t=0}^{\infty} \) satisfying equations (8), (9), (10), (11), the monetary rules (6), the debt recovery rule (7), and the exogenous process for the primary surpluses \( \{ \tau_t \}_{t=0}^{\infty} \) where \( R_{t-1} b_{t-1} \) are given and the recovery rate satisfies \( h_t \in (0,1) \).

Using equations (9), (10), (11), and some algebra – see Appendix – we obtain:

\[ h_t R_{t-1} b_{t-1} / \pi_t = \sum_{h=0}^{\infty} \beta^h E_t \tau_{t+h} = T_t \quad \forall t \] (12)

where \( T_t \) is the discounted value of present and future primary surpluses. Note that fiscal surpluses are discounted by the gross real risk-free interest rate given by \( \beta^{-1} \) – see equation (8).
Under this form, (12) is the key equation of the debate between the advocates\textsuperscript{10} of the Fiscal Theory of the Price Level determination (FTPL) and its detractors\textsuperscript{11}. If the fiscal authority is committed to honour the whole of its liabilities – and so $h_t = 1$ – then the current inflation rate, $\pi_t$, becomes determined according to the FTPL. This is because $T_t$ is exogenous and $R_{t-1}b_{t-1}$ is predetermined in period $t$. On the contrary, if $h_t$ is allowed to be less than unity, then the current value of $T_t$ may affect both current inflation and recovery rate. This may lead to the Buiter’s conclusion that any path for $h_t$ and $\pi_t$ satisfying equation (12) could be considered as an equilibrium outcome.

Using (12) to eliminate $h_tR_{t-1}b_{t-1}/\pi_t$ from equation (11) we get:

$$b_t = T_t - \tau_t = \beta E_t T_{t+1} \quad \forall t \geq 0$$

(13)

The real equilibrium value of the public debt is necessarily equal to the present value of future discounted real fiscal surpluses. Now, when $t > 0$ replaces $b_{t-1}$ by this equilibrium value in $t - 1$ into equation (12) we obtain:

$$\frac{\pi_t}{h_t} = \frac{\beta R_{t-1}}{1 + \eta_t} \quad \forall t > 0$$

(14)

where

$$\eta_t = \frac{T_t - E_{t-1}T_t}{E_{t-1}T_t}$$

is the innovation in percentage points on the present discounted value of primary surpluses. Thus, $\eta_t > 0$ if the discounted value of present and future primary surpluses is higher than the value expected for this variable in period $t - 1$. Otherwise, $\eta_t$ becomes either negative or null and void.

Equation (14) can receive the same interpretation than equation (12).\textsuperscript{12} Particularly, one may conclude, as Buiter (1999, 2001), that any path for $h_t$ and $\pi_t$ satisfying equation (14) could be considered as an equilibrium outcome. But this is not the case because equation (14) is not the only equilibrium restriction to be satisfied by both $\pi_t$ and $h_t$. The monetary rule (6) and, especially, the debt recovery rule (7) also affect the equilibrium outcome. Thus, the objective of the next section is to analyze the extent to which each of these variables may react after a shock to $T_t$. Note however that, whatever the monetary and the recovery rules, the ratio $\pi_t/h_t$ is uniquely determined by equation (14).


\textsuperscript{12}In period $t = 0$, equation (14) becomes $\pi_0/h_0 = \beta R_{-1}/1 + \eta_0$ where $\eta_0 = (T_0 - E_{-1}T_0)/E_{-1}T_0$. 

10
3 Three Scenarios for One Economy

The asymmetric form of both equations (6) and (7) may potentially imply the existence of four regimes, but assumptions 1 and 2 permit to exclude the case where the central bank naturally\(^{13}\) pegs the interest rate to \(\bar{R}\), leading the fiscal authority to default on its outstanding debt. Indeed, this scenario would require \(R_{nd}^t = \bar{R} > \bar{\pi}\) which violates the condition \(\bar{R} < \bar{R} < \bar{R}\). Three scenarios are left.

The two first scenarios correspond to the No-Default case – where \(h_t = 1\) – satisfying \(R_{nd}^t \leq \bar{R}\). Under these scenarios the fiscal authority considers that the potential cost of servicing the whole debt is affordable and so it honors its entire liabilities. The first scenario is characterized by a relatively low current inflation –say, \(\pi_t \leq \hat{\pi}\) – and so the central bank behaves passively by pegging current interest rates to the level \(\bar{R}\). This type of periods are usually called "Tranquil Times". The second scenario is characterized by a relatively high current inflation –say, \(\pi_t > \hat{\pi}\) – where the central bank behaves actively by increasing current interest rates. This scenario corresponds to "Inflation Times" described by Loyo (1999). The third one is the scenario of Sovereign Default – where \(h_t = h_d(\bar{R}_{nd}^t) < 1\) satisfying \(R_{nd}^t > \bar{\pi}\). In this case, the fiscal authority finds that the potential cost of servicing its whole debt is unaffordable. Consequently, it defaults on its liabilities by honoring only a fraction of its financial obligations.

Both "Tranquil Times" and "Inflation Times" are characterized by the absence of sovereign default. Then, the equilibrium level of inflation and interest rates are given by equations (14) and (6) with \(h_t = 1\):

\[
\pi_{nd}^t = \frac{\beta R_{t-1}}{1 + \eta_t} \quad \text{(15a)}
\]

\[
R_{nd}^t = \phi(\pi_{nd}^t) = \begin{cases} 
\bar{R} & \text{if } \pi_{nd}^t \leq \hat{\pi} \\
\bar{R} + \alpha(\bar{\pi}_{nd} - \bar{\pi}) & \text{otherwise} 
\end{cases} \quad \text{(15b)}
\]

Equation (15a) expresses that current inflation is determined by the current fiscal shock, as predicted by the Fiscal Theory of Price Level (FTPL). And equation (15b) expresses that in both No-Default scenarios, the current nominal interest rate is determined by the current inflation level.

3.1 "Tranquil Times"

When the value of the inflation rate (15a) satisfies the condition \(\pi_{nd}^t \leq \hat{\pi}\),

\(^{13}\)By "naturally", we mean: "considering the inflation rate which would be realized in the case of No Default".
Equation (15b) implies that the central bank pegs the interest rate to $R$. So, these periods are characterized by both low current inflation and interest rates. We have:

\[
T_t^T = \frac{\beta R_{t-1}}{1 + \eta_t} \leq \hat{\pi} \tag{16a}
\]
\[
R_t^T = \hat{R} \tag{16b}
\]

where $\pi_t^T$ denotes current inflation rate during Tranquil Times and $R_t^T$ denotes the risky gross nominal interest rate paid by the fiscal authority during Tranquil Times.

Time-$t$ equilibrium is determined as the FTPL determination asserts (See Woodford 1995). The central bank pegs the nominal interest rate to its target and the equilibrium price level is that level that makes the real value of nominally denominated government liabilities equal to the present value of the expected future government budget surpluses.

Both equations (16a) and (16b) are satisfied on condition that $\pi_t^T \leq \hat{\pi}$ which implies:

\[
R_{t-1} \leq (1 + \eta_t) \hat{R} \tag{17a}
\]

remembering that $\hat{R} = \beta^{-1} \hat{\pi}$; or, equivalently:

\[
\eta_t \geq \hat{\eta}_1(R_{t-1}) \tag{17b}
\]

where $\hat{\eta}_1(R)$ is defined as:

\[
\hat{\eta}_1(R) \equiv R/\hat{R} - 1 \tag{18}
\]

Note that, if this scenario applies in period $t - 1$, we have $R_{t-1} = \hat{R}$ and the condition (17b) can be simplified as: $\eta_t \geq \hat{\eta}_1(\hat{R})$ with $\hat{\eta}_1(\hat{R}) < 0$. In this case, Tranquil Times are driven by either positive or not so negative fiscal shocks. It is worth noticing that the negative fiscal shocks must be rather soft. In the case where $R_{t-1}$ verifies $R_{t-1} > \hat{R}$, and especially when $R_{t-1} > \hat{R}$, a positive fiscal shock may be necessary to restore a period of "Tranquil Times".

The deterministic steady state associated to (16a)-(16b) is given by: $\pi^T = \beta \hat{R} \equiv \bar{\pi}$ and $R^T = \hat{R}$. Of course, it verifies $\pi^T \leq \bar{\pi}$ and $R^T < \hat{R}$. This implies that the steady state inflation level is low enough to let the central bank behave passively, while the low steady state level of the interest rate enables the fiscal authority to honor the entire sovereign debt.

Starting from the steady state, the current equilibrium characterizing a Tranquil Time is described by equations (16a) and (16b) on condition that the economy were not hited by hard negative shocks. Then, if in the next period fiscal shock is void, the economy returns to its steady state.
3.2 "Inflation Times"

Compared to the previous case, these periods are characterized by both higher current interest rates and inflation levels. This is linked to the fact that the economy is hit by harder negative fiscal shocks. The current inflation remains defined like in the previous case but it now exceeds the inflation threshold $\hat{\pi}$ and the central bank behaves actively by increasing current interest rates:

$$\pi_t^I = \frac{\beta R_{t-1}}{1 + \eta_t} > \hat{\pi}$$  \hspace{1cm} (19a)

$$R_t^I = \hat{R} + \alpha \left( \frac{\beta R_{t-1}}{1 + \eta_t} - \hat{\pi} \right) < \bar{R}$$  \hspace{1cm} (19b)

where $\pi_t^I$ denotes current inflation rate during inflation times and $R_t^I$ denotes the risky gross nominal interest rate paid by the fiscal authority during inflation times.

This equilibrium is satisfied on condition that $\pi_t^I > \hat{\pi}$ and $R_t^I < \bar{R}$ which implies:

$$(1 + \eta_t) \hat{R} < R_{t-1} < (1 + \eta_t) \left( \hat{R} + \Gamma \right)$$  \hspace{1cm} (20a)

using again $\hat{R} = \beta^{-1} \hat{\pi}$; or, equivalently:

$$\bar{\eta} R_{t-1} < \eta_t < \hat{\eta} (R_{t-1})$$  \hspace{1cm} (20b)

with

$$\Gamma \equiv \frac{\bar{R} - \hat{R}}{\alpha \beta} > 0$$  \hspace{1cm} (21)

and where the function $\bar{\eta} (\cdot)$ is defined by:

$$\bar{\eta} (R) \equiv \frac{R}{R + \Gamma} - 1 < \hat{\eta} (R)$$  \hspace{1cm} (22)

Condition (20b) expresses that a period of inflation is driven by a strictly negative shock which is no longer soft, given the level of $R_{t-1}$. The shock is rather hard but not enough to drive the economy into default.

The deterministic steady state is easily obtained by putting $\eta_t = 0$ and $R_t^I = R_{t-1}$ in equation (19a) and (19b). We obtain:

$$R^I = \frac{\alpha \beta \hat{R} - \bar{R}}{\alpha \beta - 1}$$  \hspace{1cm} (23a)

$$\pi^I = \beta R^I = \frac{\alpha \beta \hat{\pi} - \bar{\pi}}{\alpha \beta - 1}$$  \hspace{1cm} (23b)
This deterministic steady state equilibrium exists on condition that $R^l < \bar{R}$ and $\pi^l > \hat{\pi}$ or, equivalently:

$$\bar{R} < R^l < \bar{R}$$

The left-hand side of the previous inequality is always verified under Assumption 1, and the right-hand side requires the following condition:

**Assumption 3:** $\bar{R} > \bar{R} + \left( \hat{R} - \bar{R} \right) / (\alpha\beta - 1)$

Assumption 3 implies that $\bar{R}$ must be high enough to satisfy $R^l < \bar{R}$. This condition is needed to ensure the existence of a deterministic steady state under a period of inflation.

The (partial) dynamics of these two scenarios is represented on Figure 1 in the case $\eta_t = 0$:

![Figure 1: The No-Default Case](image)

It is worth noticing that, while $R^T = \bar{R}$ is locally stable, given that $\alpha\beta > 1$, $R^l$ is an unstable steady state equilibrium. This means that, depending on the previous value of the nominal interest rate —at the left or at the right from $R^l$—the current interest rate will converge to $\bar{R}$ (if $\eta_t$ is void or small enough), or increase toward $\bar{R}$. Unless a big positive fiscal shocks occurs, the latter scenario inevitably leads to a sovereign default.
In the scenario of "Inflation Times", a previous value of the nominal interest rate higher than \( R \) cause the financial wealth of private agents to grow faster in nominal terms, which calls for higher inflation. Monetary authority responds to higher inflation with sufficiently higher nominal interest rates forming a vicious circle. Usually, hyperinflation is interpreted as a result of the monetary financing of serious fiscal imbalances. However, in this case a fiscalist alternative is presented in which inflation explodes because of the fiscal effects of monetary policy. Most of the action concentrates on the interest rate pays on the government debt and debt rollover instead of seigniorage. This phenomena is known as 'fiscalist hyperinflation' and is the case of Brazil in the late 1970s and early 1980s (See Loyo 1999).

3.3 "Sovereign Default Time"

According to the fiscal authority, the potential cost of servicing the whole debt becomes too high when \( R_{t}^{nd} > \bar{R} \) which implies:

\[
R_{t-1} > (1 + \eta_t) \left( \bar{R} + \Gamma \right)
\]

or, equivalently,

\[
\eta_t < \frac{\bar{R} (R_{t-1})}{R_{t-1}}
\]

This condition shows that for a given level of \( R_{t-1} \), a scenario of Default can be triggered by a hard negative shock or, for a given shock \( \eta_t \), by a high level of the previous nominal interest rate.

As a consequence, the fiscal authority defaults on its debt by honoring only a fraction \( h_t < 1 \) of its liabilities. From equations (14) and (6), current inflation and interest rate become:

\[
\pi_t^D = h_t \frac{\beta R_{t-1}}{1 + \eta_t}
\]

\[
R_t^D = \phi \left( h_t \frac{\beta R_{t-1}}{1 + \eta_t} \right)
\]

Note that without specifying the recovery rule – \( h_t = \bar{h}(R_t^{nd}) < 1 \) – the equilibrium in period \( t \) remains undetermined and defined by equation (25a) and (25b). There is a continuum of recovery rate determining the equilibrium inflation rate and so the nominal interest rate. This result is in line with Buiter's criticism.\(^{14}\) In order to avoid this indeterminacy the fiscal authority has to specify a recovery rule.

\(^{14}\)See Buiter (1999), pp. 50, Proposition 5.
Before introducing such a recovery rule, let us rewrite the system in a simplified form, using (15a) equations (25a) and (25b) can be rewritten as:

\[
\begin{align*}
\pi_t^D &= h_t \pi_t^{nd} = h_t \phi^{-1}(R_t^{nd}) \\
R_t^D &= \phi(h_t \pi_t^{nd}) = \phi(h_t \phi^{-1}(R_t^{nd}))
\end{align*}
\]  

(26a)

(26b)

where the last terms have been obtained by inverting the monetary rule (6).

We now can make the following assumption about the recovery function \(h(R_t^{nd})\):

\[
\tilde{h}(R_t^{nd}) = \frac{\beta \tilde{R}}{\phi^{-1}(R_t^{nd})}
\]  

(27)

Equation (27) shows that the higher is \(R_t^{nd}\), the potential cost of honoring the entire debt, the smaller is the recovery rate. Using the recovery function (27) and the monetary rule (6) in equations (26a) and (26b), the equilibrium values of \(\pi_t^D\) and \(R_t^D\) become:

\[
\begin{align*}
\pi_t^D &= \bar{\pi} \\
R_t^D &= \bar{R}
\end{align*}
\]

Thus, the recovery rule (27) allows the economy, by defaulting on its financial obligations, to reach the stable steady state equilibrium\(^{15}\) in the same period \(t\). The equilibrium value of the recovery rate is:

\[
h_t = \frac{(1 + \eta_t) \bar{R}}{R_{t-1}}
\]  

(28)

4 Inflation and Default Dynamics

This section illustrates the economy dynamics in two different cases of default. In the first one, the current fiscal shock is small but the initial value of the nominal interest rate, \(R_{t-1}\), is high. The economy jumps into an inflation episode which leads the central bank to rise its interest rate and, after three periods, the fiscal authority to default. In the second case, the initial interest rate is at its "Tranquil Times" stationary value: \(\bar{R}\), but the economy experiences a big negative fiscal shock\(^{16}\) which leads very rapidly to a sovereign default.

\(^{15}\) Besides, this recovery rule minimizes the probability of default after the sovereign default. See the next sections on Expected Recovery Rate and Sovereign Risk Spread.

\(^{16}\) Figure 3 and especially 4 are only illustrative because we have to expect that a negative shock - an innovation - has no reason to repeat.
Deciding to default on the government liabilities is a difficult decision for policy makers. This may explain why, in the data, the actual value of the interest rate $\bar{R}$ is greater than what one might expect. This seems to be the case of Argentina in 1989. At the end of 1989, the year in which Argentina defaulted on its debt, the inflation rate had reached a shocking 4923.6\%. Then Argentina had gradually converged to its steady state equilibrium (See Table 1):

### Table 1: Argentine Default, 1989

<table>
<thead>
<tr>
<th></th>
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<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>81,9</td>
<td>174,8</td>
<td>387,7</td>
<td><strong>4923,6</strong></td>
<td>1343,9</td>
<td>84,0</td>
<td>17,5</td>
<td>7,4</td>
<td>3,9</td>
<td>1,6</td>
</tr>
</tbody>
</table>

Source: Indec

According to our model, the fiscal authority was both too tolerant and patient, *i.e.* $\bar{R}$ was too high. Moreover, Argentina in 1989 could minimize the recovery rate on its debt in order to reach faster the (without inflation) steady state equilibrium. To explain the gradual decline of inflation, it is necessary to modify the recovery rule slightly. Suppose that the rule is now defined by:

$$h^*(R_t^{nd}) \equiv \frac{\beta R^*}{\phi^{-1}(R_t^{nd})}$$  \hspace{1cm} (29)
with $\hat{R} < R^* < R^I$.

The recovery rule (29) and the condition $\hat{R} < R^* < R^I$ are well specified in order to ensure a post-default equilibrium which drives progressively the economy to "Tranquil Times", on condition that future fiscal shocks are small enough.

This case is represented on Figure 2.4:

![Figure 4: Argentinean soft-landing](image)

This policy has the double advantage of reducing the size of the sovereign default necessary to restore the public solvency and to smooth the return toward price stability. On the other hand, this recovery rule does not minimize the probability of a new default after the first sovereign default. For the sake of simplicity, we will adopt in the rest of the paper the simpler assumption $R^* = \hat{R}$.

5 Expected Recovery Rate and Sovereign Risk Premium

In this section, we make simplifying assumptions on the fiscal shock distribution and we show that, once the Fiscal Default Rule is known\(^{17}\), the one-period Expected Recovery Rate, $E_t h_{t+1}$, and the Relative Sovereign Risk Premium, $\left( R_t - R^I_t \right) / R^I_t$, can be explicitly calculated. Note that the period-\(t\) probability of default in \(t+1\) is simply given by: $F \left( \bar{\eta} (R_t) \right)$.

The three scenarios previously described are summarized by the fol-

\(^{17}\)Which is summarized by the choice of $\bar{R}$ and $R^*$ (=$\hat{R}$ in our case).
lowing table:

Table 2: The Three Scenarios

<table>
<thead>
<tr>
<th>Tranquil Times</th>
<th>Inflation Times</th>
<th>Sovereign Default Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\eta} (R_{t-1}) &lt; \eta_t )</td>
<td>( \bar{\eta} (R_{t-1}) &lt; \eta_t &lt; \hat{\eta} (R_{t-1}) )</td>
<td>( \eta_t &lt; \bar{\eta} (R_{t-1}) )</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
 h_t : & \quad 1 \\
 \pi_t : & \quad \frac{\beta R_{t-1}}{1+\eta_t} \\
 R_t : & \quad \hat{R} \\
 & \quad \bar{R} + \alpha \beta \left( \frac{R_t}{1+\eta_t} - \hat{R} \right) \\
 & \quad \bar{R}
\end{align*}
\]

The conditions that determine the current regime are entirely defined by the couple of states variables \((R_{t-1}, \eta_t)\). Let \( F(\eta) \) defines the distribution function of the fiscal shocks, and \( \bar{\eta} > 0 \) and \( -\bar{\eta} \), respectively, the upper and lower bound of the compact set on which the shock is distributed. The Figure 2.5 summarizes these conditions:

Note that the period-\( t-1 \) probability of default in \( t \) is simply given by: \( F\left(\bar{\eta} (R_{t-1})\right) \), the probability of a "Tranquil Times" period by: \( 1 - F(\hat{\eta} (R_{t-1})) \), and the probability of an "Inflation Times" episode
by: $F(\bar{\eta}(R_{t-1})) - F(\bar{\eta}(RR_{t-1}))$. Then, if $R_{t-1} < b$, the ex ante probability of a sovereign default in period $t$ is null and, when $R_{t-1} < a$, the probability of a "Tranquil Times" period equals unity.

5.1 Expected Recovery Rate

Using Table 1, the one-period Expected Recovery Rate can be written:

$$E_t h_{t+1} = \int_{\bar{\eta}}^{\bar{\eta}(R_t)} \frac{(1 + \eta) \bar{R}}{R_t} dF(\eta) + \int_{\bar{\eta}(R_t)}^{\bar{\eta}} 1 dF(\eta)$$

$$= 1 - \frac{R_t - \bar{R}}{R_t} F(\bar{\eta}(R_t)) + \frac{\bar{R}}{R_t} \int_{\bar{\eta}}^{\bar{\eta}(R_t)} \eta dF(\eta)$$

(30)

Notice that the default probability, $F(\bar{\eta}(R_t))$, is strictly positive (Resp. null) and that $\int_{\bar{\eta}}^{\bar{\eta}(R_t)} \eta dF(\eta)$ is strictly negative (Resp. null) if $\bar{\eta}(R_t) > \bar{\eta}$ (Resp. $\bar{\eta}(R_t) \leq \bar{\eta}$). One can conclude that the one-period expected recovery rate verifies: $E_t h_{t+1} < 1$ for $\bar{\eta}(R_t) > \bar{\eta}$ and $E_t h_{t+1} = 1$ otherwise. Starting from the "Tranquil Times" steady state, i.e. $R_t = \bar{R}$, we have $\bar{\eta}(\bar{R}) = (\bar{R} + (\bar{R} - \bar{R}) / \alpha \beta)^{-1} \bar{R} - 1 < 0$ and (30) can be simplified as:

$$E_t h_{t+1} = 1 + \int_{\bar{\eta}}^{\bar{\eta}(R_t)} \eta dF(\eta)$$

which then verifies $E_t h_{t+1} < 1$ for $\bar{\eta}(\bar{R}) > \bar{\eta}$ and $E_t h_{t+1} = 1$ otherwise.

5.2 Sovereign Risk Premium

From (8) and (9), the relative sovereign risk premium can be defined by:

$$\frac{R_t - \bar{R}_t}{R_t} = \beta^{-1} \left[ E_t \frac{h_{t+1}}{\pi_{t+1}} \right]^{-1} - 1 = \frac{E_t \frac{1}{\pi_{t+1}}}{E_t \frac{h_{t+1}}{\pi_{t+1}}} - 1$$

Using again the results in Table 1, this expression becomes:

$$\frac{R_t - \bar{R}_t}{R_t} = \int_{-\bar{\eta}}^{\bar{\eta}(R_t)} (\beta \bar{R})^{-1} dF(\eta) + \int_{\bar{\eta}(R_t)}^{\bar{\eta}} (\frac{\beta \bar{R}_t}{1 + \eta})^{-1} dF(\eta)$$

$$= \int_{-\bar{\eta}}^{\bar{\eta}(R_t)} \frac{R_t}{\bar{R}} dF(\eta) + \int_{\bar{\eta}(R_t)}^{\bar{\eta}} (1 + \eta) dF(\eta) - 1$$

$$= \frac{R_t - \bar{R}}{\bar{R}} F(\bar{\eta}(R_t)) + \int_{\bar{\eta}(R_t)}^{\bar{\eta}} \eta dF(\eta)$$

(31)
with \( \int_{\bar{\eta}(R_t)}^{\bar{\eta}(R_t)} \eta dF(\eta) > 0 \) when \( \bar{\eta}(R_t) > -\bar{\eta} \) and \( \int_{\bar{\eta}(R_t)}^{\bar{\eta}(R_t)} \eta dF(\eta) = 0 \) otherwise. One concludes that the relative sovereign risk premium is strictly positive for \( \bar{\eta}(R_t) > -\bar{\eta} \) and null otherwise. At the "Tranquil Times" steady state, \( i.e. R_t = \bar{R} \), equation (31) simplifies to:

\[
\frac{\bar{R} - R^f_t}{R^f_t} = \int_{\bar{\eta}(\bar{R})}^{\bar{\eta}(R_t)} \eta dF(\eta)
\]

which is strictly positive for \( -\bar{\eta} < \bar{\eta}(\bar{R}) \) and null otherwise.

### 5.3 Calibration and Simulation

For sake of simplicity, we will assume that fiscal (relative) innovations are uniformly distributed: \( F(\eta) = (\eta + \bar{\eta}) / 2\bar{\eta} \). The one-period Expected Recovery Rate is given by equation (30) which can be rewritten, for \( -\bar{\eta} < \bar{\eta}(R_t) \) (and

\[
E_{l+1} = \frac{\bar{R}}{R_t} \left[ \frac{\bar{\eta}(R_t) + \bar{\eta}}{2\bar{\eta}} + \frac{1}{2} \left( \frac{\bar{\eta}(R_t)^2 - \bar{\eta}^2}{2\bar{\eta}} \right) \right] + 1 - \frac{\bar{\eta}(R_t) + \bar{\eta}}{2\bar{\eta}}
\]

and the one-period Sovereign Risk Premium is:

\[
R_t - R^f_t = \frac{R_t - \bar{R}}{R} \left( \frac{\bar{\eta}(R_t) + \bar{\eta}}{2\bar{\eta}} \right) + \frac{\bar{\eta}^2 - \bar{\eta}(R_t)^2}{4\bar{\eta}}
\]

We can easily illustrate our results by adopting the following annual calibration for the model’s parameters:

<table>
<thead>
<tr>
<th>Definition</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor:</td>
<td>( \beta )</td>
<td>0.95</td>
</tr>
<tr>
<td>Taylor coefficient:</td>
<td>( \alpha )</td>
<td>1.5</td>
</tr>
<tr>
<td>Interest rate target:</td>
<td>( \bar{R} )</td>
<td>1.05</td>
</tr>
<tr>
<td>Monetary threshold:</td>
<td>( \bar{R} )</td>
<td>1.10</td>
</tr>
<tr>
<td>Fiscal threshold:</td>
<td>( \bar{R} )</td>
<td>1.50</td>
</tr>
<tr>
<td>Upper bound of the distribution function:</td>
<td>( \bar{\eta} )</td>
<td>0.15</td>
</tr>
</tbody>
</table>

We can firstly calculate the lower threshold value of \( R_t \) for which \( F(\bar{\eta}(R_t)) \geq 0 \). The solution is 1.177, which is superior to \( \bar{R} = 1.05 \) (and
even superior to $\hat{R} = 1.10$). This implies that, starting from the "Tranquil Times" Steady State in period $t$, the probability for the Government to default on its debt in $t+1$ is always null. After this calibration, a Sovereign Default cannot be observed without a period of growing inflation. Consequently, an aggressive central bank fighting against inflation does not go without costs. It increases the fiscal burden of the government debt as well as the sovereign risk of default. A higher current interest rate increases the current probability of default which is captured by the current sovereign risk premium.

The resulting values for the fiscal default threshold, $\bar{\eta}(R_t)$, the default probability $F\left(\bar{\eta}(R_t)\right)$, the Expected Recovery Rate, $E_t h_{t+1}$, and the one-period Sovereign Risk Premium $\left(R_t - R_t^f\right) / R_t^f$ are represented as functions of the current interest rate $R_t$.

These Graphs show that when $R_t \leq 1.177$, even the hardest the negative fiscal shock—say $\bar{\eta} = -0.15$—does not drive the economy into default. Thus, the Probability of Default is null, the Expected Recovery Rate is equal to one, and so the Relative Risk Premium is void. Otherwise, when $R_t > 1.177$ there are (negative) values for the fiscal shock.
that might drive the economy into default. Thus, the Probability of Default becomes positive, the Expected Recovery Rate becomes lower than the unity, and the Relative Risk Premium positive. The higher is $R_t$, the higher are both the Probability of Default and Relative Risk Premium and the lower the Expected Recovery Rate.

This finding contrasts with that of Uribe (2006)’s and is in line with the empirical evidence and estimates presented on Andritsky (2005) and Sosa Navarro (2005).

6 Further Research and Discussion

This section presents a work in progress. It provides interesting findings and contributes to the discussion over the monetary policy on inflation as well as sovereign default dynamics.

As before, assume that the monetary policy takes the form of an interest-rate feedback rule whereby the short-term nominal interest rate is set as a function of inflation. But unlike the previous case, assume that the interest rate controlled by the central bank is the risk-free nominal interest rate, $R^f_t = 1/E_tQ_{t,t+1}$, and not the interest rate paid on government debt, $R_t$.

Then, the central bank behavior can be expressed as,

$$R^f_t = \phi(\pi_t) = \begin{cases} \bar{R} & \text{if } \pi_t \leq \hat{\pi} \\ \bar{R} + \alpha (\pi_t - \hat{\pi}) & \text{otherwise} \end{cases}$$

with $\alpha > \beta^{-1}$

(32)

where $\bar{R} = \beta^{-1}\bar{\pi}$ is the stationary value of the gross nominal interest rate associated to the inflation target $\bar{\pi}$, and $\hat{\pi}$ represents an inflation threshold. It will be useful to define: $\bar{R} > \bar{R}$, or, equivalently: $\hat{\pi} > \bar{\pi}$.

Therefore, the equilibrium can be defined as:

**Definition 2** A rational expectations competitive equilibrium is defined as a set of processes $\{\pi_t, b_t, R_t, R^f_t, h_t\}_{t=0}^{\infty}$ satisfying equations (8), (9), (10), (11), the monetary rules (32), the debt recovery rule (7), and the exogenous process for the primary surpluses $\{\tau_t\}_{t=0}^{\infty}$ where $R_{t-1}b_{t-1}$ are given and the recovery rate satisfies $h_t \in (0, 1)$.

---

This assumption seems to be more in accordance with the cashless economy framework that we have chosen. Indeed, this framework does not explicitly take into account the open market interventions by the central bank on the government securities market and does not facilitate an explanation of the control of the interest rate $R_t$ by monetary authorities.
6.1 The Three Scenarios for this Economy

The asymmetric form of both equations (32) and (7) may potentially imply the existence of four regimes, but assumptions 1 and 2 permit to exclude the case where the central bank naturally pegs the interest rate to $\hat{R}$, leading the fiscal authority to default on its outstanding debt. Three scenarios are left.

Both "Tranquil Times" and "Inflation Times" are characterized by the absence of sovereign default. Then, the equilibrium level of inflation and interest rates are given by equations (14) and (32) with $h_t = 1$:

\[ \pi_t^{nd} = \frac{\beta R_{t-1}}{1 + \eta_t} \quad (33a) \]

\[ R_t^f = \phi \left( \pi_t^{nd} \right) = \begin{cases} \hat{R} & \text{if } \pi_t^{nd} \leq \hat{\pi} \\ \frac{\hat{R}}{R + \alpha (\pi_t^{nd} - \hat{\pi})} & \text{otherwise} \end{cases} \quad (33b) \]

The third one is the scenario of Sovereign Default – where $h_t < 1$. In this case, the fiscal authority finds that the potential cost of servicing its whole debt is unaffordable. Consequently, it defaults on its liabilities by honoring only a fraction of its financial obligations.

6.1.1 "Tranquil Times"

When the value of the inflation rate (33a) satisfies the condition $\pi_t^{nd} \leq \hat{\pi}$, equation (33b) implies that the central bank pegs the risk-free interest rate to $\hat{R}$. Also, these periods are characterized by both low current inflation and interest rates. We have:

\[ \pi_t^T = \frac{\beta R_{t-1}}{1 + \eta_t} \leq \hat{\pi} \quad (34a) \]

\[ R_t^{f,T} = \hat{R} \quad (34b) \]

where $R_t^{f,T}$ denotes the current risk-free nominal interest rate in "Tranquil Times".

Both equations (34a) and (34b) are satisfied on condition that $\pi_t^T \leq \hat{\pi}$ which implies:

\[ R_{t-1} \leq (1 + \eta_t) \hat{R} \quad (35a) \]

remembering that $\hat{R} = \beta^{-1} \hat{\pi}$; or, equivalently:

\[ \eta_t \geq \hat{\eta} (R_{t-1}) \quad (35b) \]

where $\hat{\eta} (R)$ is defined as:

\[ \hat{\eta} (R) \equiv \frac{R}{\hat{R}} - 1 \quad (36) \]

\(^{19}\)By "naturally", we mean: "considering the inflation rate which would be realized in the case of No Default".
6.1.2 "Inflation Times"

Compared to the previous case, these periods are characterized by both higher current interest rates and inflation levels. And this is linked to the fact that the economy is hit by harder negative fiscal shocks. The current inflation remains defined like in the previous case but now it exceeds the inflation threshold $\hat{\pi}$ and the central bank behaves actively by increasing the current interest rate:

$$\pi_t^I = \frac{\beta R_{t-1}}{1 + \eta_t} > \hat{\pi}$$  \hspace{1cm} (37a)$$

$$R_t^{f,I} = \bar{R} + \alpha \left( \frac{\beta R_{t-1}}{1 + \eta_t} - \hat{\pi} \right) < \bar{R}^f$$  \hspace{1cm} (37b)$$

where $R_t^{f,I}$ denotes the current risk-free nominal interest rate in "Inflation Times".

This equilibrium is satisfied on condition that $\pi_t^I > \hat{\pi}$ and $R_t^{f,I} < \bar{R}^f$ which implies:

$$(1 + \eta_t) \hat{R} < R_{t-1} < (1 + \eta_t) \left( \bar{R} + \Gamma \right)$$  \hspace{1cm} (38a)$$

using again $\hat{R} = \beta^{-1} \hat{\pi}$; or, equivalently:

$$\eta(R_{t-1}) < \eta_t < \hat{\eta}(R_{t-1})$$  \hspace{1cm} (38b)$$

with

$$\Gamma \equiv \frac{\bar{R} - \hat{R}}{\alpha \beta} > 0$$  \hspace{1cm} (39)$$

and where the function $\bar{\eta}(\cdot)$ is defined by:

$$\bar{\eta}(R) \equiv \frac{R}{\hat{R} + \Gamma} - 1 < \hat{\eta}(R)$$  \hspace{1cm} (40)$$

Condition (38b) expresses that a period of inflation is driven by a strictly negative shock which is no longer soft, given the level of $R_{t-1}$. The shock is rather hard but not enough to drive the economy into default.

6.1.3 "Sovereign Default Time"

According to the fiscal authority, the potential cost of servicing the whole debt becomes too high when $R_t > \bar{R}$, i.e., $R_t^{f} > \bar{R}^f$ which implies:

$$R_{t-1} > (1 + \eta_t) \left( \hat{R} + \Gamma \right)$$  \hspace{1cm} (41a)$$
or, equivalently,

\[ \eta_t < \bar{\eta} (R_{t-1}) \]  \hspace{1cm} (41b)

This condition shows that for a given shock \( \eta_t \), a scenario of Default can be triggered by a high level of the previous nominal interest rate or, for a given level of \( R_{t-1} \), by a hard negative fiscal shock.

As a consequence, the fiscal authority defaults on its debt by honoring only a fraction \( h_t < 1 \) of its liabilities. From equations (14) and (32), current inflation and risk free interest rates become:

\[ \pi_t^D = h_t \pi_t^{nd} \]  \hspace{1cm} (42a)
\[ R_{t}^{f,D} = \phi \left( h_t \pi_t^{nd} \right) \]  \hspace{1cm} (42b)

where \( \pi_t^{nd} \) is always defined by (33a) as the equilibrium inflation rate in the no default case.

Note that without specifying the recovery rule \( -h_t = h(\pi_t^{nd}) < 1 \) – the equilibrium in period \( t \) remains undetermined and defined by equation (42a) and (42b).\(^{20}\) There is a continuum of recovery rate determining the equilibrium inflation rate and so the nominal interest rate. This result is line with Buiter’s critic.\(^{21}\) In order to avoid this indeterminacy the fiscal authority has to specify a recovery rule. We now can make the following assumption about the recovery function \( h(\pi_t^{nd}) \):

\[ h(\pi_t^{nd}) \equiv \frac{\pi^*}{\pi_t^{nd}} \]  \hspace{1cm} (43)

where \( \pi^* \) verifies:

**Assumption 4:** \( \pi^* < \beta \left( \bar{R} + \Gamma \right) = \bar{\pi} + \left( \bar{R}^f - \bar{R} \right) / \alpha. \)

Equation (43) shows that, for a given value of \( \pi^* \), the higher is the potential inflation rate \( \pi_t^{nd} \), the smaller is the recovery rate. Using the recovery function (43) and the monetary rule (32) in equations (42a) and (42b), the equilibrium values of \( \pi_t^D \) and \( R_{t}^{f,D} \) become:

\[ \pi_t^D = \pi^* \]
\[ R_{t}^{f,D} = \phi (\pi^*) \]

\(^{20}\)We suppose that such a rule is a (non increasing) function of the nominal interest rate, denoted \( R_{t}^{nd} \), to be determined by the monetary authority in the No-Default case. Then, \( R_{t}^{nd} = \phi (\pi_t^{nd}) \) represents the potential cost of honoring the whole debt in the future.

\(^{21}\)See Buiter (1999), pp. 50, Proposition 5.
Thus, the recovery rule (43) allows the economy, by defaulting on its financial obligations, to reach a less inflationary equilibrium in the same period \( t \). Note that if \( \pi^* \leq \hat{\pi} \), the monetary rule (32) implies \( R_{t,D}^f = \hat{R} \).

Using (33a) in equation (43), the equilibrium value of the recovery rate is:

\[
h_t = \frac{(1 + \eta_t) \pi^*}{\beta R_{t-1}}
\]  

(44)

Assumption 4 ensures that this recovery rate is always inferior to unity.

### 6.2 Expected Recovery Rate, Sovereign Risk Premium and Interest Rates

It is possible to express \( R_t \) as an invertible function of \( R_f^t \), and hence \( \bar{R}^f \) as a function of \( \bar{R} \). Let \( P_t = R_t / R_f^t \) denotes the gross sovereign risk premium. From (8) and (9), \( P_t \) can be defined by:

\[
P_t = \frac{R_t}{R_f^t} = \frac{E_t \hat{h}_{t+1}}{E_t \hat{h}_{t+1} + (1 + \eta_t) \pi^*}
\]  

(45)

Now lets make simplifying assumptions about the fiscal shock distribution and we will see that, once the Fiscal Default Rule is known - which is summarized by the choice of \( \bar{R}^f \) and \( R^* \) - the sovereign risk premium, \( P_t = R_t / R_f^t \), and the one-period expected recovery rate, \( E_t \hat{h}_{t+1} \), can be calculated. Let us begin by summarizing our results with the following table:

<table>
<thead>
<tr>
<th>Table 4: the three scenarios</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tranquil Times</strong></td>
</tr>
<tr>
<td>( \hat{h} \left( R_{t-1} \right) &lt; \eta_t )</td>
</tr>
<tr>
<td>( h_t )</td>
</tr>
<tr>
<td>( \pi_t )</td>
</tr>
<tr>
<td>( R_f^t )</td>
</tr>
<tr>
<td>( \phi (\pi^*) )</td>
</tr>
</tbody>
</table>

where \( \hat{h} (R) = R / \hat{R} - 1 \) and \( \overline{\hat{h}} (R) \equiv R / \left( \hat{R} + \frac{\bar{R}^f - \bar{R}}{\alpha \beta} \right) - 1 \) and with \( \bar{R} < \hat{R} < \bar{R}^f \) and \( \phi (\pi^*) < \bar{R}^f \).
6.2.1 Expected Recovery Rate

Using the results presented in the first row of Table 1, the one-period Expected Recovery Rate can be written:

\[ E_{t+1} = \int_{-\bar{\eta}}^{\bar{\eta}(R_t)} \frac{(1 + \eta) \pi^*}{\beta R_t} dF(\eta) + \int_{\bar{\eta}(R_t)}^{\pi} 1 dF(\eta) \]

\[ = 1 - \int_{-\bar{\eta}}^{\bar{\eta}(R_t)} \left( 1 - \frac{(1 + \eta) \pi^*}{\beta R_t} \right) dF(\eta) \]

(46)

where the last term after the sign of subtraction is positive under Assumption 4 for \( \bar{\eta}(R_t) > -\bar{\eta} \) and null otherwise.

One can conclude that the Expected Recovery Rate verifies \( E_{t+1} < 1 \) for \( \bar{\eta}(R_t) > -\bar{\eta} \) and \( E_{t+1} = 1 \) otherwise. Identically, let us define \( R_\eta \) such that \( \bar{\eta}(R_\eta) = -\bar{\eta} \). According to equation (38a), we have:

\[ R_\eta = (1 - \bar{\eta})(R + \Gamma) \]

and we know that \( E_{t+1} < 1 \) if \( R_t > R_\eta \).

6.2.2 Sovereign Risk Premium

Using now the second row of Table 4 and equation (45), the gross sovereign risk premium - or country risk spread - \( \mathcal{P}_t = R_t / R^f_t = E_{t+1} / R_t \), can be written:

\[ \mathcal{P}_t = \frac{\int_{-\bar{\eta}}^{\bar{\eta}(R_t)} (\pi^*)^{-1} dF(\eta) + \int_{\bar{\eta}(R_t)}^{0} \left( \frac{\beta R_t}{1 + \eta} \right)^{-1} dF(\eta)}{\int_{-\bar{\eta}}^{\bar{\eta}(R_t)} \left( \frac{\beta R_t}{1 + \eta} \right)^{-1} dF(\eta)} \]

\[ = 1 + \int_{-\bar{\eta}}^{\bar{\eta}(R_t)} \left( \frac{\beta R_t}{\pi^*} - (1 + \eta) \right) dF(\eta) \equiv \mathcal{P}(R_t) \]

(47)

where the last term after the sign of addition is positive under Assumption 4 for \( \bar{\eta}(R_t) > -\bar{\eta} \) and null otherwise. So, we can conclude that \( \mathcal{P}_t > 1 \) if \( R_t > R_\eta \).

We can easily obtain the derivative of the function \( \mathcal{P}(R_t) \) for \( R_t > R_\eta \):

\[ \mathcal{P}'(R_t) = \frac{\beta}{\pi^*} F\left( \bar{\eta}(R_t) \right) + \left( \frac{\beta}{\pi^*} - \frac{1}{(\hat{R} + \Gamma)} \right) R_t F'(\bar{\eta}(R_t)) > 0 \]

where we have used equation (40) and (39). Assumption 4 insures that this derivative is always positive.
Using again the definition of the gross sovereign risk premium: \( P_t = R_t / R_{tI} \) and equation (47), one can establish a link between \( R_t \) and \( R_{tI} \):
\[
R_{tI} = G(R_t) \equiv \frac{R_t}{P(R_t)} \tag{48}
\]
where the function \( G(R_t) \) has a derivative given by:
\[
G'(R_t) = \frac{1}{P(R_t)} - \frac{R_t P'(R_t)}{P(R_t)^2}
\]
which verifies: \( 0 < G'(R_t) < 1 \) if \( R_t > R_\eta \) and \( G'(R_t) = 1 \) otherwise.

Equation (48) implicitly permits to determine the interest rate on government securities as a function of the riskless interest rate set by the central bank. By inverting the function \( G(\cdot) \), one finds:
\[
R_t = G^{-1}(R_{tI}) \equiv g(R_{tI}) \tag{49}
\]
where the function \( g(R_{tI}) \) verifies: \( g'(R_{tI}) = 1 / G'(G^{-1}(R_{tI})) > 1 \) if \( R_t > R_\eta \) and \( g'(R_{tI}) = 1 \) otherwise.

### 6.2.3 Riskless and Risky Interest Rates

Using finally the last row of Table.4 and equation (49), one can express the risky sovereign debt interest rate:
\[
R_t = \begin{cases}
g\left(\hat{R}\right) & \text{if } R_{t-1} < (1 + \eta_t) \hat{R} \\ g\left(\hat{R} + \alpha \beta \left(\frac{R_{t-1}}{1+\eta_t} - \hat{R}\right)\right) & \text{if } (1 + \eta_t) \hat{R} < R_{t-1} < (1 + \eta_t) \left(\hat{R} + \Gamma\right) \\ g\left(\phi\left(\pi^*\right)\right) & \text{if } R_{t-1} > (1 + \eta_t) \left(\hat{R} + \Gamma\right)
\end{cases}
\tag{50}
\]

Let us define \( R_{tI} \) such that: \( R_{tI} = g\left(\hat{R} + \alpha \beta \left(R_{tI} - \hat{R}\right)\right) \) and suppose that \( \hat{R} < R_{tI} < \hat{R} + \Gamma \). By evaluating the derivative \( \partial R_t / R_{t-1} \) for \( R_{t-1} = R_{tI} \) when \( \eta_t = 0 \), we obtain:
\[
\left. \frac{\partial R_t}{\partial R_{t-1}} \right|_{R_{t-1}=R_{tI},\eta_t=0} = \alpha \beta g'\left(R_{tI}\right) > 1
\]

We now can represent \( R_{tI} \) and \( R_t \) as function of \( R_{t-1} \) in the case \( \eta_t = 0 \):
Note that the assumption \( \alpha \beta > 1 \) is sufficient but unnecessary to guarantee \( \alpha \beta g' (R^{f,t}) > 1 \).

**Corollary 3** In presence of a default risk, the monetary policy can be active even in the case \( \alpha \beta < 1 \).

### 7 Conclusion

The main goal of this paper is to characterize the way in which monetary policy affects the equilibrium behavior of recovery rate and sovereign risk premiums. This is an issue which has been fairly disregarded by recent monetary theory. The framework of analysis proposed in this paper offers an additional perspective to discuss the possible interrelations between monetary and fiscal policy and provides supplementary advantages as regards other settings. It allows to overcome some difficulties like the negative default rate which arises as a consequence of positive fiscal shocks – recall the ‘super-solvency premium’ in terms of Buiter. This model characterizes the way in which monetary policy affects the equilibrium behavior of price level, recovery rate and sovereign risk premiums. Indeed, in some cases, even a ‘passive’ interest rate rule might drive the economy to an unsustainable path without defaulting. It means that in presence of a default risk, the monetary policy can be active even in the case where \( \alpha \beta < 1 \). It turns out that monetary policy plays a significant role in shaping the equilibrium behavior of default and risk premiums. Both the Probability of Default and Sovereign Risk Premium are consistent with the empirical estimates presented in Andritsky (2005) and Sosa Navarro (2005).
It also underlines the fact that the size of the equilibrium default rate matters for the post-equilibrium dynamics. The size of the equilibrium default rate cannot be so high in order to ensure a post-equilibrium dynamics without defaulting. The model explicitly emphasizes the role of the government (the fiscal authority) in resolving the financial crisis.

Even though the current framework can be extended in different directions, these have been left aside to simplify the exposition. For instance, it can be assumed that a fraction of the public debt is indexed. High inflation economies tend to develop an extensive system of indexed contracts. It is worth noticing that bonds linked to price indices are not ‘real’ bonds because sampling and computing price indices involve time. The nominal value of indexed bonds is typically adjusted according to lagged inflation rates. Otherwise, it could be assumed that public debt is denominated in foreign currency. These are important characteristics of actual emerging economies that would be worthwhile incorporating.
8 Appendix

Multiplying both sides of equation (5) by \( R_t h_{t+1} \) as \( B_t R_t h_{t+1} = B_{t-1} R_{t-1} h_t R_t h_{t+1} - \tau_t P_t R_t h_{t+1} \). Then, iterating the last expression \( j \) times, it results in:

\[
R_{t+j} B_{t+j} h_{t+j+1} = R_{t-1} B_{t-1} h_t \left( \prod_{h=0}^{j} R_{t+h} h_{t+h+1} \right) - \sum_{h=0}^{j} P_{t+h} \tau_{t+h} \left( \prod_{k=h}^{j} R_{t+k} h_{t+k+1} \right) \quad (51)
\]

Dividing both sides of equation (51) by \( P_{t+j+1} \) see that \( P_{t+j+1} \) can also be written as \( P_{t+j+1} = P_t \frac{P_{t+1}}{P_t} \frac{P_{t+2}}{P_t} \ldots \frac{P_{t+j}}{P_t} \frac{P_{t+j+1}}{P_{t+j}} \) it turns out that,

\[
\frac{R_{t+j} B_{t+j}}{P_{t+j+1}} h_{t+j+1} = \frac{R_{t-1} B_{t-1}}{P_t} h_t \left( \prod_{h=0}^{j} \frac{R_{t+h} h_{t+h+1} P_{t+h}}{P_{t+h+1}} \right) - \sum_{h=0}^{j} \tau_{t+h} \left( \prod_{k=h}^{j} \frac{R_{t+k} h_{t+k+1} P_{t+k}}{P_{t+k+1}} \right) \quad (52)
\]

Applying the conditional expectations operator \( E_t \), equation (52) becomes written,

\[
E_t \frac{R_{t+j} B_{t+j}}{P_{t+j+1}} h_{t+j+1} = \frac{R_{t-1} B_{t-1}}{P_t} h_t E_t \left( \prod_{h=0}^{j} \frac{R_{t+h} h_{t+h+1} P_{t+h}}{P_{t+h+1}} \right) - E_t \sum_{h=0}^{j} \tau_{t+h} \left( \prod_{k=h}^{j} \frac{R_{t+k} h_{t+k+1} P_{t+k}}{P_{t+k+1}} \right) \quad (53)
\]

Next, by applying the law of iterated expectations and using equation (9) equation (53) remains expressed as,

\[
E_t \frac{R_{t+j} B_{t+j}}{P_{t+j+1}} h_{t+j+1} = \beta^{-j-1} \frac{R_{t-1} B_{t-1}}{P_t} h_t - \sum_{h=0}^{j} \beta^{-(j-h+1)} E_t \tau_{t+h} \quad (54)
\]

Dividing both sides of equation (54) by \( \beta^{-j} \) and then taking the limit for \( j \rightarrow \infty \) it turns out,
\[
\lim_{j \to \infty} \beta^j E_t \frac{R_{t+j} B_{t+j}}{P_{t+j+1}} h_{t+j+1} = \beta^{-1} \frac{R_{t-1} B_{t-1}}{P_t} h_t
\]

\[
- \sum_{h=0}^{\infty} \beta^h E_t \tau_{t+h}
\] (55)

See that defining \( T = t + j + 1 \), the left hand-side of equation (55) can be expressed as
\[
\lim_{T \to \infty} \beta^{T-j-1} E_t \frac{R_{T-1} B_{T-1}}{P_{T-1}} h_T = 0.
\]
Then, multiplying this expression by \( \beta \), it remains expressed as equation (10) which is equal to zero. So equation (55) results in,
\[
\frac{R_{t-1} B_{t-1}}{P_t} h_t = \sum_{h=0}^{\infty} \beta^h E_t \tau_{t+h} \quad \forall t
\]
References


