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Asset prices and monetary policy in a sticky-price economy with financial frictions

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Abstract

A recent study shows that equilibrium indeterminacy arises if monetary policy responds to asset prices, especially share prices, in a sticky-price economy. We show that equilibrium indeterminacy never arises if the working capital of firms is subject to their asset values by financial frictions.

Keywords: asset prices; financial frictions; equilibrium indeterminacy; monetary policy; sticky prices

JEL classification: C62; E32; E44; E52

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1 Introduction

Should monetary policy respond to asset prices? A large number of studies have attempted to address this question. For example, the insignificance of responding to asset prices is reminiscent of the findings of Bernanke and Gertler (2001) and Gilchrist and Leahy (2002). Iacoviello (2005) shows that little is gained by responding to asset prices, if the central bank wants to minimize output and inflation fluctuations. Faia and Monacoelli (2007) find a case where monetary policy should respond to increases in asset prices by lowering the nominal interest rate.

A recent paper by Carlstrom and Fuerst (2007) provides a negative answer: equilibrium indeterminacy arises if monetary policy responds to asset prices in a sticky-price economy. While many previous studies employ prices of capital as asset prices, Carlstrom and Fuerst (2007) focus on share prices that reflect firms’ profits. In their model, an increase in inflation reduces firms’ profits and asset prices decline. Then, monetary policy responding to asset prices, or share prices, implicitly weakens overall reactions to inflation. This is the source of equilibrium indeterminacy in their model.

In this paper, we show that equilibrium indeterminacy never arises if there is credit market imperfection. We introduce a collateral constraint to the economy of Carlstrom and Fuerst (2007). The working capital or wage payment of firms is subject to a collateral constraint in our economy. In our economy, as in the economy of Carlstrom and Fuerst (2007), an increase in inflation reduces firms’ profits. However, share prices do not change since the inefficiency of the collateral constraint increases and the premium of shares as collateral increases.

Our result implies that under credit market imperfection, there is no negative aspect of monetary policy responding to asset prices, as pointed out by Carlstrom and Fuerst (2007). Since the discussion on monetary policy responding to asset prices often arises during recessions associated with financial crises, for example, Japan’s lost decade of the 1990s and the recent financial crisis in the U.S. economy, our result for the economy with financial frictions would contribute to the literature on monetary policy.

Collateral constraints are often employed to account for the observed facts of business

The rest of this paper is organized as follows. Section 2 introduces our basic economy with a collateral constraint. Section 3 presents our main results: equilibrium indeterminacy never arises even if monetary policy responds to asset price fluctuations under credit market imperfection. Section 4 conclude the paper.

2 The model

Our model is based on that employed by Carlstrom and Fuerst (2007). One difference from their model is that the collateral constraint on working capital. In order to introduce the collateral constraint, the environment of our economy is slightly different from that of Carlstrom and Fuerst (2007). However, equilibrium system is identical to that of Carlstrom and Fuerst (2007) if the collateral constraint never binds.

2.1 Households: workers and managers

We consider households that consist of workers and managers. The household begins period $t$ with $M_t$ cash balances, $B_t$ one-period nominal bonds that pay $R_{t-1}$ gross interest rate, an $S_t$ stock of shares of stock of retailers that sell at price $Q_t$ and pay dividend $D_t$.

The utility function is

$$U(C_t, L_t, M_{t+1}/P_t) = \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{L_t^{1+\gamma}}{1+\gamma} + V(M_{t+1}/P_t),$$

(1)

where $\sigma > 0$, $\gamma > 0$, $V$ is increasing and concave, $C_t$ denotes consumption, $L_t$ denotes labor supply, and $M_{t+1}/P_t$ denotes real cash balances at the end of period $t$. 


At the beginning of the period, a household splits into worker and manager. A worker supplies labor $L_t$ and earns wage income $P_t W_t L_t$ where $P_t$ denotes the aggregate price level. A manager employs labor to produce homogenous goods and sells them to retailers at price $P_t Z_t$.

The production technology of managers is

$$Y_t = H_t,$$

where $H_t$ denotes labor demand. We assume that managers have to pay wages to workers in advance and therefore borrow working capital from banks. Banks can issue bank notes that can be circulated in our economy. Letting $N_t$ be the amount that the manager borrows, the manager’s choice of $H_t$ is constrained by

$$P_t W_t H_t \leq N_t.$$  \hspace{1cm} (3)

Since this borrowing and lending are intra-period, the gross interest rate of this is zero in equilibrium. As in Kiyotaki and Moore (1997), the manager cannot fully commit to repaying the debt. Then, the manager’s borrowing is subject to a collateral constraint

$$N_t \leq \varphi P_t Q_t S_t,$$  \hspace{1cm} (4)

where $0 < \varphi \leq 1$.\footnote{A similar setting of the credit market imperfection is employed by Kobayashi, Nakajima, and Inaba (2007); Kobayashi and Nutahara (2007); and Harrison and Wedner (2010).} In order to consider a collateral constraint, we assume that a worker cannot supply labor to a manager from the same agent.

After the production of goods, worker and manager return home to decide consumption and holdings of money and bond as a single agent: the household. The budget constraint of the household is

$$P_t C_t + M_{t+1} + P_t Q_t S_{t+1} + B_{t+1} + P_t W_t H_t$$

$$\leq P_t Z_t Y_t + P_t W_t L_t + M_t + P_t Q_t S_t + R_{t-1} B_t + P_t D_t S_t + X_t,$$  \hspace{1cm} (5)

where $Z_t$ denotes the relative price of goods produced by managers and $X_t$ denotes monetary injection.
The first order conditions of households are

\[ C_t^\sigma L_t^\gamma = W_t, \]  
(6)

\[ C_t^{-\sigma} = \beta C_{t+1}^{-\sigma} \frac{R_t}{\Pi_{t+1}}, \]  
(7)

\[ C_t^{-\sigma} Q_t = \beta C_{t+1}^{-\sigma} [Q_{t+1}(1 + \varphi \Theta_{t+1}) + D_{t+1}], \]  
(8)

\[ W_t(1 + \Theta_t) = Z_t, \]  
(9)

\[ (W_t H_t - \varphi Q_t S_t) \Theta_t = 0, \quad \Theta_t \geq 0, \]  
(10)

where \( \Pi_{t+1} \equiv P_{t+1}/P_t \) and \( \Theta_t \) denotes the ratio of the Lagrange multiplier of the collateral constraint to that of the budget constraint and can be interpreted as the inefficiency of collateral constraint. (6) is the intratemporal optimization condition, (7) is the Euler equation of consumption, (8) is the Euler equation of assets, (9) is the marginal productivity condition of labor, and (10) is the condition of the collateral constraint.

By (7) and (8), we have the more familiar asset price relationship:

\[ Q_t = [Q_{t+1}(1 + \varphi \Theta_{t+1}) + D_{t+1}] \frac{\Pi_{t+1}}{R_t}. \]  
(11)

Note that in the case of binding collateral constraint, asset price is affected by the inefficiency of collateral constraint \( \Theta \). If a shock tightens the collateral constraint, the premium of assets as collateral increases, and then it has a positive effect on asset prices.

2.2 Retailers

We assume that the retailers are monopolistically competitive, as employed by Bernanke, Gertler, and Gilchrist (1999). Retailers buy goods at price \( P_t Z_t \) from managers, produce differentiated goods using linear technology, and set prices. Under the standard Calvo-type sticky-price setting, the New Keynesian Phillips curve is

\[ \pi_t = \lambda \tilde{z}_t + \beta \pi_{t+1}, \]  
(12)

where lowercase letters denote log deviations from the steady state. Note that the real wholesale price \( Z_t \) can be interpreted as the real marginal cost of retailers. The retailers’
profits are paid out as dividends. Then, we have

\[ D_t = (1 - Z_t)Y_t. \]  

(13)

2.3 Monetary policy

We assume that monetary authority follows a simple Taylor rule:

\[ r_t = \tau \pi_t + \tau_q q_t, \]  

(14)

where lowercase letters \( r_t \) and \( q_t \) denote the log-deviations from a steady state of \( R_t \) and \( Q_t \), respectively.

2.4 Equilibrium

We focus on a symmetric equilibrium with \( H_t = L_t \). The total supply of share \( S_t = 1 \) and total supply of nominal bond \( B_t = 0 \).

The definition of a competitive equilibrium is as follows.

Definition 1. Given monetary policy rule (14), a competitive equilibrium is a sequences of prices \( \{\pi_t, Q_t, W_t, Z_t, R_t\} \) and quantities \( \{C_t, H_t, L_t, Y_t, B_t, M_t, D_t, \Theta_t\} \) such that (i) households maximize their utilities, (ii) retailers maximize their profits, and (iii) all markets clear.
The equilibrium system of this economy is

\[ C_t^\sigma H_t^\gamma = W_t, \]  
\[ C_t^{-\sigma} = \beta C_{t+1}^\sigma \frac{R_t}{\Pi_{t+1}}, \]  
\[ Q_t = [Q_{t+1}(1 + \varphi \Theta_{t+1}) + D_{t+1}] \frac{\Pi_{t+1}}{R_t}, \]  
\[ W_t(1 + \Theta_t) = Z_t, \]  
\[ (W_t H_t - \varphi Q_t) \Theta_t = 0, \quad \Theta_t \geq 0, \]  
\[ D_t = (1 - Z_t) Y_t, \]  
\[ Y_t = H_t = C_t, \]  
\[ \pi_t = \lambda z_t + \beta \pi_{t+1}, \]  
\[ r_t = \tau \pi_t + \tau q Q_t. \]

3 Main results

3.1 Equilibrium indeterminacy and credit market imperfection

The following condition is necessary and sufficient for a binding collateral constraint at a steady state.

**Proposition 1.** A collateral constraint \((4)\) is binding at a steady state if and only if

\[ \varphi < \frac{1 - \beta}{\beta} \cdot \frac{Z}{1 - Z}. \]  

**Proof.** By the steady-state equilibrium system, we obtain

\[ W = C^{\sigma+\gamma}, \]
\[ C = \left[ \frac{Z}{1 + \Theta} \right]^{1/(\sigma+\gamma)}, \]
\[ Q = \frac{(1 - Z) \left[ \frac{Z}{1 + \Theta} \right]^{1/(\sigma+\gamma)}}{1/\beta - (1 + \varphi \Theta)}. \]

Inserting these into a collateral constraint \(WC = \varphi Q\) yields

\[ \Theta = \frac{Z [1 - \beta (1 - \varphi)]}{\beta \varphi} - 1. \]
\( \Theta \) is greater than zero if and only if (24) holds.

It is clear that this economy is identical to that of Carlstrom and Fuerst (2007) if a collateral constraint never binds, that is, \( \Theta_t = 0 \). Then, the following proposition holds.

**Proposition 2.** Assume that (24) does not hold and a collateral constraint never binds.

(i) If \( \tau_q = 0 \), a necessary and sufficient condition for equilibrium determinacy is \( \tau > 1 \).

(ii) If \( \tau > 1 \), a necessary and sufficient condition for equilibrium determinacy is

\[
\tau_q < \frac{\lambda(\tau - 1)}{(1 - \beta)A},
\]

where \( A \equiv \frac{Z(1 + \sigma + \gamma) - 1}{(\sigma + \gamma)(1 - Z)} \).

**Proof.** See the proof of Proposition 1 of Carlstrom and Fuerst (2007).

Proposition 2 implies that equilibrium indeterminacy arises if \( \tau_q \) is larger than a threshold.

In this paper, we focus on a case where a collateral constraint is binding. It is convenient to log-linearize our equilibrium system for the analysis. The linearized system with a binding collateral constraint is as follows.

\[
(\sigma + \gamma)c_t = w_t, \tag{25}
\]
\[
\sigma(c_{t+1} - c_t) = r_t - \pi_{t+1}, \tag{26}
\]
\[
q_t = \beta(1 + \varphi \Theta) \left[ q_{t+1} + \frac{\varphi \Theta}{1 + \varphi \Theta} \theta_{t+1} \right] + [1 - \beta(1 + \varphi \Theta)]d_{t+1} + (\pi_{t+1} - r_t), \tag{27}
\]
\[
w_t + c_t = q_t, \tag{28}
\]
\[
d_t = c_t - \frac{Z}{1 - Z} z_t, \tag{29}
\]
\[
z_t = w_t + \frac{\Theta}{1 + \Theta} \theta_t, \tag{30}
\]
\[
\pi_t = \beta \pi_{t+1} + \lambda z_t, \tag{31}
\]
\[
r_t = \tau \pi_t + \tau_q q_t, \tag{32}
\]

where lowercase letters denote log deviations from the steady state and

\[
\Theta = \frac{Z[1 - \beta(1 - \varphi)]}{\beta \varphi} - 1. \tag{33}
\]
This system is reduced to the following matrix form:

\[
\begin{bmatrix}
1 & 0 & \Phi_1 \\
1 & 0 & \Phi_2 \\
\beta & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\pi_{t+1} \\
z_{t+1} \\
q_{t+1}
\end{bmatrix} =
\begin{bmatrix}
\tau & 0 & \Phi_1 + \tau_q \\
\tau & 0 & 1 + \tau_q \\
1 & -\lambda & 0
\end{bmatrix}
\begin{bmatrix}
\pi_t \\
z_t \\
q_t
\end{bmatrix},
\tag{34}
\]

where

\[
\Phi_1 \equiv \frac{\sigma}{1 + \sigma + \gamma},
\]

\[
\Phi_2 \equiv \frac{1 + \beta(1 - \varphi)(\sigma + \gamma)}{1 + \sigma + \gamma}.
\]

The first equation is the Euler equation of consumption (26). The second one is the Euler equation of asset price (27). The last one is the New Keynesian Phillips curve (31).

Note that this system is closed by only first and second equations with \(\pi_t\) and \(q_t\).

The main result is as follows.

**Proposition 3.** Assume \(\beta \geq Z, (24)\), and a collateral constraint is always binding. A necessary and sufficient condition for equilibrium determinacy is \(\tau > 1\).

**Proof.** Let \(x_1, x_2,\) and \(x_3\) denote three eigenvalues. It is obvious that one of them, \(x_1\), is infinity. The characteristic equation for \(x_2\) and \(x_3\) is

\[
F(x) = \frac{\lambda}{1 + \sigma + \gamma}(x - \tau)\left\{[1 - \sigma + \beta(1 - \varphi)(\sigma + \gamma)]x - (1 + \gamma)\right\}.
\]

Then, the eigenvalues are \(x_2 = \tau\) and \(x_3 = \frac{1 + \gamma}{1 - \sigma + \beta(1 - \varphi)(\sigma + \gamma)}\). The numerator, \(1 + \gamma\), of \(x_3\) is strictly positive. The denominator is

\[
1 - \sigma + \beta(1 - \varphi)(\sigma + \gamma) > 1 - \sigma + \beta \left(1 - \frac{1 - \beta}{\beta} \cdot \frac{Z}{1 - Z}\right) (\sigma + \gamma)
\]

\[
= 1 + \frac{1 - \beta}{1 - Z} \sigma + \frac{\beta - Z}{1 - Z} \gamma > 0,
\]

by (24) and \(\beta \geq Z\). Then, it is shown that \(x_3 > 1\) since

\[
(1 + \gamma) - [1 - \sigma + \beta(1 - \varphi)(\sigma + \gamma)] = [1 - \beta(1 - \varphi)](\sigma + \gamma) > 0.
\]

Finally, \(\tau > 1\) is necessary and sufficient for equilibrium determinacy. \(\square\)

Proposition 3 implies that a central bank’s stance on asset price fluctuations does not affect equilibrium determinacy if a collateral constraint is binding.
3.2 Interpretations

Why is it that equilibrium indeterminacy never arises if monetary policy responds to asset prices when the collateral constraint is binding?

Carlstrom and Fuerst (2007) explain that if the inflation increases permanently by one percent and the central bank follows a policy rule (14), the nominal interest rate increases by

\[ \tau - \frac{A(1 - \beta)}{\lambda} \tau_q. \]  

Their result is shown as follows. By the New Keynesian Phillips curve implies that a permanent increase in inflation increases real marginal cost \( Z \). By the steady-state equilibrium system where a collateral constraint never binds, we have

\[ Q = \frac{(1 - Z)Z^{1/(\sigma + \gamma)}}{1/\beta - 1}. \]  

Under reasonable calibration, it is shown that asset price \( Q \) is decreasing in \( Z \). Therefore, high inflation means low asset prices. Monetary policy responding to asset prices implicitly weakens its overall response to inflation, and this is the source of equilibrium indeterminacy in their model. This is an example of the celebrated Taylor Principle: a permanent increase in the inflation rate leads to a more than proportionate increase in the inflation rate. If (35) exceeds one, the monetary policy rule satisfies the Taylor Principle.

On the contrary, if a collateral constraint is binding, we have

\[ Q = \frac{(1 - Z)\left[ \frac{Z}{1 - \beta (1 - \varphi)} \right]^{1/(\sigma + \gamma)}}{1/\beta - (1 + \varphi \Theta)}. \]  

and

\[ \Theta = \frac{Z [1 - \beta (1 - \varphi)]}{\beta \varphi} - 1. \]  

These conditions imply that

\[ Q = \varphi^{1/(\sigma + \gamma)} \left[ \frac{\beta}{1/\beta - (1 + \varphi)} \right]^{1+\sigma+\gamma}, \]  

10
which, in turn, implies that asset price does not change if there is a permanent increase in inflation. This is because the inefficiency of the collateral constraint $\Theta$ absorbs the effects of an increase in inflation. Then, the nominal interest rate increases by $\tau$ in the economy with a binding collateral constraint. Finally, in our model with a binding collateral constraint, a central bank’s stance on asset price fluctuations does not affect equilibrium indeterminacy.

4 Concluding remarks

A recent paper by Carlstrom and Fuerst (2007) showed that equilibrium indeterminacy arises if monetary policy responds to asset prices in a sticky-price economy where asset prices reflect firms’ profits.

Since monetary policy responding to asset prices is often discussed during recessions associated with financial crises, we introduce a collateral constraint into their model and showed that equilibrium indeterminacy never arises even if monetary policy responds to asset prices. An permanent increase in inflation reduces firms’ profits and asset prices decline in a standard sticky-price model. However, asset price does not change under the credit market imperfection since the inefficiency of the collateral constraint increases and the premium of shares as collateral increases.

Our result implies that equilibrium indeterminacy, a negative aspect of monetary policy responding to asset prices, never arises under credit market imperfection. In order to determine whether monetary policy should respond to asset prices, it is a future task to investigate optimal policy in the economy with credit frictions. However, since the discussion on monetary policy responding to asset prices often arises during recessions associated with financial crises, for example, Japan’s lost decade of the 1990s and the recent financial crisis in the U.S. economy, our result would have a certain implication on the literature of monetary policy.
References


