Seasonal decomposition with a modified Hodrick-Prescott filter

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Abstract

I describe preliminary results for seasonal decomposition procedure using a modified Hodrick-Prescott (Leser) filter. The procedure is simpler to implement compared to two currently most popular seasonal decomposition procedures - X-11 filters developed by the U.S. Census Bureau and SEATS developed by the Bank of Spain. A case study for Latvia's quarterly gross domestic product shows the procedure is able to extract a stable seasonal component, yet allowing for structural changes in seasonality.

Keywords: seasonal decomposition, Hodrick-Prescott filter, quarterly GDP
JEL code: C13, C14, C22

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1 Introduction

I describe preliminary results for seasonal decomposition procedure using a modified Hodrick-Prescott (HP) filter. The procedure is simpler to implement compared to two currently most popular seasonal decomposition procedures - X-11 filters developed by the U.S. Census Bureau (see, inter alia, Bell and Monsell, 1992) and SEATS developed by the Bank of Spain (see, inter alia, Kaiser and Maravall, 2000). A case study for Latvia's quarterly gross domestic product (GDP) shows the procedure is able to extract a stable seasonal component, yet allowing for structural changes in seasonality.

Section 2 describes the procedure, and Section 3 shows its application in the seasonal decomposition of Latvia's GDP.

2 The seasonal decomposition procedure

Let

$$x_t = \tau_t + c_t + s_t + i_t$$

be an additive, or log-additive, decomposition of an observed time series x at time t, where τ_t is a trend component at t, c_t is a cyclical component at t, s_t is a seasonal component at t, and i_t is an irregular component at t. Then, the seasonal decomposition procedure that uses a modified Hodrick-Prescott (HP) filter, is as follows.

1. Estimate the trend component by the HP filter:

$$\hat{\tau}_t = \arg\min_{\{\tau_t\}} \sum_t (x_t - \tau_t)^2 + \lambda_1 \sum_t [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2, \quad (1)$$

where the default value of λ_1 for quarterly series is 1600 (see Hodrick and Prescott, 1983). Remove $\hat{\tau}_t$ from x_t to get the estimate of $c_t + s_t + i_t$.

2. Estimate the cyclical component by the HP filter :

$$\hat{c}_t = \arg\min_{\{c_t\}} \sum_t (x_t - \hat{\tau}_t - c_t)^2 + \lambda_2 \sum_t [(c_{t+1} - c_t) - (c_t - c_{t-1})]^2 \quad (2)$$

for appropriate $\lambda_2 < \lambda_1$, with $\lambda_2 \in [10, 40]$ probably being a first guess. Subtract $\hat{c}_t$ from the estimate of $c_t + s_t + i_t$ to get the estimate of $s_t + i_t$.

3. Estimate the seasonal component by a modified HP filter:

$$\hat{s}_t = \arg\min_{\{s_t\}} \sum_t (x_t - \hat{\tau}_t - \hat{c}_t - s_t)^2 + \lambda_3 \sum_t (s_t - s_{t-k})^2, \quad (3)$$

where k is the periodicity of the seasonal component, with $k = 4$ for quarterly data. The second sum in (3) penalizes unstable seasonal component. The first order condition for (3) yields

$$F(L)\hat{s}_t = x_t - \hat{\tau}_t - \hat{c}_t, \quad (4)$$

where

$$F(L) = -\lambda_3 L^k + (1 + 2\lambda_3) - \lambda_3 L^{-k} \quad (5)$$

and L is a lag operator such that $L^k z_t = z_{t-k}$. Low value of $\lambda_3 \in [1, 10]$ appears to be appropriate. For better performance at the ends of series, expression (5) can be replaced by $F(L) = 1 + \lambda_3 - \lambda_3 L^{-k}$ for the first k observations, and $F(L) = -\lambda_3 L^k + 1 + \lambda_3$ for the last k observations.

3 A case study

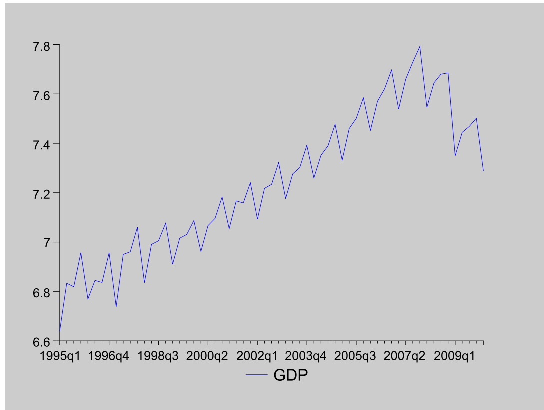
This section implements the above described seasonal decomposition method to seasonally decompose Latvia's GDP. The below results illustrate the seasonal decomposition described above, and not the full seasonal adjustment procedure, since the latter contains pre-adjustment and forecasting that I am not focusing on here.

Take Latvia's GDP (in thsd lats, chain linked with reference year 2000) from 1995Q1 till 2010Q1. Impose logarithmic transformation on the series and follow the procedure in Section 2:

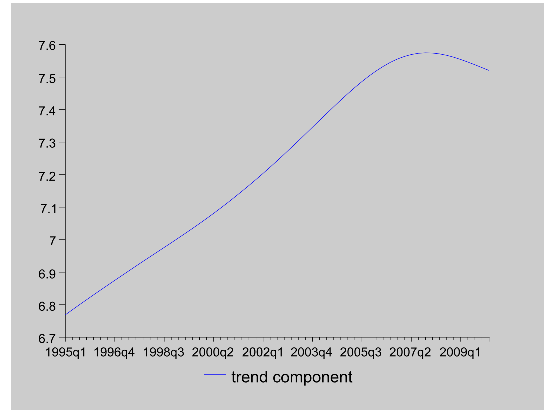
1. estimate the trend component with the HP filter, $\lambda_1 = 1600$,
2. estimate the cyclical component with the HP filter, $\lambda_2 = 20$,
3. estimate the seasonal component with a modified HP filter, $\lambda_3 = 4$.

The resulting series are plotted in Figure 1. We can see the procedure is able to extract a stable seasonal component while allowing for structural changes in seasonality between 2nd and 3rd quarters. The ends of the seasonal component can be improved by forecasting - an indispensable step in seasonal adjustment.

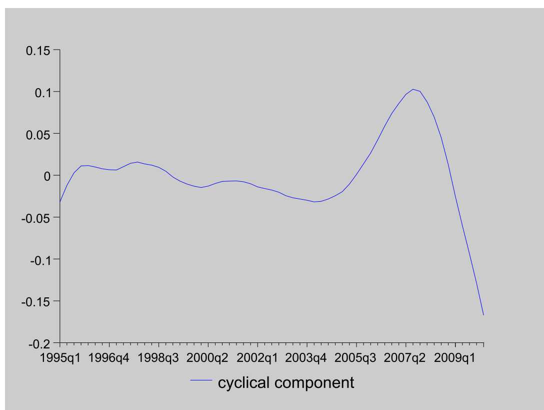
Figure 2 compares the seasonal component obtained from the procedure described in Section 2 to the one for the officially released seasonally adjusted GDP of Latvia, obtained by X-12 ARIMA method, imposing log transformation, Easter correction, SARIMA(010)(011) model with the coefficient for the seasonal moving average term -0.403, decomposed by 3x3 moving average seasonal filter and 5-term Henderson moving average trend filter. We can see the differences between the results from the two methods are small in spite of the absence of pre-adjustment and forecasting in obtaining the seasonal component from a modified HP filter.



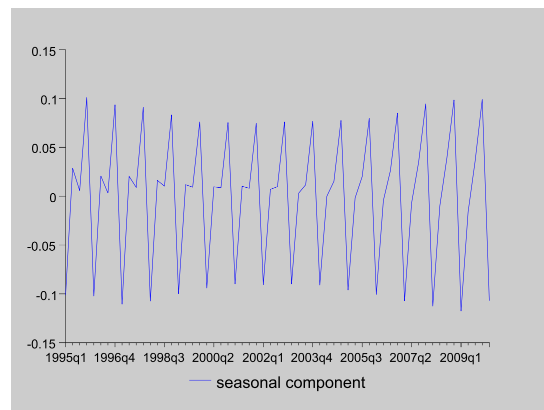
(a) Latvia's GDP (in thsd lats, logs).



(b) Trend component, estimated by HP filter, $\lambda_1 = 1600$.



(c) Cyclical component, estimated by HP filter, $\lambda_2 = 20$.



(d) Seasonal component, estimated by a modified HP filter, $\lambda_3 = 4$.

Figure 1: Results from the seasonal decomposition of Latvia's GDP using the procedure described in Section 2. We can see the procedure is able to yield stable seasonal component. The ends of the seasonal component can be improved by forecasting - an indispensable step in seasonal adjustment.

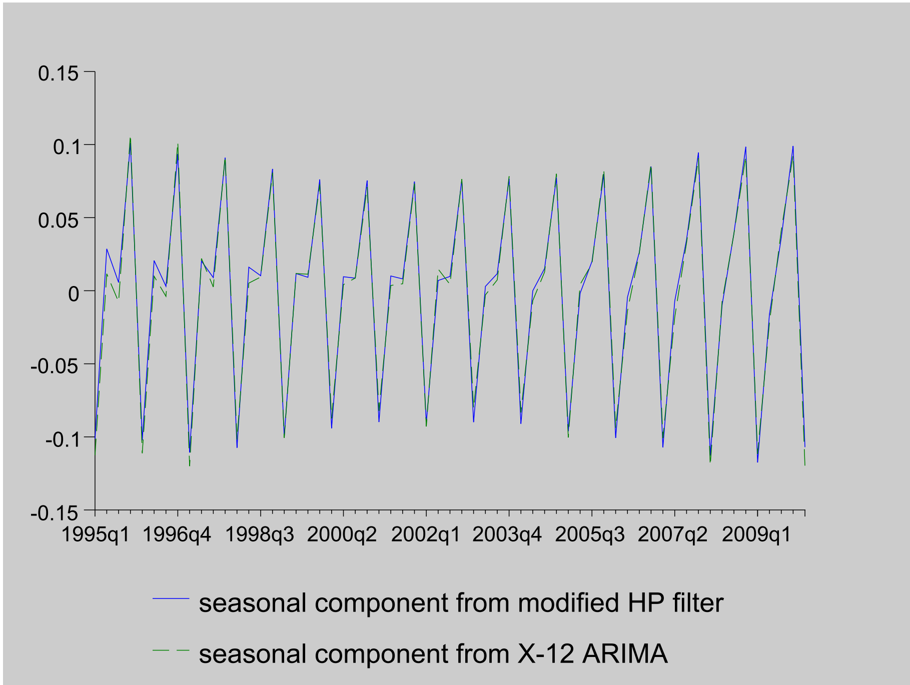


Figure 2: Seasonal component obtained by a modified HP filter versus seasonal component obtained by X-12 ARIMA method. We can see the differences of the results from the two methods are small in spite of the absence of pre-adjustment and forecasting in obtaining the former.

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