Quantity Competition, Endogenous Motives and Behavioral Heterogeneity

Alessandra Chirco and Caterina Colombo and Marcella Scrimitore

University of Salento, University of Ferrara, University of Salento (Italy)

July 2010

Quantity Competition, Endogenous Motives and Behavioral Heterogeneity

Alessandra Chirco [*]
Caterina Colombo [+]
Marcella Scrimitore [±]

Abstract. The paper shows that strategic quantity competition can be characterized by behavioral heterogeneity, once competing firms are allowed in a pre-market stage to optimally choose the behavioral rule they will follow in their strategic choice of quantities. In particular, partitions of the population of identical firms in profit maximizers and relative profit maximizers turn out to be deviation-proof equilibria, both in simultaneous and sequential game structures. Our findings that in a strategic framework heterogeneous behavioral rules are consistent with individual incentives provides a game-theoretic microfoundation of heterogeneity.

JEL Classification. L13, L21, C72.

Keywords. Behavioral Heterogeneity, Endogenous Motives, Relative Performance, Multistage Games, Quantity Competition.

Acknowledgments. We thank Corrado Benassi and Luca Lambertini for useful suggestions and comments.

* University of Salento in Lecce - Italy; e-mail alessandra.chirco@unisalento.it.
+ University of Ferrara - Italy; email: caterina.colombo@unife.it.
± University of Salento in Lecce - Italy; e-mail marcella.scrimitore@unisalento.it.

1 Introduction

This paper shows that behavioral heterogeneity can be the equilibrium outcome of the strategic interaction of ex-ante identical, fully rational agents. We analyze the phenomenon of equilibrium heterogeneity on the supply side of a oligopolistic market, by assuming that technologically identical firms, competing *à la* Cournot, may adopt two types of market behavior: they may act on the market as pure profit maximizers or rather as relative profit maximizers. The type according to which firms behave on the market is optimally chosen on the basis of profit maximizing objectives, and market interactions are modeled as a multi-stage game in which the strategic choice of the type precedes the choice of quantities. By allowing for the endogenous choice of motives, we attribute to competing firms a rationality which may imply a deviation from profit maximization at the market stage, when this ensures profit gains due to the properties of strategic interaction.

Our work can be traced back to two broad and articulated research lines. The first includes the vast and variegated literature which has critically questioned the same idea that firms behave according to a profit maximization criterion; the second concerns the individual and market performance in the presence of heterogeneous firms.

Indeed, starting from the managerial theories of the firm,[1] a large stream of the economic literature has critically investigated the classical assumption of profit maximization, by analyzing the circumstances under which alternative modes of behavior are either more plausible, or conducive to higher absolute profits, or able to guarantee a superior performance in the market. In particular, the optimality of a profit maximizing behavior has been challenged within the Industrial Organization, evolutionary and experimental literature. The IO research on strategic delegation (among others, Vickers, 1985; Fershtman and Judd, 1987; Miller and Pazgal, 2002) has shown that in a game theoretical context, managers concerned with maximizing revenues or relative performance can be allowed by the firms' owners to deviate from profit maximization. The delegation to managers of non profit maximizing decisions represents a unilaterally rational choice for the firms' profit maximizing owners, since it allows to take full advantage of the strategic interactions characterizing oligopolistic markets. Delegation strategies also turn out to reconcile the owners' profit maximization goals with the utility maximization of managers. The idea of an endogenous choice of motives also inspires the mixed duopoly model by Choi and Lu (2009), who analyze the strategic choice of a public and a private firm to be concerned with their relative or their absolute payoff.

In an alternative microfoundation set-up, namely evolutionary game theory, Schaffer (1989) demonstrates that absolute profit maximization does not ensure

[1] By focusing on the divergence of objectives due to the separation between ownership and management and on the cognitive limits of firms, these theories (Cyert and March, 1963; Marris, 1964; Williamson, 1964; Simon, 1955) consider firms as adaptively rational, rather than optimizing entities, which pursue satisfactory performance objectives, which may include sales or asset growth maximization.

firms' survival on the market, and that in this perspective the most successful behavior is to be identified in the ability of firms to maximize their relative performance. According to Schaffer, the Friedman's conjecture (Friedman, 1953) that profit maximization is required for survival, is conditional on the existence of competitive markets, while it does not hold when firms enjoy market power, as in a Cournot setting.

Non profit maximizing behavior in oligopolies is also supported by recent experimental research in the field. Georgantzís et al. (2008) test experimentally different strategic delegation contracts, and provide comparative evidence on the adoption of incentive schemes based on relative performance and on revenues. Other experimental studies show that imitative or adaptive behavior (Offerman et al, 2002; Nagel and Vriend, 1999) can emerge in oligopolistic market games in which firms are boundedly rational, with a limited perception of the environment.

The effects of behavioral heterogeneity in oligopolistic markets – and in particular the coexistence of profit and relative profit maximizing firms – have been analyzed by Riechmann (2006), who shows that in quantity competition framework the equilibrium quantities and price reflect the relative weight of the two types of firm, the Cournot and the Walrasian equilibrium emerging as limit cases, as the fraction of profit maximizers moves from one to zero. In a similar set-up, the comparative performance of behaviorally heterogeneous agents has been studied by Huang (2002), who points out that the adoption of a simple and naïve behavior (namely a price-taking rule) may lead to higher competitive gains with respect to more sophisticated strategies. The interaction between optimizers and imitators is studied, with analogous results, by Schipper (2009). While optimizers play a best response to the rivals' output choices at the earlier round, the imitators follow the rule of producing the quantity chosen in the previous period by the most profitable firm, thus mimicking the behavior of their most successful rivals. By applying stochastic stability analysis in a dynamic discrete-time setting which extends the work of Vega-Redondo (1997),[2] Schipper demonstrates a convergence to a set of states in which imitators perform better than optimizers.

Our work tries to connect these research lines and to enrich their theoretical scope. On the one hand, the literature on the microfoundation of non profit maximizing behavior has always delivered symmetric equilibria, thus characterized by behavioral homogeneity. On the other hand, the research on the effects of heterogeneity has taken the latter as given, neglecting the issue of the agents' individual incentives to maintain their assumed behavioral rule. Our aim is to microfound heterogeneity, namely the coexistence of profit maximizers and relative profit maximizers, by showing that it characterizes deviation-proof stable partitions of firms which play a non-cooperative game with respect to motives and quantities. The paper is organized as follows. Section 2 briefly illustrates some basic features of our set-up. In Sections 3 and 4 we analyze equilibrium

[2] This important work in evolutionary theory shows that the dynamic interactions of firms which imitate the rivals' success in a Cournot market result into a Walrasian competitive equilibrium.

heterogeneity in simultaneous and sequential game structures. An interpretation of the results is offered is Section 5, while Section 6 gathers some final remarks.

2 The strategic choice of behavioral rules

We consider a multi-stage game in the market for a homogeneous product, populated by technologically identical firms which compete over quantities. All firms have an ultimate objective in terms of profit maximization. However, in order to pursue this objective, in a pre-market stage they have the possibility to choose irrevocably the behavioral rule they will follow in their strategic choice of quantities – in particular, to respond to the rivals' quantity choices according to either a profit maximizing, or a relative profit maximizing reaction function. A possible interpretation of this sequence of decisions is in a strategic delegation vein. We may think of firms' profit maximizing owners delegating their quantity decisions to two alternative types of managers – managers interested in the absolute performance of the firm, or managers interested in its relative performance.[3] Delegation to a manager is indeed the simplest way to formalize the commitment to a given type of behavior in the quantity stage.

In the next sections we shall study this type-quantity strategic choice in both simultaneous and sequential game structures. In the simultaneous case we depict a traditional two-stage game, in which all firms first choose simultaneously their type, then simultaneously compete over quantities. As far as the sequential game structures are concerned, we shall allow for sequential decisions either on quantities only, or on both types and quantities. Our main interest is to verify whether the equilibria of these games generate firms' partitions across types characterized by a behavioral heterogeneity. This would amount to showing that once the behavioral rules enter the domain of strategic choices – i.e., they are endogeneized according to a profit maximizing criterion – strategic interaction may generate behavioral heterogeneity of otherwise identical firms.

All the games analyzed in the sequel are characterized by the following basic common features. The inverse market demand function is linear and given by:

$$P = a - Q$$

where Q is the total quantity produced and sold. Moreover, in order to rule out any exogenous or endogenous source of cost heterogeneity, we assume that all firms produce at a constant average and marginal cost c.

3 The simultaneous game

Assume that the total number of firms is $n \geq 2$. The structure of the game is the following: at the first stage the firms choose simultaneously whether to

[3] For an interpretation of delegation choices in terms of the type of manager hired by firms, see Miller and Pazgal, 2002.

behave as profit maximizers (PM) or as relative profit maximizers (RPM) at the market stage; at the second stage they compete simultaneously over quantities, consistently with the behavioral rule chosen at the first stage. As we clarified above, we can interpret this framework as one in which owners choose a type of manager at the first stage, and managers of possibly different types compete over quantities at the second stage. This two-stage game can be solved by backward induction.

Assume that at the first stage $k \leq n$ firms (indexed with i) chose to be profit maximizers, while the remaining $n-k$ (indexed with j) decided to be relative profit maximizers. The decisions taken at the first stage are common knowledge at the second stage. The generic PM firm maximizes:

$$\pi_i = (a - q_i - Z_i - Q_j - c)\, q_i$$

where Z_i is the quantity produced by the other PM firms, and Q_j is the quantity produced by all the RPM ones. By solving this problem and imposing symmetry over the k firms of type PM – $Z_i = (k-1)\, q_i$ – we obtain the following reaction function:

$$q_i = \frac{a - c - Q_j}{k+1} \tag{1}$$

As far as the generic RPM firm is concerned, it maximizes the difference between its profits and the average absolute profits of all firms:

$$R_j = \pi_j - \frac{\sum_{i=1}^{k} \pi_i + \sum_{r=k+1}^{n} \pi_r}{n} = \frac{n-1}{n}\pi_j - \frac{\sum_{i=1}^{k} \pi_i + \sum_{r=k+1|r\neq j|}^{n} \pi_r}{n}$$

where $\sum_{r=k+1}^{n} \pi_r$ is the sum of the profits of the RPM firms. This amounts to maximizing

$$\begin{aligned} R_j \quad = \quad & \frac{n-1}{n}\left(\left(a - Q_i - Z_j - q_j - c\right) q_j\right) + \\ & -\frac{1}{n}\left(\sum_{i=1}^{k}\left(\left(a - Q_{\bar{i}} - Z_j - q_j - c\right) q_i\right) + \sum_{r=k+1}^{n-1}\left(\left(a - Q_i - Z_j - q_j - c\right) q_r\right)\right) \end{aligned}$$

where Z_j is the quantity produced by the other RPM firms, and Q_i is the quantity produced by all the PM firms. The reaction function of the firm j, obtained by imposing symmetry over the $(n-k)$ RPM firms – $Z_j = (n-k-1)\, q_j$ – is therefore:

$$q_j = \frac{(n-2)\, Q_i - (a-c)\,(n-1)}{(n - 2k + kn - n^2)} \tag{2}$$

By substituting for $Q_j = (n-k)\, q_j$ in (1) and $Q_i = kq_i$ in (2), and solving these two equations for q_i and q_j, we obtain the following equilibrium quantities:

$$\begin{aligned} q_i^* &= \frac{(a-c)\, k}{2k + n\,(n-1)} \\ q_j^* &= \frac{(a-c)\,(k+n-1)}{2k + n\,(n-1)} \end{aligned}$$

which clearly show that RPM firms produce a larger quantity, thus earning higher profits. In particular, the profits accruing to the two types of firms are:

$$\pi_i^* = (a - kq_i^* - (n-k)\, q_j^* - c)q_i^* = \frac{(a-c)^2 k^2}{(2k + n(n-1))^2} \tag{3}$$

$$\pi_j^* = (a - kq_i^* - (n-k)\, q_j^* - c)q_j^* = \frac{k\,(a-c)^2 (k+n-1)}{(2k + n(n-1))^2} \tag{4}$$

This is the solution of the second stage, associated to a generic partition in k PM and $(n-k)$ RPM firms. In order to find the equilibria of the two-stage game, we have to identify those partitions which are indeed 'deviation-proof' at the first stage, i.e. partitions which no firm has the incentive to unilaterally deviate from, when choosing its type. A partition is an equilibrium if neither a PM nor a RPM firm perceives the unilateral incentive to commit to the alternative behavioral rule. A firm decides to deviate from a given partition, if the profits it gains by committing to the alternative type are higher than those associated to that partition.

Accordingly, the following proposition can be established.

Proposition 1 *In a simultaneous, two-stage 'type-quantity' game, the unique subgame perfect equilibrium is characterized by one firm behaving as profit maximizer, and $n-1$ firms behaving as relative profit maximizers.*

Proof. Starting from a generic partition, let us first consider the incentive to deviate for a RPM firm. Since the deviation would be observed by all the deviating firm's competitors, the latter will take it into account in their quantity optimization procedures. This implies that the profits of the deviating firm would be

$$\pi_i^D = \frac{(a-c)^2 (k+1)^2}{(2(k+1) + n(n-1))^2}$$

which is simply (3) re-calculated under the hypothesis that PM firms are now $k+1$. It can be checked that for all n, $\pi_i^D > \pi_j^*$ only if $k = 0$. The only partition of firms from which a RPM firm deviates is that in which all firms are RPM. All partitions with $k > 0$ are deviation-proof for RPM agents. The same procedure can be applied to evaluate the incentive to deviate of a PM firm. By turning into a RPM firm, it would obtain:

$$\pi_j^d = \frac{(a-c)^2 (k+n-2)(k-1)}{(2(k-1) + n(n-1))^2}$$

which is equation (4) re-calculated under the hypothesis that the PM firms are now $k-1$. The comparison between π_j^d and π_i^* shows that for any value of n, the only partition which is 'deviation-proof' for PM firms is that associated with $k = 1$, i.e. with only one PM firm operating at the quantity stage. Therefore $k = 1$ is the only equilibrium partition. ■

This outcome is not surprising, once we recall that were the market populated by RPM firms only, competition would lead to a Walrasian outcome (if $k = 0$, $\pi_j^* = 0$ in (4)). In the above two-stage game, behavioral heterogeneity has an intuitive explanation: for a RPM behavior to be profitable, there must be a benchmark PM behavior to rely upon. Notice however that once PM firms operate in the market, RPM firms obtain much higher profits than their PM rivals – and this explains why the equilibrium partition is characterized by only one PM firm, which exerts a large positive externality on its RPM competitors.

The idea that profitability of relative profit maximization requires a profit maximizing benchmark suggests us to investigate behavioral heterogeneity in more articulated market structures, which allow for sequential decisions.

4 The sequential games

When sequential decisions are introduced in the above game, two basic structures are to be investigated. In the first, sequential decisions are taken only on quantities, while firms choose simultaneously whether to be PM or RPM. In the second, we allow for sequential choices for both dimensions of the firms' strategic interaction.

4.1 Simultaneous choice of types and sequential choice of quantities

Consider a multi-stage version of the game with $n \geq 3$ firms. At the market stage firms compete sequentially on quantities, with one firm acting as first mover (leader), and the remaining $m = n - 1$ firms (followers) taking simultaneously their decisions after observing the first-mover quantity choice. All firms decide on their quantities consistently with the type chosen simultaneously at the first stage.

In order to solve the game by backward induction, we consider first the quantity decisions of the followers. In analogy with the simultaneous game we assume that $k \leq m$ followers (indexed with i) chose at the first stage to be PM, and the remaining $m - k$ (indexed with j) chose to be RPM. By maximizing the objective functions of the PM and RPM followers, we obtain the following reaction functions:

$$\begin{aligned} q_i &= \frac{a - c - (m-k)\, q_j - q_0}{k+1} \\ q_j &= \frac{(a-c)\, m - (kq_i + q_0)\,(m-1)}{(k - km + m^2 + 1)} \end{aligned}$$

where q_0 denotes the quantity produced by the leader. Solving for q_i and q_j in terms of q_0 yields

$$q_i = \frac{(a-c)(k+1) - q_0(m+1)}{2k+m^2+1} \tag{5}$$

$$q_j = \frac{(a-c)(k+m) - q_0(m-1)}{2k+m^2+1} \tag{6}$$

Accordingly, the followers' profits in terms of q_0 are

$$\pi_i = \frac{(q_0(m+1) - (a-c)(k+1))^2}{(2k+m^2+1)^2} \tag{7}$$

$$\pi_j = \frac{(q_0(m+1) - (a-c)(k+1))(q_0(m-1) - (a-c)(k+m))}{(2k+m^2+1)^2} \tag{8}$$

The quantity produced by the leader clearly depends on its being PM or RPM. A PM leader maximizes:

$$\pi_0 = (a - q_0 - kq_i - (m-k)q_j - c)\, q_0 \tag{9}$$

where q_i and q_j are given by (5) and (6), and its optimal quantity is[4]

$$q_0^{PM} = \frac{(k+1)(a-c)}{2(m+1)} \tag{10}$$

By substituting (10) into (7), (8), and (9) we obtain the profits $\overline{\pi}$ which accrue to the various firms in a market structure characterized by a PM quantity leader, and by a generic partition with k PM and $(m-k)$ RPM followers:

$$\overline{\pi}_0 = \frac{1}{4}\frac{(a-c)^2(k+1)^2}{(2k+m^2+1)(m+1)}$$

$$\overline{\pi}_i = \frac{1}{4}\frac{(a-c)^2(k+1)^2}{(2k+m^2+1)^2}$$

$$\overline{\pi}_j = \frac{1}{4}\frac{(a-c)^2\left(3k+m+km+2m^2+1\right)(k+1)}{(2k+m^2+1)^2(m+1)}$$

Alternatively, a RPM leader maximizes

$$R_0 = \frac{m}{m+1}\pi_0 - \frac{k\pi_i + (m-k)\pi_j}{m+1}$$

where π_i, π_j and π_0 are given by (7), (8) and (9). The corresponding maximizing quantity is therefore:

$$q_0^{RPM} = \frac{1}{2}\frac{(a-c)\left(3k+2k^2+2m^2+km^2\right)}{(2k+m^2)(m+1)} \tag{11}$$

[4] Notice that q_0 is increasing in k, since RPM agents are more aggressive than PM agents.

By substituting (11) into (7), (8), and (9) we get the firms' profits $\widehat{\pi}$ associated with the alternative market structure with a RPM quantity leader and the above generic partition with k PM and $m-k$ RPM followers:

$$
\begin{aligned}
\widehat{\pi}_0 &= \frac{1}{4}\frac{(a-c)^2 k\left(3k+2k^2+2m^2+km^2\right)}{(2k+m^2)^2(m+1)} \\
\widehat{\pi}_i &= \frac{1}{4}\frac{(a-c)^2 k^2}{(2k+m^2)^2} \\
\widehat{\pi}_j &= \frac{1}{4}\frac{(a-c)^2 k\left(3k+km+2m^2\right)}{(2k+m^2)^2(m+1)}
\end{aligned}
$$

Again, the solution of the multi-stage game is a partition across types of the population of firms, which is stable in the sense that neither the leader, nor the followers perceive the incentive to unilaterally change their type, given the type chosen by the others. The result is synthesized in the following proposition.

Proposition 2 *In a 'type-quantity' game with simultaneous choice of types and sequential choice of quantities the unique subgame perfect equilibrium is characterized by the quantity leader behaving as profit maximizer, and all the followers behaving as relative profit maximizers.*

Proof. We proceed to identify the deviation-proof configurations in two steps. First, for any given choice of the leader we identify the partitions of the followers from which the latter have no incentive to deviate. With respect to these partitions we then check the incentives to deviate for the leader. Let us start from the case in which the leader is a PM. Consider first the incentive to unilateral deviation perceived by a RPM follower. By solving again the quantity stage of the game under the hypothesis that there are now $k+1$ PM followers, we obtain that the profits of the deviating firm would be:

$$
\overline{\pi}_i^D = \frac{1}{4}\frac{(a-c)^2(k+2)^2}{(2k+m^2+3)^2}
$$

The comparison between $\overline{\pi}_j$ and $\overline{\pi}_i^D$ shows that $\overline{\pi}_j > \overline{\pi}_i^D$ for all m and $k<m$: with a PM leader, all partitions are deviation-proof for a RPM follower. As far as PM followers are concerned, the profits they would earn in case of deviation to a RPM behavior are given by:

$$
\overline{\pi}_j^d = \frac{1}{4}\frac{(a-c)^2\left(3k+km+2m^2-2\right)k}{(2k+m^2-1)^2(m+1)}
$$

It can be checked that $\overline{\pi}_j^d > \overline{\pi}_i$ for all m and $k>0$: if the leader is PM, a PM follower always finds it convenient to deviate and to become a RPM agent. We can therefore establish that with a PM leader, the only stable partition of the followers implies that all followers are RPM, i.e. $k=0$.

We now turn to the case in which the leader is a RPM. Following the same procedure applied above, we find that the profits gained by a RPM follower unilaterally deviating to a PM behavior are

$$\widehat{\pi}_i^D = \frac{1}{4}\frac{(a-c)^2 (k+1)^2}{(2k+m^2+2)^2}$$

It is immediate to verify that $\widehat{\pi}_i^D < \widehat{\pi}_j$ for all m and $k > 0$. The incentive to deviate arises only when $k = 0$: with a RPM leader, a deviation of a RPM follower occurs only when in the market there are no PM firms. Consider now the incentives of PM followers. If one of them deviated into a RPM behavior, it would earn

$$\widehat{\pi}_j^d = \frac{1}{4}\frac{(a-c)^2 (k-1)\left(3k-m+km+2m^2-3\right)}{(2k+m^2-2)^2 (m+1)}$$

A PM follower deviates if $\widehat{\pi}_j^d > \widehat{\pi}_i$, which for all m occurs if $k > 1$: it always deviates, unless it is the only PM firm active in the market. Therefore, if the leader is a RPM the only stable configuration for the followers is the one in which $m-1$ of them are RPM, and only one behaves as PM.

Having identified the configurations which are stable from the followers' perspective, we have now to check whether they are deviation-proof for the leader. We start by considering the partition in which the quantity leader is RPM, $m-1$ followers are RPM and one follower is PM. In this configuration the quantity leader's profits are

$$\widehat{\pi}_{0|k=1|} = \frac{1}{4}\frac{(a-c)^2\left(5+3m^2\right)}{(2+m^2)^2 (m+1)}$$

This expression must be compared with the profits in case of unilateral deviation:

$$\overline{\pi}_{0|k=1|} = \frac{(a-c)^2}{(3+m^2)(m+1)}$$

Since $\overline{\pi}_{0|k=1|} > \widehat{\pi}_{0|k=1|}$ for all m, this partition is not stable and cannot be a solution of the multi-stage game. In the alternative case, with a PM leader and m RPM followers, the leader's profits are

$$\overline{\pi}_{0|k=0|} = \frac{1}{4}\frac{(a-c)^2}{(m^2+1)(m+1)}$$

Should the leader unilaterally deviate from a PM to a RPM behavior, at the quantity stage all firms would be RPM and the equilibrium profits would be equal to zero for both the leader and the followers ($\widehat{\pi}_{0|k=0|} = \widehat{\pi}_{j|k=0|} = 0$). Since $\overline{\pi}_{0|k=0|} > 0$, this partition is overall stable. Therefore, the only equilibrium partition is the one in which the leader is PM, while all followers are RPM ($k = 0$). ■

Notice that in this equilibrium configuration the profits earned by the RPM followers are higher than those of the PM leader: $\overline{\pi}_{0|k=0|} < \overline{\pi}_{j|k=0|}$. Once we allow for a simultaneous choice of types, the advantages of being first-mover at the quantity stage vanish: the leader takes up the role of the benchmark profit maximizer, thus creating a proper environment for the aggressiveness of its RPM followers.

4.2 Sequential choice of types and sequential choice of quantities

In this section we study the equilibrium of a multi-stage game in which we allow for sequentiality of decisions with respect to both the behavioral rules and the quantity decisions.

In particular, the first case we analyze is characterized by the property that all decisions upon types precede those upon quantities: the leader and the followers choose sequentially their types; then the leader and the followers take sequentially the quantity decisions. The key difference between this game and the previous sequential game is that now the leader can take a commitment on its type, which is irrevocable not only with respect to the quantity stage, but also with respect to the choice of type of the followers. This implies that we can identify its solution by simply comparing the profits which accrue to the leader in the two market configurations which are deviation-proof for the followers: if the leader is RPM, $k = 1$; if the leader is PM, $k = 0$. Since $\widehat{\pi}_{0|k=1|} > \overline{\pi}_{0|k=0|}$, in the sequential decision on types the leader will choose to be RPM. This leads to the following proposition:

Proposition 3 *In a sequential 'type-quantity' game, where sequential decisions on types take place before sequential decision on quantities, the unique subgame perfect equilibrium entails a relative profit maximizing leader, and a partition of followers with one profit maximizer and* $(m-1)$ *relative profit maximizers.*

Once the quantity leader is given the opportunity of being first-mover also on types, its optimal choice is to be RPM and to leave to (one of) the followers the burden to be the benchmark profit maximizer. In this case the first mover advantage at the quantity stage is fully recovered: $\widehat{\pi}_{0|k=1|} > \widehat{\pi}_{j|k=1|} > \widehat{\pi}_{i|k=1|}$.

We finally consider the case in which both decisions of the leader – on type and quantity – precede those of the followers. The leader, by formulating an expectation about the distribution of followers across types, takes a commitment both on its type and on its quantity. Followers decide, given all choices of the leader. A rational-expectation equilibrium is defined as a partition of the followers and a choice of type of the leader such that the partition is deviation-proof, the leader's expectations are realized and his choice guarantees the maximum level of profits consistent with his expectations.

The solution of the game under the above sequence of decisions is given in the following proposition:

Proposition 4 *In a sequential 'type-quantity' game, where the leader's decisions on type and quantity precede the type and quantity decisions of the followers, the subgame perfect equilibria are characterized as follows:(a) if* $m = 2$ *and*

$m = 3$, two equilibria exist, both with a PM leader, with $k = 2$ or $k = 1$; (b) if $m \geq 4$, three equilibria exist, two with a PM leader, with $k = 0$ or $k = 1$ and one with a RPM leader, with $k = 2$.

Proof. See the Appendix. ■

Therefore, the equilibria of this game may be characterized by heterogeneity in behavioral rules not only between leader and followers, but also, and more significantly, among followers.

5 The heterogeneity in behavioral rules: a comment

The last case analyzed in the previous section is of particular interest: in that game, heterogeneity in followers' behavior does not simply rely upon the property that RPM agents realize positive profits if at least one PM agent exists. Heterogeneity of followers may occur in many circumstances, in which the alternative is not the simple zero-profit outcome. If the number of followers is equal to three, for example, the equilibria are characterized by a PM leader and a partition of followers, with two of them choosing one type and one choosing the other type. Why is it that heterogeneous partitions of identical agents turn out to be stable? What we have to explain is why the lack of incentives for a, say, RPM agent to move into the PM group may coexist with the incentives of a PM agent to stick to its type. The only possibility for this to occur is that the decision of an agent to unilaterally deviate from one group to the other significantly alters the payoff of any agent of the destination group.

More formally, heterogeneity is a deviation-proof equilibrium if two conditions are satisfied:

$$\begin{aligned} \pi_i &> \pi_j^d \\ \pi_j &> \pi_i^D \end{aligned}$$

where the first inequality states that there is no incentive to deviate from PM to a RPM behavior, and the second excludes the incentive to move in the opposite direction. Notice that we use a generic notation for profits to nest all cases discussed above. For these two inequalities to be simultaneously met, the values of π_i and π_i^D must be sufficiently different, and the same must hold for π_j and π_j^d. Indeed, should $\pi_i \approx \pi_i^D$ and $\pi_j \approx \pi_j^d$, then the above inequalities would yield $\pi_i > \pi_j^d \approx \pi_j$ and $\pi_j > \pi_i^D \approx \pi_i$, with an obvious inconsistency. Therefore, in our set-up explaining behavioral heterogeneity amounts to identifying the conditions under which unilateral deviations from one type to the other cause sufficiently large changes of the individual payoff of the latter. To this purpose, two key elements must be considered. The first is the total number of agents: if the number of agents is high, the impact of individual choices on aggregate variables is relatively low; it is therefore unlikely that a unilateral deviation might alter significantly the rivals' payoffs. The second element is the reaction of the other agents to the deviating behavior. The deviation occurs at the type-stage; therefore, it is observed, and taken into account, by all the firms setting their quantities after its implementation. In a strategic substitutability environment, the rivals partially compensate the quantity implications of any change of type, by moving in the opposite direction. The stronger this compensation, the smaller the impact of the deviation on aggregate variables and on the rivals' payoffs. The stability of heterogeneous partitions of agents depends on the interplay of these two factors.

In particular, in our model we observe heterogeneity of followers (with a PM leader) only in the framework of the game of Proposition 4, in which the leader is locked in his quantity decisions, which take place before the followers' choice of type. Indeed, when all agents, included the PM leader, observe the hypothetical deviation and take it into account at the quantity-stage, the only stable partition entails homogeneity of the followers (Proposition 2). Moreover, we obtain a more articulated pattern of heterogeneity when the number of agents in the above set-up is small (two or three): this is a clear example of a game in which on the one side a unilateral decision to change the behavioral rule has a relevant impact on the rivals' payoffs, on the other side the assumed time-sequence of moves does not allow, at the quantity stage, for a sufficient compensation of the effects of the deviation.

Finally, notice that similar considerations also apply to the cases in which the deviation of the unique PM agent active in the market would bring to zero the profits of the rival RPM agents (Proposition 1 and Proposition 3): the difference between π_j and $\pi_j^d = 0$ is so high to support this peculiar kind of heterogeneity.

6 Conclusions

This paper shows that in a strategic environment behavioral heterogeneity can be an equilibrium outcome of the interaction of profit maximizing agents. In particular, we have shown that this kind of heterogeneity can be a stable configuration of a market in which identical, profit-concerned agents are allowed to strategically choose the behavioral rule they will follow at the market stage. Though all agents have an ultimate objective in terms of profits, partitions in which at the market stage some of them behave as profit maximizers and others as relative profit maximizers turn out to be equilibria, in the sense that no agent perceives the incentive to change his behavioral rule.

Our analysis goes beyond the investigation of rational motives for 'irrational behavior', and it goes beyond the evaluation of the performance of different behavioral rules. It represents a first attempt to trace back behavioral heterogeneity into the individual incentives perceived by rational agents in a strategic environment. In our microfoundation of heterogeneity, both the properties of the relative performance maximization and the sequence of decisions play a crucial role. However, we believe that the basic mechanism supporting the heterogeneity result could be exploited also in different set-ups, relying on the key idea that the stability of a heterogeneous partition of agents requires that individual deviations from one group to the other produce (relatively) large changes in the payoffs of the destination group. A deeper investigation of the robustness of this analysis to different strategic environments is left to future research.

References

[1] Choi, K. and Lu, Y. (2009). A model of endogenous payoff motives and endogenous timing in a mixed duopoly. *Australian Economic Papers*, 48(3), 203–223.

[2] Cyert, R.M. and March, J.G. (1963). *A Behavioral Theory of the Firm.* Englewood Cliffs: Prentice-Hall.

[3] Fershtman, C. and Judd, K.L. (1987). Equilibrium incentives in oligopoly. *American Economic Review*, 77(5), 926–940.

[4] Friedman, M. (1953), The methodology of positive economics. In M. Friedman, *Essays in positive economics* (pp. 1-43). Chicago: University of Chicago Press.

[5] Georgantzís, N., Manasakis, C., Mitrokostas, E., & Petrakis, E. (2008), Strategic delegation in experimental duopolies with endogenous incentive contracts. Working Paper No. 0809, Department of Economics, University of Crete.

[6] Huang, W. (2003). A naive but optimal route to Walrasian behaviour in oligopolies. *Journal of Economic Behavior & Organization*, 52(4), 553–571.

[7] Miller, N. and Pazgal, A. (2002). Relative performance as a strategic commitment mechanism. *Managerial and Decision Economics*, 23(2), 51–68.

[8] Marris, R. (1964). *The Economic Theory of 'Managerial' Capitalism.* London: Macmillan.

[9] Nagel R. and Vriend, N.J. (1999). An experimental study of adaptive behavior in an oligopolistic market game. *Journal of Evolutionary Economics*, 9(1), 27–65.

[10] Offerman, T., Potters, J., & Sonnemans, J. (2002). Imitation and belief learning in an oligopoly experiment. *The Review of Economic Studies*, 69(4), 973–997.

[11] Riechmann, T. (2006). Mixed motives in a Cournot game. *Economics Bulletin*, 29(4), 1– 8.

[12] Schaffer, M.E. (1989). Are profit maximizers the best survivors? *Journal of Economic Behavior & Organization*, 12(1), 29–45.

[13] Schipper, B.C. (2009), Imitators and optimizers in Cournot oligopoly. J*ournal of Economic Dynamics and Control*, 33(12), 1981–1990.

[14] Simon, H.A. (1955). A behavioural model of rational choice. *Quarterly Journal of Economics*, 69(1), 129–138.

[15] Vega-Redondo, F. (1997). The evolution of Walrasian behavior. *Econometrica*, 65(2), 375–384.

[16] Vickers, J. (1985). Delegation and the Theory of the Firm. *The Economic Journal*, 95 (Supplement), 138–147.

[17] Williamson, O.E. (1964). *The Economics of Discretionary Behavior: Managerial Objectives in a Theory of the Firm.* Englewood Cliffs: Prentice-Hall.

Appendix

The proof of Proposition 4 is developed in two steps. We first assume a given choice of the leader and evaluate the incentives to deviate of the followers; then, given the followers' behavior we identify the optimal, rational-expectation choice of the leader.

Step 1a. Assume that the leader is PM and conjectures a partition of the followers in k PM and $(m-k)$ RPM. If these conjectures are realized, the followers' quantities are given by (5) and (6) and the associated profits by (7) and (8), all expressed as a function of the quantity of the leader. In order to check the stability of this partition, we proceed to verify the incentive to unilateral deviation perceived by the followers. Consider first a RPM follower. For any given quantity of the leader, should it deviate into a PM firm, it would earn:

$$\widetilde{\pi}_i^D = \frac{\left(q_0\left(m+1\right)-\left(a-c\right)\left(k+2\right)\right)^2}{\left(2k+m^2+3\right)^2}$$

By substituting in the above expression the quantity the leader committed to, given by (10), we obtain that the profits of a RPM deviating firm are:[5]

$$\widetilde{\pi}_i^D = \frac{1}{4}\frac{\left(a-c\right)^2\left(k+3\right)^2}{\left(2k+m^2+3\right)^2}$$

It can be checked that $\widetilde{\pi}_i^D > \overline{\pi}_j$ only for $k=0$ and $m<4$; therefore, we may conclude that a RPM follower perceives an incentive to unilateral deviation only in the case in which all followers are RPM and the number of followers is limited to two or three. Following a similar procedure we can calculate the profits of a PM deviating follower:

$$\widetilde{\pi}_j^d = \frac{1}{4}\frac{\left(a-c\right)^2\left(3k+m\left(k-1\right)+2m^2-1\right)\left(k-1\right)}{\left(m+1\right)\left(2k+m^2-1\right)^2}$$

In this case the incentive to deviation exhibits a more complicate pattern. For $m=2$ deviation never occurs; for $m=3$, $\widetilde{\pi}_j^d > \overline{\pi}_i$ for $k=3$, while for $m \geq 4$, $\widetilde{\pi}_j^d > \overline{\pi}_i$ for all $k>1$.

We can therefore establish that with a PM leader: (a) if $m=2$ or $m=3$ two stable partitions of the followers exist, with $k=2$ or $k=1$; (b) if $m \geq 4$, the two stable partitions entail $k=0$ or $k=1$.

Step 1b. We turn now to the case of a RPM leader, conjecturing the usual partition of the followers in k PM and $(m-k)$ RPM. Again we use (5) and

[5] Notice that $\widetilde{\pi}_i^D$ differs from $\overline{\pi}_i^D$ (the profits of a deviating RPM firm used in Propositions 2 and 3) since the sequence of decisions has been significantly altered: now we are assuming that deviation occurs once the leader's quantity has already been irrevocably set. While $\overline{\pi}_i^D$ gives the deviation profits of the follower embodying the quantity adjustment of the leader, $\widetilde{\pi}_i^D$ and $\overline{\pi}_j$ are calculated for the same value of q_0.

(6) for the followers' quantities and (7) and (8) for their profits. The quantity produced by the leader is now given by (11). Consider the incentive to deviate of a RPM follower. Should it deviate into a PM firm, it would earn:

$$\underline{\pi}_i^D = \frac{\left(q_0\,(m+1) - (a-c)\,(k+2)\right)^2}{\left(2k+m^2+3\right)^2}$$

By substituting in the above expression the quantity already set by the leader, we obtain the profits of a RPM deviating firm:

$$\underline{\pi}_i^D = \frac{1}{4}\frac{(a-c)^2\left(5k+2k^2+2m^2+km^2\right)^2}{\left(2k+m^2\right)^2\left(2k+m^2+3\right)^2}$$

The comparison between $\underline{\pi}_i^D$ and $\widehat{\pi}_j$ shows that for $m=2$, a RPM follower always deviates, since $\underline{\pi}_i^D > \widehat{\pi}_j$ for $k=0,1$, while for all $m>2$, $\underline{\pi}_i^D > \widehat{\pi}_j$ only if $k=0$. Provided that $m>2$, a RPM firm deviates only if all followers are RPM. By adopting the same procedure, we can show that the profits of a PM deviating firm are:

$$\underline{\pi}_j^d = \frac{1}{4}\frac{(a-c)^2\left(k\,(2k-3)+m^2\,(k-2)\right)w}{\left(2k+m^2\right)^2(m+1)\left(2k+m^2-1\right)^2}$$

where $w = \left(km\,(2k-3)+k\,(6k-1)+m^3\,(k-2)+7km^2+2m^4\right)$. The comparison between $\underline{\pi}_j^d$ and $\widehat{\pi}_i$ shows that for all m, $\underline{\pi}_j^d > \widehat{\pi}_i$ only if $k>2$. Therefore, a PM firm does not deviate only if $k=1$ and $k=2$; for $k>2$ a PM firm always deviates.

We can therefore establish that with a RPM leader, (a) if $m=2$ there is only one stable partition of the followers, with $k=2$; (b) if $m>2$, two stable partitions of the followers exist, with $k=1$ or $k=2$.

Step 2. Consider now the choice of the leader. We recall that if the leader's expectations are fulfilled, his profits as PM or RPM are given respectively by:

$$\overline{\pi}_0 = \frac{1}{4}\frac{(a-c)^2(k+1)^2}{\left(2k+m^2+1\right)(m+1)} \tag{A1}$$

$$\widehat{\pi}_0 = \frac{1}{4}\frac{(a-c)^2\,k\left(3k+2k^2+2m^2+km^2\right)}{\left(2k+m^2\right)^2(m+1)} \tag{A2}$$

Since expectations must be fulfilled at equilibrium, for all m we must exclude all those partitions which are not deviation-proof for the followers under both possible choices of the leader. We are therefore left with the following alternatives:

- if $m=2$ or $m=3$, $k=1$ or $k=2$;
- if $m \geqslant 4$, $k=0$, $k=1$ or $k=2$.

However, some of these partitions are deviation-proof only for a given choice of the leader. In particular:

- if $m = 2$, $k = 1$ is an equilibrium only if the leader is PM;
- if $m \geqslant 4$, $k = 0$ is an equilibrium only if the leader is PM;
- if $m \geqslant 4$, $k = 2$ is an equilibrium only if the leader is RPM.

In the remaining cases – $m = 2$ and $k = 2$; $m = 3$ and $k = 1, 2$; $m \geqslant 4$, $k = 1$ – the leader chooses its type, given his expectations, through a comparison of (A1) and (A2). In all these cases, $\overline{\pi}_0 > \widehat{\pi}_0$ so that the leader chooses to be PM. These results allow us to establish Proposition 4.