Opacity of Banks and Runs with Solvency

Carmela D’Avino and Marcella Lucchetta

University of Venice Cà Foscari

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C. D’Avino and M. Lucchetta

University Cà Foscari in Venice

Abstract

In absence of bank risk-taking behavior, opacity is defined as the inability of depositors, speculators and central banker to disentangle default risk and asset’s return from the asset’s value. We show the conditions under which opacity leads to runs on a solvent bank in equilibrium and uncertainty on fundamental values of the asset. The main repercussion of the opacity is, however, on the central bank’s policy response which is inefficient during a banking crisis.

Key words: Opacity, Bank Runs, Central Bank Intervention, Cash-in-Market Pricing.

JEL Classification: E5, E61, G01, G1, G21.

1 Introduction

The opacity of banks is conventionally perceived as the investors’ and any other interested agent inability to assess the effective riskiness embodied in the banks’ assets portfolio. The difficulty in quantifying risk arises from either the bank’s engagement in less-transparent and non-traditional activities -Myers and Rajan (1995), Morgan (2002), Wagner (2007)- or from limited accounting disclosures -Cordella and Yeyati (1998), Estrella (2004)-. Asymmetric information and/or moral hazard are, thus, the prerequisites to the opacity existence.

To many observers, opacity has had a key role during the recent banking crisis. The uncertainty regarding the actual solvency status of banks at the
crisis’ outburst was essentially due to the discretionary accounting standards used by banks in their assets’ valuation. Specifically, fair value standards\textsuperscript{1} were only applied to trading books of banks and to brokerage firms’ holdings valuation. Illiquid assets were, instead, valued at each bank’s discretion using internal models. Such internal models make hardly valuable, eventually from outsiders of the bank, the real level of assets’ risk. Moreover, missing markets and uncertainty on fundamental values for the ‘toxic’ products have considerably intensified the concerns on the actual solvency of many banks.

The response of the US authorities to this concern has been somehow controversial. The Financial Accounting Standard Board (FASB) on April the 2\textsuperscript{nd} 2009 decided to relax mark-to-market valuation rules, giving, thus, more discretion to banks when evaluating whether a permanent loss has occurred and how to measure it. Moreover, the announcement of the details of the results of the stress test carried out on the top 19 US banks has occurred after a long debate on whether the public disclosure was appropriate. In reply, notwithstanding investors were soliciting for more transparency in order to restore market confidence.

Probably, this policy support to "opacity", through relaxed valuation rules, finds its support also in the existing theoretical models. Cordella and Yeyati (1998), for instance, argue that portfolio risk disclosure increases the probability of bank failure when the bank manager does not have control over the volatility of the assets’ return. In Myers and Rajan (1995) opacity\textsuperscript{2} is desirable for the investors as it leaves limited scope for trading assets and asset substitution undertaken by managers. Wagner (2007) shows that it is optimal for banking managers to be less-transparent, especially during periods of

\textsuperscript{1}In order to assess their ‘fair’ solvency status, banks should have recognized their marked-to-market losses which imply the unveiling of their opaque balance sheet.

\textsuperscript{2}Opacity in this paper implies “less liquid assets”.
increased financial development.

These existing models, thus, by focusing on the conditions under which opacity is optimal by the bank manager, leave unexplored the effects on opacity on several interesting issues such as the run/no-run decision of depositors, asset market pricing and central banker’s intervention. Understanding the existence of opacity affects other agents, other than the bank’s profit-maximizers managers, is obviously relevant for policy-makers.

The aim of this work is to account for all these issues in a simple theoretical model in which opacity has an enhanced role. In particular, we consider opacity as a per-se phenomenon which is not implied by moral hazard behaviors or asymmetric information, as found in the existing theoretical models. In this regard we re-define opacity as the inability of depositors, speculators and central banker to disentangle default risk and asset’s return from the asset’s expected return. We abstract from asymmetric information since the bank faces the same uncertainty when proposing to depositors a standard deposit contract. The signal on the asset’s expected return, which is true and accurate, is determined by the nature and announced by the bank in an intermediate period, when all the agents have the same information set. Moreover, we assume that the contract offered to depositors solves the optimal risk-sharing problem (Allen and Gale (1998)) in which the riskiness of the illiquid asset is irrelevant for the optimal portfolio allocation chosen by the bank. In this way, we are able to abstract from a situation in which the bank has incentives to undertake a moral hazard-type of behavior.

Our modified version of the benchmark model of Allen and Gale (1998) implies the inclusion of default risk of the risky asset; this added feature allows us to draw interesting implications of opacity for bank-runs and fire-sale pricing.
when speculators are either risk-neutral or risk-averse. We show that with opacity a bank run may occur with positive probability in equilibrium even if the bank turns out to be solvent when uncertainty unveils. Moreover, we argue that opacity leads to uncertainty on the fundamental value of the risky asset when speculators in the asset market are risk-averse. Intervention by a central banker has many interesting features when opacity occurs. Intervention is desirable since the central banker bears the eventual losses from the risky asset, ensuring a fixed level of consumption to depositors. However, the central bank lends either more or less than the bank should be entitled to, given the quality of its assets.

The paper is organized as follows. In section 2 we propose the theoretical framework of the paper in which we define the standard deposit contract offered by the bank to consumers and the asset market in which the risky asset might be traded. Moreover, we specify the information set of the bank, consumers and speculators. Section 3 looks at the risky asset market pricing given the opaque signal sent by the nature in the interim period. We distinguish between two cases: one in which speculators are risk-neutral and another in which they are risk-averse. In section 4 we introduce the central banker and we analyze the welfare effects for the consumers following an intervention. We draw different welfare implication depending on whether the speculators are risk-neutral or risk-averse. Section 6 concludes.
2 The Model

2.1 Framework

The model comprises a four-periods economy, \( t = 0, 1, \frac{3}{2}, 2 \), with one consumption good (withdrawals). The agents in this framework are: one representative risk-neutral bank, a continuum of rational depositors/consumers and speculators. In section 4 we will introduce the central banker.

2.1.1 Depositors

Depositors are uninsured with initial endowment \( E \) normalized to 1, i.e. \( E = 1 \). They will deposit all their endowment in \( t = 0 \) at the bank, which offers them insurance against idiosyncratic liquidity shock\(^3\). Indeed, at period 0, depositors do not know when they will be hit by an idiosyncratic liquidity shock: with probability \( \mu \) a given consumer will be withdrawing \( C_1 \) at \( t = 1 \), thus, being early consumer, and with probability \( 1 - \mu \) he will withdraw \( C_2 \) in \( t = 2 \) being a late consumer. Ex-ante, the size of \( \mu \) is known, however, each consumer does not know which type (early/late) he will be at \( t = 1 \). The continuum of depositors is normalized to one such that \( \mu \) is the proportion of early consumers. The utility arising from the consumption of each type in each period is described by a concave and continuous consumers’ utility function \( u(C_t) \).

\(^3\) The bank invests on behalf of consumers given its expertise in recognizing valuable risky assets. Deposits allow consumers that are hit in the last date by a liquidity shock to enjoy the return of the investment made by the bank. Depositors that are hit by the liquidity shock in the earlier period are assured a given level of consumption.
2.1.2 The Bank

At \( t = 0 \) the risk-neutral bank issues demand deposit liabilities equal to one unit of consumption, collecting the whole consumers’ endowment. The bank operates in a competitive market, maximizing the expected utility of consumers; moreover, the bank knows the size of \( \mu \).

At date 0 the bank can invest the deposits in a safe and in a risky asset. The safe asset, \( y \), is in variable supply and can be considered as a storage technology whose price at \( t = 0 \) is normalized to one and can be liquidated at no cost both at \( t = 1 \) and at \( t = 2 \); it has a risk-free gross return equal to 1. The amount of investment in risky asset is denoted \( x \) and is such that \( x + y = 1 \). \( x \) is in fixed supply and yields a random return \( R \) only in \( t = 2 \). In \( t = 2 \) \( R \) yields \( R^h \) with probability \( p \) or zero with probability \( 1 - p \).

2.1.2.1 Information set of the Bank and Consumers

At \( t = 0 \) and \( t = 1 \) both the bank and the consumers face the same uncertainty regarding the random variable \( R \). More specifically, they do not know both the probability density function of \( R \) and the exact value that \( R \) might take in the good state, that is \( R^h \).

Therefore, these agents in \( t = 0 \) and \( t = 1 \) know that in \( t = 2 \) \( R \) yields \( R^h \) with probability \( p^i \) or zero with probability \( 1 - p^i \) where \( i = l, h \). If \( p = p^l \) then, the asset carries an high default risk; if \( p = p^h \) then, the default risk is low. The probability \( p \) allows us to model the default risk of the risky asset, which is, in any case equal to \( 1 - p^i \). With probability \( \alpha \) \( p = p^h \) while \( p = p^l \) occurs with probability \( 1 - \alpha \). In other words, the agents could be facing one of the two cases:
Case 1 Low default risk. With probability $\alpha$:

$$R_{p^h} = \begin{cases} 
\tilde{R}^h & \text{with prob. } p^h \\
0 & \text{with prob. } 1 - p^h
\end{cases}$$

with $E[R_{p^h}] = p^h \tilde{R}^h$.

Case 2 High default risk. With probability $1 - \alpha$:

$$R_{p^l} = \begin{cases} 
\tilde{R}^h & \text{with prob. } p^l \\
0 & \text{with prob. } 1 - p^l
\end{cases}$$

with $E[R_{p^l}] = p^l \tilde{R}^h$.

$\tilde{R}^h$ is also a random variable which is assumed to be distributed according to a normal distribution with mean $\tilde{R}^h$ and finite variance $\sigma_{\tilde{R}^h}$. The distribution of $\tilde{R}^h$ is ex-ante common knowledge.

We further assume that $E[R_{p^l}] > 1$ this implies that investment in risky asset dominates in terms of expected value the investment in storage technology.

2.1.2.2 The Deposit Contract The bank offers non-state contingent contracts that allow depositors to withdraw their funds on demand in either $t = 1$ or $t = 2$ in the fashion of Allen and Gale (1998).

The bank promises a fixed level of consumptions $C_1 = \bar{c}$ to early consumers and $C_2 \geq \bar{c}$ to late consumers. If it is unfeasible to give $\bar{c}$ to all consumers then there is risky asset liquidation and pro-rata distribution among all depositors. The size of $\bar{c}$ is computed from a state-contingent optimal risk-sharing problem.
where no asset liquidation takes place of the type. Since there is aggregate uncertainty in both the return and of its probability density function of the risky asset, the optimal risk sharing problem will yield an optimal portfolio choice \((y^*, x^*)\) which is independent of \(R, R^h\) and of the probabilities attached to it.

The problem can be formalized as follows (see Allen and Gale (1998) for details):

\[
Max_{x, y} E[\mu U(C_1) + (1 - \mu) U(C_2)]
\]

(subject to:

\[
y + x \leq 1 \quad \text{(i)}
\]

\[
\mu C_1(R) \leq y \quad \text{(ii)}
\]

\[
\mu C_1(R) + (1 - \mu) C_2(R) \leq y + Rx \quad \text{(iii)}
\]

The solution to the above problem \((y^*, x^*)\) will determine the consumption levels of early and late consumers. In particular, the bank will promise \(c\) to early consumers such that:

\[
C_1 = \bar{c} = \frac{y^*}{\mu}
\]

Late consumers will receive:
\[ C_2 = \frac{Rx^*}{1-\mu} \]  \hspace{1cm} (2)

In the benchmark model aggregate uncertainty only concerns the return on the risky asset and is accurately revealed at \( t = 1 \); there, runs only happen on a truly *insolvent* banks\(^4\) (i.e. when \( R \) is low enough so that \( C_2 < \bar{c} \)). However, as we will show in the section 4, our stochastic structure of \( p \) and \( R^h \) yields to different implications for the run decisions of consumers, as it causes uncertainty on the size of \( C_2 \) (i.e. (2) is not accurately observed).

### 2.1.3 Speculators and Asset Market

There exists an asset market in which the bank can liquidate the risky asset in the intermediate period \( t = 1 \) whenever the withdraw of early consumers exceeds \( y^* \). In this market there are some identical speculators that will want to purchase the risky asset whenever speculative profits can be made, i.e. when its price falls below its fundamental value. Speculators hold some of the \( y \) asset, \( y_s \), which will be exchanged for the risky asset at a *fire-sale* price. This price will be determined by the size of \( y_s \); indeed, speculators will find it profitable to exchange all their storage technology for the risky asset. Thus, the market price (cash-in-market pricing) will be:

\[ P_x = \frac{y_s}{x} \]  \hspace{1cm} (3)

Throughout the paper we will only consider the case in which \( y_s < y^* \); this assumption allows us to rule out many cases that are not of interest given the aim of our paper. It is, however, reasonable to think that the portfolio of the

\(^4\) Throughout the paper, we refer to insolvent bank as a bank which is not able to guarantee at least \( \bar{c} \) to all consumers.
bank is by far larger than that of speculators, so that can invest in a larger portion of the safe asset.

2.1.3.1 Information set of Speculators. We assume that speculators have the same information set of banks and consumers. However, the size of $y_s$ is speculator’s private information in $t = 1$: it is revealed only if cash in market pricing takes place after than a run has occurred.

2.2 Timing, Signal and Runs on a Solvent Bank

2.2.1 Timing and Signal

In the previous section we have outlined the uncertainty regarding $p^i$ and $R^h$ faced by all agents in the model in both $t = 0$ and $t = 1$. The main implication of the above framework is that late consumers can only observe the expected value of their level of consumption in the final period, i.e. $C_2$. That is:

$$E[C_2] = E[R|x^*] = \frac{R^h x^*}{1 - \mu} (\alpha p^h + (1 - \alpha)p^i)$$ (4)

However, we assume that in $t = 1$ the nature reveals a true and accurate signal on the expected value of the risky asset. That is,

$$\phi = E[R] = pR^h$$ (5)

**Definition** An accurate signal on the asset’s expected return is opaque because it does not enable agents to disentangle default risk and asset’s return.

The main implication of the above opaque signal is that depositors cannot
assess with certainty how much of the observed $\phi$ is due to default risk and asset return.

The uncertainty regarding $p^i$ and $R^h$ is solved in $t = \frac{3}{2}$ while the uncertainty regarding whether $R$ is $R^h$ or zero is solved in the last period, $t = 2$. Therefore, the expected consumption of late consumers becomes:

$$E[C_2] = \frac{\phi x^*}{1 - \mu}$$

Late consumers, given $\mu = 1 - \mu$, will run only if the following condition holds:

$$E[C_2] < \bar{c}$$

that is, if:

$$\phi < \frac{y^*}{x^*}$$

Since $\phi > 1$ then it must also be that a run can only occur when $y^* > x^*$. Clearly, values of $\phi$ sufficiently low can imply very opposite outcomes: very high returns associated with very high default risk or very low returns and low default risk.

If the above condition holds, then, the run will cause costly liquidation on the asset market. As stated in the previous section, when consumers decide to run they do not know the exact size of $y_s$ and so what the market price will be in case of liquidation. While formal asset pricing is derived in the following section, we now summarize the timing of the framework in figure 1.
2.2.2 Runs on a Solvent Bank

The problem of runs dictated by the expected values of future consumptions is mainly that there can be equilibriums in which a run has occurred on what turns out to be a solvent bank. In particular, for a given portfolio choice of the bank, \((y^*, x^*)\), inefficient runs in these terms will depend on the sizes of \(R^{hi}\).

Let’s assume that \(\phi < \frac{y^*}{x^*}\) so that a run occurs. The bank recur to the asset market for costly liquidation, as we will model in detail in section 3. When default risk is low \((\alpha = 1)\)\(^5\) and the good state of the world unveils in \(t = 2\) \((p^i = p^h = 1)\) the bank is solvent if:

\[
R^{hi} > \frac{y^*}{x^*} \tag{9}
\]

Or, if:

\[
\phi < \frac{y^*}{x^*} < R^{hi} \tag{10}
\]

**Proposition 1** In the presence of an opaque signal such that \(\phi \rightarrow 1\) and \(\phi < \frac{y^*}{x^*}\), there might be in equilibrium a run on a potentially solvent bank if

\(^5\) We are implicitly assuming that \(R^{hi,x^*} > y^*\).
the good state of the world materializes with low default risk. This will occur whenever $R^{hl} > \frac{y}{x^2}$.

However, the opposite does not hold true: if a bank is insolvent, a run will never occur with an opaque signal. This is because $\phi > \frac{y}{x^2}$ and $R^{hl} < \frac{y}{x^2}$ (such that $R^{hl} < \frac{y}{x^2} < \phi$) can never occur jointly given that $\phi < R^{hl}$.

3 Risky Asset Market Pricing

3.1 Risk-Neutral Speculators

In this section we consider the pricing of the risky asset in the market when identical speculators are risk-neutral. If at date 1 the bank receives an higher level of withdraws than its available liquidity promised to early consumers, then it is obliged by its contract terms to liquidate all its risky asset and distribute the revenues on a pro-rate basis to all consumers.

The speculators in this market will observe the signal $\phi$ before carrying out any purchase of the risky asset. In particular, the signal $\phi = E[R]$ will perfectly reflect the fundamental value of the asset, given the risk neutrality of speculators. Indeed, the risk-neutrality of these agents implies that their spending decisions are not affected by the default risk or the relative return implied in the signal. Speculators, then, once observed $\phi$ will purchase the risky asset if its market price, $P_x$, is below its fundamental value, i.e. $\phi^6$.

The pricing in the market happens through a cash-in-market mechanism (Allen and Gale 1998). That is, since speculators will want to exchange all

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6 The safe asset is held by speculators in order to exchange it with profitable purchases of the risky asset.
their safe asset for the risky, given $\phi > 1$, then the price of the risky asset will simply be the ratio of the safe asset of the speculators, $y_s$, to the risky asset of the bank, $x^*$. In other words, it is the amount of safe asset, readily exchangeable to cash, to determine the market price of the risky asset. However, speculators will only buy if speculative profits can be made, that is, if the liquidity in their hands is such that prices are below fundamentals, that is:

$$P_x = \frac{y_s}{x^*} \leq E(R) = \phi \quad (11)$$

Given that (8) must hold, in order to a run to ever occur, then it must be that speculators will purchase the risky asset whenever the observed signal satisfies the following condition:

$$\frac{y_s}{x^*} \leq \phi \leq \frac{y^*}{x^*} \quad (12)$$

The associated consumption levels will be:

$$C_1 = C_2 = \frac{y^* + y_s}{2} \quad (13)$$

Figures A1 and A2 depict the asset market pricing of the risky asset and the (expected and actual) late consumption levels for all signal levels signal respectively. In figure 1A it can be seen that for $\phi < \frac{y_s}{x^*}$ there does not exist a market for the risky asset, thus, its market price is zero as speculators are not willing to buy the risky asset. In this case, as shown in 2A, early and late consumers share equally the available safe asset in the bank’s portfolio, i.e. $y^*$. It is clearly seen from the pictures that when (12) is satisfied, then the late (realized) consumption level is as specified in (13). For high enough signals, i.e. $\phi > \frac{y^*}{x^*}$, then no run occurs and expected late consumption, as perceived
in $t=1$, is equal to $E[C_2] = \phi x^\ast$.

3.2 Risk-Averse Speculators

In this section we relax the assumption of risk-neutrality of speculators, by assuming that they are risk-averse. The main implication of this modified setting is that the observed signal $\phi$ does not reveal anymore the fundamental value of the risky asset. Therefore, speculators now face uncertainty regarding the intrinsic value of the asset. Indeed, now the fundamental value has to reflect the default premia that speculators require to take on more risk. At date 1, if the risky asset has a higher default risk, i.e. $p^i = p^l$, then its fundamental value will be lower than the fundamental value of the asset with the lower default risk, i.e. $p^i = p^l$. The fundamental values of the asset in each state of the world can be written as:

$$F^h_v = \frac{E(R)}{1 + \pi^l}$$ (14)

$$F^l_v = \frac{E(R)}{1 + \pi^h}$$ (15)

where $\pi^l$ and $\pi^h$ are the discounts which reflect the default premia of the asset in each state with $\pi^h > \pi^l$. Given $F^h_v > F^l_v$, $F^h_v$ is the fundamental value of the asset for which $\phi = p^h R^{hl}$ is true; while $F^l_v$ is the fundamental value of the asset for which $\phi = p^l R^{hh}$ is true.

Speculators, will buy the risky asset only if (8) occurs and if the two conditions below are satisfied:
\[ E(F_v) = \alpha F^h_v + (1 - \alpha) F^l_v > 1 \quad (16) \]

\[ P_x = \frac{y_s}{x} < E(F_v) \quad (17) \]

Condition (16) implies that the expected fundamental value corresponding to the observed \( \phi \) has a gross return higher than that of the safe asset. (17), instead, states that the liquidity (safe asset, \( y_s \)) in the hands of speculators has to be such that the market price of the risky asset is less than the expected fundamental value. Indeed, buying only if \( \frac{y_s}{x} < F^l_v \), would prevent speculators to make potential speculative profits if \( F^l_v < \frac{y_s}{x} < E(F_v) \). Solving (17) with respect to \( \phi \), we find that:

\[ \theta \frac{y_s}{x} < \phi \quad (18) \]

Where:

\[ \theta = \frac{1}{\psi} = \frac{1}{\frac{\alpha}{1 + \pi} + \frac{(1 - \alpha)}{1 + \pi}} > 1 \quad (19) \]

Combining (8) with (18), we find that the buy-condition for risk-averse speculators is:

\[ \theta \frac{y_s}{x^*} < \phi \leq \frac{y^*}{x^*} \quad (20) \]

Or:
\[
\frac{y_s'}{x^*} < \phi \leq \frac{y_s}{x^*}
\]  
(21)

with \(\theta y_s = y_s'\).

The market price of the risky asset, if speculators buy is always \(\frac{y_s}{x}\). However, now, contrarily to what seen in the previous section, there is the chance that speculators might not make speculative profits. Figures 3A and 4A in the appendix show how this might occur. Let’s consider the case in which speculators purchase the risky asset as condition (21) holds for an observed \(\phi\). In 3A speculators hold a larger amount of \(y_s\); at a market price \(P_x = \frac{y_s}{x}\) speculative profits will be made only if uncertainty unveils in \(t = \frac{3}{2}\) that \(\phi = p^h R^{hl}\) so that \(F_v = F^h_v\). If in \(t = \frac{3}{2}\), indeed, turns out that \(\phi = p^l R^{hh}\), then the asset has been overpriced by the cash-in-market mechanism, i.e. speculators have paid too much for the risky asset. If, instead, \(y_s\) is lower, as depicted in 4A, then speculative profits can be made even if uncertainty unveils in \(t = \frac{3}{2}\) that \(\phi = p^l R^{hh}\) (i.e. \(F_v = F^l_v\)) given that the signal is at least \(s\). If, instead, the signal is such that \(\frac{y_s'}{x^*} < \phi \leq s\) then again speculators have paid too much for the risky asset. It is worth noting that a buying strategy for speculators which implies buying if \(s < \phi \leq \frac{y_s}{x}\) is not desirable since it would preclude speculators to make considerable profits if \(F_v = F^h_v\).

A last case should also be considered here; that is, the possibility that the safe asset in the hands of speculators could be so low that they would make speculative profits whatever the signal. In this case the market prices would much smaller than the so-far considered cases and speculators will price the risky asset at a price lower than \(F^l_v\) for all signal included in \(\frac{y_s'}{x^*} < \phi \leq \frac{y_s}{x^*}\).

If there is no central banker’s intervention, late consumers will be better off the higher \(y_s\) in the speculators’ portfolio, given that it is proportional to market
price paid for the asset.

We can formalize these results as follows:

**Proposition 2** *With risk-averse speculators an opaque signal causes uncertainty on the fundamental value of the risky asset. When speculators hold enough safe asset they may overprice the risky asset if the nature unveils a state of the world with high default risk. In this instance, late consumers are better off than if the safe asset in the hands of speculators was lower. Therefore, consumers benefit at the speculators’ expenses from speculators’ higher amounts of safe asset holdings with higher default risk.*

4 Central Banker’s Intervention

In this section we consider the welfare effects of an intervention by the central banker. The central bank cannot restore consumption levels of a no-run equilibrium but can guarantee higher levels of late and early consumption than if cash-in-market pricing had taken place.

The central banker in this model has the same information set of consumers. That is, he observes the signal $\phi$ at $t = 1$. Depending on the market price of the risky asset, whenever, a bank run occurs, the central banker might decide to intervene in order to sustain asset prices. If intervenes, he enters a repurchase agreement with the bank in which he purchase the risky asset. The price paid for the risky asset in the repo agreement is equal to its fundamental if investors are risk-neutral. If, instead, investors are risk-averse then the central bank faces uncertainty on the fundamental value of the risky asset and might over/under price the asset. The terms of the repurchase agreement oblige the
bank to re-pay the central banker in $t = 2$ whatever it gets from the risky asset. The central banker will enter the repo agreement only if its expected net gain is greater than zero:

$$E[NG^{cb}] = \phi x^* - M[\alpha(1 - p^h) + (1 - \alpha)(1 - p^l)] > 0 \quad (22)$$

Where $M = P^s x^*$ is the price paid by the central banker to the bank for the purchase of the risky asset at the support price $P^s$.

4.1 Risk-Neutral Speculators

If the asset market is populated by risk-neutral investors, then the fundamental price of the asset is equal to the observed signal $\phi$. The central banker might decide to enter the repo agreement when the liquidity (safe asset) in the hands of speculators is low enough to drive market prices below fundamentals and when there is no market for the risky asset. Therefore, he will lend $M = \phi x^*$ to the bank with $P^s = \phi$, i.e. he will sustain prices to fundamentals. It can be easily noticed that in this setting the central bank will enter the repo agreement at every level of $\phi$. Indeed, (22) becomes:

$$E[NG^{cb}] = \phi x^* \{1 - [\alpha(1 - p^h) + (1 - \alpha)(1 - p^l)]\} > 0 \quad \forall \phi \quad (23)$$

The resulting consumption levels for early and late consumers will therefore, be:

$$C_1 = C_2 = \frac{y^* + \phi x^*}{2} \quad (24)$$
This is greater that what consumers would have received if fire-sale had occurred:

\[
\frac{y^* + \phi x^*}{2} > \frac{y^* + P_x x^*}{2}
\]  \hspace{1cm} (25)

The main implication of the above intervention is that the central banker that engages in the rescue intervention is not certain about the solvency of the bank. Insolvency can be due to either the occurrence of the bad state of the world, i.e. \( R^l = 0 \), or to the fact that in the good state of the world late consumers get less than early consumers (this will depend on the size of \( R^{hl} \)).

The inability to distinguish a solvent from an insolvent bank renders the intervention by the central bank risky, in the sense that the central bank could bear the loss if either the bad state of the world materializes or \( R^{hl} \) is low enough so that the realized (i.e. in \( t = 2 \)) \( NG^{cb} \) is less than zero. In the former case, then the bank in \( t = 2 \) will be unable to pay anything to the central banker, which will bear a loss equal to, the whole \( M \). If instead, the good state of the world materializes and \( R^{hl} < \frac{y^*}{x^*} \), then the loss faced by the central banker will be:

\[
NG^{cb} = (R^{hl}x^*) - M < 0
\]  \hspace{1cm} (26)

The intervention by the central banker, moreover, avoids late consumers to bear the losses incurred in the bad state of the world with \( R^l = 0 \). In fact, it guarantees a fixed level of consumption for late/early consumer equal to \( \frac{y^* + \phi x^*}{2} \) which is in any case higher than what they would have received if the bank had gone to the asset market. This is shown in figure 6A in which is depicted the consumption levels (actual and expected) by late consumers following the
central banker’s intervention when $\phi < \frac{\nu}{x}$ (blue line). Figure 6A, instead, shows the effect on the pricing of the risky asset of an intervention of this kind (blue line): the price is equal to its fundamental for every level of the signal.

**Proposition 3** With risk-neutral speculators the central bank will intervene to support prices to fundamentals at every $\phi < \frac{\nu}{x}$. The central bank will carry both the default risk and the risk that the bad state of the world materializes. Consumers are guaranteed a sure and fixed consumption level equal to $\frac{\nu + \phi x^*}{2}$.

### 4.2 Risk-Averse Speculators

If the fundamental value of the risky asset is uncertain, then, it becomes more problematic for the central bank to pursue an intervention aimed to support fundamental prices. Reasonably, the central banker’s intervention when there is opacity in fundamental values will be such that (1) consumers get more than they would do from the cash-in-market pricing and (2) the expected net gain of the central banker are maximized. The risky asset price that the central bank will support is, thus, dependent on these two conditions. However, it will on a first place depend on the cash-in-market price in the asset’s market which is determined by $y_s$. Indeed, a fits-for-all policy that sustain prices at the expected fundamental level (i.e. $P^s = E(F_v)$ for $\forall \phi < \frac{\nu}{x}$) could decrease the expected net gains of the central bank. Let’s see this in more details.

Let’s assume, for simplicity, that the central bank has three possible intervention strategies. That is, it can lend to the bank:

$$M_1 = E(F_v)x^*$$ (27)
\[ M_2 = F^h_v x^* \]  

(28)

\[ M_3 = F^l_v x^* \]  

(29)

The corresponding expected net gains are:

\[ E[N G^{cb}_1] = x^*(\phi - E(F_v))[\alpha (1 - p^h) + (1 - \alpha)(1 - p^l)] \]  

(30)

\[ E[N G^{cb}_2] = x^*(\phi - F^h_v)[\alpha (1 - p^h) + (1 - \alpha)(1 - p^l)] \]  

(31)

\[ E[N G^{cb}_3] = x^*(\phi - F^l_v)[\alpha (1 - p^h) + (1 - \alpha)(1 - p^l)] \]  

(32)

Given that \( \phi > E(F_v), \phi > F^i_v \) and that \( 0 \leq \alpha (1 - p^h) + (1 - \alpha)(1 - p^l) \leq 1 \) then it must be that:


(33)

Also note that (30), (31) and (32) are all greater than zero \( \forall \phi \), therefore the central banker always wishes to intervene and lend to the bank.

4.2.1 Intervention with high levels of \( y_s \)

If speculators hold abundant levels of \( y_s \) in their portfolio, as described in figure 3.A, as we have already seen, they will make speculative profits only if he fundamental value turns out to be high (low default risk) when \( \frac{y^l}{x^2} < \phi \leq \frac{y^u}{x^2} \).
Sustaining asset price to low fundamental values, i.e. $P^* = F^l_v$ and $M_3 = F^l_v x^*$, although maximizes the expected net gain of the central banker, would not be a sustainable intervention. This is because early and late consumers would get less than if speculators were purchasing the asset, that is:

\[
\frac{y^* + y_s}{2} > \frac{y^* + F^l_v x^*}{2}
\]  

(34)

Therefore, when $\frac{y^*}{x^*} < \phi < \frac{y_s}{x^*}$ the central bank will support prices to its expected fundamental values since $E[NG_{cb}^3] < E[NG_{cb}^1]$. The actual consumption level are, thus:

\[
C_1 = C_2 = \frac{y^* + E[F_v] x^*}{2}
\]  

(35)

However, when the signal is low enough so that no market for the risky asset exists, that is when $\phi < \frac{y_s}{x^*}$, then the central banker can support prices to low fundamental values, that is $P^* = F^l_v$. In this case, early and late consumers will get more than if they were sharing equally the available $y^*$:

\[
C_1 = C_2 = \frac{y^* + F^l_v x^*}{2} > \frac{y^*}{2}
\]  

(36)

The pricing of the risky asset with central bank’s intervention and high levels of $y_s$ is depicted in figure 7A in the appendix.

**Proposition 4** With risk-averse speculators and high enough market prices (and $y_s$) the central bank intervenes to support prices at every $\phi < \frac{y_s}{x^*}$. The central bank will carry both the default risk and the risk that the bad state of the world materializes. Consumers are guaranteed a fixed consumption level equal to $\frac{y^* + P^* x^*}{2}$. 

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A central banker’s intervention of this kind (i.e. with opacity) can cause inefficient asset pricing, that is, asset pricing different from fundamentals. Indeed, when the signal is very low such as $\phi < \frac{\nu}{x^2}$ the central bank might under-price the asset, lending to the bank less than it should have received if in $t = \frac{3}{2}$ it occurs that $\phi = \phi^h R^h$ (so that $F_v = F^h_v$). For higher levels of the signals such that $\frac{\nu}{x^2} < \phi \leq \frac{\nu^c}{x^2}$ the central bank is surely either over-pricing or under-pricing the asset. In other words, the central bank is lending either more or less than the bank should be entitled to, given the quality of its assets.

**Proposition 5** Opacity leads to inefficient policy responses. The central bank can lend either more or less than the bank should be entitled to, given the quality of its assets.

**Proposition 6** Given high values of the signal (but always less than $\frac{\nu}{x^2}$), risk-averse speculators and high enough market prices (i.e. high $y_s$), the policy response is surely inefficient.

### 4.2.2 Intervention with low levels of $y_s$

If speculators hold relatively low levels of safe asset as in figure 4A, we have already shown that there exist a boundary signal $s$ which determines two different outcomes for speculators. If the signal is such that $s < \phi \leq \frac{\nu^c}{x^2}$, then, speculative profits can be made whatever the fundamental value unveils (although clearly $F^h_v$ is associated with higher profits). If, instead, the signal is such that $\frac{\nu}{x^2} < \phi \leq s$ then again speculators make profits only if the default risk attached to the asset is low, that is, if $F_v = F^h_v$.

The central banker, thus, will adopt three different intervention strategies, depending on the observed signal. If there is no market for the risky asset as $\phi < \frac{\nu}{x^2}$, as before, the central banker will support prices to $F^l_v$, lending to the
bank $M_3$ and achieving the consumption levels as in (37). If the signal is such that $\frac{y^s}{x^s} < \phi \leq s$ then for the same reasoning as in the previous section, the central banker lends $M_1$ to the bank. If, instead, $s < \phi \leq \frac{y^s}{x^s}$ then the central bank will maximize its expected net gain by lending $M_3$ to the bank, which implies $P^s = F^d$ with the following consumption levels:

$$C_1 = C_2 = \frac{y^s + F^d x^*}{2} > \frac{y^s + y_s}{2}$$

(37)

The pricing of the risky asset with central bank’s intervention and low levels of $y_s$ is depicted in figure 8A in the appendix.

The safe asset in the hands of speculators, however, could be so low that they would make speculative profits whatever the signal (in this case the signal $s$ would not exist). In this case, clearly the central bank would support the prices of the asset at its low fundamental value.

From these results we can formalize the following proposition:

**Proposition 7** When speculators hold low levels of safe asset, so that market prices are relatively lower, the central bank tends to support asset prices as if they carried a high default risk. For a small interval of signals, however, the central bank might over-price/under-price the asset if the cash-in-market mechanism yields a higher pricing than $F^d$.

5 Conclusions

In this paper we have included opacity in a simple model in which a representative bank, solving an optimal risk-sharing problem when offering a standard deposit contract, is subject to runs by depositors. Opacity is modeled through
the inclusion of unobservable default risk on the bank’s portfolio, as well as unobservable return on the risky asset. The inability of the agents to distinguish between the two given a signal sent by the nature on their product has many interesting implications. Firstly, we show that run decisions based on expected consumption levels can cause a run on a solvent bank. Secondly, we model the asset market pricing that occurs through a cash-in-market mechanism. In this regard, we stress that if that opacity leads to uncertainty on the fundamental value of the risky asset when speculators in the asset market are risk-averse. Lastly, we analyze the welfare implications of a central banker’s intervention which is unable to prevent the run but ensures a fixed level of consumption higher than if speculators were purchasing the asset during a run. The central banker, with the aim to minimize its loss function, supports prices to the low fundamental values. In general, the lower the market prices, the more likely the central banker will enter a repo agreement with the bank by offering a price for the risky asset equal to the lowest fundamental level that it can take. Therefore, opacity can cause inefficient policy responses. This is because the central bank lends either more or less than the bank should be entitled to, given the quality of its assets.
References


Figures

Figure 1A: Risky asset pricing and observed signal with risk-neutral speculators (without central banker’s intervention)
Figure 2A: Expected late consumption and observed signal with risk-neutral speculators (without central banker’s intervention)

A bank run associated with speculators purchase of the risky asset occurs if the observed signal at date 1 is such that \( \frac{y_s}{x} \leq \phi \leq \frac{y^*}{x} \). Realized late consumption in this case is equal to \( C_2 = \frac{y^* + y_s}{2} \). It is easily seen that at this consumption level, late consumers receive more than they would have got if they did not run if \( \frac{y_s}{x} \leq \phi \leq s = \frac{y^* + y_s}{2x^*} \). Otherwise (i.e. if \( s \leq \phi \leq \frac{y^*}{x^*} \)) late consumers would have received more if they did not run and cash-in-market pricing did not take place, even if \( E[C_2] < y^* \). Indeed, recall that when a run takes place, consumers are unaware of the size of \( y_s \). When the signal is so low that speculators are not willing to buy, i.e. \( \phi \leq \frac{y_s}{x^*} \), the bank will share equally among early and late consumers the available \( y^* \). Also in this case, late consumers might have received more if they did not run, in particular as \( \phi \to \frac{y_s}{x^*} \).
Figure 3A: Buying decision and observed signal with risk-averse speculators- high levels of $y_s$

Note that $\frac{\mu_{y} s}{x^2} = \theta \frac{y_s}{x^2}$. 
Figure 4A: Buying decision and observed signal with risk-averse speculators—low levels of $y_s$.

Note that $\frac{y_{is}}{x^*} = \theta \frac{y_s}{x^*}$.

Figure 5A: Risky asset pricing and observed signal with risk-neutral speculators (with central banker’s intervention).
Figure 6A: Expected late consumption and observed signal with risk-neutral speculators (with central banker’s intervention)

![Graph showing expected late consumption and observed signal with risk-neutral speculators]

Figure 7A: Risky asset pricing and observed signal with risk-averse speculators (with central banker’s intervention)- high levels of $y_s$

![Graph showing risky asset pricing and observed signal with risk-averse speculators]

The red lines refer to asset market pricing without intervention. That is, when $\phi < \frac{y_s}{x^*}$ there is no market for the risky asset; when $\frac{y_s}{x^*} < \phi \leq \frac{y^*}{x^*}$ there is cash-
in-market asset pricing. In the former case, the central bank will support prices to low fundamentals (blue line). In the latter case, it will support prices to expected fundamental values (blue line).
Figure 8A: Risky asset pricing and observed signal with risk-averse speculators (with central banker’s intervention)- low levels of $y_s$

The red lines refer to asset market pricing without intervention. That is, when $\phi < \frac{y'}{x^*}$ there is no market for the risky asset; when $\frac{y'}{x^*} < \phi \leq \frac{y^*}{x^*}$ there is cash-in-market asset pricing. In the former case, the central bank will support prices to low fundamentals (blue line). When $\frac{y'}{x^*} < \phi \leq s$ the central banker support prices at expected fundamental values (blue line). When $s < \phi \leq \frac{y^*}{x^*}$ the central bank will support prices to low fundamentals (blue line).