Testing Goodwin's Growth Cycle Disaggregated Models: Evidence from the Input-Output Table of the Greek Economy for the Year 1988

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ABSTRACT

This paper tests empirically the well-known Goodwin's 'growth cycle' disaggregated models, using data from the symmetric input-output table of the Greek economy for the year 1988. It is found that from qualitative as well as quantitative point of view, both models are not adequate to describe the long-run workers' share-employment rate trajectories of the Greek economy. However, in the medium-run analysis the evidence presented here are more encouraging: at qualitative level, one of the two Goodwin's models is found to be adequate to describe the dynamic behavior of the workers' share and employment rate.

Keywords: Business cycles, Disaggregated models, Input-output tables, Principal coordinates

JEL Classification: C67, D57, E24, E32

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1. Introduction

As is known, there have been a great number of attempts to evaluate the original Goodwin's 'growth cycle' model [5] empirically.[1] However, there has not been a parallel empirical development of the two Goodwin's 'growth cycle' disaggregated models: (i) the model which was originally developed by Goodwin in *Use of Normalised General Co-ordinates in Linear Value and Distribution Theory* [6], and re-printed in [8: ch. 7] (hereafter, GDM1= Goodwin's Disaggregated Model 1); and (ii) the model which was previously developed by Goodwin [9] and re-proposed again in a set of papers [10, 11] and in his book with Punzo [12] (hereafter, GDM2).

The purpose of this paper is to test empirically these Goodwin's models using data from the symmetric input-output table (SIOT) of the Greek economy for the year 1988. For this purpose, we also use data for OECD Department of Economics and Statistics' publications National Accounts and Labour Force Statistics for various years.

The remainder of the paper is organized as follows. Section 2 presents the analytic framework. Section 3 provides the results of the empirical analysis. Section 4 concludes.

2. Goodwin's Models

The model which was proposed in [6] may be described by the following relations [6, 7, 8: ch. 7]:

$$y_j \equiv (1-\lambda_j)x_j, \lambda_j < 1 \ , \ j = 1,2,...,n \tag{1}$$

$$L_j / x_j = a_j, \hat{a}_j = -b \tag{2}$$

$$\hat{N}_j = n \tag{3}$$

$$\hat{m}_j \equiv \hat{w}_j - \hat{p}_j = \varepsilon (L_j / N_j) - \zeta, \zeta < \varepsilon \tag{4}$$

$$g_j = r_j \equiv r_n - \hat{p}_j \tag{5}$$

$$\hat{p}_j = (\lambda_j + \theta_j)(1 + r_n) - 1 \tag{6}$$

where b, n, ε, ζ are positive constants. As usual, a 'dot' ('hat') above a variable denotes time derivative (logarithmic derivative), *i.e.,* $\dot{z} = dz/dt$ ($\hat{z} = \dot{z}/z$). Furthermore, y_j (x_j), L_j, a_j, N_j, m_j (w_j), g_j, r_j, p_j and θ_j denote the net (gross)

[1] See [1], [2], [21], [3], [13], [16], [4], *inter alia.*

output, employment, direct labour coefficient, labour force, money (real) wage rate, growth rate, real rate of profit, 'price level', and the unit labour cost of the jth eigensector, respectively. Finally, λ_j denotes the j eigenvalue of the diagonalizable $n \times n$ matrix of material input coefficients, $\mathbf{A}$.[2] It is also assumed that the system is viable, *i.e.*, the Perron-Frobenius (hereafter, P-F) eigenvalue, $\lambda_{\mathbf{A}}$, is less than 1 (for more details, see [14: chs 3-4]).

Relation (1) captures the assumption that capital lasts for one period of production. Relations (2) and (3) capture the assumption of steady ('disembodied') technical progress and steady growth of the labour force, respectively. Relation (4) captures the assumption that the money wage rate rise in the neighborhood of full employment. Relation (5) captures the assumption that 'profit rate is proportional to growth rate' and 'the proportion is being taken as unity for simplicity' [8: 147]. Finally, the nominal profit rate, r_n, is assumed to be fixed and *uniform* in all eigensectors, whilst the cost in one period determines price in the next, *i.e.*, $p_j(t+1) = (p_j(t)\lambda_j + m_j(t)a_j(t))(1+r_n)$.[3] Hence, from the last relation, 'if we ignore the difference between differentials and differences' (Goodwin, *ibid.*) we get relation (6).

Relations (1)-(6) reduce to n *2D*-systems of differential equations in state variables l_j and β_j:

$$\hat{l}_j = \varepsilon\beta_j - (b+\zeta) - [\lambda_j + (1-\lambda_j)l_j](1+r_n) + 1 \tag{7}$$

$$\hat{\beta}_j = (1+r_n)(1-\lambda_j)(1-l_j) - (b+n) \tag{8}$$

where $l_j (\equiv (w_j L_j)/y_j)$ and $\beta_j (\equiv L_j / N_j)$ denote the state variables, workers' share and employment rate of the jth eigensector, respectively.

[2] Matrices (and vectors) are denoted by boldface letters.

[3] It should be mentioned that Goodwin considered: (i) n independent labour markets, each with its particular, given growth rates of productivity and labour force (or $b_j, n_j, \varepsilon_j, \zeta_j$); and (ii) a particular nominal profit rate in every eigensector. However, the present analysis is based on the assumption of a uniform labour market and nominal profit rate in all eigensectors. It is not too difficult to show that if the eigenvalues of the material input coefficients matrix are complex, then Goodwin's assumptions lead to a system (of coupled equations) which is economically insignificant. See Appendix A for details.

Linearizing these systems around the non-zero equilibrium points $\left(l_j^*, \beta_j^*\right) = \left(\alpha_j / e_j, c_j / d_j\right)$, where $\alpha_j \equiv (1+r_n)(1-\lambda_j)-(b+n)$, $e_j \equiv (1+r_n)(1-\lambda_j)$, $c_j \equiv r_n + \zeta - n$, $d_j \equiv \varepsilon$, and using algebra, we obtain

$$\ddot{\mathbf{l}} = -<\mathbf{K}>\dot{\mathbf{l}} - <\mathbf{N}>\mathbf{l}', \mathbf{l}' \equiv \mathbf{l} - \mathbf{l}^* \tag{9}$$

$$\dot{\boldsymbol{\beta}} = -<\mathbf{\Omega}>\mathbf{l}' \tag{10}$$

where $\mathbf{l} \equiv [l_j]$ ($\mathbf{l}^* \equiv [l_j^*]$) and $\boldsymbol{\beta} \equiv [\beta_j]$ denote the vectors of (equilibrium) workers' shares and employment rates of the disaggregated system, respectively. Finally, $<\mathbf{K}> \equiv <\alpha_j>$, $<\mathbf{N}> \equiv <\alpha_j c_j>$ and $<\mathbf{\Omega}> \equiv <e_j(c_j/d_j)>$ are the $n \times n$ diagonal matrices of the parameters of the system (9)-(10).

Going back to the 'original coordinates'.[4] Thus, pre-multiplying relations (9) and (10) by $\mathbf{Q}$ and taking into account that the vector $\mathbf{u} \equiv [u_j] \equiv \mathbf{Ql}$ ($\mathbf{v} \equiv [v_j] \equiv \mathbf{Q}\boldsymbol{\beta}$) denotes the *sectoral* workers' shares (the *sectoral* employment rates) of the 'original system', we obtain

$$\ddot{\mathbf{u}} = -\mathbf{K}\dot{\mathbf{u}} - \mathbf{N}\mathbf{u}', \mathbf{u}' \equiv \mathbf{u} - \mathbf{u}^* \tag{11}$$

$$\dot{\mathbf{v}} = -\mathbf{\Omega}\mathbf{u}' \tag{12}$$

where $\mathbf{u}^* \equiv [u_j^*] \equiv \mathbf{Ql}^*$ is the vector of the equilibrium workers' shares of the 'original system'. The matrices $\mathbf{K} \equiv \mathbf{Q}<\mathbf{K}>\mathbf{Q}^{-1}$, $\mathbf{N} \equiv \mathbf{Q}<\mathbf{N}>\mathbf{Q}^{-1}$ and $\mathbf{\Omega} \equiv \mathbf{Q}<\mathbf{\Omega}>\mathbf{Q}^{-1}$ give the parameters of the system (11)-(12). Finally, $\mathbf{Q} \equiv [q_j]$ ($\mathbf{Q}^{-1} \equiv [q_j']$) is the $n \times n$ matrix which is formed from the right-hand side (left-hand side) eigenvectors of $\mathbf{A}$, *i.e.*, it denotes the 'modal matrix' of $\mathbf{A}$.

The system (11) is easily recognizable as a 'free vibration of damped multiple degree of freedom system'. Hence, from system (11) we obtain the solutions u_j', in terms of t. Then substituting $\mathbf{u}'$ in (12), we obtain $\mathbf{v}'$. Therefore, as mentioned by Goodwin, the above system exhibits the following dynamic: 'A given initial condition chooses one of the curves, and each sector spiral on to its stable equilibrium point.' [8: 148].

[4] It is important to note that 'these eigensectors do not exist-they are mere accounting devices: no decisions are taken by such fictitious units' [12: 60]. Regarding this, Goodwin observes that we can always go back to the 'original coordinates'. Through a 'coordinate transformation', the n independent single degree of freedom systems (see, relations (9)-(10)) are transformed back to a multiple degree of freedom system (see, relations (11)-(12)) and *vice versa*.

Ceteris paribus, we assume that the speed of adjustment of output to excess demand of the jth eigensector is given by the following relation [10, 11, 12: 106-112]

$$\dot{x}_j = \mu[k\dot{x}_j - (x_j - \lambda_j x_j - w_j a_j x_j)], \mu k > 1 \tag{13}$$

where μ denotes the speed of adjustment of output to excess demand and k the desired capital output ratio, both of which are taken to be the *same* in all eigensectors.[5] Furthermore, we assume that $\hat{p} = 0$, and, therefore relation (4) is substituted by the following relation

$$\hat{w}_j = \rho (L_j / N_j) - \gamma, \gamma < \rho \tag{4a}$$

where ρ, γ are positive constants.

Relations (1)-(3), (4a) and (13) reduce to n *2D*-dynamical systems in l_j and β_j:

$$\hat{l}_j = \rho \beta_j - (b + \gamma) \tag{14}$$

$$\hat{\beta}_j = [\mu / (k\mu - 1)](1 - \lambda_j)(1 - l_j) - (b + n) \tag{15}$$

Linearizing these systems around the non-zero equilibrium point $\left(l_j^*, \beta_j^*\right) = \left(\phi_j / \chi_j, s_j / \iota_j\right)$, where $\phi_j \equiv [\mu / (k\mu - 1)](1 - \lambda_j) - (b + n)$, $\chi_j \equiv [\mu / (k\mu - 1)](1 - \lambda_j)$, $s_j \equiv b + \gamma$, $\iota_j \equiv \rho$, and using algebra, we obtain

$$\dot{\mathbf{l}} = - < \mathbf{M} > \mathbf{l}' \tag{16}$$

$$\dot{\boldsymbol{\beta}} = - < \boldsymbol{\Phi} > \boldsymbol{\beta}', \boldsymbol{\beta}' \equiv \boldsymbol{\beta} - \boldsymbol{\beta}^* \tag{17}$$

where $< \mathbf{M} > \equiv < \phi_j s_j >$ and $< \boldsymbol{\Phi} > \equiv < \chi_j (s_j / \iota_j) >$ are the $n \times n$ diagonal matrices of the parameters of the system (16)-(17), and $\boldsymbol{\beta}^* \equiv [\beta_j^*]$ the vector of the equilibrium employment rates of the disaggregated system.

Therefore, going back to the 'original coordinates', we obtain

$$\ddot{\mathbf{u}} = -\mathbf{M}\mathbf{u}' \tag{18}$$

$$\dot{\mathbf{v}} = -\boldsymbol{\Phi}\mathbf{u}' \tag{19}$$

[5] Note that Goodwin considered a particular $\mu(k)$ in every eigensector. However, it is not too difficult to show that if a particular $\mu(k)$ is assumed and the eigenvalues of the material input coefficients matrix are complex then the results are economically insignificant. See Appendix A for details.

where the $n \times n$ matrices $\mathbf{M} \equiv \mathbf{Q} < \mathbf{M} > \mathbf{Q}^{-1}$ and $\mathbf{\Phi} \equiv \mathbf{Q} < \mathbf{\Phi} > \mathbf{Q}^{-1}$ give the parameters of the system (18)-(19).

The system (18) is easily recognizable as a 'free vibration of undamped multiple degree of freedom system'. Whereas, solving the system (18) we obtain $\mathbf{u}'$, and, then substituting $\mathbf{u}'$ in (19), we obtain $\mathbf{v}'$. We conclude the analysis of the GDM2 by noting that, the theoretical investigation of the system (14)-(15) (alternatively (18)-(19)) has shown that there are two possibilities regarding the eigenvalues of matrix $\mathbf{A}$ [20]. These possibilities are: (i) if all the eigenvalues are real then the result is a '$2n$ dimensional system, with n Lotka Volterra oscillating pairs', where the motion of each sector is a linear combination of these pairs; and (ii) if some eigenvalues are complex then there do not exist mathematical theorems to be applied for an appropriate analysis of the properties of our system. Therefore it is studied by means of *ad hoc* numerical simulations methods, which gave explosive oscillations.

3. Results and their evaluation

The application of the previous analysis to the data of the Greek economy, for the year 1988, gives the results summarized in Tables 1-2 and Figures 1-4.[6] Table 1 shows the parameters of the models. The parameters n, b, ρ, γ, which have been estimated econometrically using ordinary least squares (OLS) regression, are taken from [13]. The parameter r_n, which is estimated on the basis of the available input-output data, is taken from [15]. Finally, following Harvie [13: 356], we estimate the parameters ε and ζ.[7]

Table 1.The parameters $n, b, \rho, \gamma, \varepsilon, \zeta$ and r_n; Greek economy

Parameter	
n	0.003568
b	0.0401
ρ	53.48
γ	46.02
ε	1.44764
ζ	1.19995
r_n	0.272

[6] *Mathematica 7.0* is used in the calculations. The analytical results are available on request from the author.
[7] See Appendix for details B.

Table 2 shows the eigenvalues of the material coefficient matrix.[8]

Table 2. The eigenvalues of the material coefficient matrix; Greek economy; 1988

the eigenvalues of **A**
0550129
0.44024
0.337418
0.33321
0.30684
0.240883
0.181501+0.474397i
0.181501-0.474397i
-0.0653024+0.0798211i
-0.0653024-0.0798211i
0.0770817+0.0539659i
0.0770817-0.0539659i
0.0792281+0.0133834i
0.0792281-0.0133834i
-0.020519+0.0275169i
-0.020519-0.0275169i
0.0237064
-0.0114637
0.0069544

Unfortunately, the parameters μ and k cannot be estimated from the given data. Therefore, we have to study the system by means of *ad hoc* values of μ and k. In what follows we investigate the cases: [9] $[\mu/(k\mu-1)]$ =0.5, 1, 1.5.

[8] Sectoral Classification: **[1]** Agriculture, hunting and related service activities, products of forestry: logging related service; **[2]** Fish and other fishing products; **[3]** Mining of coal and lignite; extraction of peat, extraction of crude oil and natural gas, mining of nuclear materials; **[4]** Mining of metal ores, other mining and quarrying products; **[5]** Manufacture of food products and beverages, tobacco products; **[6]** Manufacture of textiles, manufacture of clothes process and dyeing of fur, manufacture of tanning and dressing of leather; **[7]** Wood and wood products; **[8]** Pulp, paper and paper products publishing printing and reproduction of recorded media; **[9]** Manufacture of coke: refined petroleum products and nuclear fuel; **[10]** Manufacture of chemicals and chemical products, manufacture of rubber and plastic products; **[11]** Manufacture of other non-metallic mineral products; **[12]** Basic metals and fabricated metal products; **[13]** Fabricated metal products except machinery and equipment; **[14]** Machinery and equipment, office machinery and computers, electrical machinery and apparatus, radio, television and communication equipment and apparatus, medical precision and optical instruments, watches and clocks, motor vehicles trailers and semi-trailers; **[15]** Electricity, gas, steam and hot water, collection purification and distribution of water; **[16]** Construction work; **[17]** Whole sale and retail sale of motor vehicles, whole sale and retail sale except vehicles and retail trade; **[18]** Hotel and restaurant services; and **[19]** Transports, water transport services, air transport services, post and telecommunications.

[9] Note that since $[\mu/(k\mu-1)]$ can be smaller, equal or bigger than 1, we have also to investigate the effect of $[\mu/(k\mu-1)]$ on the dynamic behavior of the system.

For reasons of clarity of presentation and economy of space, the following set of figures is only associated with the sector 1: Figure 1 displays the solution path of $u_1'(v_1')$, which is associated with the GDM1. Figures 2 , 3 and 4 display the solution paths of $u_1'(v_1')$, which are associated with the GDM2 and the cases (i), (ii) and (iii), respectively.

Figure 1. The solution paths of u_1' and v_1' ; GDM1

Figure 1.a The path of u_1' ; long-run, t_0 =1988

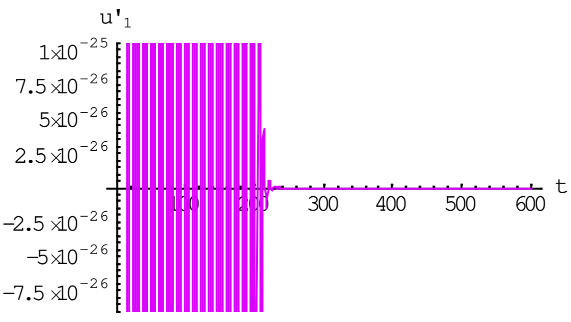


Figure 1.b The path of u_1' ; medium-run, t_0 =1988

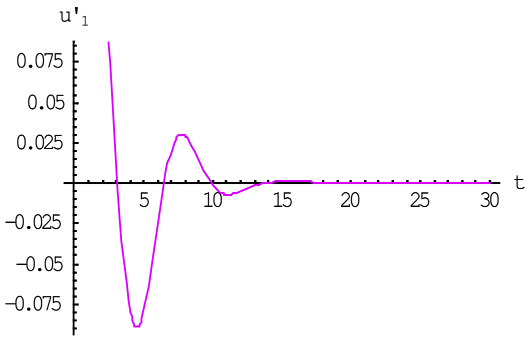


Figure 1.c The path of v_1' ; long-run, t_0 =1988

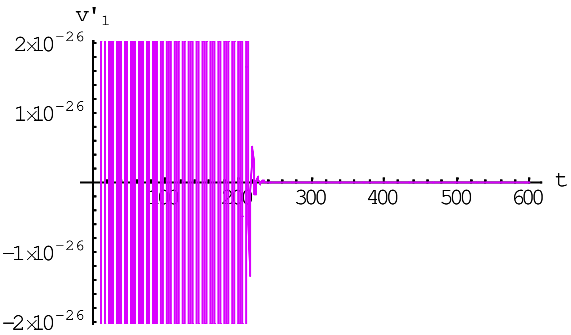


Figure 1.d The path of v_1' ; medium-run, t_0 =1988

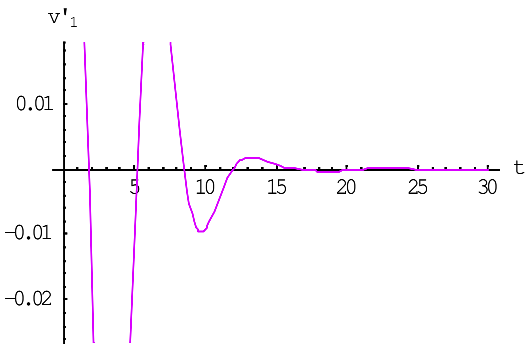
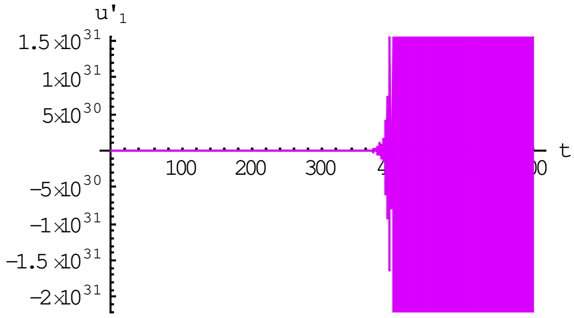


Figure 2. The solution paths of u_1' and v_1'; GDM2, Case (i)

Figure 2.a The path of u_1'; long-run, t_0 =1988 **Figure 2.b** The path of u_1'; medium-run, t_0 =1988



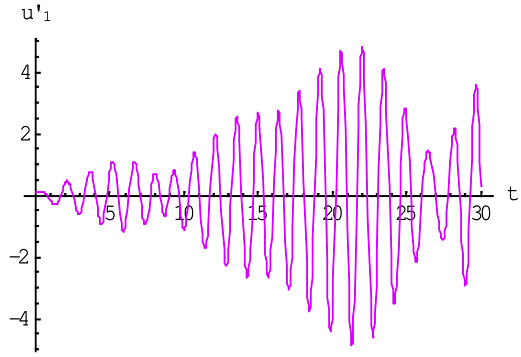
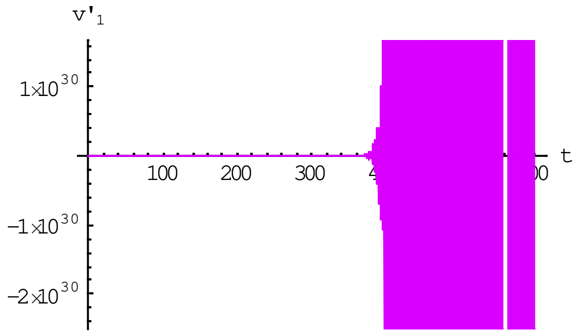


Figure 2.c The path of v_1'; long-run, t_0 =1988 **Figure 2.d** The path of v_1'; medium-run, t_0 =1988



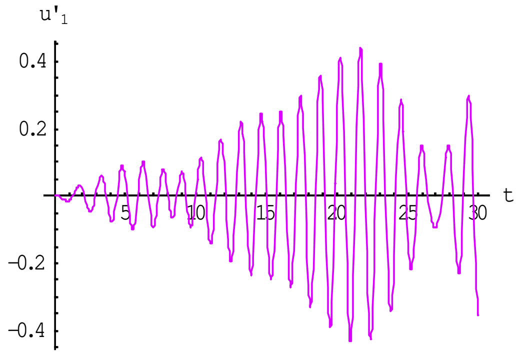
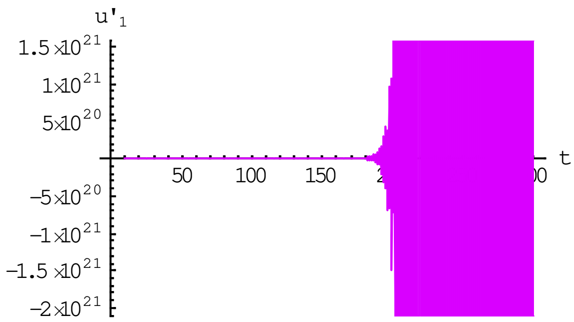


Figure 3. The solution paths of u_1' and v_1'; GDM2, Case (ii)

Figure 3.a The path of u_1'; long-run, t_0 =1988 **Figure 3.b** The path of u_1'; medium-run, t_0 =1988



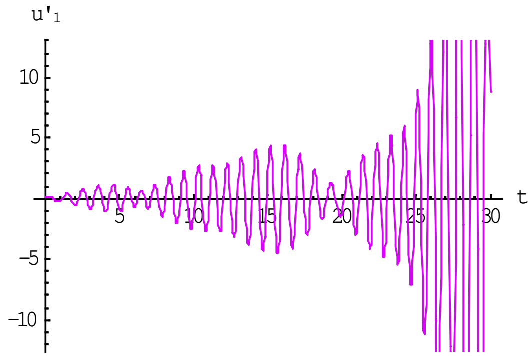
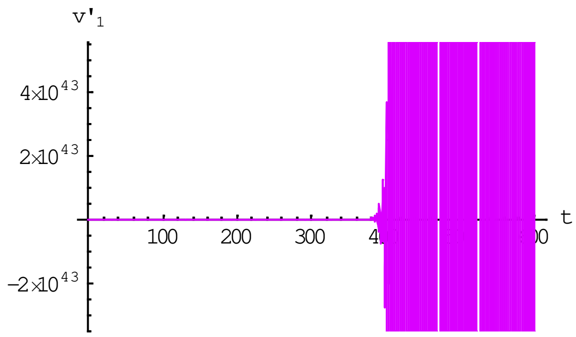


Figure 3.c The path of v_1'; long-run, t_0 =1988 **Figure 3.d** The path of v_1'; medium-run, t_0 =1988



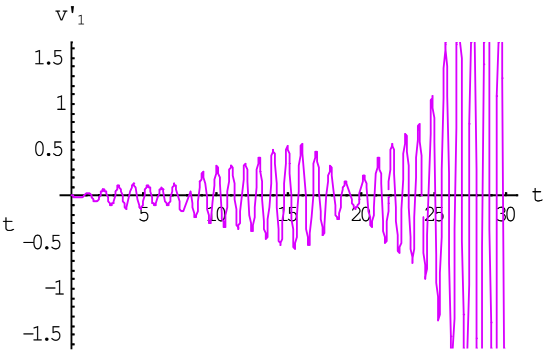
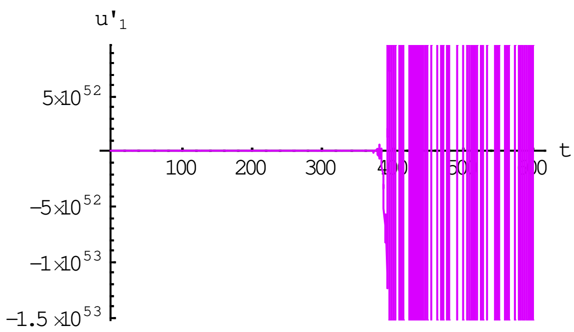


Figure 4. The solution paths of u_1' and v'; GDM2, Case (iii)

Figure 4.a The path of u_1'; long-run, t_0 =1988 **Figure 4.b** The path of u_1'; medium-run, t_0 =1988



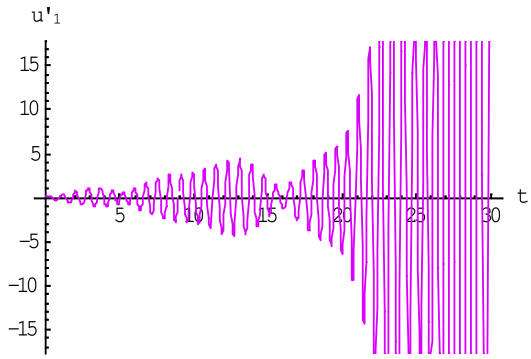
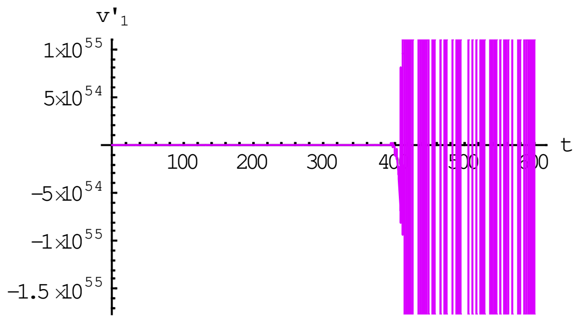
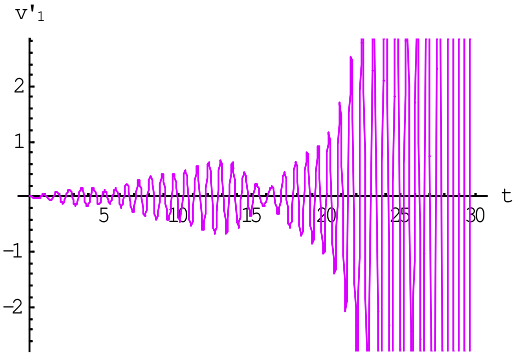


Figure 4.c The path of v_1'; long-run, t_0 =1988 **Figure 4.d** The path of v_1'; medium-run, t_0 =1988





Finally, the *actual* trajectories of the workers' share of national income, u^a, and employment rate, v^a, over the period 1959-2007 are shown in Figure 5. The evidence presented in Figure 5 suggests the existence of a large cycle over the period 1959-2007 (Figures 5a), and a slightly shorter one staring in the 1990s (Figure 5b).

Figure5. The trajectories of the actual workers' share and employment rate

Figure5a.i The $u^a v^a$ -trajectories of the for the Greek economy; 1959-2007[10]

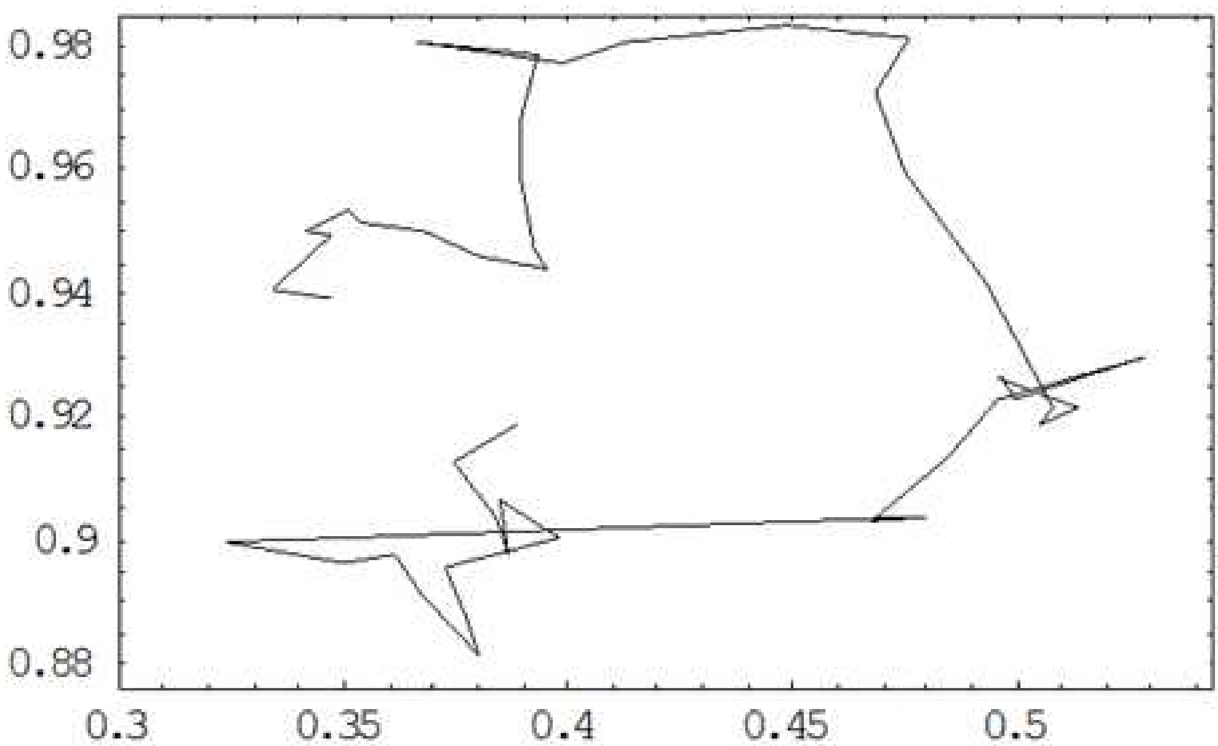
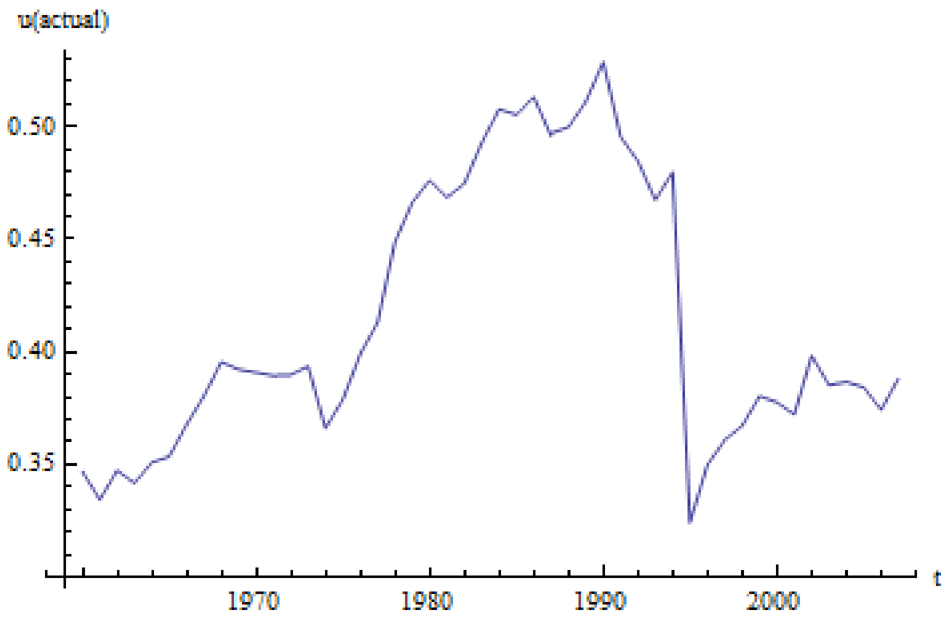


Figure5.a.ii The paths of actual workers' share; 1959-2007



[10] The horizontal axis represents employment rate and the vertical axis represents workers' share.

Figure5.a.iii The paths of the actual employment rate; 1959-2007

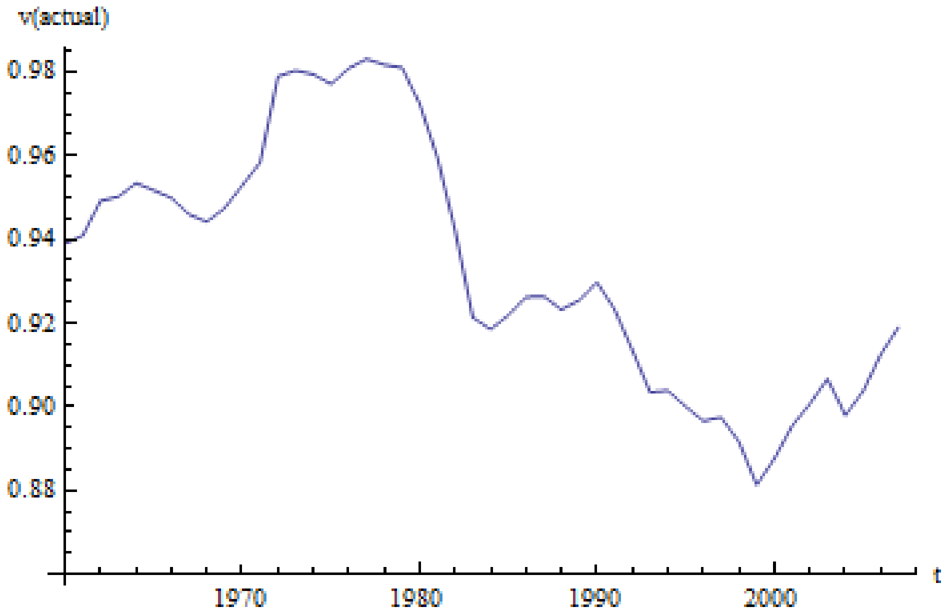
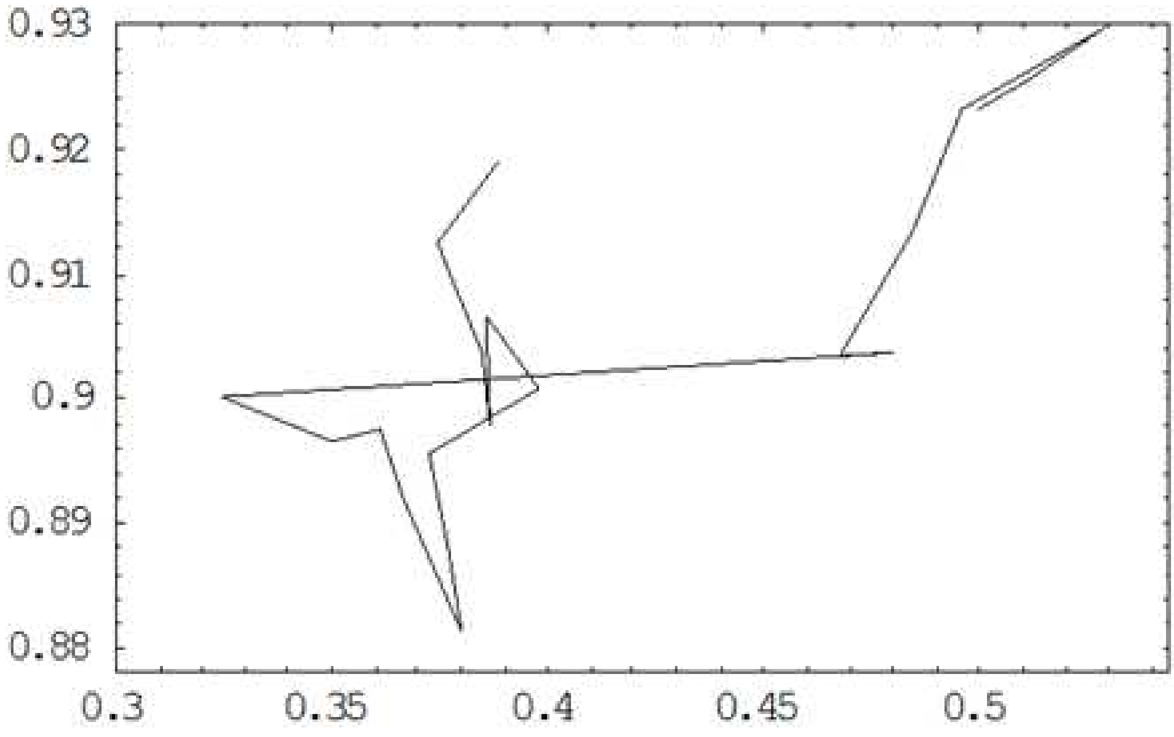


Figure5b.i The $u^a v^a$ -trajectories of the for the Greek economy; 1988-2007[11]



[11] See footnote 10.

Figure5.b.ii The paths of actual workers' share; 1988-2007

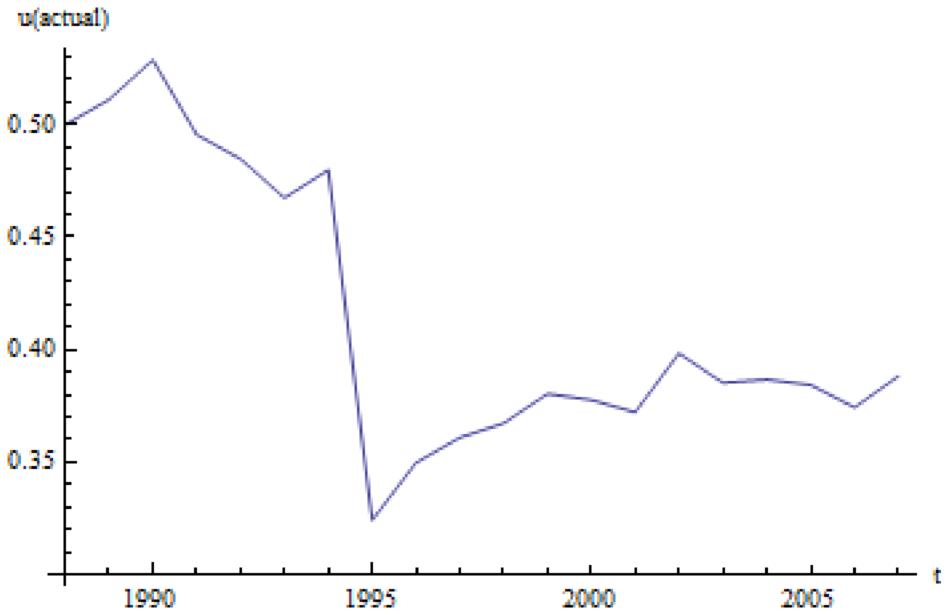
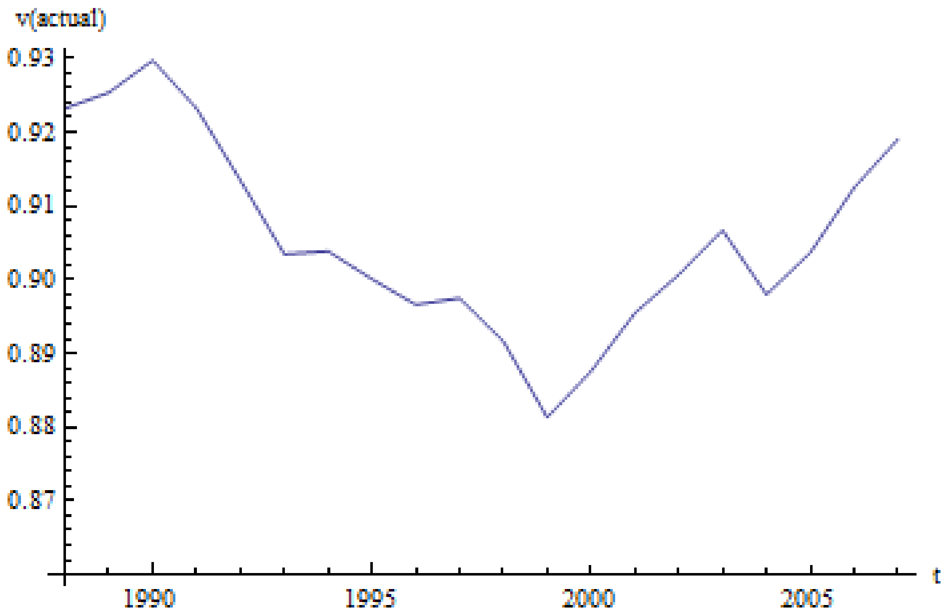


Figure5b.iii The paths of actual workers' share; 1988-2007



On the basis of these estimates we may remark the following: For all of the sectors, the evidence of the investigation of the GDM1 suggest damped oscillations, *i.e.*, both u'_j and v'_j become smaller and smaller over time, tending to zero in the limit (see, *e.g.*, Figure 1). Since all u'_j and v'_j tend to zero, *i.e.*, each sector tends to its equilibrium point, it can be expected that the whole system will tend to equilibrium. This evidence can be compared with the actual aggregative workers' share-employment rate trajectories for the Greek economy (see Figure 5). The evaluation of the results shows that from qualitative as well as quantitative point of view, the model is found not to be adequate: the GDM1 predicts damped oscillations and, therefore, cannot exhibit the cyclical behavior of the actual system.

The result obtained with the GDM2 can be summarized as follows. For all 19 of the sectors being investigated, the solution paths show clearly, that both workers' share and employment rate start with a low fluctuation and increase with time (see, *e.g.*, Figures 2, 3, 4). Especially, at first they exhibit cyclical oscillations which become more and more explosive (see, *e.g.*, Figure 2b, 3b, 4b), then a critical value of t is reached for which the oscillations become monotonically explosive (see, *e.g.*, Figure 2a, 3a, 4a). As before, one can say that the dynamic behavior (the motion) of the whole system is 'similar' to the motions of the 19 sectors of Greek economy. Comparing now this motion with the motion of the actual system over the period 1988-2007 (see Figurer 5b), it is found that, at qualitative level, the model is adequate to exhibit the cyclical movements of workers' share and employment rate. However, as mentioned before, there is a critical value of t for which the oscillations become monotonically explosive. So, the model cannot describe long-run business cycles. Moreover, at quantitative level, the model is found not to be adequate: both workers' share and employment rate exceed unity and if the 'investigated period' is long then the values of u'_j and v'_j become 'exotic' (see, *e.g.*, Figure 2a, 3a, 4a). Finally, it must be mentioned that, the evaluation of the results shows clearly that, the dynamic behaviour of the model does not depend on the values of the parameters μ and k (see, *e.g.*, Figures 2, 3, 4).

4. Concluding Remark

This paper tested the two Goodwin's 'growth cycle' disaggregated models using data for the Greek economy. It has been found that, none of the two Goodwin's models can describe a long-run cycling behavior of the phase variables, workers' share and employment rate. So, from qualitative as well as quantitative point of view, both models are found not to be adequate to describe the long-run workers' share-employment rate trajectories of the Greek economy. These results are likely to have been derived from limitations of the present models, *e.g.*, Goodwin's neglect of effective demand considerations in his models. However, in the medium-run analysis the evidence presented here are more encouraging: at qualitative level, the GDM2 is found to be adequate to describe the cycling behavior of the phase variables, workers' share and employment rate. Future work should (i) test these models for various countries and years; and (ii) investigate the possibility of improving these models by taking effective demand considerations into account.

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Appendix A: Proofs

Let us consider a 3×3 matrix $\mathbf{A}$ with one real eigenvalue (the P-F eigenvalue), $\lambda_1 (<1)$, and a pair of complex conjugate eigenvalues λ_2, λ_3. From (11) we get

$$\mathbf{K} \equiv \mathbf{Q} < \mathbf{K} > \mathbf{Q}^{-1}$$

where

$$<\mathbf{K}>=\begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{pmatrix}$$

From the above relation after rearrangement, we obtain

$$\mathbf{K}=\begin{pmatrix} \alpha_1 q_{11}q'_{11}+\alpha_2 q_{12}q'_{21}+\alpha_3 q_{13}q'_{31} & \alpha_1 q_{11}q'_{12}+\alpha_2 q_{12}q'_{22}+\alpha_3 q_{13}q'_{32} & \alpha_1 q_{11}q'_{13}+\alpha_2 q_{12}q'_{23}+\alpha_3 q_{13}q'_{33} \\ \alpha_1 q_{21}q'_{11}+\alpha_2 q_{22}q'_{21}+\alpha_3 q_{23}q'_{31} & \alpha_1 q_{21}q'_{12}+\alpha_2 q_{22}q'_{22}+\alpha_3 q_{23}q'_{32} & \alpha_1 q_{21}q'_{13}+\alpha_2 q_{22}q'_{23}+\alpha_3 q_{23}q'_{33} \\ \alpha_1 q_{31}q'_{11}+\alpha_2 q_{32}q'_{21}+\alpha_3 q_{33}q'_{31} & \alpha_1 q_{31}q'_{12}+\alpha_2 q_{32}q'_{22}+\alpha_3 q_{33}q'_{32} & \alpha_1 q_{31}q'_{13}+\alpha_2 q_{32}q'_{23}+\alpha_3 q_{33}q'_{33} \end{pmatrix}$$

Since λ_1 is the P-F eigenvalue, the first column (row) of $\mathbf{Q}$ (of $\mathbf{Q}^{-1}$), is real and positive. On the other hand, the corresponding eigenvectors to λ_2, λ_3 will ordinary involve negative and complex numbers. Therefore, the element κ_{11} of $\mathbf{K}$ can be expressed as:

$$\kappa_{11} = \alpha_1 q_{11} q'_{11} + (\pm\tau \pm \eta i)(\pm\sigma \pm \omega i)(\pm\xi \pm \delta i) + (\pm\tau \mp \eta i)(\pm\sigma \mp \omega i)(\pm\xi \mp \delta i)$$

where $\sigma, \omega, \xi, \delta \geq 0$ and $\tau, \eta, \alpha_1 q_{11} q'_{11} > 0$.

The above relation is a sum of real numbers, and, therefore the element κ_{11} is real. By contrast, if $n_2 \neq n_3$ or $b_2 \neq b_3$ or $r_{n2} \neq r_{n3}$ then $\alpha_2 \neq \alpha_3$ and then κ_{11} is complex (and the same holds true for any κ_{ij} of a $n \times n$ matrix). Hence, if each eigensector has its particular n_j, b_j, r_{nj}, then the elements of matrix $\mathbf{K}$ will be complex.

In the same way, it can be proved that the elements of the matrices $\mathbf{N}$ and $\mathbf{\Omega}$ ($\mathbf{M}$ and $\mathbf{\Phi}$) are real *iff* the parameters $b_j, n_j, \zeta_j, r_{nj}$ ($b_j, n_j, \gamma_j, k_j, \mu_j$) are the same in all eigensectors.

Appendix B: A Note on the Data

The symmetric input-output table of the Greek economy for the years 1988 is provided at the 19×19 sector detail and is taken from Rodousakis, 2006. Furthermore, in our estimation of parameters ε and ζ we follow Harvie [13: 356], *i.e.*, the long-run relationship between money wage growth, $\hat{m}_t \equiv (m_{t+1} - m_t)/m_t$, and employment rate, is estimated by the following relation

$$\hat{m}_t \equiv \sum_{j=0}^{n} \varepsilon_j v_{t-j} + \sum_{j=1}^{n} \sigma_j \hat{m}_{t-j} - \eta$$

where $\varepsilon = (1-\varepsilon_0-\varepsilon_1-\varepsilon_2-...-\varepsilon_n)/(1-\sigma_1-\sigma_2-...-\sigma_n)$, $\zeta = \eta/(1-\sigma_1-\sigma_2-...-\sigma_n)$ and n is the number of lags necessary to ensure the model is dynamic well specified. The dada for this estimation are taken from OECD Department of Economics and Statistics' publications National Accounts and Labour Force Statistics for the period 1959-1994 [17,18]. Finally, once again, following Harvie [13: 356] we estimate our state variables. We define the actual aggregative workers' share of national income and employment rate to be $u^a =$ compensation of employs/(compensation of employs + operating surplus) and $v^a =$ total employment/ total labour force, respectively. The dada for this estimation are taken from OECD Department of Economics and Statistics' publications National Accounts and Labour Force Statistics for the period 1959-2007 [17, 18].