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Abstract

The extent to which the money supply affects the aggregate cash balance demanded at a certain level of nominal income and interest rates is determined by the interest-rate-elasticity and stability of the money demand. An actuarial approach is adopted in this paper for dealing with investors facing liquidity constraints and maintaining different expectations about risks. Under such circumstances, a level of surplus exists which maximises expected value. Moreover, when the distorted probability principle is introduced, the optimal liquidity demand is expressed as a Value at Risk and the comonotonic dependence structure determines the amount of money demanded by the economy. As a consequence, the more unstable the economy, the greater the interest-rate-elasticity of the money demand. Moreover, for different parametric characterisation of risks, market parameters are expressed as the weighted average of sectorial or individual estimations, in such a way that multiple equilibria of the economy are possible.

Key words: Money demand; Monetary policy; Economic capital; Distorted risk principle; Value-at-Risk.

JEL-Classification: E41, E44, E52, G11.

1 Introduction

According to the Keynes’s liquidity preference proposition (Keynes, 1935), the demand for cash balances is positively affected by the level of income and negatively affected by the return offered by a class of money substitutes. The first part of the proposition is a consequence of the assumption that the amount of transactions is proportional to the level of income. To explain the effect of the interest rate, Keynes emphasises the influence of capital fluctuations in decision-making. Thus, investors expecting interest rates to rise demand fewer risk-free securities in order to avoid capital losses — since the price of such instruments is expected to diminish in this case. By contrast, when interest rates are expected to fall, more bonds are
demanded — in this way, capital gains can be attained after the collapse of interest rates. Therefore, fewer provisions are maintained for high levels of the interest rate and vice versa.

In macroeconomic analysis, the level of prices establishes the connection between nominal magnitudes, expressed in monetary units, and real quantities, which represent flows of goods and services. Accordingly, \( Y = P \cdot y \), where \( P, Y \) and \( y \) respectively denote the level of prices, the nominal income and the real income. Let us additionally denote by \( M \) the total money supply. Therefore, the short-run monetary equilibrium is given by the quantity equation:

\[
M = P \cdot y \cdot l(r) = Y \cdot l(r) \quad (1)
\]

The liquidity preference function \( l(r) \) expresses the ratio between demanded cash balances and nominal income. It is not likely to be constant but it may change slowly over time. The inverse ratio of the liquidity preference function is called velocity of money.

Any change in the money supply will require a change in one or more of the variables determining the liquidity demand (i.e. \( P, y \) or \( r \)) in order to reestablish the monetary equilibrium. When prices are rigid for short-run fluctuations and the real product remain stable in short-terms, the whole adjustment is performed in \( l(r) \). In addition, if liquidity preference is absolute, i.e. if investors are satisfied at a single level of the interest rate,\(^1\) the amount of money can change without a change in either nominal income or interest rates. Under such circumstances, monetary policy is useless for dealing with short-run fluctuations. The situation is different if prices are flexible and liquidity preference is non-absolute. Then a monetary expansion produces a new equilibrium involving a higher price for the same quantity, the higher this response the more inelastic the money demand. In the short-run, production is encouraged until prices are reestablished at their original level. In the long-run, new producers enter the market and existing plants are expanded (Friedman, 1970).

Under such circumstances, the efficacy of monetary policy depends on the degree of rigidity of prices and the elasticity of the money demand, as well as on the stability of liquidity preference. There is a consensus among researchers about the existence of a stable long-run relationship, though fluctuations of cash balances in the short-run remain unexplained. Episodes like the missing money in the mid-seventies, the great velocity decline in the early eighties, followed by the expansion of narrow money in the mid-eighties, or the velocity puzzle of the mid-nineties, still lack a satisfactory explanation (Ball, 2001 and 2002; Carpenter and Lange, 2002; Teles and Zhou, 2005). In accounting for such drawbacks, recent literature has focussed on uncertainty, which is supposed to have been incremented after 1980 due to deregulation and financial innovations (Atta-Mensah, 2004; Baum et al., 2005; Carpenter and Lange, 2002; Choi and Oh, 2003; Greiber and Lemke, 2005). Deregulation and financial innovation are also given as arguments to support the role of the opportunity cost in accounting for unexplained fluctuations (Ball, 2002; Collins and Edwards, 1994; Duca, 2000; Dreger and Wolters, 2006; Teles and Zhou, 2005). According to this view, a stable long-run relationship exits and movements of the interest rate can explain all short-run episodes, as long as the right monetary aggregate is used (Ball, 2002).

\(^1\)Absolute liquidity preference corresponds to the case when the liquidity demand is perfectly elastic with respect to the interest rate. According to Keynes, the degree of elasticity depends on how homogeneous expectations are, where perfect elasticity is obtained when expected and actual values are the same. In this case, money and risk-free securities are perfect substitutes — since no capital gains or losses are expected.
In this paper, an extended model is proposed according to which liquidity preference is explicitly determined by uncertainty and information. First, the cash demand of a single representative investor is obtained. Investors are supposed to face liquidity constraints and consequently, in Section 2 equity is treated as an additional liability. In addition, the behaviour towards risk is determined by the transformation of probabilities according to an informational parameter. Then the expected return of the fund is maximised when the mathematical expectation of the residual exposure (a measure of the cost of assuming bankruptcy) plus the opportunity cost of capital is minimised. In this way, I follow Dhaene et al. (2003), who on these terms develop a mechanism for capital allocation (see also Goovaerts et al., 2005). When looking for the aggregated surplus in Section 3, capital is supposed to be provided by a central authority or financial intermediaries acting in a competitive market, in such a way that a single interest rate is required for lending. Hence the situation is similar to the case of a centralised conglomerate distributing capital among subsidiaries (Dhaene et al., 2003; Goovaerts et al., 2005; Mierzejewski, 2006) and the opportunity cost of money is related to the average return over a class of money substitutes. Thus, monetary aggregates are determinants of liquidity preference in the model. Finally, within a Gaussian setting, the aggregate exposure is normally distributed and its volatility is equal to the weighted average of individual volatilities. Therefore, aggregation plays a role in the determination and stability of the liquidity demand. The same results are obtained when marginal risks are Exponentially and Pareto distributed. The final remarks are given in Section 4.

2 The Rational Money Demand

Since in frictionless markets the amount of cash maintained for precautionary purposes can be modified at any time by lending and borrowing, managers who maximise value demand no equity — which is actually the proposition established by Modigliani and Miller (1958). However, averse-to-risk customers are sensible to fluctuations and, as long as the business activities of financial intermediaries — which accordingly are said to be opaque — are not observed by outsiders, a pressure is established to be perceived as default-free (Merton, 1997). In the model developed by Tobin (1958), averse-to-risk investors show liquidity preference as behaviour towards uncertainty. Assuming that risks follow Gaussian distributions, a linear relationship is established between the expected returns and volatilities of the portfolios containing a proportion of a certain fund and a cash guarantee, which determines the set of efficient portfolios — in the sense that for any combination outside the line, it is always possible to build a new fund providing the same expected return and a lower risk, or the same risk but a higher return. The way preferences affect portfolio decisions can then be analysed in the plane of expected returns and volatilities, where the indifference curves of risk-lovers should present a negative slope, as long as such individuals accept a lower expected return if there is a chance to obtain additional gains. By contrast, averse-to-risk investors do not take more risk unless they are compensated by a greater expected return and consequently, their indifference curves have positive slopes. Therefore, for any risk-aversion profile, the optimal combination is determined by the (tangency point of) intersection between the unique indifference curve representing preferences and the line of efficient portfolios.

Let us analyse in the following how the Tobin’s model is affected by the hypothesis of im-
perfect competition, a case where risks belong to a general class of probability distributions which economic agents distort according to their information and knowledge when making decisions. Moreover, liquidity constraints are faced when borrowing and lending and managers have to expend effort to correctly assess prices. Let the parameter $\theta$ denote the state of information of an investor holding a mutual fund whose percentage return is represented by the random variable $X$. Because of the precautionary motive, a guarantee $L$ is maintained for a determined period of time to avoid bankruptcy. In order to introduce in the model the effect of liquidity constraints, equity is regarded as an additional liability and the size of the guarantee is expressed as a proportion of the level of income $Y$, such that $L = Y \cdot l$, where $l$ represents the proportion of income assigned to the non-risky asset. Hence, if $r_0$ denotes the risk-free interest rate, the percentage capital return of the total portfolio can be expressed as $Y = X - l - r_0 \cdot l$ and decisions are affected by the percentage return on income:

$$\mu_{\theta,Y} = E_{\theta}[Y] = (\mu_{\theta,X} - l) - r_0 \cdot l$$

In giving a meaning to the informational parameter $\theta$, let us stress the fact that expectations are wanted to be modified. Then probability beliefs are transformed by a distortion parameter which is supposed to be determined by information and knowledge and the proportional hazards distortion is introduced (Wang, 1995):

$$E_{\theta} [X] = \int x \ dF_{\theta,X}(x) = \int G_{\theta,X}(x) \ dx := \int G_X(x)^{\theta} \ dx$$

The cumulative and decumulative (also known as survival) probability distribution functions have been introduced, $F_{\theta,X}(x) = P_{\theta} [X \leq x] = 1 - P_{\theta} [X > x] = 1 - G_{\theta,X}(x)$. When $\theta > 1$, the expected value of risk is overestimated and underestimated when $\theta < 1$, in this way respectively accounting for the behaviour of adverse-to-risk and risk-lover investors.

Notice, however, that individuals react differently depending on the sign of the capital return. In fact, when a loss is suffered, cash is demanded to avoid default, while in the case a gain is obtained the surplus can be used to pay current liabilities or assigned to new investments. Hence, decision-makers mainly concerned about the speculative and the precautionary motives respectively focus on the terms $E_{\theta} [(X - l)_+]$ and $E_{\theta} [(X + l)_-]$. Let us accordingly assume that capital decisions are taken by risk managers who minimise bankruptcy and rely on the average value of the insured return:

$$E_{\theta} [ (X - l)_+] \approx E_{\theta} [X_+] - r_{\theta,X} \cdot l$$

Since the term $r_{\theta,X} > 0$ represents the absolute value of the marginal reduction in insured capital gains produced when attracting an additional unit of equity, it can be regarded as a premium for solvency. Hence the following expression is obtained for the expected percentage income:

$$\mu_{\theta,Y} = E_{\theta} [X_+] - E_{\theta} [ (X + l)_-] - (r_0 + r_{\theta,X}) \cdot l$$

Under such conditions, precautionary investors that maximise value minimise bankruptcy costs. Applying Lagrange optimisation, we obtain that decision makers attract funds until the marginal return of risk equals the total cost of capital:
\[-\frac{\partial}{\partial l} E_{\theta} \left[ (X + l)_- \right] - (r_0 + r_{\theta,X}) = G_{\theta,-X}^1 (l^*) - (r_0 + r_{\theta,X}) = 0\]

Equivalently, it can be said that investors stop demanding money at the level at which the marginal expected gain in solvency equals its opportunity cost. Thus the optimal cash demand is given by:

\[ l_{\theta,X} (r_0 + r_{\theta,X}) = G_{\theta,-X}^1 (r_0 + r_{\theta,X}) \]  

(2)

From this expression, the money demand follows a decreasing and — as long as the distribution function describing uncertainty is continuous — continuous path, whatever the kind of risks and distortions. The minimum and maximum levels of surplus are respectively demanded when \((r_0 + r_{\theta,X}) \geq 1\) and \((r_0 + r_{\theta,X}) \leq 0\).

In practical applications, intermediaries face operational and administrative costs, at the time that a premium over the risk-free interest rate is asked for lending in secondary markets. Hence, the return \(r_0 + r_{\theta,X}\) can be interpreted as a net opportunity cost. Though environmental facts, such as the perception of credit quality and gains in efficiency because of improvements on analysis and administration, are expected to evolve on time, we can regard them as softly modified — and not a matter of speculation. Also the risk attitude of managers is supposed to remain more or less unchanged. Therefore, the parameter \(\theta\) is expected to remain stable and consequently, as long as the probability distribution of the random variable \(X\) is also stable, the capital decisions of investors should remain more or less the same and the economy as a whole should behave accordingly.

However, if probability distributions are allowed to evolve on time — i.e. if the processes of capital gains and losses are not stationary — so does the premium for solvency \(r_{\theta,X}\). Actually, this can be the case after a monetary expansion — which can be performed by the central bank as well as by the entrance of new investors — since as long as part of the extra money is used to buy financial securities and the increment in demand is high and persistent enough to induce the price to rise more frequently, the term \(E_{\theta} [(X - l)_-]\) is pushed to increase. In a similar way, a monetary contraction can press the insured return to decrease. This situation might in turn impel decision-makers to actualise expectations and so the informational parameter \(\theta\) might be modified. But this adjustment is supposed to be produced with a certain delay — for time is required for analysis — while the opportunity cost may be instantaneously altered. Therefore, changes in the stock of money may induce instability from within in secondary markets. Adjustments are performed along a stable money demand relationship, though the process may be reinforced by structural modifications once expectations are actualised.

3 Short-Run Monetary Equilibrium

In order to obtain an expression for the cash balance demanded by the whole economy, let us assume that economic agents hold aggregate exposures characterised by the random variables \(X_1, \ldots, X_n\). Capital is supplied by a central authority at a single interest rate \(r\) (or, equivalently, secondary markets are regarded as competitive and financial intermediaries are price takers) relying on the informational parameter \(\theta\) and the uncertainty introduced
by the market portfolio \( X \). When differing expectations are allowed among decision makers, the aggregate money demand is given by:

\[
l_{\theta_1, \ldots, \theta_n, -X}(r) = \sum_{i=1}^{n} G_{\theta_i}^{-1}(X_i)(r) = G_{\theta_1, \ldots, \theta_n, -X}(r)
\]

The second equality is a mathematical identity as long as the process of capital gains and losses of the market portfolio is described by the comonotonic sum \( X = X^c_1 + \cdots + X^c_n \), where \( G_{\theta_1, \ldots, \theta_n, -X} = \left( \sum_{i=1}^{n} G_{\theta_i}^{-1}(X_i) \right)^{-1} \) denotes the distribution function of the comonotonic sum when marginal distributions are given by \( (G_{\theta_1, -X_1}, \ldots, G_{\theta_n, -X_n}) \). Comonotonicity characterises an extreme case of dependence, when no benefit can be obtained from diversification.\(^2\) Thus precautionary investors rely on the most pessimistic case, when the failure in any single firm spreads all over the market.

The dependence of the liquidity demand on the variability of income becomes explicit in a Gaussian setting. Let us assume in the following that individual exposures are distributed as Gaussians with means \( \mu_1, \ldots, \mu_n \) and volatilities \( \sigma_1, \ldots, \sigma_n \), while the contributions of individual exposures to the market portfolio are given by the coefficients \( \lambda_1, \ldots, \lambda_n \), with \( 0 \leq \lambda_i \leq 1 \ \forall i \), such that \( \bar{Y}_i = \lambda_i \cdot \bar{Y} \) and \( \bar{Y} = \bar{Y}_1 + \cdots + \bar{Y}_n \). Volatilities are expressed as proportions of the levels of income and can be interpreted as the volatilities of different funds as well as the distorted volatilities of the same Gaussian exposure — or some intermediate case. Under such conditions, the comonotonic sum is also a Gaussian random variable whose mean and volatility are respectively given by (Dhaene et al., 2002):

\[
\mu = \sum_{i=1}^{n} \lambda_i \cdot \mu_i \quad & \quad \sigma = \sum_{i=1}^{n} \lambda_i \cdot \sigma_i
\]

On these grounds, the weighted average mean and volatility describe the uncertainty of the market portfolio. In particular, high volatility may be induced by a single group, as a negative externality to more efficient companies and so the possibility of contagion naturally arises in the model. In the same way, stability may be inherited by less efficient institutions when low volatility predominates.

Since the quantile function of a Gaussian random variable can be express in terms of the standard Normal distribution \( \Phi \) (Dhaene et al., 2002), the short-run monetary equilibrium is described by the following equation:

\[
M = \bar{Y} \cdot l_{\mu, \sigma}(r) = \bar{Y} \cdot \left[ -\left( \mu + \sigma \cdot \Phi^{-1}(r) \right) \right]
\]

Therefore, the monetary equilibrium can be reestablished by modifying the level of nominal income \( \bar{Y} \), the average return \( \mu \), the market volatility \( \sigma \) or the interest rate \( r \). As already stated, only \( r \) is expected to change in the short-run. Monitoring and analysis induce investors to eventually incorporate the new regime of \( X \) in decision making and possibly modify expectations, both determinants of \( \mu \) and \( \sigma \).

The difference between the classic and the extended model can be noticed by comparing Equations 1 and 4. Thus, while in Equation 1 the elasticity of income with respect to the

\(^2\)The inverse probability distribution of the comonotonic sum is given by the sum of the inverse marginal distributions (Dhaene et al., 2002).
stock of money exclusively depends on the interest rate through the liquidity preference function, in Equation 4 it is also affected by uncertainty. In addition, if $\bar{y}$ represents the level of real income, the new short-run equilibrium can be written in real terms as:

$$M = P \bar{y} \cdot \left[ -\left(\mu + \sigma \Phi^{-1}(r)\right) \right]$$

Therefore, to stabilise the product it is also required to control the market risk. A proper monetary policy should then consider a combination of $P$, $\mu$, $\sigma$ and $r$ compatible with a given level of income. The level of $\sigma$ that preserves the monetary equilibrium for given values of $M$, $\bar{Y}$, $\mu$ and $r$ can be regarded as the induced volatility. A tentative criterion for monetary policy may then involve the determination of the level of interest rates ensuring a given inflation and induced market volatility. Additionally, the non-distorted volatility can be estimated by the standard deviation of the random variable $X$ representing the capital losses of the market portfolio. A measure of the degree of distortion performed by the market is thus determined by the difference between the induce and the non-distorted volatility.

An alternative representation is obtained by considering that individual exposures are exponentially distributed. In this case, the comonotonic sum is also exponentially distributed (see Dhaene et al., 2002), such that if $\beta_1, \ldots, \beta_n$ denote the informational types of investors, with $\beta_i \geq 0 \forall i$, and, as before, $\lambda_1, \ldots, \lambda_n$ represent the marginal contributions to the aggregate income, with $0 \leq \lambda_i \leq 1 \forall i$, then the exponential parameter of the market portfolio is expressed as the weighted average of the exponential parameters of marginal risks:

$$\beta = \sum_{i=1}^{n} \lambda_i \cdot \beta_i$$

Therefore, the liquidity preference function of the economy is given by:

$$l_{\beta}(r) = -\beta \cdot \ln(r) \quad \text{with} \quad \beta > 0$$

Thus, the higher the parameter $\beta$, which within this framework completely characterises risk, the more sensitive is liquidity preference to the cost of capital. In this way, uncertainty is explicitly related to the monetary equilibrium and hence to the terms of liquidity — determined by the money supply.

When marginal risks are Pareto distributed, the survival probability distributions as estimated by decision-makers are given by:

$$G_{-X_i}(x) = x^{-\frac{1}{\alpha_i}}, \quad \text{with} \quad \alpha_i > 0 \quad \text{and} \quad x > 1$$

The parameters $\alpha_1, \ldots, \alpha_n$ correspond to the states of information of investors. As long as they agree on a single value $\alpha$, the comonotonic sum is also Pareto distributed (Dhaene et al., 2002) and liquidity preference is given by:

$$l_{\alpha}(r) = n \cdot r^{-\alpha}$$

Under differing expectations, the comonotonic sum is not necessarily Pareto distributed. However, an estimation of the parameter $\alpha$ can be found such that Equation 6 determines the monetary equilibrium. In this case, the point interest-rate-elasticity is constant and
equal to \( \alpha \). Many models for the estimation of the money demand are supported on this assumption.

## 4 Conclusions

An extended model is presented in this paper — also referred to as the imperfect competition model — to characterise the liquidity preference of investors facing liquidity constraints. Under such circumstances, a level of surplus exists that maximises value and the rational money demand is determined by the quantile function — a measure of the probability accumulated in the tail of the distribution function — of the random variable representing the series of capital profits and losses of the residual exposure (Equation 2). In this way, an equivalence is established between a confidence level and the opportunity cost of capital and the optimal amount of cash is determined by the exchange of a sure return and a flow of probability. An informational parameter, affecting the opportunity cost of money, represents the expectations of decision makers. Averse-to-risk and risk-lover investors respectively under and overestimate the cost of capital and so they respectively demand more and less equity.

The importance attached to liquidity preference in macroeconomic analysis is a consequence of the fact that it determines the short-run monetary equilibrium of the economy. In the classic approach (Friedman, 1970), the amount of money which is compatible with given levels of nominal income and interest rates can be obtained from Equation 1. According to the extended model presented in this paper, the aggregate money demand of the economy is given by the sum of the liquidity preferences of investors, mathematically characterised by the comonotonic sum of individual exposures. The aggregate money demand is thus expressed as a Value-at-Risk but referred to a market portfolio which relies on the most pessimistic case, when no gain can be obtained from diversification. In a Gaussian setting, the comonotonic sum is also a Gaussian variable, whose volatility is equal to the weighted average of individual volatilities (Equation 4). In this way, the classic model is extended allowing a correction for risk.

Within the imperfect competition framework, the total stock of money \( M \), the level of income \( Y \), the interest rate \( r \), the mean \( \mu \) and the market volatility \( \sigma \) are all determinants of the short-run monetary equilibrium (Equation 4). Thus, as long as part of the funds available in the economy are spent on capital assets, an adjustment in the opportunity cost \( r \) is expected in the short-run — stimulated by the modification of the stochastic nature of capital gains — which is supposed to instantaneously affect liquidity preference. In the medium-term, investors correct their expectations and so part of the adjustment may be performed through informational shocks affecting the aggregate mean or the market volatility. An important feature of the mechanism is that the evolution of risks, motivated by flows of funds, determine expectations and not the opposite, though liquidity preference might also be affected by a purely informational shock.

As pointed out in Section 2, liquidity preference is not affected in the same way by capital gains and losses. Thus, while positive returns affect the opportunity cost of money and so determine a movement along a stable relationship, the precautionary attitude of decision makers depends on negative returns, as does the shape of the money demand (see Equation 2). The first adjustment is supposed to occur instantaneously, while the second one
is performed gradually, for it takes time for investors to internalise new market conditions. In practice, both decisions are related to different markets. Accordingly, the cost of equity $r$ is represented by the average return over a class of securities, other than cash, that can be regarded as substitutes to money. On the other hand, the liquidity preference function depends on the series of returns over a set of instruments representative of assumed exposures. Then the variability showed by a representative index of this class determines the market volatility $\sigma$.

Finally, as stated in Equation 3, in a Gaussian setting the expected value of the market portfolio and the market variability are respectively given by the weighted average of individual means and volatilities. Hence, the market uncertainty will be mainly determined by a single institution or sector, in the case it contributes more to the aggregate exposure. Stability can be induced in the whole market in this way. The same results are obtained when individual exposures are exponentially or Pareto distributed, for in both cases the risk parameters are aggregated when accounting for market behaviour. Moreover, the model accepts multiple equilibria, since different combinations of the risk parameters may lead to the same market characterisation.

The terms under which market shocks affect individual expectations about risks will be determined by specific conditions, such as the state of aggregation, restrictions in the access to credit, the distribution of information within the market and the skills and knowledge of investors. Thus, changes in the aggregate monetary stock may induce intermediaries to prefer bigger or more efficient companies — flight to quality — a situation that may become more difficult the availability of funds and possibly increment more the riskiness of less productive sectors of the economy. In this way, within the imperfect competition framework, a broader meaning is attached to instability.

References


