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AXIOMATIC BASICS OF E·ECONOMICS

Egmont Kakarot-Handtke

Abstract: Standard economic models are based on an axiom set that epitomizes the fundamental behavioral assumptions. The present treatise moves these assumptions from the foreground to the background. The suggested change of perspective is guided by the question: what is the minimum set of foundational propositions for a consistent reconstruction of the evolving money economy? We start with four non-behavioral axioms. Subsequently their logical and factual implications are explored and the building blocks of the general axiomatic model are determined. The switch of the unifying principle resolves the profit conundrum – 'one of the most convoluted and muddled areas in economy theory'. Hence structural axiomatization has ramifications on larger parts of standard economics. By virtue of the axiom set evolution supersedes equilibrium as central organizing idea.

Key Words: Framework of Concepts – Structure-centric – Axiom Set – Propensity Function – General Axiomatic Model – Stochastic Processes – Evolutionary Economics – Evolving Money Economy – e-Economics

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AXIOMS, OR THE LAWS OF MOTION



Law 1 *Every body perseveres in its state of being at rest or of moving ^auniformly straight forward,^a except insofar as ^bit^b is compelled to change ^cits^c state by forces impressed.*

Projectiles persevere in their motions, except insofar as they are retarded by the resistance of the air and are impelled downward by the force of gravity. A spinning hoop,^d which has parts that by their cohesion continually draw one another back from rectilinear motions, does not cease to rotate, except insofar as it is retarded by the air. And larger bodies—planets and comets—preserve for a longer time both their progressive and their circular motions, which take place in spaces having less resistance.

Law 2 *A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed.*

If some force generates any motion, twice the force will generate twice the motion, and three times the force will generate three times the motion, whether the force is impressed all at once or successively by degrees. And if the body was previously moving, the new motion (since motion is always in the same direction as the generative force) is added to the original motion if that motion was in the same direction or is subtracted from the original motion if it was in the opposite direction or, if it was in an oblique direction,

But it was a second and more important quality that struck readers of the *Principia*. At the head of Book I stand the famous *Axioms, or the Laws of motion*: ... For readers of that day, it was this deductive, mathematical aspect that was the great achievement. Truesdell, quoted in (Schmiechen, 2009 p. 213), original emphases

Walras's early reading consolidated his boundless admiration for Newtonian astronomy and the solid edifice of classical mechanics, which he regarded as unequalled models of scientific knowledge throughout his life. (Ingrao, et al., 1990 p. 88)

In political economy, Ricardo and James Mill compared the certainty of the propositions they were advancing to the certainty of the propositions of Euclid. (Halévy, 1960 p. 494)

The impression that one could build price theory up from basics in the image of Euclid was much more important than commitment to any particular proposed formalization.

(Mirowski, 2004 pp. 348-349)

Mathematical form powerfully contributes to defining a philosophy of economic analysis whose major tenets include rigor, generality and simplicity.
Gérard Debreu

Those who take the foundations of their speculations from hypotheses, even if they then proceed most rigorously according to mechanical laws, are merely putting together a romance, elegant perhaps and charming, but nevertheless a romance. Roger Cotes, Preface to the second edition of Newton's *Principia*

My way is to begin with the beginning. Lord Byron

1 Euclid's Rigorous Disciples

1.1 The Framework of Concepts

Criticism of the standard economic research program relates to its theoretical, empirical, methodological, and political aspects. Part of the pleas amounts to not much more than 'a bombardment of soap bubbles' (Hahn, 1984 p. 78). Though, if it is acknowledged that the serious scholars' central objections are valid, and if it is acknowledged on the other hand that the conclusions of standard economics are formally correct, unsatisfactory results must – modus tollens – be due to the basic assumptions.

It would be uneconomical to challenge any of these assumptions here in any detail. This has been done at length in the literature and the results are common knowledge^{1, 2}.

Moreover, as Blaug has noted:

The moral of the story is simply this: it takes a new theory, and not just the destructive exposure of assumptions or the collection of new facts, to beat an old theory. (Blaug, 1998 p. 703)

At present standard economics is not firmly enough based on positive knowledge of economically relevant human behavior. As Binmore recapitulates:

I do not understand why we cannot just accept that the empirical evidence shows that traditional economics works well in some situations and not at all in others – and that the interesting cases for current research all lie somewhere in between.

(Binmore, 2008 p. 249); see also (Kahneman, 1996), (Sugden, 1991)

This contingency, though, begs the underlying methodological question whether behavioral hypotheses *per se* are capable of supporting a sophisticated formal

superstructure that corresponds reasonably well with real world phenomena³. As Rosenberg resumes:

The notion that microeconomics is a branch of applied mathematics does economists more credit than several possible alternative explanations for its empirical weakness. (...) It isolates the limitations of the theory in a factual supposition about the determinants of human behavior, one that economists share with all of us. But the supposition we all share is false, and so economics rests on a purely contingent, though nevertheless central, mistaken belief (...). (Rosenberg, 1992 p. 247)

For the moment there is no need to take sides in the discussion of the status of the folk psychology (Hands, 1993 pp. 171-183) that underlies standard behavioral assumptions because:

The failure of microeconomic theory to uncover laws of human behavior is due to its wrongly assuming that these laws will trade in desires, beliefs or their cognates. And the system of propositions about markets and economies that economist have constructed on the basis of its assumptions about human behavior is deprived of improving explanatory and predictive power because its assumptions cannot be improved in a way that transmits improved precision to their consequences. (Rosenberg, 1994a p. 224)

If one shares the outlook that 'anything based on this mock-up is unlikely to fly' (Hahn, 1981 p. 1036) then the cognitive investment decision between standard economics and the exploration of alternative approaches is a straightforward one⁴. The present treatise makes the attempt to reconstruct a coherent formal superstructure on a non-behavioral foundation. When the market economy is conceived as a complex maze of structural and behavioral interdependencies then the probability of eventually finding some strong regularity is greater in the structural than in the behavioral domain. And since the individual 'neither intends to promote the public interest, nor knows how much he is promoting it' (Smith, 2008 / 1776 pp. 291-292) there is a sizeable *a priori* probability that the beneficial operations of the invisible hand will have the character of structural interdependencies, which of course are not apparent to the naked eye but have to be abstracted from readily accessible phenomena on the surface. This analytical venture requires a unique coordinate system of primitive concepts to start with. This framework of concepts is formalized by a set of axioms^{5, 6}.

Axiomatization is, again, a serious issue in economics since von Neumann, Debreu, Arrow and Hahn (Mirowski, 2002 pp. 104-113, 303, 408-409), (Weintraub, 1998), (Leonhard, 1995 pp. 755-756), (Ingrao, et al., 1990) and it is supposed to guarantee the coherence and consistency of all parts of a theoretical edifice. Axiomatization alone, however, leads merely to 'rigorous rubbish' (Clower, 1994 p. 409) if the first principles that govern the subject matter are not accurately identified. Hence axiomatization is a necessary but not a sufficient condition for any theoretical approach concerned with the complex real and nominal interdependencies of the evolving economic system and their eventual outcomes. It is important to recognize that axiomatization is neither an entirely neutral nor an infallible vehicle for abstract thought but may just as well produce unintended or unexpected side effects (Mirowski, 1986 pp. 179-204).

For a comprehensive perspective it is worth to recall that in 1677 Spinoza demonstrated his *Ethics* 'ordine geometrico' that is, by applying the axiomatic method, and proved the existence of God. Why could this proof not attain the same status of impeccability as Euclid's proofs, as was certainly intended? After all, as Peirce observed, 'metaphysics has always been the ape of mathematics'.

It is a well-known jest that 'a mathematician is a scientist who knows neither what he is talking about nor whether whatever he is talking about exists or not' (Cartan, quoted in (Ronan, 2006 p. 70); see also (Kline, 1982 p. 251)). What Spinoza apparently could not discern is that proofs cannot prove the existence of anything beyond the confines of the chosen formal system. The fact that products of deductive reasoning correspond in numerous cases admirably to the objects and processes of reality has puzzled scientists and philosophers since the Greeks (Kline, 1982 pp. 328-354). Spinoza, counting on what Wigner (1979 pp. 222-237) much later called the 'unreasonable effectiveness of mathematics' ventured a pioneer transplant of the axiomatic method into a quite different body of knowledge. It did not work in his case but it worked ten years later with an achievement rare in the history of science. In 1687 Newton made his *Principia* known to the world, laying out the axiomatic groundwork for most of classical mechanics in the next centuries (1999 / 1687 pp. 416-430). The two unlike disciples of Euclid were followed by J. S. Mill⁷:

In the definition which we have attempted to frame of the science of Political Economy, we have characterized it as essentially an *abstract* science, and its method as the method *à priori*. Such is undoubtedly its character as it has been

understood and taught by all its most distinguished teachers. It reasons, and, as we contend, must necessarily reason, from assumptions, not from facts. It is built upon hypotheses, strictly analogous to those which, under the name of definitions, are the foundations of other abstract sciences. (Mill, 2004 / 1844 p. 110), original emphases; for an elaborate account of Mill's method see (Hausman, 2001)

And without undue delay Mill states the fundamental behavioral principle:

Just in the same manner [as geometry] does Political Economy presuppose an arbitrary definition of man, as a being who invariably does that by which he may obtain the greatest amount of necessaries, conveniences, and luxuries, with the smallest quantity of labour and physical self-denial with which they can be obtained in the existing state of knowledge. (Mill, 2004 / 1844 p. 110)

The degree of formalization was certainly not up to contemporary set-theoretical standards but there can be no doubt that the axiomatic method has been the crucial element of theoretical economics since Adam Smith:

His [Smith's] method is always the method of Newton, which we have already seen applied to psychology and morals: to attain, by generalization, certain simple truths, from which it will be possible to reconstruct, synthetically, the world of experience. (Halévy, 1960 pp. 100, 494); see also (Hollander, 1977)

This, however, does not imply that Mill's behavioral axiom is the only or the best point of departure or, for that matter, that axiomatization is at all productively applicable to human behavior⁸. With regard to the first concern, the appropriateness of the set of basic concepts, Cournot was quite explicit and stupendously clairvoyant:

The abstract idea of wealth or value in exchange (...) must be carefully distinguished from accessory ideas of utility, scarcity and suitability to the needs and enjoyment of mankind (...). These ideas are variable, and by nature indeterminate and consequently ill suited for the foundation of a scientific theory (...). Cournot 1897, quoted in (Mirowski, 1995 p. 208); see also (Ingrao, et al., 1990 pp. 38, 41, 47, 81)

The second concern, the applicability of the hypothetico-deductive method to human behavior in the economic realm, led around 1890 to the fierce controversy known as *Methodenstreit* (Hodgson, 2001 pp. 93-94), (Hands, 1993 pp. 39-49, 81), (Georgescu-Roegen, 1971 p. 325). Although the Historical School had valid arguments it was eventually split off from the mainstream whose proponents preferred to look to the

triumphant physical sciences for methodological inspiration⁹. Hahn summarized the effect of the ensuing conceptual monoculture a century later:

History dependence stares us in the face (...), but it is not the stuff of pure theory.
(Hahn, 1991 p. 48)

The stance taken in this treatise is a pragmatic one. Euclid's 5th postulate, Hilbert's program and Gödel's proof are of no direct concern. Or, to be more specific: Hilbert's 'Finitistic Program for the Foundations of Arithmetic' (FPFA) is of no concern outside mathematics yet Hilbert's 'Axiomatic Approach' (AA) indeed has a bearing on theoretical economics (Weintraub, 2002 pp. 72-100); for comments see (Rosser Jr., 2003 pp. 584-587), (Davidson, 2003 pp. 533-535).

Hilbert described the axiomatic approach as a fairly general method of research:

When we assemble the facts of a definite, more-or-less comprehensive field of knowledge, we soon notice that these facts are capable of being ordered. This ordering always comes about with the help of a certain *framework of concepts* [*Fachwerk von Begriffen*] (...). The framework of concepts is nothing other than the *theory* of the field of knowledge. (...) If we consider a particular theory more closely, we always see that a few distinguished propositions of the field of knowledge underlie the construction of the framework of concepts, and these propositions then suffice by themselves for the construction, in accordance with logical principles, of the entire framework. (...) The procedure of the axiomatic method, as it is expressed here, amounts to a *deepening of the foundations* of the individual domains of knowledge – a deepening that is necessary for every edifice that one wishes to expand and to build higher while preserving its stability. (Hilbert, 2005 / 1918 pp. 1107-1109), original emphases

Axiomatization and formalism are closely related but distinct notions¹⁰.

(...) one can axiomatize, without being committed to a formalist reading of the axiomatic system. Axiomatization is as old as Euclid, whereas formalism is a much later development. (...) Poincaré, the most outstanding mathematician at the turn of the twentieth century, while acknowledging the value of axiomatic systems, rejected Hilbert's formalism. Similarly Frege, the leading logician of the period, while rejecting Hilbert's formalism, extensively used the AA. (Boylan, et al., 2007 pp. 430, 432)

So, Hilbert's FPFA-program did not go unchallenged. As a matter of fact, the mathematicians had a fierce *Methodenstreit* of their own:

Brower blasted away the formalists. Of course, he said, axiomatic, formalistic treatment may avoid contradictions, but nothing of mathematical value will be obtained in this way. "An incorrect theory, even if it cannot be rejected by any contradiction that would refute it, is nevertheless incorrect, just as a criminal act in nonetheless criminal whether or not any court could prevent it." (...) "To the question, where shall mathematical rigor be found, the two parties give different answers. The intuitionist says, in the human intellect; the formalist says, on paper." (Kline, 1982 pp. 252-253)

As a result of the methodological discourse within economics axiomatization has been scaled down to the human dimension of proper communication:

The procedural defense hinges upon an ideal of rigour, which is independent of the uses and purposes in which the mathematics is embedded. My favourite advocate of this position is Tjalling Koopmans, if only because, for him, mathematical rigour was the only thing standing between us and the chaos of shoddy argumentation, duplicity and the 'noise' in communication due to human frailty. (Mirowski, 1994 p. 62)

This apparent rightsizing of the axiomatic method seems justified in respect of the somewhat disappointing results of the state of art formalization of general equilibrium theory (Ackerman, 2004); it goes too far indeed because it neglects the constructive capacity of axiomatization:

Formalisation was not to be merely a mechanical check on the integrity of scientific reasoning, but through the process of axiomatisation, mathematics was to be an engine of discovery. (Weintraub, 1998 p. 1844); see also (Suppes, 1968 p. 653)

The proper working of this engine is vitally dependent on the underlying framework of concepts. Axiomatization is not exempt from the trivial rule 'garbage in, garbage out' (Blatt, 1983 p. 167). Hence, it is the predominantly intuitive ensemble of primitive concepts that is crucial and not formalization *per se* (Woo, 1986 p. 66). Admittedly, since nobody can see or portray The Economy one has first to rely on concepts and their logical relations. And these have to be, in somewhat coarse terms, total and abstract rather than partial and concrete or, in one word, general. And, not to forget, the logical implications of the axiom set must have empirical counterparts¹¹.

It is true, as McCloskey (1994 p. 166) put it: “Consistency is *not* the chief scientific virtue”. Logical consistency, whether encapsulated in an axiomatized framework of concepts or not, is but a matter of course. The longstanding quarrel about excessive formalization in economics (Katzner, 1991) therefore boils down to the 'trite injunction' (Baumol) that powerful tools should be applied with a sense of proportion:

As a personal matter, I have long believed that in dealing with M-Worlds, axiomatization is useful as well as safe (...) and I have not changed my mind, even though my faith in formalization has been sorely tested (...). My opinion continues to be that axiomatics, like every other tool of science, is no better than its user, and not all users are skilled. (Clower, 1995 p. 308)

As a common methodological denominator one can therefore agree upon the following conclusion about the approach of standard economics:

All this suggests that, like geometry, economics is best viewed as a branch of mathematics somewhere on the intersection between pure and applied axiomatic systems. (Rosenberg, 1994a p. 230)

With this consensus we come full circle back to where Adam Smith and J. S. Mill started, though with the wrong – behavioral – foot¹². Let us resume the main conclusion as Cournot’s Unfitness Proposition¹³: Human behavior is indeterminate and behavioral hypotheses, even if utterly plausible, are 'ill suited for the foundation of a scientific theory'. A behavioral axiom therefore is a methodological oxymoron.

This gives one a clear-cut choice: 1) Forget about axiomatization and build mathematical models instead (Niehans, 1994 pp. 313-317), each conveniently endowed with its own basic assumptions. 2) Scale down vacuous theory and rely on down-to-earth empirical research. 3) Restart with a new set of axioms.

Choosing the first option makes economics a pile of incoherent and inconclusive partial models (Morishima, 1984 p. 58) and involves some mild schizophrenia because the mathematics used derives its merits straight from being axiomatized. Either way, the proliferation of models is a 'good intellectual game' (Hicks)¹⁴ but lacking a common formal ground the only remaining commonality defining the subject matter is a behavioral *passpartout* tool, the calculus of constrained optimization¹⁵. The second option produces a relatively small number of historical snapshots that can stand for themselves yet need a theoretical framework for generalization (Feyerabend, 2002 p. 27). The third option is at

all times worth a trial according to J. S. Mill’s principle of proliferation¹⁶ (Feyerabend, 1995 pp. 139-143).

The goal of the present treatise is to establish a formalism of maximum structural simplicity and generality. We start with an axiom set that is free of any behavioral specifications and subsequently approach the complexity of the real world by a process of consistent differentiation, that is, by applying the method of decreasing abstraction (Klant, 1988 p. 90). While progressing from the big picture to the details the coherence of whole and parts is maintained at every level of differentiation. It has to be shown that on the new axiomatic foundation a general theory in the strict sense can be reconstructed.

With a minimum expense of words Figure 1 summarizes that the present treatise is about a switch of the unifying principle from the behavioral axioms represented by homo economicus to structural axioms.

	Foreground	Background
Standard economics	Behavioral axioms – homo economicus	Restrictive structural assumptions
e-Economics	General structural axioms	Appropriate behavioral assumptions

Figure 1 Switch of the Unifying Principle

It should therefore cause no great surprise that the terminology used in the following necessarily differs to some extent from the subjective-marginalistic terminology. The phenomena to be studied, naturally, are the same. However, since the perspective is structure-centric instead of behavior-centric and since the formal premises are entirely new, the following inquiry leads in some relevant cases to new insights and conclusions.

1.2 Implicit World Views

“Life is short. Nature is niggardly. Our fellows have other objectives “, Robbins (1935 p. 13) was quite explicit about his ontological stance. It is widely admitted that each theoretical endeavor is influenced by the researcher’s implicit world view (Suppe, 1977 pp. 217-221). These preconceptions are understood to be *subjectively* self-evident characteristics of the natural order (Klant, 1988). Robbins distilled his value-charged preconception about the human condition into the neutral notions of choice, scarcity, and competition. The original meaning of these concepts became further blurred with progressive formalization but it is invariably present as a subtext in the economic discourse.

Ontology is educated guesswork about what is truly real or merely accidental or even illusory (Mäki, 2001), (Searle, 1995), (Lawson, 1994). Hence the contention that Nature is niggardly would not be of any importance were it not for the fact that ontology, however trivial, is crucial for the acceptance or rejection of theories (Mäki, 2001 p. 9). Einstein did not come to terms with quantum theory because of his deterministic ontology (Atmanspacher, 2002 p. 50) and not because he regarded quantum theory as logically deficient.

Ontology meanders, in interaction with the distinct scientific subfields, between antipodes like: form–substance, being–becoming, determinism–randomness, order–chaos, causality–finality, indestructibility–degradation, unity–diversity, equilibrium–process, atomistic–systemic, quantitative–qualitative, continuous–discrete, natural–social, rational–emotional, cooperative–competitive, finite–infinite, perfect (Supreme Being)–corrupt (human being), and so on.

That the ontological profile of standard economics closely resembles that of classical physics (Mirowski, 1995), (Ingrao, et al., 1990 pp. 33-37) and is therefore misleading in the domain of economics has been an ongoing critique since Walras and Jevons started their extensive borrowing from the hard sciences (Cohen, 1994 p. 79).

It has been repeatedly suggested that scientists come ontologically in two types: Platonists and Heraclitians. The first type is comfortable with stasis and order and sees the world basically in equilibrium. The second type likes process and change and sees the world in intermittent continuous and discontinuous flux (Waldrop, 1993 pp. 334-335). There is, though, somewhat more to ontology than this simplistic binary scheme covers. The Durkheim-Mauss-Douglas Thesis asserts:

Theories of the physical world are shaped by the social relations within the culture that generates them, and these are used in turn to express in reified format the essence of that culture's ideal of order. (Mirowski, 1988 p. 110)

Economics is not outside the sphere of social belief structures. It has often been noticed that its persuasive metaphors: progress, individual self-improvement, efficiency, the invisible hand, timeless equilibrium, and the Pareto optimum as summum bonum of social well-being, are secularized gospels (Nelson, 2006). Depending on their ontology economic theories therefore have social merits that are independent of their scientific merits (however assessed by the peer group). Proofs, either mathematical or dialectical, either unintended or intended, are unfailingly scientific *and* political proofs (Clower,

1995 p. 317). The Austrians have always been quite outspoken about this ontological nexus:

In what was perhaps their finest hour, the Austrians (...) argued that this vision of market socialism was impossible and that it was based on a fundamentally misguided vision of markets and prices. (Rosen, 1997 p. 144)

It was certainly the finest hour of neoclassical economics and a 'major intellectual achievement' (Hahn, 1984 p. 114) when Arrow and Debreu (1954) following Wald (1951 / 1936) proved the existence of a general equilibrium by applying the axiomatic method about 80 years after Walras had posed the problem (and incidentally about 280 years after Spinoza had delivered a work of 'remarkable resemblance' (Morishima, 1984 p. 51). The social benefits have been considerable:

(...) general equilibrium theory seems to consign ideological controversy to the past; the theory of value is now the province of disinterested research. (Mandler, 1999 p. 46); see also (Porter, 1994 p. 160)

This disinterested research (Mas-Colell, et al., 1995 pp. 598-606) accompanied by the ever present hunch among economists that general equilibrium theory might be a 'blind alley' (Blaug, 2001 p. 160) or the 'wrong peak' (Kirman, 2006 p. 248) has led to the real challenge:

It is good to have [the technically best study of equilibria], but perhaps the time has now come to see whether it can serve in an analysis of how economies behave. The most intellectually exciting question of our subject remains: is it true that the pursuit of private interest produces not chaos but coherence, and if so, how is it done? (Hahn, 1984 p. 102)

To answer this question research has gone in various directions (Rizvi, 2006 p. 231). The present treatise is, in a broad sense, unanimous with the ontological stance and concerns of evolutionary economics¹⁷. A programmatic definition has been given by Witt:

Evolution is the self-transformation over time of a system under consideration. In this definition, the term 'transformation' means a process of change governed by regularities. The prefix in 'self-transformation' points to the endogenous sources and causes of novelty. (Witt, 2003 p. 13), original emphases, for detailed criteria see (Nelson, 1995 p. 56), for an evolutionary ontology see (Dopfer, et al., 2004)

The notion of evolution has different meanings in different contexts. One can sensibly speak about the evolution of ideas (Luhmann), of an evolutionary epistemology (Popper)

or about the evolution of physical laws (Peirce), and, of course, of biological evolution (Darwin, Wallace, Lamarck). The term evolution was in wide use long before it acquired a pronounced Darwinistic flavor (Hodgson, 1996 pp. 18-21) and its most general connotations are: cumulative change, randomness, irreversibility, population variety, self-organization, autopoiesis, circular causation, increasing complexity and open-endedness. Darwinian evolution is far more specific and embraces the inseparable causal interaction of variation, replication, and selection (Hodgson, 2002), (Vromen, 2001). Social Darwinism, which developed at the political and intellectual fringe, has no logical connection to Darwin's theory (Hodgson, 2006 p. 57)¹⁸. In the most general sense, and apart from rather intricate details, evolution is basically an ontological contraposition to mechanics. It would be misleading to personalize the two paradigms in the platitudinous antagonism Darwin vs. Newton. For better or worse neither scholars nor paradigms are unambiguous entities:

An amusing point is that *Newton was not Newtonian*. He, on the contrary, believed in an evolving world. The world would go into 'confusion' and the 'agent' (God?) would have to repair it. (Prigogine, 2005 p. 63), original emphases; see also (Gleick, 2003 p. 47), (Westfall, 2008 pp. 646-647)

On the other hand it has been observed:

In a real sense Darwin produced a theory of proximal cause, evolution by natural selection, designed to exorcise the specter of supernatural design. In doing so, he moved biology into a Newtonian framework. (Brooks, et al., 1986 p. x); see also (Gingerezer, et al., 1997 p. 136)

The interlinking explanandum of economics is the phenomenon of accelerated 'technophysico evolution' (Fogel, 1999 p. 2), (Day, 2008) that goes along with an even faster development of the financial sphere. In the present treatise the adjective evolutionary embraces five specific tenets with regard to the subject matter of economics¹⁹:

- The money economy is – ontologically – the real economy.
- The economy operates at the margin of time.
- The basic systemic characteristics are: change, chance, and variety²⁰.
- Neither the agent nor the theoretical economist *knows* that the sun will rise tomorrow²¹.
- The principle of hierarchical ontological consistency applies.

This principle demands:

(...) theories of each ontological level must be consistent with all other theories pertaining to that reality, including those at other levels. Although each theory and mode of theorizing is different, no theory can overturn an acceptable theory at another ontological level. For example, reigning socio-economic principles cannot overturn the known and received laws of biology or physics. (...) This meta-theoretical principle (...) is required to avoid contradictions within a theoretical structure. (Hodgson, 2001 p. 328)

A rather straightforward application of this principle asserts: since the unsurpassable speed of light is finite, the speed of economic agent's action and reaction is even more finite; therefore the behavioral assumption of simultaneous mutual adaptation is pointless, even in pure as-if models. Physical impossibility is different from mere idealization.

Hierarchical ontological consistency is independent of formal consistency. The opaque interaction between advanced formalization and ontology has been elucidated by Woo:

By the time a sophisticated logical system is build up, it is no longer easy to relax [the] originally *instrumental* but now *ontological* assumptions on a one-by-one basis, because the relaxed assumptions (even those of a more realist kind), still have to be blended into the rest of the system in order to derive new theorems. This, the system can hardly escape being contaminated by the unreal ontology unwittingly formed in the rest of the system, an ontology that no longer has any conceivable counterpart in reality (...).(Woo, 1992 p. 37), original emphases

The ontological underworld is not a harmless virtual reality. It imparts legitimacy and it busily manufactures insiders and outsiders. The ontological stance is pre-theoretical but neither neutral nor useless because it implicitly determines the positive and negative heuristics of a research program. The evolutionary paradigm denies the ontological legitimacy of the equilibrium paradigm.

1.3 Strong Arguments

The ontological stance provides criteria for the appraisal of alternative approaches. With regard to the distinct currents of economic thought Weintraub asserts:

General systems theory (G.S.T.), of which general equilibrium theory is but a specification to certain economic problems, has existed for many years. (...) G.S.T., then, looks for, and finds, many structural similarities among fields of scientific analysis. To the extent that G.S.T. is a constructive approach to inquiry, general

equilibrium theory in economics becomes rooted not just in the particular tradition that have generated the multi-fold extensions of the Arrow-Debreu-McKenzie [ADM] model, but in the very structural unities of science itself. To attack general equilibrium theory in economics as a legitimate model of reasoning is to *simultaneously* deny homeostatic reasoning to psychologists and morphogenetic analysis to the biologist. To argue that G.S.T. is inapplicable to economics is to negate claims that economics is a science. (Weintraub, 1979 pp. 71-72), original emphases

From the fact that G.S.T. is a metatheory therefore follows that GET as a subdiscipline is the metatheory in economics:

General equilibrium analysis is not *the* theory of the microfoundations of macroeconomics. Instead, the ADM structure is a metatheory, or an investigative logic which, since it is used to construct all economic theories, must necessarily be used to examine “microfoundations of macroeconomics” models. (...) This argument is a fairly strong one, (...) (Weintraub, 1979 p. 73), original emphases

When a system is defined as a set of elements standing in interrelation then we can in fact see 'systems everywhere' from the atom to the universe (Bertalanffy, 1969 p. 55). So we can readily agree that the economy is a system.

Now, general equilibrium theory as the core of standard economics rests on the notion of simultaneous equilibrium. This, indeed, is a concept foreign to systems theory.

Stationary or steady states of systems are a completely different notion:

So, (...), much of cybernetics is concerned with the study of mechanisms which maintain equilibrium. But this is emphatically not the formal notion of equilibrium employed in neoclassical economics. (Denis, 2007 p. 274)

Moreover, in contrast to standard economics with its almost exclusive preoccupation with static equilibrium, systems theory does not see in steady states, i.e. in systemic equilibrium, the primary object of interest:

Concepts of equilibrium, homeostasis, adjustment, etc., are suitable for the maintenance of systems, but inadequate for the phenomena of change, differentiation, evolution, negentropy, production of improbable states, creativity, building-up of tensions, self-realization, emergence, etc; (Bertalanffy, 1969 p. 23)

The seeming affinity of G.S.T. and GET originates from the equivocation of equilibrium. As Hicks (1939 p. 23) once remarked: “Pure economics has a remarkable

way of producing rabbits out of a hat". Actually G.S.T. and GET are disjunct. Moreover, even if they were not, from the fact that the economy is a system does not logically follow that GET is the sole possible application of systemic principles to economics. And in conclusion, a metatheory may very well establish deeper structural similarities between different domains of science but this does not corroborate any domain specific theory. Hence GET cannot derive scientific credentials from G.S.T.

The intellectual founding fathers of G.S.T. were well aware of vacuousness as the major pitfall of the hypothetico-deductive method:

Advantages and shortcomings of mathematical models in the social sciences are well known (...). Every mathematical model is an oversimplification, and it remains questionable whether it strips actual events to the bones or cuts away vital parts of their anatomy. (Bertalanffy, 1969 p. 113)

Once sensitized, we can in fact see 'systems everywhere' and we can also see 'evolution everywhere'. Should there emerge a metatheory as 'grand synthesis' (Laszlo, 1987) any time soon, economics as a specific subdiscipline would be an essential constituent. This, though, would preclude GE; and this argument, too, is a fairly strong one.

1.4 The Importance of Being Ignorant

Theory has aptly been compared to a map that reduces the overwhelming complexity of the real thing to manageable proportions. As a first step to a useful map naïve empiricism translates the real thing into commonplace economics (Niehans, 1994 p. 15) which asserts:

- Successive units of a commodity are less and less urgently needed. (...)
- Abundance makes prices fall a scarcity makes them rise (...).
- A decline in price stimulates demand but reduces supply (...).
- Higher fertility and better location result in higher land rent.
- An increase in the money supply raises prices.
- Voluntary exchange is advantageous to both parties.
- Trade occurs because different regions have different natural endowments (...).

The salient trait of this pre-theoretical raw material is that it reflects a proximal glance at a fairly developed economy with prices, rent, and money, which is historically located somewhere between a pure barter economy and a globalized economy:

For the history of economic science, the concept of commonplace economics is useful mainly as a benchmark from which analytical achievement can be measured. It marks the zero point of scientific progress. (Niehans, 1994 p. 15)

What we need, then, is a corresponding analytical zero point. The crucial requirement thereby is that this point must embrace the *whole* economy. To insist on this requirement is the distinguishing merit of general equilibrium theory in comparison to the Marshallian approach that has to protect its open flanks with *ceteris paribus*²². This analytical expedient is unwarranted because circular interdependencies are the defining characteristic of the economy. Therefore no partial approach of any kind will do in the long run.

The task is to create a map of the whole economy without firsthand experience because The Economy is invisible and intangible. That is, one has to leap from commonplace economics to an extremely abstract set of basic propositions about the economy as a whole²³. This set has to reduce the vast complexity of the real thing to almost nothing. From this almost-nothingness the real world complexity then has to be logically reconstructed. This requires the successive introduction of more and more specific assumptions and historical contingencies. The consequence of which can be made clear with an analogy to physics. The law of gravitation allows a fairly good prediction of a cannonball's parabolic trajectory but does not help much with a flying feather that is more exposed to the innumerable contingencies of the moment. It is obvious, however, that this kind of complexity is altogether independent from the simple general laws of motion. Hence the complexities of real life by no means preclude the simplicity of theory. Admittedly, they severely hamper the straightforward application of the theory in the 'abyss of disorder' (Quesnay). For *individual* practical purposes commonplace economics is therefore true and sufficient. By the same token is Aristotle's commonplace physics still empirically valid for all who remain close to the earth's surface and do not care about Galileo's acrimonious refutation of parochial realism (Feyerabend, 1995 pp. 176-181). Yet theory aims at generality²⁴.

It may well turn out, though, that it is impossible to analytically reduce the complexity of the whole economy. In this case a faithful description and interpretation of the state of affairs by ideal types is the alternative to axiomatization. Whether it is possible to capture the essence of a money economy with a few axioms is unknowable in advance yet we may find it out with successive trials:

Whether an axiom is or is not valid can be ascertained either through direct experimentation or by verification through the result of observations, or, if such a thing is impossible, the correctness of the axiom can be judged through the indirect method of verifying the laws which proceed from the axiom by observation or experimentation. (If the axiom is deemed to be incorrect it must be modified or instead a correct axiom must be found.) (Morishima, 1984 p. 53)

Contradicting Morgenstern's credo 'I believe that it is quite possible to axiomatize economics.' (Ingrao, et al., 1990 p. 193), it has been argued more than once that this approach is futile in principle:

(...) theory means a logical filing of *all* extant knowledge in some particular domain such that every known proposition be either contained in the logical foundation or deducible from it. That such a filing has the unique merit of affording *comprehensibility* is a leitmotiv inherited from Aristotle. However, hardly any attention has been paid to the fact that there can be no comprehensibility without the *compressibility* of extant knowledge into only a relatively few ω -propositions. (...) It is then the evolutionary nature of the economic process that precludes a grasping of all its relevant aspects by an arithmomorphic scheme, even by a dynamic one. (...) Given that the "chemical" doctrine fails to work in the chemical domain, it would be foolhardy to count on its success in social sciences, where the number of compounds is almost limitless and quality dominates the scene to an incomparably greater degree than in the domain of elementary matter. (Georgescu-Roegen, 1971 pp. 322, 330, 327), original emphases

True, axiomatization is concerned with the analytical reduction of real world complexity. This gargantuan simplification requires, yet is at the same time *more* than the shuffling of a small set of symbols according to the rules of logic. The key words in the foregoing quote are *all* and *filing of extant knowledge*. It would be foolhardy indeed to make such a claim and this is emphatically not what axiomatization is about in the first place. To begin with, axiomatization is not primarily concerned with extant knowledge but should be seen as an 'engine of discovery' (Weintraub, 1998 p. 1844). The 'problem situation' (Popper, 1994 p. 155) is rather defined by *non-extant* knowledge. Second, it is obvious that not *all* phenomena are compressible and paying heed to the obvious physicist *completely ignored* the feather's complex trajectory and busied themselves with falling apples and cannonballs thrown from a tower²⁵. But readily affirming with

Cournot's Unfitness Proposition that human behavior is formally incompressible does not entail that *all* economic phenomena defy axiomatization. Georgescu-Roegen himself pointed the way:

The more complicated the model and the greater the number of the variables involved, the further it moves beyond our mental control, which in social sciences is the only possible control. (...) A "simple-minded" model may after all be the more enlightening representation of the economic process provided that the economist has developed his skill to the point of being able to pick up a few but significant elements from the multitude of cluttering facts. The choice of relevant facts is the main problem of any science, as Poincaré and Bridgman insisted. (Georgescu-Roegen, 1971 pp. 340-341)

With regard to irrelevant complexity axiomatization may therefore alternatively be defined as the art of ignorance.

I mean by this that formalization eliminates provincial and inessential features of the way in which a scientific theory has been thought about. (...) Formalization is a way of setting off from the forest of implicit assumptions and the surrounding thickness of confusion, the ground that is required for the theory being considered. (...) In areas of science where great controversy exists about even the most elementary concepts, the value of such formalization can be substantial. (Suppes, 1968 pp. 654-655)

1.5 Sarcey on Axiomatization

The goal is to better understand how the economy generates favorable conditions for its accelerated self-transformation. The analytical zero point for the quest is given with the structural axiom set in sections 2.1 and 2.3. The axioms that contain ten measurable variables constitute an evolving consumption economy. The route then leads quite lineally towards the general axiomatic model in section 14. Since the axiomatic variables are measurable, the logical implications have the format of 'laws of algebra' (Shaik, 1980) that have an empirical counterpart.

The behavioral building block, the propensity function, is introduced in section 2.4. This general equation compactifies all logical variants from uncertainty to determinism and from it follows the drifting economy as the minimalistic evolutionary benchmark process. The market outcomes of the pure random processes are analyzed in section 4 and the distinction between stochastic and deterministic supersymmetry on the one hand and

behavioral equilibrium on the other is exemplified. It is shown that the microeconomic details of a behavioral equilibrium or disequilibrium can be readily mapped onto the elementary axiom set. This compactification is then also carried out for an arbitrary number of periods.

In section 5 the development of real and nominal stocks including the quantity of money is directly derived from the axiom set.

Human behavior is epitomized by a new economic man and his bundle of signum functions. The signum function is introduced in section 6 and first applied to analyze the clearing of the product market and the balancing of the household sector's budget with a minimum of behavioral assumptions. The signum function, which determines the directionality of the propensity function, is derived as the product of the information and the action function. In section 6.1, 6.4, and 6.5 it is demonstrated for the product market how directed randomness leads to stochastic stability and optimality in a random environment.

Section 7 is devoted to the introduction of the axioms of profit and saving and the consistent derivation of net worth. The substantial conceptual differences with standard profit theory are elaborated at length. Standard profit theory needs neither empirical nor logical falsification because it is known to be incoherent. What is actually needed is a consistent alternative, which is provided by the structural axiom set.

In section 8 the behavioral and structural interplay is explored in detail in order to specify the favorable conditions that are indispensable for the proper functioning of the market economy. In this straightforward mechanical analysis the concepts of employment multiplier and price multiplier are put to work. It turns out that there exists a significant algebraic relation between the employment multiplier and the *original* Phillips curve.

The extended axiomatic base is the formal precondition for the logical emergence of new markets: the secondary commodity market and the various financial markets that are dealt with in section 9. In this context, as in others, we are led to the conclusion that there is no such thing as a generic market.

The distribution of output between the wage earners and the receivers of distributed profits and the mechanism of redistribution is analyzed in section 10 together with the distribution of profits between firms. It is shown that the distribution of output does not depend on a well-behaved production function or on diminishing returns.

Applying the method of rational reconstruction an account is given in section 11 of how the initial economy and the supporting institutions come into being by the rational choice of individuals. The analytical aim is to set the development of the economy in motion from initial conditions that are entirely transparent and to oversee without difficulty. Non-human production factors and an evolutionary analogue to the production function are added and then formally compactified, i.e. mapped onto the axiom set.

With the consistent integration of the labor market in sections 12 the basic two-market system is completed and the probability of full employment is formally established. The crucial conditions that are conducive to virtuous feedback loops are discussed in section 12.5.

In section 13 the formal system is completed and closed. Given the signum functions of economic man the question to be addressed is how the reference values of the variables of the axiom set can be endogenously determined. Economic man is portrayed as a goal setter. Self-interest in a money economy is defined uniformly for the representative household and the representative firm with regard to the growth of net worth. The Principle of Valuation is expressed as a function that connects the cardinal valuation price with net worth, i.e. the stock of money and the valued stock of not yet consumed (not yet sold) commodities. All variables follow in direct lineage from the axiom set. No foreign entity like utility enters the self-contained formal core. It's all in the axioms.

The formal differentiation of the axiom set is carried out for an arbitrary number of firms in section 14. In this main part the structural value theorem is derived and applied first to two firms that produce different consumption goods and then to the central bank and the land agency. The harmonic production structure is defined as the perfect congruence of the allocation of labor input and the partitioning of consumption expenditures between several firms. An important property of the harmonic structure is the equality of profit ratios if the average wage rates in two firms are equal. For the special case of market clearing and budget balancing in one period the optimal partitioning of total expenditures can be formally connected to the marginal rate of substitution. The harmonic structure allows for the determination of the rate of interest and of the ratio of the lease price for land to the product price. The classical notions of factor income and distribution are restated in structural axiomatic terms to clarify fundamental differences and to pinpoint the interconnection of productivity differentials, institutional settings, choice, and distribution.

The household sector is differentiated in section 15 and the analysis of the direct lender-borrower relationship leads to the formal integration of liquidity, time preference, risk, and insurance.

At germane junctures the formal interfaces to established approaches are highlighted. In some cases it can be demonstrated immediately that an established approach is a limiting case of the general axiomatic model. For the Keynesian approach this is done at length in section 16. The structural axiomatic approach makes it possible to precisely locate the conceptual and logical flaws of the Keynesian formalism. The analysis of the investment cycle is then carried further in section 17. The focus is on the interrelation of profit and the real rate of interest, i.e. on the interrelation of the nominal side and the productivity effect of investment. We arrive at the general result that the structural stability of the economic system is vitally dependent on prolonged asymmetric growth.

Section 18 highlights the long term conditions for a thriving market economy in structural axiomatic terms. The clues of the previous investigation are resumed and brought to the conclusion that there are two invisible hands, the right one giving and the left one taking, and that we have in the past mostly felt the right one.

Section 19 concludes.

The simulation details of the evolving money economy are gathered and illustrated in the appendix. A link for the download of the Excel simulation file from Google Docs is forthcoming in section 21.1.

In his famous *Science and Method* Henri Poincaré gave a felicitous characterization of the axiomatic enterprise:

The essential thing is to learn to reason with the axioms once admitted. Uncle Sarcey, who loved to repeat himself, often said that the audience at a theatre willingly accepts all the postulates imposed at the start, but that once the curtain has gone up it becomes inexorable on the score of logic. Well, it is just the same in mathematics. (Poincaré, 2007 / 1914 p. 136)

As it happens, it is just the same in economics.

[The following sections are under review. The complete manuscript shall be available presumably in three month.]

22 Notes

¹ For different aspects see as a small historical selection: (Stiglitz, 2010), (Akerlof, et al., 2009), (Ackerman, 2004), (Kanth, 2004), (Boland, 2003), (Downward, et al., 2002), (Chick, et al., 2001), (Backhouse, 1998), (Lawson, 1997), (Perelman, 1996), (Heilbroner, et al., 1995), (Mirowski, 1995), (Clower, 1994), (Ormerod, 1994), (Blaug, 1992), (Morishima, 1991), (Stiglitz, 1991), (Wiseman, 1991), (Ansari, 1991), (von Hayek, 1989), (Kirman, 1989), (Minsky, 2008 / 1986), (O'Driscoll, et al., 1985), (Wiles, et al., 1984), (Bell, et al., 1981), (Deane, 1983), (Eichner, 1983), (Harcourt, 1972), (Hunt, et al., 1972), (Kaldor, 1972), (Phelps Brown, 1972), (Shackle, 1972), (Ward, 1972), (Worswick, 1972), (Georgescu-Roegen, 1971), (Kornai, 1971), (Boulding, 1970).

² Roughly, the argument follows these main strands:

– 'Thousands upon thousands of scholars, as well as thousands of statesmen and men of affairs, have contributed their efforts to the attempt to understand the course of events of the economic world. And today this field of investigation is being cultivated more extensively, than ever before. How is it, then, that in all these years, and with all the undoubted talent that has been lavished upon it, the subject of economics has advanced so little?' (Schoeffler, 1955 p. 2)

'I am talking about the evident bankruptcy of economic theory which for the second time has nothing to say on the questions that, to everyone except economists, appear to be most in need of an answer.' (Robinson, 1972 p. 9)

– 'Now, at any rate, we have an explanation for why the assumptions of economic theory about individual action have not been improved, corrected, sharpened, specified, or conditioned in ways that would improve the predictive power of the theory. None of these things have been done by economists because they cannot be done. The intentional of the fundamental explanatory variables of economic theory prohibits such improvement.' (Rosenberg, 1992 p. 149); 'The predictive weakness of theories couched in intensional vocabulary do not correlate in a manageable way with the vocabulary of other successful scientific theories; they do not divide 'nature at the joints'; (...)' (Rosenberg, 1994a p. 224), for comments see (Hands, 2001 pp. 334-341)

– 'Thus many are inclined to blame inappropriate copying of physics for the willingness of neoclassicals to tolerate bizarrely unrealistic assumptions and to place everything historical, cultural, institutional, and even psychological outside the framework of economic analysis.' (Porter, 1994 p. 128)

– 'None of these scientist-critics aimed to deny the legitimacy of theory, not even in political economy. Nor did they commonly denounce a premature use of mathematics. They objected, rather, to "loose" theorizing. The precision and rigor of quantitative methods were held up as a cure for this looseness.' (Porter, 1994 p. 130)

– 'General Equilibrium Theory regarded by many as the summum of the 'grand neo-classical synthesis' has throughout its development been systematically attacked by a wide variety of critics from many different angles. Yet, curiously, these criticisms have been largely ineffective and it would not be unfair to say that this theory still furnishes the basic foundations of what many are pleased to call 'mainstream economics'. Indeed such theory as is used by practical men to justify their economic recommendations is derived from this underlying framework, albeit with unwarranted appendages. There seems to be a quiet confidence in the profession that we are moving, if only slowly, towards a more

scientific basis for economics. (...) Paradoxically many of those who have contributed much to the development of general equilibrium theory are less complacent.' (Kirman, 1989 p. 126)

– 'As will become evident, there is more agreement on the defects of orthodox theory than there is on what theory is to replace it: but all agreed that the point of the criticism is to clear the ground for construction.' (Nell, 1980 p. 1)

– '(...) if you think you can do better with a non-neoclassical model (...), then you are quite welcome to try.' (Boland, 1992 p. 19)

– 'If one calls those individuals working in the field of microeconomic foundations of Keynesian economics Keynesian-economic theorist, then, as Hahn has said, these 'Keynesians were not much better'.' (Morishima, 1984 p. 57)

– 'Suffice it to say that, in my opinion, what we presently possess by way of so-called pure economic theory is objectively indistinguishable from what the physicist Richard Feynman, in an unflattering sketch of nonsense "science," called "cargo cult science" (Clower, 1994 p. 809)

– 'We should also like to underline Debreu's effective reference to Bacon when he says that "citius emergit veritas ex errore quam ex confusione." It would be a mistake to lower the level of analysis and clarification. The only way possible is a thorough reexamination of the theory's basic hypotheses, i.e., a true paradigmatic revolution.' (Ingrao, et al., 1990 p. 362), original emphasis

³ 'Just as one of the great contributions of twentieth-century neoclassical economics was to make clear why considerations which they had excluded from their analyses – such as information and transactions costs – simply had to be brought into the analysis, so too one of the central contributions of game theory has been to make it clear that the 'rational' actor model is not only descriptively inaccurate (as earlier economists had charged), but internally incomplete and/or inconsistent (...). The hope of game theory that some simple version of rationality could lead to well-defined, let alone reasonable, predictions of behaviour has been dashed. Game theorists have increasingly relied in their analyses on 'small' degrees of irrationality, while at the same time showing that the exact nature of the equilibrium depends precisely on the nature of these small irrationalities (...). This research makes it clear (if it was not already so) that economists must study how individuals actually behave, whether that conforms to some economists' preconception of rationality or not.' (Stiglitz, 1991 p. 138)

⁴ '(...) we may say that the long-lasting success of our categories and the omnipresence of a certain point of view is not a sign of excellence or an indication that the truth or part of the truth has at last been found. It is, rather, the indication of a *failure of reason* to find suitable alternatives which might be used to transcend an accidental intermediate stage of our knowledge.' (Feyerabend, 2004 p. 72), original emphasis

⁵ 'It would be amusing to put the question whether formalization in science is desirable to Archimedes in Sicily or some three hundred years later to Ptolemy in Alexandria. I can imagine Archimedes, in a characteristic turn of phrase, saying that no man of eminence in philosophy would ask such a question.' (Suppes, 1968 p. 651)

⁶ 'The often heard rule that concepts are to be defined before they are used in a discussion is much too simple minded pre-Hilbertian. The only way to arrive at coherent languages is to set up axiomatic systems implicitly defining the basic concepts. (Schmiechen, 2009 p. 344)

⁷ 'As we shall see, general economic equilibrium theory originated and developed in the context of a project put forward in varying forms by different scholars to repeat Newton's titanic achievement – i.e., the fulfillment of Galileo's program for a *quantitative* (mathematical) study of physical processes – in the field of the social sciences.' (Ingrao, et al., 1990 pp. 33-34), original emphasis

⁸ 'Not that any political economist was ever so absurd as to suppose that mankind are really thus constituted, but because this is the mode in which science must necessarily proceed.' (Mill, 2004 / 1844 p. 106)

⁹ 'A picture of science has developed since the Renaissance, or perhaps more precisely since the seventeenth century that was formed entirely by mathematics and physics. Philosophers from Bacon and Descartes to Locke and Kant were of the same opinion as the physical scientists from Galileo and Newton to Lavoisier and Laplace, that it should be the ideal of science to propose mathematically formulated theories that are based on universal laws. Proof and the capacity for exact prediction were considered the yardsticks for the quality of a scientific explanation. (...) The theoreticians of science prior to 1859 were unable to incorporate history into the physical sciences. One could not experiment with it, and one could reconstruct it only by indirect inference. How could this be reconciled with the objectivity of science? There was no room for history in the classical philosophy of science from the Vienna positivists to Hempel-Oppenheim and Nagel. And this was even true for Karl Popper until the end of the 1970s. There can be no doubt, as was shown by Ghiselin (1969), Mayr (1982), and Gould (1986), that Darwin had founded a new methodology, the methodology of the historical sciences.' (Mayr, 1991 pp. 134-135)

¹⁰ 'The sense of formalization I shall use in the subsequent discussion is just that of a standard set-theoretical formulation.' (Suppes, 1968 p. 653).

'This tendency to identify mathematics with formalism rather than formulas became all the more dominant in the 1930s and 1940s, when general equilibrium theory was established as the most prestigious research field in the economic discipline.' (Porter, 1994 p. 159)

It is important to note that the present treatise rests on the conviction that economics as applied axiomatics may well start above the set-theoretical level. In this sense we axiomatize but do not formalize.

'Thus not all axiomatic theories need to be phrased in terms of set theory but much more conveniently and intelligibly rather in terms of some advanced mathematical structures.'

(Schmiechen, 2009 p. 367)

'Under the influence of recent mathematical fashion, some authors have developed axiomatic formulations of mechanics using set theory. But set theory is not the right mathematical tool because it is too general. Consequently, theorems and proofs in this approach are inordinately unwieldy.' Hestens, quoted in (Schmiechen, 2009 p. 368)

¹¹ General equilibrium theory did not find much acceptance among physicists, engineers, and mathematicians because of its lack of empirical content: '(...) it was in part because they had been brought up to think even less of theory without measurement than of measurement without theory.' (Porter, 1994 p. 161)
'Walras approached Poincaré for his approval. (...) But Poincaré was devoutly committed to applied mathematics and did not fail to notice that utility is a nonmeasurable magnitude. (...) He also wondered about the premises of Walras's mathematics: It might be reasonable, as a first approximation, to regard men as completely self-interested, but the assumption of perfect foreknowledge "perhaps requires a certain reserve."' (Porter, 1994 p. 154)

¹² 'Now the rationality principle, which in the social sciences plays a role somewhat analogous to the universal laws of the natural sciences, is false, and if in addition the situational models are also false, then both the constituent elements of social theory are false.' (Popper, 1994 p. 173)

¹³ Cournot is here meant to act as a symbolic representative of a larger and sometimes quite radical group:
'A remarkable discussion has been lately going on in the revues and journals concerning the logical method of the science, touching even the question whether there exists such a science at all. Attention was drawn to the matter by Mr. T. E. Cliffe Leslie's remarkable article "On the Philosophical Method of Political Economy," in which he endeavours to dissipate the deductive science of Ricardo. Mr. W. T. Thornton's writings have a somewhat similar tendency. (...) Many would be glad if the supposed science collapsed altogether, and became a matter of history, like astrology, alchemy, and the occult sciences generally. (...) But as regards the fate of the deductive method, I disagree altogether with my friend Mr. Leslie; he is in favor of simple deletion; I am for thorough reform and reconstruction.' (Jevons, 2006 / 1911 pp. xv-xvi)

¹⁴ 'Some fields of economics consist mainly of interesting possibilities. The hundredth possible world of international trade theory gives the impression of an allegorical poesy gone wacko.' (McCloskey, 1990 p. 31)

¹⁵ 'Economics is the science which studies human behavior as a relationship between ends and scarce means which have alternative uses.' (Robbins, 1935 p. 16).
'This definition, while it is still popular in introductory textbooks, is really a definition that reduces all of economics to a particular kind of microeconomics; (...)' (Hands, 2001

p. 36)

'Yet, in spite of these methodological triumphs, the subject does not bear all the hallmarks of some of the other sciences. Most strikingly, while economists of many persuasions may agree about the tools to be employed, there is no agreement about the basic economic model for describing the economy: while in many circles, the competitive model, with perfectly informed agents, rational consumers and value maximising firms, is believed to provide the foundations for understanding both the aggregative behaviour of the economy and its components, in other circles, that model is viewed with some circumspection. Evidently, the tools are not strong enough to discriminate among fundamentally different hypotheses, or at least not strong enough to overcome differences in prior beliefs, beliefs which are often influenced by ideological concerns.' (Stiglitz, 1991 p. 134)

In the present treatise the subject matter is delineated by the non-behavioral axiom set and the propensity function. Constrained optimization is thereby not excluded but regarded as a limiting case.

¹⁶ See also Popper's conjectures and refutations (Popper, 1981 p. 53).

¹⁷ For diverse concepts see (Dopfer, et al., 2008), (Lehmann-Waffenschmidt, 2007), (North, 2005), (Metcalf, 2005), (Witt, 2003), (Cantner, et al., 2002), (Hodgson, 1996), (Vromen, 1995), (Dosi, et al., 1994), (Faber, et al., 1990), (Anderson, et al., 1988), (Hanusch, 1988), (Foster, 1987), (Nelson, et al., 1982), (Boulding, 1978), (Veblen, 1961). (Witt, 2008 p. 555) provides a succinct categorization of the diverse evolutionary ontologies and heuristics.

¹⁸ Logic, however, is not the sole criterion:
'Moreover, Darwin's work was so thoroughly saturated with the rhetoric and imagery of British political economy, Malthus in particular, that any distinction between science and ideology is meaningless.' (Bannister, 1979)

¹⁹ The straightforward adaptation of the Darwinian metaphor is not intended:
'The role of metaphors in science is not well understood. Indeed the role of metaphors is still controversial on its home ground in language. It should be no surprise that when we metaphorically or otherwise extend literary metaphor to scientific practice, matters become quickly obscure. Darwin's notion of blind variation and natural selection has been one of the most tempting of metaphors in the social sciences. Whether it has been a source of fruitful stimulation is debatable.' (Rosenberg, 1994 pp. 407-408), for a more general impact of Darwin's main ideas see (Mayr, 1995)

²⁰ Variety, diversity, heterogeneity has a biological, cultural, economic, and political dimension:

'Darwin, by contrast, introduced an entirely new way of thinking, when he maintained that species are not classes but variable populations composed of uniquely different

individuals. One can almost say that this view is an upside down version of the axioms of essentialism. At this view is an upside down version of the axioms of essentialism. For Darwin the real thing in nature was the uniqueness of the individual, while the mean value of the population was only an abstraction. For the essentialist on the other hand, the idea was the only thing that was real, and variation simply an "error" or "accident." (Mayr, 1991 p. 127)

'Individuals, classes, nations, have been extremely unlike one another: they have struck out a great variety of paths, each leading to something valuable; and although at every period those who travelled in different paths have been intolerant of one another, and each would have thought it an excellent thing if all the rest could have been compelled to travel his road, their attempts to thwart each other's development have rarely had any permanent success, and each has in time endured to receive the good which the others have offered.' (Mill, 1998 / 1859 p. 85)

²¹ 'Since the time when Adam Smith's friend David Hume observed that there was no logical justification for the common belief that much of our empirical knowledge was based on inductive proof (...), methodologists and philosophers have been plagued with what they call the 'Problem of Induction'. The paradigmatic instance of the Problem of Induction is the realization that we cannot provide an inductive proof that 'the sun will rise tomorrow'. (...) Several writers have claimed to have solved this famous problem (...) – which is quite surprising, since it is impossible to solve.' (Boland, 2003 p. 13) 'In Adam Smith's time, inductive generalization was the paradigm of rational thinking; Newton's physics was the paradigm of inductive generalization.' (Boland, 2003 pp. 14, 15)

²² In a certain sense all economic analysis is, of course, partial analysis: 'Economic data are not ultimate data, like the speed of light in physics. Rather they are provisional in nature. This is expressed by means of the *ceteris paribus* clause. All factors not explicitly considered as variables are assumed to be fixed within an argument. This clause is used, explicitly or implicitly, throughout economics. The *ceteris paribus* clause is particularly restrictive in those cases where only a narrowly limited issue is analysed, such as price formation in a single market. Effects on other markets, and possible repercussions, are excluded. Price changes in one market might lead to price changes in other markets, however, and these work back on the market under consideration. The assumption that all other prices are given is certainly wrong here. But the same reservation applies also to more comprehensive theories, since these have to presuppose data, too, and repercussions of the processes under study on the data of the analysis cannot be excluded, irrespective of how we try to delimit the problem! Economics is bound to perform *partial analysis* rather than *total analysis*: It considers phenomena in an economic system which is only a part of the wider complex and interdependent social system, and fixes its demarcations by means of the *ceteris paribus* clause. (Schlicht, 1985 p. 3), original emphases

²³ 'Wer sich mit der Forschung beschäftigt hat, wird schwerlich glauben, daß die Entdeckungen nach dem Aristotelischen oder Bacon'schen Schema der Induktion (...)

zustande kommen. Da wäre ja das Entdecken ein behagliches Handwerk. Die Tatsachen, deren Erkenntnis eine Entdeckung vorstellt, werden vielmehr erschaut.' Mach, quoted in (Schmiechen, 2009 pp. 197-198)

'This indicates that any attempt logically to derive the basic concepts and laws of mechanics from the ultimate data of experience is doomed to failure. If then it is the case that the axiomatic basis of theoretical physics cannot be an inference from experience, but must be free invention, have we any hope that we shall find the correct way?' (Einstein, 1934 pp. 166-167)

²⁴ '(...) that those who are called practical men require specific experience, and argue wholly upwards from particular facts to a general conclusion; while those who are called theorists aim at embracing a wider field of experience, and, having argued upwards from particular facts to a general principle including a much wider range than that of the question under discussion, then argue downwards from that general principle to a variety of specific conclusions.' (Mill, 2004 / 1844 p. 109)

'Since, therefore, it is vain to hope that truth can be arrived at, either in Political Economy or in any other department of the social science, while we look at the facts in the concrete, clothed in all the complexity with which nature has surrounded them, and endeavour to elicit a general law by a process of induction from a comparison of details; there remains no other method than the à priori one, or that of "abstract speculation."' (Mill, 2004 / 1844 pp. 113-114)

²⁵ 'The *Principia* begins with an idealized world, a simple mental construct, a "system" of a single mathematical particle and a centrally directed force in a mathematical space. Under these idealized conditions, Newton freely develops the mathematical consequences of the laws of motion that are the axioms of the *Principia*. At a later stage, after contrasting this ideal world with the world of physics, he will add further conditions to his intellectual construct – for example, by introducing a second body that will interact with the first one and then exploring further mathematical consequences. (...) In this way he can approach by stages nearer and nearer to the condition of the world of experiment and observation, introducing bodies of different shapes and composition and finally bodies moving in variant types of resistant mediums rather than in free space.' (Cohen, 1994 p. 77)

As true epigones the social scientists borrowed a lot from Newton but did not grasp the 'Newtonian style'. By consequence it cannot be said that Newton's method has failed in the social sciences. It has never been applied properly but was soon made redundant by Hamilton's reformulation of rational mechanics (Cohen, 1994 pp. 71-75). The crucial point of the Newtonian style is the undissolvable combination of axiomatics and empiricism. Despite heavy borrowing, these essential points never got across: 'Did anyone ever attempt to found a system of social science or economics on the level of identity with Newtonian rational mechanics or the Newtonian system of the world? In my research I have never found such an example. (...) In the Newtonian system, furthermore, there is no equilibrium, no balancing of contrary forces as in the case of a lever.' (Cohen, 1994 p. 61)