The Value of Commitment in Auctions with Matching

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Abstract

In many auctions, a good match between the bidder and seller raises the value of the contract for both parties although information about the quality of the match may be incomplete. This paper examines the case in which the bidder is better informed about the quality of his match with the seller than the seller is. We derive the optimal mechanism for this setting and investigate whether the seller requires commitment power to implement it. It is shown that once the reserve price is set, it is optimal for the seller to do away with any matching considerations and allocate the contract on the basis of price alone. If matching is sufficiently important to the seller, the optimal mechanism can be implemented without commitment. However, if matching is not sufficiently important, the seller suffers a loss when he is unable to commit. The magnitude of this loss increases as the importance of matching decreases.

Keywords Asymmetries, Auctions, Auction Theory, Bidding, Matching, Mechanism Design, Signaling

JEL Classification C72, C78, D44, D82

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1 Introduction

The real value of a contract lies beyond its financial components: the degree to which the parties are compatible also matters. According to a KPMG study, "83% of all mergers and acquisitions (M&As) failed to produce any benefit for the shareholders and over half actually destroyed value." Interviews of over 100 senior executives revealed that the overwhelming cause of failure "is the people and the cultural differences" (Gitelson et al., 2001).

Compatibility between a buyer and seller matters for more than just mergers and acquisitions. It is an issue in publishing and team sports as well: a good match between an author and his editor may generate a better book, and an athletic team is more likely to win games if the players have compatible skills. Given the impact of matching on contract value, it is not surprising that we observe sellers using matching as a factor in their choice of buyer. During the 2002 auction for the rights to his second novel, Charles Frazier, author of *Cold Mountain*, asserted that "money was not the only consideration and that he was keen to choose the right editor to help him shape the book from the beginning" (Gumbel, 2002). Venezuela's state-owned oil company, Petróleos de Venezuela (PDVSA), accounted for technological compatibility when it selected private partners for the development of marginal fields in the early 1990s (Chalot, 1996).

In this paper, we consider a setting in which a good match raises the value of the contract for both the seller and bidder but information about the quality of the match is incomplete. In particular, we assume each bidder is better informed about the quality of his match with the seller than the seller is. The paper begins by examining whether it is in the seller's interest to use matching as a factor in his allocation decision. It is shown that once the reserve price is set, the seller need not account for matching in his allocation decision; that is, the seller behaves optimally by doing away with any matching considerations and allocating the contract on the basis of price alone.

We then turn our attention to the issue of commitment. A seller is said to have *commitment power* if he can allocate the contract in accordance with the allocation rule announced at the outset of the auction – even when he prefers a different allocation after observing the bids. In contrast, a seller without commitment power cannot refrain from allocating the contract to the bidder whose combination of bid payment and expected match is most attractive. It is shown that if matching is sufficiently important to the seller, the optimal mechanism can be implemented without commitment. However, if matching is not important, the seller suffers a loss when he is unable to commit. The lesser the importance of matching, the greater the magnitude of the loss.

A stylized model is developed in which a single seller seeks to contract with one of several bidders. Each pairing of seller and bidder is characterized by a match. The better the match, the more each party values the contract. Each bidder observes his match with the seller (but not the matches of his opponents with the seller). In contrast, the seller does not observe his matches with the bidders.[1]

[1] The reader will note that the notion of matching advanced here is different from the notion advanced in the two-sided matching literature (e.g., Gale and Shapley, 1962). While the latter use the term to refer to the pairing of agents in a two-sided market, our paper uses the term to refer to the compatibility between a bidder and seller. Since this compatibility induces a positive correlation between the valuations of the bidder and seller, our notion of matching is more closely related to the literatures on affiliated values (e.g., Milgrom and Weber, 1982) and interdependent valuations (e.g., Jehiel and Moldovanu, 2001), but while these literatures are more concerned with linking the bidders' valuations, our paper focuses on linking the valuations of the bidder and seller.

We solve for the optimal mechanism and find that it can be implemented via a standard first-price auction with an appropriate choice of reserve price. Since a better match implies a higher contract value, well matched bidders face a higher opportunity cost of not raising their bids. As a result, bids increase in match. By awarding the contract to the highest bidder, the seller finds himself automatically contracting with the best matched bidder.

To the extent that the seller allocates the contract to the bidder who submits the highest *price* offer, the seller need not account for matching in his allocation decision. However, matching does factor into the reserve price prescribed by the optimal mechanism. As is generally the case, the optimal reserve price is artificially high: allocative efficiency is sacrificed in favor of extracting informational rents. Introducing matching into the seller's utility function affects this trade-off, thereby inducing a corresponding adjustment in the optimal reserve price relative to the standard independent private values framework. Hence, the seller should account for matching in setting the reserve price, but once the reserve price is set, the seller can do away with any matching considerations and allocate the contract on the basis of price alone.

The question naturally arises: can the optimal mechanism be implemented without commitment? In order to answer this question, we examine the equilibria of a *first-score* auction. In a first-score auction, each bidder submits a price offer. After observing these offers, the seller updates his beliefs about the quality of his match with each of the bidders and identifies the bidder whose combination of price and expected match is most attractive. If the seller expects that contracting with that bidder will yield as much utility as not contracting at all, the contract is awarded to that bidder.[2] Since this allocation rule reflects the seller's true preferences, there should be no incentive for the seller to deviate from the rule ex post.

We find that the equilibrium bidding strategies in the first-score auction are identical to those in the first-price auction. In a first-score auction, well matched bidders have an incentive to convey their information to the seller in order to raise their probability of winning. Since well matched bidders have a higher value for the contract, they can credibly signal their favorable matches by raising their bids beyond the point at which it is profitable for poorly matched bidders to mimic them.[3] Given that higher bids signal better matches, the contract goes to the bidder submitting the highest bid – which is precisely the allocation rule in a first-price auction.

We then address the feasibility of adhering to the prescribed reserve price. We find that in order to sustain an elevated reserve price, the seller must (1) associate bids that fall short of the reserve with a poor match and (2) find contracting with a poorly matched bidder to be sufficiently unfavorable. When these two conditions are met, the seller prefers retaining the contract to contracting with a bidder whose bid does not meet the reserve. It follows that if

[2]Che (1993), Branco (1997), Zheng (2000), and Asker and Cantillon (2008) analyze a similar auction format in which the winning bidder is selected on the basis of price and quality. But while the bidders in these papers bid directly on both factors, the bidders in our paper bid only on price, leaving the seller to estimate the quality of the matches on his own. The mechanism in our paper is more closely linked to the biased procurement problem studied by Rezende (forthcoming), in which each bidder submits a price offer and the seller selects the winner on the basis of both price and some pre-existing bias. But while the bias in Rezende's paper is determined by the seller's private information, the bias in our paper is determined by the bidders' private information.

[3]Bikhchandani and Huang (1989), Katzman and Rhodes-Kropf (2002), Das Varma (2003), Goeree (2003), Haile (2003), and Molnár and Virág (2008) examine signaling in auctions, but these papers are concerned with bidders signaling their private information to other bidders so as to affect future strategic interactions. In contrast, the signaling behavior in our paper is motivated by the structure of the auction game itself: bidders are interested in signaling their private information to the seller in order to influence the seller's choice of winner. In this sense, our paper is more similar to Avery (1998), which addresses the use of jump bids to signal a high valuation and encourage competing bidders to withdraw.

matching is sufficiently important to the seller, the optimal mechanism *can* be implemented without commitment.

If matching is not sufficiently important, a lack of commitment causes the seller to suffer a loss. As the importance of matching decreases, the impact of associating low bids with a poor match is diminished. Consequently, the maximum reserve price the seller can credibly adhere to falls. As the gap between the prescribed reserve and the actual reserve widens, the seller's expected utility decreases. Hence, reducing the importance of matching raises the seller's value for commitment.

In sum, we find that once the reserve price has been set, the seller need not account for matching in his allocation decision since the optimal outcome can be achieved via a first-price auction. If matching is sufficiently important to the seller, the optimal mechanism can be implemented without commitment. However, if matching is not sufficiently important, commitment power is needed in order to adhere to the prescribed reserve price. As the importance of matching decreases, the power to commit becomes more valuable.

The remainder of this paper is organized as follows. Section 2 introduces the model. In Section 3, we solve for the optimal mechanism and show that it can be implemented by a first-price auction. In Section 4, we derive a set of conditions under which the optimal outcome can be achieved without commitment. Section 5 characterizes the relationship between the importance of matching and the value of commitment. Concluding remarks are offered in Section 6. All proofs are relegated to the appendices.

2 The Model

A seller offers a contract n risk-neutral bidders ($n \geq 2$). Every potential pairing of seller and bidder has an associated match. We denote the match between the seller and bidder i by $\theta_i \in \left[\underline{\theta}, \overline{\theta}\right] \subset \mathbb{R}$, where $\theta_i > \theta_j$ indicates that bidder i has a better match than bidder j does. We assume the θ_i's are independently and identically distributed according to a commonly known cumulative distribution function (cdf) F with $F\left(\underline{\theta}\right) = 0$ and $F\left(\overline{\theta}\right) = 1$.

Assumption 1 *F has positive density f at every $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$.*

Bidder i's utility from contracting with the seller is

$$\theta_i - b_i,$$

where θ_i is bidder i's value for the contract and $b_i \in \mathbb{R}$ is the bid submitted by bidder i. Bidder i's utility is zero if he does not win the contract.

The seller derives utility from both the bid payment and his match with the winning bidder. We assume the seller's utility from contracting with bidder i is

$$V\left(\theta_i\right) + b_i,$$

where $V\left(\theta_i\right)$ represents the seller's value for his match with bidder i. The following assumption captures the notion that a good match raises the value of the contract for both the seller and the bidder:

Assumption 2 *$V : \left[\underline{\theta}, \overline{\theta}\right] \to \mathbb{R}$ is continuous and strictly increasing over $\left[\underline{\theta}, \overline{\theta}\right]$.*

The seller's utility is zero if he does not contract with any bidder.[4]

The following assumption is imposed so as not to rule out the possibility of a mutually beneficial trade:

Assumption 3 (participation condition) *$V\left(\overline{\theta}\right)+\overline{\theta}$ is positive.*

Additionally, we impose the following regularity condition:

Assumption 4 (regularity condition) *The function*

$$V\left(\theta_i\right)+\theta_i-\frac{1-F\left(\theta_i\right)}{f\left(\theta_i\right)}$$

is strictly increasing over $\left[\underline{\theta},\overline{\theta}\right]$.[5]

We assume bidder i is better informed about his match than the seller is. Bidder i observes his match θ_i but not the matches of his opponents.[6] The seller does not observe the matches directly, and therefore, his beliefs about the matches are determined by the prior, F, and the observed bids.

3 The Optimal Mechanism

Intuition suggests that if matching affects the seller's expected utility, the seller should account for matching in his allocation decision. In fact, we observe this behavior in a number

[4]When $V(\theta_i)=v\theta_i$, where v is a positive constant, our model can be mapped to the interdependent valuations framework outlined in Section 5 of Jehiel and Moldovanu (2001): simply let the agents be indexed by $i\in\{0,1,2,...,n\}$, where agent i is the seller if $i=0$ and bidder i otherwise; let p_k^i be the probability the contract is awarded to agent i in alternative k; and let $s^0=0$, $s^i=\theta_i$, $a_{k0}^i=vp_k^i$, $a_{ki}^i=p_k^i$, and $a_{ki}^j=0$ for all $j\neq i$.

[5]Note that Assumption 4 is less restrictive than the monotone hazard rate condition.

[6]Bidder i's beliefs about θ_j, $j\neq i$, are determined by the prior, F.

of environments. For example, PDVSA accounted for technological complementarities in its selection of partners for the development of marginal oil fields while novelist Charles Frazier accounted for his compatibility with the editor in his choice of a publisher. In this section, we ask whether the seller should account for matching in allocating the contract. Our approach will be to solve for the optimal mechanism and then identify which allocation rules implement it.

By appealing to the revelation principle (Myerson, 1981), we restrict attention to direct revelation mechanisms $\{p(\cdot), t(\cdot)\}$, where $p_i : \left[\underline{\theta}, \overline{\theta}\right]^n \rightarrow [0,1]$ is the probability that bidder i is awarded the contract and $t_i : \left[\underline{\theta}, \overline{\theta}\right]^n \rightarrow \mathbb{R}$ is the expected transfer from bidder i to the seller.

If bidder i observes his type θ_i but announces that his type is x, his expected utility from the mechanism $\{p(\cdot), t(\cdot)\}$ is

$$U_i\left(x, \theta_i\right) \equiv E_{\theta_{-i}}\left[\theta_i p_i\left(x, \theta_{-i}\right) - t_i\left(x, \theta_{-i}\right)\right]. \tag{3.1}$$

Similarly, the seller's expected utility from the mechanism is

$$U_0 \equiv E_\theta\left[\sum_{i=1}^{n} V\left(\theta_i\right) p_i(\theta) + \sum_{i=1}^{n} t_i(\theta)\right]. \tag{3.2}$$

The mechanism is optimal if it maximizes U_0 subject to

Incentive compatibility (IC): $U_i\left(\theta_i, \theta_i\right) \geq U_i\left(x, \theta_i\right) \quad \forall i, \forall \theta_i, \forall x;$

Individual rationality (IR): $U_i\left(\theta_i, \theta_i\right) \geq 0 \quad \forall i, \forall \theta_i;$

and

$p_i(\theta) \geq 0 \quad \text{and} \quad \sum_{i=1}^{n} p_i(\theta) \leq 1 \quad \forall i, \forall \theta.$

Proposition 1 *The optimal mechanism satisfies*

$$p_i(\theta) = \begin{cases} 1 & \text{if } \theta_i \geq \theta_j \ \forall j \ \text{ and } \ \theta_i \geq \theta_* \\ 0 & \text{otherwise} \end{cases}$$

and

$$E_{\theta_{-i}}\left[t_i\left(\theta_i, \theta_{-i}\right)\right] = \begin{cases} \theta_i \, F^{n-1}(\theta_i) - \int_{\theta_*}^{\theta_i} F^{n-1}(x)\,\mathrm{d}x & \text{if } \theta_i \geq \theta_* \\ 0 & \text{otherwise} \end{cases}$$

where

$$\theta_* = \begin{cases} \left\{ x \in (\underline{\theta}, \overline{\theta}\,) : V(x) + x = \dfrac{1 - F(x)}{f(x)} \right\} & \text{if } V(\underline{\theta}) + \underline{\theta} < \dfrac{1}{f(\underline{\theta})} \\ \underline{\theta} & \text{otherwise} \end{cases}$$

Proof See Appendix A.

The proof follows Myerson (1981). It develops a relaxed optimization program by reducing the number of choice variables from two to one. The unique solution of the relaxed program is identified and shown to satisfy the constraints of the original program. The regularity condition plays a key role.

As in Myerson, the optimal mechanism can be implemented via a standard first-price sealed-bid auction with an appropriate choice of reserve price. Since a bidder with a higher match has a higher value for the contract, well matched bidders can afford to bid more than poorly matched bidders. Hence, a rule that allocates the contract to the highest bidder also allocates the contract to the bidder with the best match.

To the extent that the seller allocates the contract to the bidder who submits the highest *price* offer, matching does not figure into the allocation decision. Note that this result holds no matter how important matching may be to the seller.

However, matching does factor into the reserve price prescribed by the optimal mechanism, and this is where our result differs from Myerson's. There are two reasons why introducing matching into the seller's utility function affects the optimal reserve price. First, matching may affect the minimum type for which a mutually beneficial trade exists. In Myerson's framework, that type is simply $\theta = 0$, but in our framework, that type is given by $V(\theta) + \theta = 0$. If $V(0) > 0$, the minimum type is lower in our framework, and hence, the optimal reserve price is lower as well. The converse is true if $V(0) < 0$.

Second, matching may affect the trade-off between allocative efficiency and extraction of informational rents. Suppose the minimum type for which there exists a mutually beneficial trade is the same in our framework as it is in Myerson's framework; that is, suppose $V(0) = 0$. Since V is strictly increasing, $V(\theta) + \theta > \theta$ for all $\theta > 0$. In other words, the surplus lost by failing to contract when a mutually beneficial trade exists is greater in our framework than in Myerson's. It follows that the optimal reserve price is lower in our framework. Moreover, the more the seller cares about matching (i.e., the higher V'), the greater the surplus lost and the lower the optimal reserve price.

In sum, the seller should account for matching in allocating the contract but only with respect to the choice of reserve price. Once the reserve price is set, the seller can do away with any matching considerations and allocate the contract on the basis of price alone.

4 Implementation under No Commitment

We now turn our attention to the case in which the seller lacks commitment power. Our objective is to determine whether the optimal mechanism can be implemented under such conditions. In the previous section, we showed that a first-price auction with an elevated reserve price is optimal. However, there are several reasons to believe that commitment is needed to carry out this auction. First, it is not apparent that the elevated reserve can be sustained: the seller may prefer contracting with a well-matched bidder whose offer falls short of the reserve to not contracting at all.

The second reason is more subtle. In the previous section, we saw that awarding the contract to the highest bidder automatically delivers the bidder with the best match. Consequently, the seller has no incentive to contract with anyone other than the highest bidder. However, this result is predicated on the bidders believing the seller is committed to carrying out the rules of the first-price auction. If the seller's inability to commit is common knowledge, the bidders may alter their bidding strategies accordingly. Once this adjustment takes place, the highest bidder may no longer be the one with the best match. In this case, the seller may find that he prefers to contract with another bidder whose bid is lower but expected match is higher. In other words, the seller may find himself reneging on the rules of the first-price auction.

The important point is that if it is known that the seller lacks commitment power, bidders ignore the announced allocation rule and assume the seller will award the contract to the bidder who offers the most attractive combination of price and expected match, provided that offer exceeds the seller's reservation utility. That is, bidders select their strategies as if they are playing the following game:

1. Each bidder submits a price offer independently and simultaneously.

2. The seller contracts with the bidder whose combination of price and expected match maximizes the seller's expected utility provided that the combined value is not less than the seller's reservation utility of zero. That is, bidder i wins the contract if

$$E\left[V(\theta_i) \mid b_i\right] + b_i \geq 0$$

and

$$E\left[V(\theta_i) \mid b_i\right] + b_i > E\left[V(\theta_j) \mid b_j\right] + b_j \quad \forall j \neq i,$$

where b_i denotes the price offer (bid) submitted by bidder i. Ties are resolved by a random draw with equal probability.

3. If the contract is allocated to bidder i, θ_i is revealed. The seller's utility is $V(\theta_i) + b_i$, bidder i's utility is $\theta_i - b_i$, and all other bidders get zero utility. If the contract is not allocated, every agent gets zero utility.

We call this game a *first-score* auction, where the term "score" refers to the combination of price and expected match. For instance, bidder i's score is given by $E\left[V(\theta_i) \mid b_i\right] + b_i$. As indicated in the timeline above, the contract is allocated to the bidder with the highest score provided that the score is nonnegative. The winning bidder pays the price he offered b_i, thereby delivering his true score $V(\theta_i) + b_i$.

A first-score auction is similar to a first-price auction in that bidders make price offers and the winning bidder pays his bid. However, while the winner in a first-price auction is the bidder who submits the highest price offer, the winner in a first-score auction is the bidder who submits the most attractive combination of price and expected match. This allocation rule allows the seller to reject an offer made by the highest bidder and allocate the contract to another bidder whose pairing of bid and expected match is more attractive than the pairing offered by the highest bidder. Moreover, the first-score auction does not feature an elevated reserve price: the seller retains the contract only when every offer falls short of the seller's reservation utility. Since the allocation rule of the first-score auction reflects the seller's true preferences, there is no incentive for the seller to deviate from it after observing the bids.

Our use of a first-score auction to represent the seller's lack of commitment is consistent with the literature. Che (1993) states that in the absence of commitment "the only feasible scoring rule is one that reflects the seller's [true] preference ordering," and Rezende (forthcoming) allows a seller without commitment power to renege on the announced allocation rule and select the auction winner arbitrarily. Our representation is also consistent with the principal-agent model in Bester and Strausz (2000): they define imperfect commitment in terms of a two-stage game, in which agents select messages in the first stage and the principal updates his beliefs and selects an allocation in the second stage.

Our approach will be to investigate the perfect Bayesian equilibria of the first-score auction to see whether the optimal mechanism outlined in Proposition 1 can be implemented in the absence of commitment. Recall that the optimal mechanism allocates the contract to the bidder with the best match. Hence, if the first-score auction implements the optimal mechanism, the equilibrium bidding strategies must be such that the seller can distinguish between bidders of different types. For this reason, we elect to focus on separating equilibria.

The following lemma characterizes the symmetric separating equilibria of the first-score auction game.

Lemma 1 *In any symmetric separating equilibrium of the first-score auction game, there exists a* $t_* \in [\,\underline{\theta}, \overline{\theta}\,]$ *such that*

(1) *any bidder with type* $\theta \in (t_*, \overline{\theta}\,]$ *bids according to the function*

$$b(\theta) \;=\; \theta - \frac{\int_{t_*}^{\theta} F^{n-1}(x)\,\mathrm{d}x}{F^{n-1}(\theta)}$$

and wins with positive probability.

(2) *any bidder with type* $\theta \in [\,\underline{\theta}, t_*)$ *wins with zero probability.*

Proof See Lamping (2008).

Lemma 1 indicates that equilibrium bids increase with the quality of the match over the range $[t_*, \overline{\theta}\,]$. Since a better match implies a higher value for the contract, well matched bidders can credibly signal their favored status by offering higher bids. The contract is thus awarded to the bidder with the highest type – as prescribed by the optimal mechanism.

Moreover, the bidding function specified by Lemma 1 is identical to the equilibrium bidding function in a first-price auction with a reserve price of t_*. The equivalency follows from the fact that when higher bids signal higher types, the allocation rule for the first-score auction coincides with the allocation rule for the first-price auction. Since the optimal mechanism can be implemented by a first-price auction, it appears that the optimal mechanism may be implementable by a first-score auction as well.

The issue is that the absence of commitment power may affect the feasibility of adhering to the threshold prescribed by the optimal mechanism:

$$\theta_* = \begin{cases} \left\{ x \in (\underline{\theta}, \overline{\theta}\,) : V(x) + x = \dfrac{1 - F(x)}{f(x)} \right\} & \text{if } V(\underline{\theta}) + \underline{\theta} < \dfrac{1}{f(\underline{\theta})} \\ \underline{\theta} & \text{otherwise} \end{cases} \tag{4.1}$$

Suppose $V(\underline{\theta}) + \underline{\theta} < \dfrac{1}{f(\underline{\theta})}$. It follows from Assumptions 3 and 4 that $V(\theta_*) + \theta_* > 0$. Hence, we encounter the usual problem associated with an elevated threshold: if a bidder with type $\theta_* - \epsilon$ has the highest type and bids his valuation, the seller will prefer to award the contract and earn $V(\theta_* - \epsilon) + \theta_* - \epsilon$ than to retain the contract and earn zero.

Note that the preceding argument hinges on the seller knowing the bidder's type. Suppose, once again, that a bidder with type $\theta_* - \epsilon$ has the highest type and bids his valuation but imagine that the seller believes (incorrectly) that this bidder has type $\underline{\theta}$. In this case, the seller expects that contracting with this bidder will yield $V(\underline{\theta}) + \theta_* - \epsilon$. If $V(\underline{\theta}) \leq -\theta_*$, the seller will prefer to retain the contract and earn zero than to award the contract and earn $V(\underline{\theta}) + \theta_* - \epsilon < 0$. This suggests that an elevated threshold *can* be sustained if the seller (1) associates bids that fall short of the threshold with a poor match and (2) finds contracting with a poorly matched bidder to be sufficiently unfavorable. We formalize this notion in the following proposition.

Proposition 2 *Let θ_* be defined as in Proposition 1.*

(1) *If either $V(\underline{\theta}) + \underline{\theta} \geq \dfrac{1}{f(\underline{\theta})}$ or $V(\underline{\theta}) + \theta_* \leq 0$, there is a symmetric separating equilibrium of the first-score auction game that implements the optimal mechanism.*

(2) *If* $V(\underline{\theta}) + \underline{\theta} < \frac{1}{f(\underline{\theta})}$ *and* $V(\underline{\theta}) + \theta_* > 0$*, there is no symmetric separating equilibrium of the first-score auction game that implements the optimal mechanism. The symmetric separating equilibrium that maximizes the seller's expected utility has* $t_* = \max\{\underline{\theta}, -V(\underline{\theta})\}$.

Proof See Appendix B.

If $V(\underline{\theta})+\underline{\theta} \geq \frac{1}{f(\underline{\theta})}$, the surplus generated by contracting with the lowest type $\underline{\theta}$ is so large that it is optimal for the seller to allow every bidder to participate. Since there is no elevated threshold to be sustained, the optimal mechanism can be implemented without commitment. If $V(\underline{\theta}) + \underline{\theta} < \frac{1}{f(\underline{\theta})}$, the optimal mechanism does prescribe an elevated threshold. Suppose $V(\underline{\theta}) + \theta_* \leq 0$ and note that this condition can be rewritten as

$$V(\theta_*) - V(\underline{\theta}) \geq \frac{1 - F(\theta_*)}{f(\theta_*)}. \tag{4.2}$$

Proposition 2 establishes that if matching is sufficiently important to the seller (i.e., if the range of V is sufficiently large), the impact of punishing off-equilibrium-path beliefs is substantial enough to sustain the prescribed threshold. Hence, the optimal mechanism can be implemented without commitment.

However, if neither $V(\underline{\theta})+\underline{\theta} \geq \frac{1}{f(\underline{\theta})}$ nor $V(\underline{\theta})+\theta_* \leq 0$, the optimal mechanism cannot be implemented by a first-score auction. In this case, the prescribed threshold is elevated, but matching is not important enough for even the most punishing off-equilibrium-path beliefs to sustain it. The highest threshold that can be sustained is $t_* = \max\{\underline{\theta}, -V(\underline{\theta})\}$.

In sum, if matching is sufficiently important to the seller, the optimal mechanism can be implemented without commitment.[7] This may explain why we observe agents, such as PDVSA and Charles Frazier, who are able to commit to a different allocation mechanism, simply implementing a first-score auction. If matching is not sufficiently important, the seller may be unable to implement the optimal mechanism without commitment.

5 The Value of Commitment

In this section, we seek to more explicitly characterize the relationship between the importance of matching and the value of commitment. In order to do so, we introduce some additional notation. Let $V^A : [\underline{\theta}, \overline{\theta}] \to \mathbb{R}$ and $V^B : [\underline{\theta}, \overline{\theta}] \to \mathbb{R}$ be two functions satisfying Assumptions 2, 3, and 4. Let θ_*^A and θ_*^B be the optimal thresholds associated with V^A and V^B respectively. In addition, let

$$t_*^k = \begin{cases} \theta_*^k & \text{if } V^k(\underline{\theta}) + \underline{\theta} \geq \dfrac{1}{f(\underline{\theta})} \text{ or } V^k(\underline{\theta}) + \theta_* \leq 0 \\ \max\left\{\underline{\theta}\,, -V^k(\underline{\theta})\right\} & \text{otherwise} \end{cases} \tag{5.1}$$

and

$$\underline{t}^k = \begin{cases} \left\{x \in (\underline{\theta}, \overline{\theta}\,) : V^k(x) + x = 0\right\} & \text{if } V^k(\underline{\theta}) + \underline{\theta} < 0 \\ \underline{\theta} & \text{otherwise} \end{cases} \tag{5.2}$$

[7]This result is predicated on the game having a single period. In an intertemporal model, the Coase conjecture would apply (Coase, 1972). If every bid were to fall short of the reserve price, the seller would retain the contract and update his beliefs about the distribution of types. In the following period, he would offer the contract again but set a lower reserve price. If the bidders were sufficiently patient, they would simply wait for the reserve to fall to a suitable level. Thus, in an intertemporal model, the seller does suffer a loss due to his inability to commit. Note that the issue is the reserve price and not the scoring system. Even in an intertemporal model, a higher bid signals a better match. Therefore, there is no loss associated with selecting the winner on the basis of both price and match instead of price alone. For a more formal treatment of the Coase conjecture, see Stokey (1981), Bulow (1982), Gul, Sonnenschein, and Wilson (1986), and Hart and Tirole (1988). McAfee and Vincent (1997), Skreta (2007), and Vartiainen (2007) specifically address the application of the Coase conjecture to auctions.

where $k \in \{A, B\}$. Note that t_*^k represents the optimal *sustainable* threshold and that $\underline{t}^k$ represents the lowest type for which there exists a mutually beneficial trade. A straightforward application of the methodology in Riley and Samuelson (1981) yields the following expression for the seller's expected utility when the lowest participating type is t_*^k:

$$n \int_{t_*^k}^{\overline{\theta}} \left[V^k(x) + x - \frac{1 - F(x)}{f(x)} \right] F^{n-1}(x)\, f(x)\, \mathrm{d}x\,, \quad k \in \{A, B\}. \tag{5.3}$$

The loss associated with the seller's lack of commitment power can then be written as

$$C^k \equiv -n \int_{t_*^k}^{\theta_*^k} \left[V^k(x) + x - \frac{1 - F(x)}{f(x)} \right] F^{n-1}(x)\, f(x)\, \mathrm{d}x\,, \quad k \in \{A, B\}. \tag{5.4}$$

We are now ready to specify the relationship between the importance of matching and the value of commitment.

Proposition 3 *Suppose V^A and V^B satisfy the following three conditions:*

(1) $V^A(\underline{t}^A) = V^B(\underline{t}^B)$,

(2) $\dfrac{\mathrm{d}V^A(\theta)}{\mathrm{d}\theta} < \dfrac{\mathrm{d}V^B(\theta)}{\mathrm{d}\theta}$ *for all* $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, *and*

(3) $V^A(\underline{\theta}) + \underline{\theta} < \dfrac{1}{f(\underline{\theta})}$ *and* $V^A(\underline{\theta}) + \theta_*^A > 0$.

Then $C^A > C^B$.

Proof See Appendix C.

Proposition 3 establishes that if we (1) hold fixed the range of types for which there exists a mutually beneficial trade and (2) increase the importance of matching, the seller's value for commitment decreases. Increasing the importance of matching has several effects. First, it magnifies the impact of associating low bids with a poor match. As a result, the optimal sustainable threshold increases ($t_*^A < t_*^B$). Second, it magnifies the loss associated with retaining the contract when a mutually beneficial trade exists. This reduces the threshold prescribed by the optimal mechanism ($\theta_*^A > \theta_*^B$). Finally, it raises the seller's value for any particular match provided that a mutually beneficial trade with that match exists ($V^A(\theta) < V^B(\theta)$ for all $\theta > \underline{t}$). Every one of these three effects serves to reduce the loss associated with the seller's lack of commitment power.

6 Concluding Remarks

For a wide range of commercial arrangements, a good match between the buyer and seller raises the value of the contract for both parties. However, at the time the terms of the contract are set, the parties may not be fully informed about the degree to which they match. In this paper, we have addressed the case in which the quality of the match is the private information of the bidder.

The paper opened by asking whether the seller should account for matching in his allocation decision. It is shown that once the reserve price is set, the seller can do away with any matching considerations and allocate the contract on the basis of price alone. Since the bidder's value for the contract increases with the quality of his match with the seller, allocating the contract to the highest bidder is equivalent to selecting the bidder with the best match.

We then ask whether commitment is required to implement the optimal mechanism. We argue that a seller who lacks commitment power simply awards the contract to the bidder who offers the most attractive combination of price and expected match. Since well matched bidders have a higher value for the contract, higher bids signal higher matches. As a result, the seller has no incentive to contract with anyone other than the highest bidder.

The issue is that commitment may be necessary to sustain the elevated threshold prescribed by the optimal mechanism. We show that if matching is sufficiently important to the seller, the prescribed threshold can be sustained by associating bids that fall below the threshold with a poor match. However, if matching is not sufficiently important, the seller cannot adhere to the prescribed threshold and suffers a loss. As the importance of matching decreases, the magnitude of the loss increases, and commitment becomes more valuable.

We conclude with a few remarks on potential avenues for further research. Given the attention paid to commitment, one logical extension would be to allow for multiple periods. Clearly, our results would be unaffected for the case in which the seller can commit. However, our results for the case in which the seller lacks commitment power would change. In Section 4, we found that the optimal mechanism could be implemented without commitment if (1) the seller associated bids that fell short of the prescribed threshold with a poor match and (2) matching was sufficiently important to the seller. However, with multiple periods, the seller has an opportunity to update his beliefs about the matches over time. In the absence of commitment, the Coase conjecture applies making it impossible to sustain the prescribed threshold in the long run. Hence, if bidders are sufficiently patient, the optimal mechanism cannot be implemented.[8]

[8]For a more detailed discussion of this issue, see Footnote 7.

Another desirable extension would be to allow both the bidder and seller to observe a signal about the quality of their match. In the commitment case, we would have an informed principal problem with common values along the lines of Maskin and Tirole (1992) but with multiple heterogeneous agents. Solving the mechanism design problem would be particularly challenging because one would have to contend with both an informed principal problem and an asymmetric auctions problem. For the non-commitment case, this paper provides a starting point in that it offers a characterization of non-commitment (the first-score auction) and shows that in this setting, bidders have an incentive to reveal favorable information via higher bids. Developing these ideas further would be an interesting area for future research.

A Proof of Proposition 1

The proof follows Myerson (1981). We first show that if the mechanism $\{p(\cdot), t(\cdot)\}$ is optimal, then $U_i(\underline{\theta}, \underline{\theta}) = 0$. We then develop a relaxed optimization program and find its solution. After noting that the solution of the relaxed program coincides with the mechanism outlined in Proposition 1, we demonstrate that the mechanism satisfies the constraints of the original program.

Suppose the mechanism $\{p(\cdot), t(\cdot)\}$ satisfies the IC and IR constraints for all $\theta_i \in [\underline{\theta}, \overline{\theta}]$ but that $U_i(\underline{\theta}, \underline{\theta}) = \epsilon > 0$. Now consider a different mechanism $\{p(\cdot), \hat{t}(\cdot)\}$, where $\hat{t}_i(\cdot) \equiv t_i(\cdot) + \epsilon$. The mechanism $\{p(\cdot), \hat{t}(\cdot)\}$ satisfies the IR constraint for all $\theta_i \in [\underline{\theta}, \overline{\theta}]$ since

$$\begin{aligned} \widehat{U}_i(\theta_i, \theta_i) &= E_{\theta_{-i}}\left[\theta_i p_i(\theta_i, \theta_{-i}) - \hat{t}_i(\theta_i, \theta_{-i})\right] \\ &= E_{\theta_{-i}}\left[\theta_i p_i(\theta_i, \theta_{-i}) - t_i(\theta_i, \theta_{-i})\right] - \epsilon \\ &\geq E_{\theta_{-i}}\left[\theta_i p_i(\underline{\theta}, \theta_{-i}) - t_i(\underline{\theta}, \theta_{-i})\right] - \epsilon \\ &\geq E_{\theta_{-i}}\left[\underline{\theta} p_i(\underline{\theta}, \theta_{-i}) - t_i(\underline{\theta}, \theta_{-i})\right] - \epsilon \\ &= 0. \end{aligned} \tag{A.1}$$

The first inequality holds because $\{p(\cdot), t(\cdot)\}$ satisfies the IC constraint; the second inequality holds because $p_i \in [0, 1]$. The mechanism $\{p(\cdot), \hat{t}(\cdot)\}$ also satisfies the IC constraint for all $\theta_i \in [\underline{\theta}, \overline{\theta}]$ since

$$\begin{aligned} \widehat{U}_i(\theta_i, \theta_i) &= E_{\theta_{-i}}\left[\theta_i p_i(\theta_i, \theta_{-i}) - t_i(\theta_i, \theta_{-i})\right] - \epsilon \\ &\geq E_{\theta_{-i}}\left[\theta_i p_i(x, \theta_{-i}) - t_i(x, \theta_{-i})\right] - \epsilon \\ &= \hat{U}_i(x, \theta_i) \end{aligned} \tag{A.2}$$

for all $x \in [\underline{\theta}, \overline{\theta}]$. The inequality follows from the fact that $\{p(\cdot), t(\cdot)\}$ satisfies the IC constraint. Since $\epsilon > 0$ and $\hat{t}_i(\cdot) \equiv t_i(\cdot) + \epsilon$, the seller's expected utility is strictly greater under $\{p(\cdot), \hat{t}(\cdot)\}$ than it is under $\{p(\cdot), t(\cdot)\}$. Therefore, the original mechanism $\{p(\cdot), t(\cdot)\}$ cannot be optimal – a contradiction.

Having established that $U_i(\underline{\theta}, \underline{\theta}) = 0$, we turn our attention to developing a relaxed optimization program. Our approach is to modify the original program so as to reduce the number of choice variables from two, $p(\cdot)$ and $t(\cdot)$, to one, $p(\cdot)$. From equation (3.1), we have

$$E_{\theta_{-i}}\left[t_i(\theta_i, \theta_{-i})\right] = E_{\theta_{-i}}\left[\theta_i p_i(\theta_i, \theta_{-i})\right] - U_i(\theta_i, \theta_i), \tag{A.3}$$

and after substituting for $E_{\theta_{-i}}\left[t_i\left(\theta_i,\theta_{-i}\right)\right]$ in equation (3.2), we obtain

$$U_0 = E_\theta\left[\sum_{i=1}^{n}\left(V\left(\theta_i\right)+\theta_i\right)p_i(\theta)\right] - \sum_{i=1}^{n}E_{\theta_i}\left[U_i\left(\theta_i,\theta_i\right)\right]. \tag{A.4}$$

It remains to develop an expression for $E_{\theta_i}\left[U_i\left(\theta_i,\theta_i\right)\right]$ in terms of $p_i(\theta)$.

Incentive compatibility requires that $U_i\left(\theta_i,\theta_i\right) \geq U_i\left(x,\theta_i\right)$ for all $\theta_i \in \left[\underline{\theta},\overline{\theta}\right]$ and all $x \in \left[\underline{\theta},\overline{\theta}\right]$. Subtracting $U_i(x,x)$ from both sides and applying equation (3.1) yields

$$U_i\left(\theta_i,\theta_i\right) - U_i\left(x,x\right) \geq \left(\theta_i - x\right)E_{\theta_{-i}}\left[p_i\left(x,\theta_{-i}\right)\right]. \tag{A.5}$$

Since this inequality holds for all $\theta_i \in \left[\underline{\theta},\overline{\theta}\right]$ and all $x \in \left[\underline{\theta},\overline{\theta}\right]$, it must be the case that

$$\left(\theta_i - x\right)R_i(x) \leq \widetilde{U}_i\left(\theta_i\right) - \widetilde{U}_i\left(x\right) \leq \left(\theta_i - x\right)R_i(\theta_i), \tag{A.6}$$

where $R_i(x) \equiv E_{\theta_{-i}}\left[p_i\left(x,\theta_{-i}\right)\right]$ and $\widetilde{U}_i(x) \equiv U_i\left(x,x\right)$.

Suppose $\theta_i > x$ and let $\delta \equiv \theta_i - x$. Then inequality (A.6) can be rewritten as follows:

$$R_i(x) \leq \frac{\widetilde{U}_i\left(x+\delta\right) - \widetilde{U}_i\left(x\right)}{\delta} \leq R_i(x+\delta). \tag{A.7}$$

Since $R_i(\cdot)$ is nondecreasing, it is Riemann integrable. Hence,

$$\int_{\underline{\theta}}^{\theta_i} R_i\left(x\right)\,\mathrm{d}x = \widetilde{U}_i\left(\theta_i\right) - \widetilde{U}_i\left(\underline{\theta}\right). \tag{A.8}$$

After substituting for $R_i(x)$, $\widetilde{U}_i\left(\theta_i\right)$, and $\widetilde{U}_i\left(\underline{\theta}\right)$ and applying our earlier result, $U_i(\underline{\theta},\underline{\theta}) = 0$, we obtain

$$U_i\left(\theta_i,\theta_i\right) = \int_{\underline{\theta}}^{\theta_i} E_{\theta_{-i}}\left[p_i\left(x,\theta_{-i}\right)\right]\,\mathrm{d}x. \tag{A.9}$$

Taking the expectation of $U_i\left(\theta_i,\theta_i\right)$ and integrating by parts yields the expression we seek:

$$E_{\theta_i}\left[U_i\left(\theta_i,\theta_i\right)\right] = E_\theta\left[\frac{1-F\left(\theta_i\right)}{f\left(\theta_i\right)}p_i(\theta)\right]. \tag{A.10}$$

Using equation (A.10), we substitute for $E_{\theta_i}\left[U_i\left(\theta_i,\theta_i\right)\right]$ in equation (A.4) and obtain an expression for U_0 in terms of $p(\cdot)$. The resulting relaxed program is as follows:

$$\max_{\{p(\cdot)\}} E_\theta\left[\sum_{i=1}^{n}\left(V\left(\theta_i\right)+\theta_i-\frac{1-F\left(\theta_i\right)}{f\left(\theta_i\right)}\right)p_i(\theta)\right]$$

subject to

$$p_i(\theta)\geq 0 \quad \text{and} \quad \sum_{i=1}^{n} p_i(\theta)\leq 1 \quad \forall i,\forall\theta.$$

Since $V\left(\theta_i\right)+\theta_i-\dfrac{1-F\left(\theta_i\right)}{f\left(\theta_i\right)}$ is strictly increasing by Assumption 4, the unique solution of the relaxed program is

$$p_i(\theta)=\begin{cases}1 & \text{if } \theta_i\geq\theta_j \ \forall j \ \text{ and } \ V(\theta_i)+\theta_i-\dfrac{1-F(\theta_i)}{f(\theta_i)} \ \geq \ 0\\ 0 & \text{otherwise}\end{cases}$$

Note that the expression above coincides with the $p_i(\theta)$ outlined in Proposition 1. We proceed by deriving the associated $t(\cdot)$ and verifying that it too coincides with Proposition 1. From equation (A.3) we have

$$E_{\theta_{-i}}\left[t_i\left(\theta_i,\theta_{-i}\right)\right]=\theta_i E_{\theta_{-i}}\left[p_i\left(\theta_i,\theta_{-i}\right)\right]-U_i\left(\theta_i,\theta_i\right). \tag{A.11}$$

After substituting for $U_i\left(\theta_i,\theta_i\right)$ using equation (A.9) and replacing $p_i\left(\theta_i,\theta_{-i}\right)$ with the solution of the relaxed program, we obtain

$$E_{\theta_{-i}}\left[t_i\left(\theta_i,\theta_{-i}\right)\right]=\begin{cases}\theta_i F^{n-1}\left(\theta_i\right)-\int_{\theta_*}^{\theta_i}F^{n-1}(x)\,\mathrm{d}x & \text{if } V\left(\theta_i\right)+\theta_i\geq\dfrac{1-F\left(\theta_i\right)}{f\left(\theta_i\right)}\\ 0 & \text{otherwise}\end{cases}$$

which clearly coincides with the corresponding expression in Proposition 1.

It remains to show that the mechanism $\{p(\cdot), t(\cdot)\}$ satisfies the constraints of the original program. The IR constraint, $p_i(\theta) \geq 0$, and $\sum_{i=1}^{n} p_i(\theta) \leq 1$ are satisfied trivially. To verify that the IC constraint holds, we must show that

$$U_i\left(\theta_i, \theta_i\right) \geq U_i\left(x, \theta_i\right) \tag{A.12}$$

for all $\theta_i \in \left[\underline{\theta}, \overline{\theta}\right]$ and all $x \in \left[\underline{\theta}, \overline{\theta}\right]$. Subtracting $U_i(x, x)$ from both sides and applying equation (3.1) yields

$$U_i\left(\theta_i, \theta_i\right) - U_i\left(x, x\right) \geq \left(\theta_i - x\right) E_{\theta_{-i}}\left[p_i\left(x, \theta_{-i}\right)\right]. \tag{A.13}$$

If θ_i and x are less than θ_*, the inequality is trivially satisfied. If $\theta_i < \theta_*$ and $x \geq \theta_*$, the left-hand side is

$$\begin{aligned} U_i\left(\theta_i, \theta_i\right) - U_i\left(x, x\right) &= -\int_{\theta_*}^{x} F^{n-1}(y)\,\mathrm{d}y \\ &\geq -\left(x - \theta_*\right) F^{n-1}(x) \\ &\geq -\left(x - \theta_i\right) F^{n-1}(x) \\ &= \left(\theta_i - x\right) E_{\theta_{-i}}\left[p_i\left(x, \theta_{-i}\right)\right] \end{aligned} \tag{A.14}$$

and the inequality is again satisfied. If $\theta_i \geq \theta_*$ and $x < \theta_*$, the inequality reduces to $\int_{\theta_*}^{\theta_i} F^{n-1}(y)\,\mathrm{d}y \geq 0$, which is trivially satisfied. If $\theta_i \geq x \geq \theta_*$,

$$\begin{aligned} U_i\left(\theta_i, \theta_i\right) - U_i(x, x) &= \int_{x}^{\theta_i} F^{n-1}(y)\,\mathrm{d}y \\ &\geq \left(\theta_i - x\right) F^{n-1}(x) \\ &= \left(\theta_i - x\right) E_{\theta_{-i}}\left[p_i\left(x, \theta_{-i}\right)\right] \end{aligned} \tag{A.15}$$

An analogous argument can be used to verify that the inequality holds if $x \geq \theta_i \geq \theta_*$. ■

B Proof of Proposition 2

The proof exploits the following result from Lamping (2008):

Lemma 2 *A symmetric separating equilibrium exists for every* $t_* \in [\underline{t}, \overline{t}\,]$, *where*

$$\underline{t} = \begin{cases} \{x \in (\underline{\theta}, \overline{\theta}\,) : V(x) + x = 0\} & \text{if } V(\underline{\theta}) + \underline{\theta} < 0 \\ \underline{\theta} & \text{otherwise} \end{cases}$$

and

$$\overline{t} = \min\left\{\underline{t} + [V(\underline{t}) - V(\underline{\theta})]\,, \overline{\theta}\,\right\}.$$

Moreover, there are no symmetric separating equilibria such that $t_* \notin [\underline{t}, \overline{t}\,]$.

To verify part (1) of Proposition 2, we will show that $\theta_* \in [\underline{t}, \overline{t}\,]$ whenever $V(\underline{\theta}) + \underline{\theta} \geq \dfrac{1}{f(\underline{\theta})}$ or $V(\underline{\theta}) + \theta_* \leq 0$. Suppose first that $V(\underline{\theta}) + \underline{\theta} \geq \dfrac{1}{f(\underline{\theta})}$. By Assumption 1, $\dfrac{1}{f(\underline{\theta})} > 0$. It follows that $\underline{t} = \overline{t} = \underline{\theta}$ and that $\theta_* = \underline{\theta}$. Clearly, $\theta_* \in [\underline{t}, \overline{t}\,]$. Now suppose that $V(\underline{\theta}) + \theta_* \leq 0$. $V(\underline{\theta}) + \theta_* \leq 0$ implies $V(\underline{\theta}) + \underline{\theta} \leq 0$. It follows that $\underline{t}$ satisfies $V(\underline{t}) + \underline{t} = 0$. Similarly, $V(\underline{\theta}) + \underline{\theta} \leq 0$ implies $V(\underline{\theta}) + \underline{\theta} < \dfrac{1}{f(\underline{\theta})}$. It follows that θ_* satisfies

$$V(\theta_*) + \theta_* = \frac{1 - F(\theta_*)}{f(\theta_*)}\,. \tag{B.1}$$

By Assumption 1, $\dfrac{1 - F(\theta_*)}{f(\theta_*)} > 0$. Since $V(\underline{t}) + \underline{t} = 0$, $V(\theta_*) + \theta_* > 0$, and $V(x) + x$ is increasing in x (by Assumption 2), it must be the case that $\theta_* > \underline{t}$. In addition, $V(\underline{t}) + \underline{t} = 0$ implies that $\overline{t} = \min\{-V(\underline{\theta}), \overline{\theta}\,\}$. If $\overline{t} = \overline{\theta}$, θ_* must be less than or equal to $\overline{t}$. If $\overline{t} = -V(\underline{\theta})$, it is still the case that $\theta_* \leq \overline{t}$ since $V(\underline{\theta}) + \theta_* \leq 0$. Having shown that $\theta_* > \underline{t}$ and $\theta_* \leq \overline{t}$, we assert that $\theta_* \in [\underline{t}, \overline{t}\,]$.

We will now verify part (2) of Proposition 2. We begin by showing that $\theta_* > \overline{t}$ whenever $V(\underline{\theta}) + \underline{\theta} < \dfrac{1}{f(\underline{\theta})}$ and $V(\underline{\theta}) + \theta_* > 0$. Suppose first that $V(\underline{\theta}) + \underline{\theta} \leq 0$ so that $\underline{t}$ satisfies $V(\underline{t}) + \underline{t} = 0$. In this case, $\overline{t} = \min\{-V(\underline{\theta}), \overline{\theta}\,\}$. Since $V(\underline{\theta}) + \theta_* > 0$, it must be the case that $V(\underline{\theta}) + \overline{\theta} > 0$. It follows that $\overline{t} = -V(\underline{\theta})$. Since $V(\underline{\theta}) + \theta_* > 0$, $\theta_* > \overline{t}$. Now suppose that $V(\underline{\theta}) + \underline{\theta} > 0$ so that $\underline{t} = \underline{\theta}$. In this case, $\overline{t} = \underline{\theta}$. Since $V(\underline{\theta}) + \underline{\theta} < \dfrac{1}{f(\underline{\theta})}$, $\theta_* > \underline{\theta} = \overline{t}$.

Having shown that $\theta_* > \overline{t}$, we will now show that the seller's expected utility increases in t_* over the range $[\underline{t}, \overline{t}]$. By applying the methodology in Riley and Samuelson (1981) to Lemma 1, we obtain the following expression for the seller's expected utility:

$$n \int_{t_*}^{\overline{\theta}} \left[V(x) + x - \frac{1 - F(x)}{f(x)} \right] F^{n-1}(x)\, f(x)\, \mathrm{d}x\,. \tag{B.2}$$

Taking the derivative with respect to t_* yields

$$-n \left[V(t_*) + t_* - \frac{1 - F(t_*)}{f(t_*)} \right] F^{n-1}(t_*)\, f(t_*)\,. \tag{B.3}$$

Since $\theta_* > \overline{t}$, the expression $V(t_*) + t_* - \dfrac{1 - F(t_*)}{f(t_*)}$ is negative for all $t_* \in [\underline{t}, \overline{t}]$. Hence, expression (B.3) is positive if $t_* \in (\underline{t}, \overline{t}]$ or $t_* = \underline{t} > \underline{\theta}$ and zero if $t_* = \underline{t} = \underline{\theta}$. It follows that the symmetric separating equilibrium that maximizes the seller's expected utility has $t_* = \overline{t}$.

To complete the proof, we derive an expression for $\overline{t}$ given that $V(\underline{\theta}) + \underline{\theta} < \dfrac{1}{f(\underline{\theta})}$ and $V(\underline{\theta}) + \theta_* > 0$. It was shown earlier that $\overline{t} = -V(\underline{\theta})$ when $V(\underline{\theta}) + \underline{\theta} \leq 0$ and that $\overline{t} = \underline{\theta}$ when $V(\underline{\theta}) + \underline{\theta} > 0$. These observations can be summarized as follows: $\overline{t} = \max\{\underline{\theta}, -V(\underline{\theta})\}$. ■

C Proof of Proposition 3

The proof proceeds in three parts. We first show that $V^A(\underline{t}^A) = V^B(\underline{t}^B)$ implies $\underline{t}^A = \underline{t}^B$. It follows that $V^A(\theta) < V^B(\theta)$ for all $\theta \in (\underline{t}^A, \overline{\theta}]$. We then show that $\theta_*^A > \theta_*^B$ and that either $t_*^B \geq t_*^A$ or $t_*^B = \theta_*^B$. The final part combines these results to demonstrate that $C^A > C^B$.

Suppose that $V^k(\underline{\theta}) + \underline{\theta} \leq 0$ for all $k \in \{A, B\}$. In this case, $V^A(\underline{t}^A) + \underline{t}^A = 0$ and $V^B(\underline{t}^B) + \underline{t}^B = 0$. Since $V^A(\underline{t}^A) = V^B(\underline{t}^B)$, it must be the case that $\underline{t}^A = \underline{t}^B$. Now suppose that $V^k(\underline{\theta}) + \underline{\theta} < 0$ for all $k \in \{A, B\}$. In this case, $\underline{t}^A = \underline{\theta}$ and $\underline{t}^B = \underline{\theta}$. Hence, $\underline{t}^A = \underline{t}^B$. Finally, suppose that $V^k(\underline{\theta}) + \underline{\theta} \leq 0$ for some $k \in \{A, B\}$ but that $V^\ell(\underline{\theta}) + \underline{\theta} > 0$ for $\ell \neq k$. In this case, $\underline{t}^\ell = \underline{\theta}$ while $\underline{t}^k$ satisfies $V^k(\underline{t}^k) + \underline{t}^k = 0$. Since $V^k(\underline{t}^k) = V^\ell(\underline{t}^\ell)$ and $\underline{t}^k \geq \underline{t}^\ell$, we have $V^k(\underline{t}^k) + \underline{t}^k \geq V^\ell(\underline{t}^\ell) + \underline{t}^\ell$. However, since $V^k(\underline{t}^k) + \underline{t}^k = 0$, it must be the case that $V^\ell(\underline{t}^\ell) + \underline{t}^\ell \leq 0$, which contradicts our initial assumption that $V^\ell(\underline{\theta}) + \underline{\theta} > 0$. The contradiction allows us to rule out this final case. Having shown that $\underline{t}^A = \underline{t}^B$, we drop the superscript and let $\underline{t}$ denote both $\underline{t}^A$ and $\underline{t}^B$. The combination of $V^A(\underline{t}) = V^B(\underline{t})$ and $\frac{\mathrm{d}V^A(\theta)}{\mathrm{d}\theta} < \frac{\mathrm{d}V^B(\theta)}{\mathrm{d}\theta}$ imply that $V^A(\theta) < V^B(\theta)$ for all $\theta \in (\underline{t}, \overline{\theta}\,]$.

We will now show that $\theta_*^A > \theta_*^B$. Since $V^A(\underline{\theta}) + \underline{\theta} < \frac{1}{f(\underline{\theta})}$, θ_*^A exceeds $\underline{\theta}$ and satisfies

$$V^A(\theta_*^A) + \theta_*^A - \frac{1 - F(\theta_*^A)}{f(\theta_*^A)} = 0. \tag{C.1}$$

By applying Assumption 1, we obtain $\theta_*^A > \underline{t}$. Suppose $V^B(\underline{\theta}) + \underline{\theta} < \frac{1}{f(\underline{\theta})}$. It follows that θ_*^B exceeds $\underline{\theta}$ and satisfies

$$V^B(\theta_*^B) + \theta_*^B - \frac{1 - F(\theta_*^B)}{f(\theta_*^B)} = 0. \tag{C.2}$$

By applying Assumption 1, we obtain $\theta_*^B > \underline{t}$. Since $V^A(\theta) < V^B(\theta)$ for all $\theta \in (\underline{t}, \overline{\theta}\,]$ and since both V^A and V^B satisfy Assumption 4, we have $\theta_*^A > \theta_*^B$. Now suppose $V^B(\underline{\theta}) + \underline{\theta} \geq \frac{1}{f(\underline{\theta})}$. In this case, $\theta_*^B = \underline{\theta}$. Since $\theta_*^A > \underline{\theta}$, we have $\theta_*^A > \theta_*^B$.

Having established that $\theta_*^A > \theta_*^B$, we show that either $t_*^B \geq t_*^A$ or $t_*^B = \theta_*^B$. Since $V^A(\underline{\theta}) + \underline{\theta} < \frac{1}{f(\underline{\theta})}$ and $V^A(\underline{\theta}) + \theta_*^A > 0$, we have $t_*^A = \max\{\underline{\theta}, -V^A(\underline{\theta})\}$. If $t_*^A = \underline{\theta}$, $t_*^B \geq t_*^A$.

Now suppose $t_*^A = -V^A(\underline{\theta})$. Note that this implies $V^A(\underline{\theta}) + \underline{\theta} \leq 0$. If $t_*^B \neq \theta_*^B$, we have $t_*^B = \max\{\underline{\theta}, -V^B(\underline{\theta})\}$. Since $V^A(\underline{t}) = V^B(\underline{t})$ and $\dfrac{\mathrm{d}V^A(\theta)}{\mathrm{d}\theta} < \dfrac{\mathrm{d}V^B(\theta)}{\mathrm{d}\theta}$ for all $\theta \in [\underline{\theta}, \overline{\theta}\,]$, it must be the case that $V^A(\underline{\theta}) \geq V^B(\underline{\theta})$. $V^A(\underline{\theta}) + \underline{\theta} \leq 0$ then implies $V^B(\underline{\theta}) + \underline{\theta} \leq 0$, which implies $t_*^B = -V^B(\underline{\theta})$. Since $-V^B(\underline{\theta}) \geq -V^A(\underline{\theta})$, $t_*^B \geq t_*^A$.

We have established that (1) $V^B(\theta) > V^A(\theta)$ for all $\theta \in (\underline{t}, \overline{\theta}\,]$, (2) $\theta_*^B < \theta_*^A$, and (3) either $t_*^B \geq t_*^A$ or $t_*^B = \theta_*^B$. We will now exploit these results to demonstrate that $C^A > C^B$. Recall that

$$C^k \equiv -n \int_{t_*^k}^{\theta_*^k} \left[V^k(x) + x - \frac{1 - F(x)}{f(x)} \right] F^{n-1}(x)\, f(x)\, \mathrm{d}x\,, \quad k \in \{A, B\}. \tag{C.3}$$

Since $V^A(\underline{\theta}) + \underline{\theta} < \dfrac{1}{f(\underline{\theta})}$ and $V^A(\underline{\theta}) + \theta_*^A > 0$, we have $t_*^A = \max\{\underline{\theta}, -V^A(\underline{\theta})\}$. In addition, $V^A(\underline{\theta}) + \underline{\theta} < \dfrac{1}{f(\underline{\theta})}$ implies $\theta_*^A > \underline{\theta}$, and $V^A(\underline{\theta}) + \theta_*^A > 0$ implies $\theta_*^A > -V^A(\underline{\theta})$. It follows that $\theta_*^A > t_*^A$. Since $V^A(x) + x - \dfrac{1 - F(x)}{f(x)}$ is negative for all $x \in [t_*^A, \theta_*^A)$, $C^A > 0$.

Suppose $t_*^B \neq \theta_*^B$. In this case, $t_*^B \geq t_*^A$, which allows us to write $[t_*^B, \theta_*^B] \subset [t_*^A, \theta_*^A]$. Recall that $t_*^A = \max\{\underline{\theta}, -V^A(\underline{\theta})\}$. If $V^A(\underline{\theta}) + \underline{\theta} \geq 0$, $t_*^A = \underline{\theta}$ and $\underline{t} = \underline{\theta}$. If $V^A(\underline{\theta}) + \underline{\theta} < 0$, $t_*^A = -V^A(\underline{\theta})$ and $\underline{t} = -V^A(\underline{t})$. In either case, $t_*^A \geq \underline{t}$. It follows from Condition (1) that $V^B(\theta) > V^A(\theta)$ for all $\theta \in (t_*^A, \theta_*^A]$. Since $[t_*^B, \theta_*^B] \subset [t_*^A, \theta_*^A]$ and $V^B(\theta) > V^A(\theta)$ for all $\theta \in (t_*^A, \theta_*^A]$, we have $C^A > C^B$. Now suppose $t_*^B = \theta_*^B$. In this case, $C^B = 0$. Since $C^A > 0$, we have $C^A > C^B$. ■

References

Asker, J., Cantillon, E.: Properties of scoring auctions. RAND Journal of Economics **39**, 69-85 (2008)

Avery, C.: Strategic jump bidding in English auctions. Review of Economic Studies **65**, 185-210 (1998)

Bester, H., Strausz, R.: Imperfect commitment and the revelation principle: The multi-agent case. Economics Letters **69**, 165-171 (2000)

Bikhchandani, S., Huang, C.-F.: Auctions with resale markets: An exploratory model of treasury bill markets. Review of Financial Studies **2**, 311-339 (1989)

Branco, F.: The design of multidimensional auctions. RAND Journal of Economics **28**, 63-81 (1997)

Bulow, J.: Durable-goods monopolists. Journal of Political Economy **90**, 314-332 (1982)

Chalot, J.P.: Personal interview. Petróleos de Venezuela (1996, October 31)

Che, Y.-K.: Design competition through multidimensional auctions. RAND Journal of Economics **24**, 668-680 (1993)

Coase, R.H.: Durability and monopoly. Journal of Law and Economics **15**, 143-149 (1972)

Das Varma, G.: Bidding for a process innovation under alternative modes of competition. International Journal of Industrial Organization **21**, 15-37 (2003)

Gale, D., Shapley, L.S.: College admissions and the stability of marriage. American Mathematical Monthly **69**, 9-15 (1962)

Gitelson, G., Bing, J.W., Laroche, L.: The impact of culture on mergers and acquisitions. CMA Management (2001, March)

Goeree, J.: Bidding for the future: Signaling in auctions with an aftermarket. Journal of Economic Theory **108**, 345-364 (2003)

Gul, F., Sonnenschein, H., Wilson, R.: Foundations of dynamic monopoly and the Coase conjecture. Journal of Economic Theory **39**, 155-190 (1986)

Gumbel, A.: Civil War writer's one-page outline earns him record $11m book and film contract. The Independent (2002, April 8)

Haile, P.A.: Auctions with private uncertainty and resale opportunities. Journal of Economic Theory **108**, 72-110 (2003)

Hart, O.D., Tirole, J.: Contract renegotiation and Coasian dynamics. Review of Economic Studies **55**, 509-540 (1988)

Jehiel, P., Moldovanu, B.: Efficient design with interdependent valuations. Econometrica **69**, 1237-1259 (2001)

Katzman, B., Rhodes-Kropf, M.: The consequences of information revealed in auctions. Mimeo, Columbia University, Graduate School of Business (2002)

Lamping, J.: Ignorance is bliss: Matching in auctions with an uninformed seller. Mimeo, University of Colorado, Department of Economics (2008)

Maskin, E., Tirole, J.: The principal-agent relationship with an informed principal, II: Common values. Econometrica **60**, 1-42 (1992)

McAfee, R.P., Vincent, D.: Sequentially optimal auctions. Games and Economic Behavior **18**, 246-276 (1997)

Milgrom, P.R., Weber, R.J.: A theory of auctions and competitive bidding. Econometrica **50**, 1089-1122 (1982)

Molnár, J., Virág, G.: Revenue maximizing auctions with market interaction and signaling. Economics Letters **99**, 360-363 (2008)

Myerson, R.B.: Optimal auction design. Mathematics of Operations Research **6**, 58-73 (1981)

Rezende, L.: Biased procurement auctions. Economic Theory (forthcoming)

Riley, J.G., Samuelson, W.F.: Optimal auctions. American Economic Review **71**, 381-392 (1981)

Skreta, V.: Optimal auction design under non-commitment. Mimeo, New York University, Stern School of Business (2007)

Stokey, N.: Rational expectations and durable goods pricing. Bell Journal of Economics **12**, 112-128 (1981)

Vartiainen, H.: Auction design without commitment. Mimeo, Yrjö Jahnsson Foundation (2007)

Zheng, C.Z.: Optimal auction in a multidimensional world. Northwestern University, Center for Mathematical Studies in Economics and Management Science Discussion Paper 1282 (2000)