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Abstract

In many industries, firms pre-order input and forward sell output prior to the actual production period. It is known that forward buying input induces a "Cournot-Stackelberg endogeneity" (both Cournot and Stackelberg outcomes may result in equilibrium) and forward selling output induces a convergence to the Bertrand solution. I analyze the generalized model where firms pre-order input *and* forward sell output. First, I analyze oligopolists producing homogenous goods, generalize the Cournot-Stackelberg endogeneity to oligopoly, and show that it additionally includes Bertrand in the generalized model. This shows that the "mode of competition" between firms may be entirely endogenous. Second, I consider heterogenous goods in duopolies, which generalizes existing results on forward sales of output, and derive the outcome set in general duopolies. This set does not contain the Bertrand solution anymore, but it is well-defined and shows that forward sales increase welfare also when goods are complements.

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Keywords: forward sales, capacity accumulation, Cournot, Stackelberg, Bertrand

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1 Introduction

The paper analyzes oligopolistic industries in a model that explicitly contains planning periods prior the production period. The model sets in T periods prior to the production period, e.g. $T = 52$ weeks prior to the year 2011. In these T preliminary stages, firms may pre-order input (i.e. pre-build capacity for 2011) and conclude forward contracts to sell the output that they will produce. In the production period, firms set production quantities and sell the output that was not sold via forward contracts. Capacities can be extended in the production period, possibly at incremental costs.

This model unifies two streams of literature—the studies of production timing following Saloner (1987) and those of sales timing following Allaz and Vila (1993). I consider production timing in the sense of capacity pre-building, e.g. pre-ordering machinery or raw materials, and allow for sales timing assuming efficient forward contracts (e.g. to retailers). In many markets, these timing issues interact, but their interaction has not yet been analyzed and it is therefore unclear whether sales or production timing dominate from a strategic point of view.

If production timing dominates, then the results on the "Cournot-Stackelberg endogeneity" derived by Saloner (1987), Pal (1991), and Romano and Yildirim (2005) apply. They have shown that in two-stage games of (quantity) accumulation, a continuum of outcomes may result in equilibrium that contains both Stackelberg outcomes and the Cournot outcome. The continuum exists if the costs of production do not change between first and second period. Their analyses assumed that quantity produced in stage 1 cannot be withheld from being sold in the final stage (stage 2). This assumption is relaxed in the model of "capacity accumulation" analyzed here, interestingly without notable implications with respect to the equilibrium set.[1] From a more general point of view, their work has shown that industries need not converge to Cournot equilibrium, that non-cost-related size differences may persist in equilibrium, and that Stackelberg leadership can be sustained without asynchronous timing and without retaliations against deviations of followers (i.e. in stationary equilibria of

[1]Following Kreps and Scheinkman (1983), Saloner (1987), and many subsequent studies, I assume that the costs of pre-building capacity are sunk in the short term. This implies that capacity is either constant or accumulates along the path of play and relates the present study to "games of accumulation" (Romano and Yildirim, 2005).

repeated games).[2]

If sales timing dominates, then the results on forward trading in oligopoly, following Allaz and Vila (1993), apply. These results differ strikingly from the Cournot-Stackelberg endogeneity. Allaz and Vila consider T-stage games where the firms may sell forward (some of) their eventual output in stages $t < T$ and they set production quantities in stage $t = T$. Contrary to the implications of production timing, the possibility of sales timing does not affect the dimensionality of the equilibrium set. The equilibrium outcome is unique, but competition is intensified in relation to Cournot and the outcome actually converges to Bertrand as T tends to infinity.[3] Mahenc and Salanié (2004) show that forward trading has the opposite effect—to weaken competition—if firms compete in prices.

My analysis shows that neither production timing nor sales timing dominates the other. Rather, the equilibrium structure merges results from both streams of literature. The outcome set is a continuum that extends the Cournot-Stackelberg endogeneity to additionally include the Allaz-Vila outcome, and in case the goods are homogenous, the Allaz-Vila outcome converges to the Bertrand outcome as T approaches infinity. I derive the outcome set for oligopolies producing homogenous goods, which also shows how the Cournot-Stackelberg endogeneity generalizes to oligopoly, and for duopolies producing heterogenous goods, which additionally provides the Allaz-Vila prices for heterogenous goods.

These results highlight that the mode of competition may be entirely endogenous in oligopolistic industries. The various equilibrium outcomes may result in ex-ante equivalent industries if sales timing and production timing interact. Thus, if firms anticipate Cournot, then they are best off playing according to Cournot, if firms an-

[2]Another branch of literature, including e.g. Hamilton and Slutsky (1990), Robson (1990), and van Damme and Hurkens (1999), studies endogenous timing in duopoly. As Matsumura (1999) shows, endogenous Stackelberg does typically not result if there are more than two firms, and in general, models of endogenous timing are restrictive in the sense that firms can produce only in one of two or more initially feasible periods. Romano and Yildirim (2005) discuss this in more detail.

[3]Independently, Bolle (1993) and Powell (1993) reached similar conclusions for $T = 2$, and to name a few subsequent studies, Ferreira (2003) derives a Folk theorem for the case that there is no final trading period, and Liski and Montero (2006) show that forward trades simplify penal strategies and tacit collusion in repeated oligopoly.

ticipate Stackelberg (with an arbitrary leader-follower assignment), then Stackelberg results, and so on. The firms' anticipations, in turn, may be given by historical precedents or social norms. The set of equilibrium outcomes will be characterized using a novel indexation of oligopoly equilibria that is derived from the first order conditions. The indexation links the classic modes of competition—Cournot, Bertrand, Stackelberg, and Allaz-Vila—in terms of conjectural variations and the equilibrium analysis rationalizes the corresponding conjectures. Finally, the analysis shows that forward trading of quantity setting firms is socially efficient in general (i.e. also in case the goods are complements), which was questionable after Mahenc and Salanié (2004) had shown that it softens competition between price setting firms.

Section 2 defines the notation. Section 3 derives requisite preliminary results, introduces the equilibrium index, and extends the basic Cournot-Stackelberg endogeneity to oligopoly. Section 4 analyzes the general model of oligopolists producing homogenous goods, Section 5 concerns the case of duopolists producing heterogenous goods. Section 6 concludes. The proofs are relegated to the appendix.

2 The base model

Initially we focus on two-stage oligopoly games in markets for homogenous goods. Further notation will be introduced when we augment the base model. Firms are denoted as $i \in N = \{1, \ldots, n\}$. In stage 1, the planning phase, the firms choose "capacities" z_i (i.e. they order the respective amounts of input factors) and they conclude forward contracts for y_i units of output (e.g. with retailers). In stage 2, the production phase, they choose the quantities x_i to be produced. The players act simultaneously in each stage, and the choices made in stage 1 are common knowledge in stage 2. The unit costs of pre-building capacity are $\gamma_i \geq 00$. In case a quantity $x_i > z_i$ is chosen in stage 2, the pre-built capacity is extended at unit costs $c_i \geq \gamma_i$. There are no costs of production besides the costs of capacity. The inverse demand function is $p(\mathbf{x}) = a - b \sum_{i \in N} x_i$. The forward sales are priced competitively in that the eventually resulting market price is anticipated correctly. Throughout this paper scalar values and functions are set in italics, e.g. capacities z_i, vectors are set in boldface type, e.g. $\mathbf{z} = (z_i)_{i \in N}$, sets of scalars are denoted by capital letters, e.g. $Z_i \ni z_i$, and sets of vectors are denoted by capital letters set in boldface type, e.g. $\mathbf{Z} = \times_{i \in N} Z_i$.

Definition 2.1 (Base game)**.** Strategy profiles are triples $(\mathbf{z}, \mathbf{y}, \mathbf{x}) = (z_i, y_i, x_i)_{i \in N}$. In stage 1, firms ($i \in N$) set capacities $z_i \in Z_i \subseteq \mathbb{R}_+$ and forward sales $y_i \in Y_i \subseteq \mathbb{R}_+$, and in stage 2, they set quantities $x_i : \mathbf{Z} \times \mathbf{Y} \to X_i \subseteq \mathbb{R}_+$. In stage 2, the forward trades have been concluded at price p^f and the short-term profit of i is

$$\Pi_i^S(\mathbf{x}|\mathbf{z}, \mathbf{y}) = (x_i - y_i) \cdot \left(a - b \sum_j x_j\right) + p^f \cdot y_i - c_i \cdot \max\{x_i(\mathbf{z}, \mathbf{y}) - z_i, 0\} - \gamma_i z_i. \quad (1)$$

The assumption that profitable arbitrage is impossible in equilibrium follows Allaz and Vila (1993) and implies that the market price for forward trades equates with the anticipated market price conditional on the choices of $(\mathbf{z}, \mathbf{y})$, i.e.

$$p^f(\mathbf{x}|\mathbf{z}, \mathbf{y}) = a - b \cdot \sum_{j \in N} x_j(\mathbf{z}, \mathbf{y}). \quad (2)$$

Substituting p^f in Eq. (1), the stage-1 (long term) profit function of $i \in N$ becomes

$$\Pi_i^L(\mathbf{z}, \mathbf{y}, \mathbf{x}) = x_i(\mathbf{z}, \mathbf{y}) * p\left(\mathbf{x}|\mathbf{z}, \mathbf{y}\right) - c_i \cdot \max\{x_i(\mathbf{z}, \mathbf{y}) - z_i, 0\} - \gamma_i z_i. \quad (3)$$

We focus on subgame-perfect equilibria (SPEs) in pure strategies.

3 Preliminary analysis and benchmark results

Outcome uniqueness in production phase

In the last round, the *production phase*, the firms choose quantities (x_i) contingent on their capacity pre-builds (z_i) and forward sales (y_i). In standard Cournot models with linear demands, the quantities chosen in equilibrium are unique. Our model assumes quantity competition and linear demands, too, but the discontinuity at the capacity limit implies that uniqueness in the production phase is less obvious than in standard models. Establishing outcome uniqueness in the production phase is important, however, to understand that the indeterminacy of the mode of competition originates in the planning phase (as one would expect) rather than the production phase.

Our first result establishes outcome uniqueness in the production phase, and in addition it characterizes the equilibrium outcome. To gain intuition, define the indicator $I_{x_i > z_i}$, i.e. it evaluates to 1 iff $x_i > z_i$, and consider the marginal profit of i. This derivative is well defined for all $x_i \neq z_i$.

$$\frac{\partial \Pi_i^S}{\partial x_i} = p - b(x_i - y_i) - c_i \cdot I_{x_i > z_i} \tag{4}$$

The marginal profit is piecewise linear in x_i and discontinuous at $x_i = z_i$. The non-standard characteristic of capacity pre-builds is that, if $x_i = z_i$, the marginal revenue $MR_i = p - b(x_i - y_i)$ may attain any value in $[0, c_i]$ without violating individual rationality. If $MR_i \geq 0$, firm i cannot gain by reducing quantity (no costs are saved by doing so if $x_i \leq z_i$), and if $MR_i \leq c_i$, increasing quantity does not pay off either (to this end, the capacity would have to be extended, but the respective unit costs c_i would not be covered). Thus, the equilibrium condition is not $p - bx_i = c_i$ as in standard Cournot models or $p - b(x_i - y_i) = c_i$ as in standard Allaz-Vila models, but

$$p - b(x_i - y_i) = 0, \qquad \text{if } x_i < z_i \tag{5}$$

$$p - b(x_i - y_i) \in [0, c_i], \qquad \text{if } x_i = z_i \tag{6}$$

$$p - b(x_i - y_i) = c_i, \qquad \text{if } x_i > z_i \tag{7}$$

for all $i \in N$. Overall, the first order conditions are less restrictive than those of models without capacity pre-builds, but as I show in the appendix, outcome uniqueness in the production phase can be established nonetheless. As we will see below, the relaxation of equilibrium conditions implies that outcome non-uniqueness may result in the planning phase.

Lemma 3.1. *Fix any* $(\mathbf{z}, \mathbf{y})$. *The equilibrium quantities* $x_i^*(\mathbf{z}, \mathbf{y})$ *are unique for all* $i \in N$ *and satisfy, using* $r_i = a - b\left(2z_i - y_i + x_{-i}^*\right)$ *and* $x_{-i}^* = \sum_{j \neq i} x_j^*$,

$$x_i^*(\mathbf{z}, \mathbf{y}) = \begin{cases} z_i + \frac{r_i}{2b}, & \text{if } r_i < 0 \\ z_i, & \text{if } 0 \leq r_i \leq c_i \\ z_i + \frac{r_i - c_i}{2b}, & \text{if } r_i > c_i. \end{cases} \tag{8}$$

The characterization of the stage 2 equilibrium rests on (r_i), which denotes the hypothetical short-term marginal revenue at $x_i = z_i$. If $r_i < 0$, then i does not exploit the pre-built capacity ($x_i^* < z_i$), i chooses $x_i^* = z_i$ if $r_i \in [0, c_i]$, and it extends capacity ($x_i^* > z_i$) if $r_i > c_i$.

No excess capacity

Varying the capacity z_i of firm i in Lemma 3.1, we observe a *capacity effect*, i.e. a tendency to exactly use the pre-built capacity. For, $x_i = z_i$ is optimal whenever r_i is in $[0, c_i]$, and if r_i is in the interior of $[0, c_i]$, then small increments of z_i keep r_i in the interior $[0, c_i]$—which implies that the incremented capacity would still be exploited. To further illustrate, let $MR_i^S = \partial R_i^S / \partial x_i$ denote i's short-term marginal revenue (as above), and let $MR_i^L = \partial R_i^L / \partial z_i$ be i's long-term marginal revenue.

$$MR_i^S = a - b\sum_j x_j - b(x_i - y_i) \qquad\qquad MR_i^L = a - b\sum_j x_j - bx_i \tag{9}$$

The difference between MR_i^S and MR_i^L is the *forward trade effect* originally described by Allaz and Vila (1993). As y_i increases, the quantity that is left to be sold in stage 2 shrinks and hence the MR in the short term increases. This effect implies $MR_i^S \geq MR_i^L$ in general and $MR_i^S > MR_i^L$ if $y_i > 0$. In light of this, one might suspect that if y_i is large enough, then $MR^L \leq \gamma_i$ and $MR^S > c_i$ could hold true simultaneously. In this case, firm i would be best off delaying capacity investments until stage 2. The next result shows that capacity investments are neither delayed nor excessive in equilibrium (if $c_i > \gamma_i$). Along the equilibrium path, firms exactly exploit their pre-built capacity.

Lemma 3.2. *Fix any SPE* $(\mathbf{z}^*, \mathbf{y}^*, \mathbf{x}^*)$. *For all* $i \in N$, *the quantity chosen along the equilibrium path satisfies* $x_i^*(\mathbf{z}^*, \mathbf{y}^*) \geq z_i^*$, *and in case* $c_i > \gamma_i$ *it satisfies* $x_i^*(\mathbf{z}^*, \mathbf{y}^*) = z_i^*$.

The case $c_i = \gamma_i$ is a little more complex. As the formulation of Lemma 3.2 suggests, capacity may be extended in stage 2 if $c_i = \gamma_i$. Lemma 4.2 proved below shows that the set of equilibrium outcomes is unaffected by this effect.

Basic oligopoly models and implied conjectural derivatives

If the pre-built capacity cannot be extended in stage 2, i.e. if $c_i = \infty$ for all $i \in N$, then the forward trade effect disappears. In this case, the Cournot outcome results. This shows that a necessary condition for the competition-enhancing effect of forward trades is that capacity can be extended after output had been sold forward. The first order conditions in the Cournot model and the Cournot price are, using $p = a - b\sum_j z_j$ and assuming an interior solution exists (i.e. $p > \gamma_i$ for all $i \in N$),

$$p - bz_i - \gamma_i = 0 \quad \forall i \in N, \qquad \Rightarrow \qquad p^C = \frac{a + \sum_{j\in N}\gamma_j}{1+n}. \tag{10}$$

Secondly, if capacity cannot be pre-built, the framework of Allaz and Vila (1993) results. If we assume, for notational clarity, that $c_i = \gamma_i$ applies (since γ_i as used before is irrelevant when capacity cannot be pre-built), then the first order conditions on quantities (x_i) contingent on (y_i) are $p - b(x_i - y_i) - \gamma_i = 0$ for all $i \in N$. If we solve these conditions for (x_i), and represent the resulting conditions for optimal (y_i) in terms of the induced capacities z_i, then the following reduced-form first order conditions of the Allaz-Vila model are obtained.

$$p - \frac{1}{n} \cdot bz_i - \gamma_i = 0 \quad \forall i \in N, \qquad \Rightarrow \qquad p^{AV} = \frac{a + n\sum_{i\in N}\gamma_i}{1+n^2}. \tag{11}$$

These two sets of first order conditions, Eqs. (10) and (11), can be represented as

$$p - \lambda_i bz_i - \gamma_i = 0 \quad \forall i \in N \tag{12}$$

for certain $(\lambda_i) \in \mathbb{R}^N_+$. The first order conditions in the Cournot-model are obtained for $\lambda_1 = \cdots = \lambda_n = 1$, and the Allaz-Vila conditions correspond with $\lambda_1 = \cdots = \lambda_n = 1/n$. The equilibrium price and profits associated with arbitrary (λ_i) are

$$p = \frac{a + \sum_i \lambda_i^{-1}\gamma_i}{1 + \sum_i \lambda_i^{-1}}, \qquad \Pi_i = \frac{1}{\lambda_i b} \cdot \left(\frac{a - \gamma_i + \sum_j \lambda_j^{-1}(\gamma_j - \gamma_i)}{1 + \sum_j \lambda_j^{-1}} \right)^2. \tag{13}$$

Throughout this paper, I will represent models and equilibrium outcomes by such profiles $(\lambda_i) \in \mathbb{R}^N_+$. These (λ_i) relate straightforwardly to conjectural derivatives: λ_i describes how the aggregate market quantity increases if i increases x_i by a unit. In the Cournot model, the aggregate quantity increases by a unit, but in the Allaz-Vila model, an increase of the amount of forward trades induces an increase of the own quantity which in turn crowds out the opponents' quantities. The ratio of the resulting increase of the overall quantity to the increase of the own quantity is $\lambda_i = 1/n$ in the Allaz-Vila model.

To clarify the relation to conjectural derivatives, let x_i denote the quantity of i and x_{-i} the aggregate quantity of i's opponents. If players compete by choosing quantities in a market with inverse demand $P(x_i + x_{-i})$, then i's first order condition is (assuming constant marginal costs γ_i)

$$P - x_i \cdot P'(x_i + x_{-i}) \cdot \left(1 + \frac{dx_{-i}}{dx_i}\right) - \gamma_i = 0. \tag{14}$$

$P = a - b\sum_i x_i$ implies $P'(q) = -b$, and hence $\lambda_i = 1 + \frac{dx_{-i}}{dx_i} = \frac{d(x_{-i}+x_i)}{dx_i}$.

To conclude this overview, consider the (λ_i) implied in Stackelberg games.

Definition 3.3 (Stackelberg games)**.** For any partition $(N_t)_{t\leq T} = (N_1, N_2, \ldots, N_T)$ of N, define the $(N_t)_{t\leq T}$-Stackelberg game as the T-round extensive form game of perfect information where the players $i \in N_t$ simultaneously move (choosing quantities) in round t, for all $t = 1, \ldots, T$.

Lemma 3.4. *Assume $\gamma_i = \gamma_j$ for all $i, j \in N$. The equilibrium profits of the players in any $(N_t)_{t\leq T}$-Stackelberg game are given by Eq.* (13) *using $\lambda_i = \prod_{t'=t+1}^{T} \frac{1}{|N_{t'}|+1}$ for all $i \in N_t$ and all $t \leq T$.*

Thus, Stackelberg followers $i \in N_T$ have conjectural derivatives $\lambda_i = 1$ that correspond with those of Cournot oligopolists (they play their best responses). Allaz-Vila oligopolists have conjectural derivatives that are equivalent to those of singleton first movers, i.e. to those in $\big(\{i\}, N\setminus\{i\}\big)$-Stackelberg games where all opponents of i are equally ranked followers.[4]

The Cournot-Stackelberg endogeneity

The "Cournot-Stackelberg endogeneity" (Romano and Yildirim, 2005) was originally derived by Saloner (1987). He assumed that quantity can be pre-built and showed that a continuum of outcomes containing the Stackelberg outcomes and the Cournot outcome may result if $\boldsymbol{c} = \boldsymbol{\gamma}$. Pal (1991) showed that the Cournot-Stackelberg endogeneity disappears as $\boldsymbol{c} \neq \boldsymbol{\gamma}$. Correspondingly, I distinguish the cases (i) $c_i > \gamma_i$ for all i and (ii) $c_i = \gamma_i$ for all i. The first main result, Proposition 3.5, shows how the observations of Saloner and Pal generalize to the oligopoly case, and that they hold true even if

[4] As for two-player games, this link was previously established by Allaz and Vila (1993, Prop. 2.2). They showed that if only one oligopolist may sell forward in their two-period game, then the Stackelberg outcome results.

quantity produced in stage 1 can be withheld from being sold on the market in stage 2 (i.e. if we consider “capacity accumulation” rather than “quantity accumulation”). This is not obvious, and in fact questioned by Saloner (1987, p. 186f).

Prior to stating the result, let me illustrate why a continuum of equilibria exists in case $\boldsymbol{c} = \boldsymbol{\gamma}$. Saloner considered the case of two players, and expressed in terms of (λ_i) as defined above, he showed that all outcomes associated with (λ_1, λ_2) where $\lambda_1 \in [1/2, 1)$ and $\lambda_2 = 1$ can result in equilibrium. In this case, $1/2 \leq \lambda_1 < 1$ implies that firm 1 chooses a capacity somewhere between that of a Stackelberg leader and that of a Cournot duopolist, and $\lambda_2 = 1$ implies that 2 plays the best response. In a pure Cournot framework, firm 1 would benefit by decreasing his quantity, but in the two-stage game considered here, firm 2 would respond by increasing the capacity in stage 2 in case $c_2 = \gamma_2$. Hence, decreasing capacity pays off for firm 1 only if it pays off for a Stackelberg leader. The latter, however, cannot be the case if Eq. (12) holds true for $\lambda_1 \geq 1/2$. In turn, player 1 does not benefit from increasing capacity when Eq. (12) is satisfied for some $\lambda_1 \leq 1$. For, firm 2 does not respond to a capacity increase of 1 by decreasing his quantity in stage 2 as long as $\lambda_2 > 0$ (i.e. as long as i’s marginal revenue is positive). To summarize: 1’s incentives correspond with those of a Cournot duopolist with respect to capacity increases and with those of Stackelberg followers with respect to capacity decreases. Hence, λ_1 may attain any value in $[1/2, 1]$ in equilibrium.

Proposition 3.5 (Zero forward trades)**.** *Assume “sufficiently similar” (see below) marginal costs* (γ_i) *and* $Y_i = \{0\}$ *for all* $i \in N$.

1. *If* $c_i > \gamma_i$ *for all* $i \in N$*, then the Cournot outcome results in the unique SPE.*
2. *If* $c_i = \gamma_i$ *for all* $i \in N$*, then an outcome* $\langle p, (\Pi_i) \rangle$ *can result if and only if there exists a partition* (N_1, N_2) *of* N *such that* $\langle p, (\Pi_i) \rangle$ *satisfies Eq.* (13) *for some* (λ_i) *where* $\lambda_i \in \left[\frac{1}{|N_2|+1}, 1\right]$ *for all* $i \in N_1$ *and* $\lambda_j = 1$ *for all* $j \in N_2$.

The (γ_i) *are “sufficiently similar” to sustain price* p *if* $p > \gamma_i$ *for all* $i \in N$.

The set of equilibrium outcomes is a continuum containing the outcomes of all *two-stage* Stackelberg games[5] and the Cournot outcome. The set is not convex in

[5]Any two-stage Stackelberg game is defined by a partition (N_1, N_2) of N where all $i \in N_1$ move

the payoff space, but it is the union of finitely many hyperrectangles in (λ_i)-space. Namely, it is the union, over all (N_1, N_2)-Stackelberg games, of the hyperrectangles containing the respective (N_1, N_2)-Stackelberg outcome and the Cournot outcome. For example, in three-player games, the set of outcomes is the union of six hyperrectangles (as there are six two-stage Stackelberg games between three players), where each hyperrectangle has a Stackelberg solution and the Cournot solution at its corner points.

The set of two-stage Stackelberg games contains all Stackelberg games if we focus on duopolies, as Saloner (1987), Pal (1991), and Romano and Yildirim (2005) did, but it is incomplete if there are more than two firms. Proposition 3.5 thus shows that the Cournot-Stackelberg endogeneity is incomplete in n-player games of accumulation. The following section shows that the Cournot-Stackelberg endogeneity can be reestablished (slightly weakened) in the more general game allowing for both capacity accumulation and forward trades.

4 Oligopoly with homogenous goods

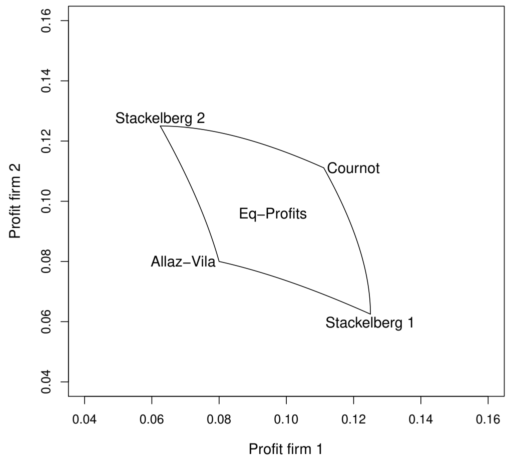
The two-stage game

Initially we focus on the "generic" case $c_i > \gamma_i$ for all i. The extension toward $c_i = \gamma_i$ for all i is covered below. The main result, Proposition 4.1, shows that the set of equilibrium outcomes is a hypercube in (λ_i)-space, with the Cournot solution ($\lambda_i = 1$ for all i) and the Allaz-Vila solution ($\lambda_i = 1/n$ for all i) at its opposite corner points. It contains all two-stage Stackelberg outcomes and many intermediate outcomes. The set of equilibrium outcomes is only "quasi-hyperrectangular" in the payoff space, i.e. neither convex nor hyperrectangular (see also Figure 1).

Proposition 4.1. *Assume $c_i > \gamma_i$ for all $i \in N$ and fix a price $p \in \mathbb{R}$ such that $p > c_i > \gamma_i$ for all $i \in N$. This price can result in an SPE if and only if there exists $(\lambda_i) \in \left[\frac{1}{n}, 1\right]^N$ such that p satisfies Eq.* (13)*, and the corresponding profit profile is* Π *according to Eq.* (13).

in round 1 and all $j \in N_2$ move in round 2. By Lemma 3.4, the solution is characterized by $\lambda_i = 1/(|N_2|+1)$ for all first-movers $i \in N_1$ and $\lambda_j = 1$ for all followers $j \in N_2$.

Figure 1: Range of profit profiles that may result in equilibrium (two players, zero costs)



To capture the intuition, let us look at the structure of the equilibria constructed in the proof (further equilibria exist, but they do not induce alternative outcomes). Fix $(\lambda_i) \in \left[\frac{1}{n}, 1\right]^N$ and find the unique capacities $(z_i)_{i \in N}$ such that

$$p - \lambda_i b z_i - \gamma_i = 0 \quad \forall i \in N, \tag{15}$$

at the respective market price $p = a - b \sum_i z_i$. See e.g. Eq. (40). Set $(y_i)_{i \in N}$ such that

$$p - b(z_i - y_i) - c_i = 0 \quad \forall i \in N. \tag{16}$$

Since $\lambda_i > 0$, corresponding y_i exist even if $c_i > \gamma_i$.[6] All such strategy profiles $(\mathbf{z}, \mathbf{y})$ can be extended to SPEs through the appropriate $\mathbf{x}^*$ (see Lemma 3.1). By Lemma 3.2, $x_i^* = z_i^*$ results along the path of play for all $i \in N$, and as a result of Eq. (16), the stage 2 marginal revenue is $r_i = c_i$ for all $i \in N$. When any $i \in N$ deviates unilaterally by increasing z_i in stage 1, then he will be best off exploiting the extended capacity in stage 2 (his marginal revenue falls below c_i but remains positive). Anticipating this quantity increase after observing the capacity "increase" of i, the opponents' marginal revenues fall below marginal costs c_j, but they remain positive, too. Hence,

[6]To be precise, such (y_i) exist if $c_i \leq p^{AV}$ for all $i \in N$. Eq. (16) cannot be satisfied if $c_i > p$. Assuming $c_i = c$ for all i, $p^{AV} \leq c \leq p^C$ implies that the equilibrium price range is the interval $\left[c, p^C\right]$. If $c > p^C$, short-term capacity are effectively prohibitive, and the Cournot outcome results.

the opponents' quantities are constant in response to i's capacity increase, and thus, i's capacity increase pays off if and only if it would pay off for a Cournot oligopolist. The latter applies iff

$$p - bz_i - \gamma_i \geq 0 \quad \forall i \in N. \tag{17}$$

Alternatively, i may cut capacity. The most profitable capacity cut implies that i simultaneously adjusts y_i so that he will not be best off extending capacity in stage 2 again. Regardless of y_i, however, the opponents' marginal revenues in stage 2 rise above c_j due to the capacity cut (i.e. due to correctly anticipating the quantity cut that follows), and hence they all respond by extending their capacities in stage 2. In turn, capacity cuts pay off if and only if a quantity cut pays for a Stackelberg leader to which all $n-1$ opponents respond by acting simultaneously. This applies iff

$$p - \frac{1}{n}bz_i - \gamma_i \leq 0 \quad \forall i \in N. \tag{18}$$

Since Eq. (15) is satisfied for some $\lambda_i \in \left[1/n, 1\right]$, neither Eq. (17) nor Eq. (18) can be satisfied, i.e. neither capacity cuts nor capacity extensions pay off if $\lambda_i \in \left[1/n, 1\right]$.

The generalized Cournot-Stackelberg endogeneity

Lemma 4.2 establishes that Proposition 4.1 extends to the degenerate case $\boldsymbol{c} = \boldsymbol{\gamma}$.

Lemma 4.2. *Assume (γ_i) are sufficiently similar. The set of outcomes that can be sustained in equilibrium is upper hemicontinuous in (c_i) if $c_i \geq \gamma_i$ for all $i \in N$.*

This result shows that the Cournot-Stackelberg endogeneity is general, i.e. neither degenerate nor generic, in the games discussed presently.

Additionally, by Lemma 3.4, an equilibrium of a general $(N_1, \ldots, N_T)$-Stackelberg game corresponds with an equilibrium according to Proposition 4.1 if and only if $\prod_{t=2}^{T} \frac{1}{|N_t|+1} \geq 1/n$. This includes many Stackelberg outcomes, even of Stackelberg games with more than two stages, but not all of them. As an example of a Stackelberg outcome that is included, consider the $(\{1\}, \{2\}, \{3, 4\})$-Stackelberg game, i.e. the game where 1 moves first, 2 moves second, while 3 and 4 simultaneously move last. By Lemma 3.4, its equilibrium is characterized as $\lambda_1 = \frac{1}{4}$, $\lambda_2 = \frac{1}{2}$, and $\lambda_3 = \lambda_4 = 1$, which satisfies $\lambda_i \geq 1/n$ for all four players. In turn, the Stackelberg outcome of the game where $n = 3$ players move strictly sequential (player 1 moves first, 2 moves second, 3 moves third) is not included. By Lemma 3.4, its equilibrium is characterized as $\lambda_1 = \frac{1}{4}$, $\lambda_2 = \frac{1}{2}$, and $\lambda_3 = 1$, which in this case violates $\lambda_i \geq \frac{1}{n}$ for all i. Then again, its outcome is weakly Pareto dominated by the Cournot outcome and strictly Pareto dominated by other equilibrium outcomes compatible with Proposition 4.1—and based on the Pareto criterion, it is possible to generalize the Cournot-Stackelberg endogeneity.

The following result establishes the corresponding generalization: equilibria of general $(N_1, \ldots, N_T)$-Stackelberg games are either compatible with Proposition 4.1 or they are Pareto dominated by an outcome compatible with Proposition 4.1.

Lemma 4.3. *Assume $\gamma_i = \gamma_j$ for all $i, j \in N$. Not all equilibrium outcomes of general $(N_t)_{t \leq T}$-Stackelberg games correspond with equilibrium outcomes according to Proposition 4.1. All those that do not are Pareto dominated by some equilibrium outcome compatible with Proposition 4.1.*

The T-stage game

Now assume that the number T of rounds in the planning phase prior to the production phase is increased. For example, production in 2020 may not just be planned in 2019 (by forward buying input and forward selling output), but already in 2018, or even 2017, and so on. Alternatively, T increases if the length of the preliminary planning periods is shortened, from years to say quarters. Allaz and Vila (1993) show that the forward trade effect intensifies as T increases, with the consequence that the equilibrium price converges to the Bertrand price as T approaches infinity. Romano and Yildirim (2005), in turn, show that solutions of accumulation games are invariant with respect to T, i.e. the set of equilibrium outcomes is independent of the number of accumulation periods T. The game analyzed in this paper allows for both of these effects. The following results show that neither of these effects dominates the other in the sense that the Pareto frontier of the equilibrium set is invariant with respect to T, whereas the lower bound converges to the Bertrand outcome.

In the following, the game with T planning periods is denoted as $\Gamma(T)$. We focus on Markov perfect equilibria (MPEs),[7] and make two additional assumptions. The marginal costs are identical $\gamma_1 = \cdots = \gamma_n$, as competitive pricing is not well-defined in the case of heterogenous marginal costs, and $c_i = \gamma_i$ for all $i \in N$ for notational convenience (otherwise, the price is bounded below by c, see also Footnote 6).

Our first result relates to the T-invariance observed by Romano and Yildirim (2005). It shows that the set of outcomes of $\Gamma(T)$ is a *subset* of the outcomes of $\Gamma(T+1)$. Hence, the outcomes of $\Gamma(T)$ are included in the set of outcomes of $\Gamma(T+l)$ for all $l \geq 1$, i.e. the earlier the firms start planning the production period, the larger the set of possible equilibrium outcomes.

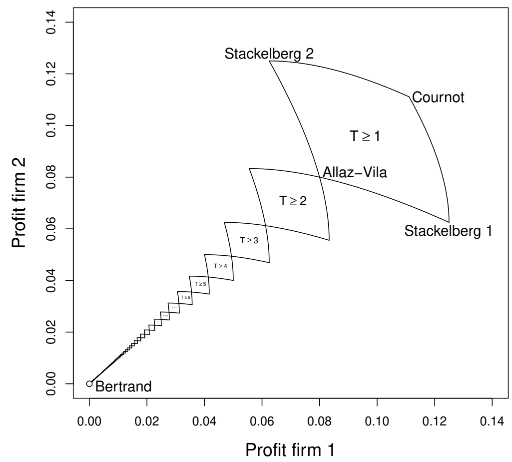
Lemma 4.4. *For all $T \geq 1$ and any payoff profile $\Pi \in \mathbb{R}^N$ that results in an MPE of $\Gamma(T)$, there exists an MPE of $\Gamma(T+1)$ that results in the same payoff profile.*

Hence, T-invariance of accumulation games remains partially intact in capacity accumulation—equilibria do not disappear as T increases. To gain intuition, consider an MPE of $\Gamma(T)$. At the end of round T, all firms have concluded their planning phase, i.e. their plans (forward trades and pre-built capacities) are mutual best responses. We can now construct a strategy profile of $\Gamma(T+1)$ that replicates the moves in all rounds $t \leq T$ of the considered MPE, and loosely speaking everything is held constant in round $T+1$. The proof of Lemma 4.4 shows that the mutual optimality of the plans in $\Gamma(T)$ implies that the players may not gain by deviating *unilaterally* in round $T+1$ of $\Gamma(T+1)$, and based on this, we can construct an outcome-invariant strategy profile that is an MPE of $\Gamma(T+1)$.

However, while the players are best off not to deviate unilaterally from an equilibrium of $\Gamma(T)$ in round $T+1$, they may well be best off deviating from the $\Gamma(T)$-plans in $\Gamma(T+1)$ if all opponents are doing so. That is, either all firms effectively conclude their planning phase after round T or they do so after round $T+1$. This implies that all equilibria of $\Gamma(T)$ can be characterized by an integer $T^* \leq T$ which denotes the *effective* duration of the planning phase and a vector (λ_i) of implicit conjectural derivatives. By varying T^* and (λ_i), the general continuum of equilibrium

[7] The players' strategies depend on the cumulative amounts of pre-built capacity and forward trades, and on the current round index $t \leq T$, but they do not depend on the actual move sequence detailing how the cumulative amounts have been reached. By definition, all MPEs are also SPEs, and thus the set of outcomes that can result in SPEs includes at least the outcomes derived below.

Figure 2: Set of equilibrium profit profiles for $T \geq 1$ (assuming $a = b = 1$ and $\gamma = 0$)



outcomes is obtained.

Proposition 4.5. *Fix $T \geq 1$ and $\gamma = \gamma_i = c_i$ for all $i \in N$. The Pareto frontier of the equilibrium profits in $\Gamma(T)$ equates with the one of Proposition 4.1, and as T tends to infinity the minimal equilibrium price converges to marginal costs γ. Price p and profit profile $\Pi \in \mathbb{R}^N$ can result in an MPE of $\Gamma(T)$ if and only if there exist $T^* \leq T$ and $\lambda \in \left[\frac{1}{n}, 1\right]^N$ such that*

$$p = \frac{a + \beta^1 \gamma}{1 + \beta^1} \qquad \text{and} \qquad \Pi_i = \frac{\alpha_i^1 + \lambda_i^{-1}}{b} (p - \gamma)^2 \tag{19}$$

where $\beta^{T^*} = \sum_i \lambda_i^{-1}$ and for all $t \leq T^*$,

$$\beta^t = \beta^{T^*} + \left[n + (n-2)\beta^{T^*}\right] \sum_{\tau=1}^{T^*-t} (n-1)^{\tau-1}, \tag{20}$$

$$\alpha_i^t = \sum_{\tau=t+1}^{T^*} (1 + \beta^\tau - 2\lambda_i^{-1}) * (-1)^{T^*-\tau+1}. \tag{21}$$

The respective capacities/quantities and amounts of forward trades can be computed straightforwardly, as a function of $\langle T^*, (\lambda_i)\rangle$, as detailed in the proof of Proposition 4.5. A graphical representation of the set of equilibrium outcomes in a two-player case is provided in Figure 2. It is rather easy to distinguish the various components of the outcome set, i.e. the components that relate to equilibria with effective duration of the planning phase $T^* = 1$, $T^* = 2$, and so on. The set of equilibrium outcomes corresponding with any $T^* \leq T$ form a hyperrectangle in (λ_i)-space, and the intersection of succeeding hyperrectangles consists of exactly one point (i.e. the components are not disconnected nor do they overlap).[8]

Proposition 4.5 shows that the mode of competition is fully endogenous if the firms produce homogeneous goods. The time invariance of accumulation implies that the Pareto-frontier establishing the Cournot-Stackelberg endogeneity remains in the equilibrium set, while the the competitiveness of forward trades implies that the lower bound approaches the Bertrand equilibrium as T grows. The next section examines whether this continues to hold true as we allow for heterogeneous goods.

5 Duopoly with heterogenous goods

The assumption that goods be perfect substitutes is invalid if mere transportation costs are taken into account, and obviously it is outright wrong if complements such as coal and iron ore are considered. Interactions of firms forward selling heterogenous goods are unexplored in the existing literature, however. By a similar token, the robustness of the convergence toward Bertrand competition as T grows, as described by Allaz and Vila, with respect to relaxations of perfect homogeneity is an open question. I will analyze these issues by assuming the following inverse demands for $i = 1, 2$ and $j = 3 - i$.[9]

$$p_i(x_i, x_j) = a - b_1 x_i - b_2 x_j \tag{22}$$

The case of complementary goods ($b_2 < 0$) induces a constituent game that exhibits strategic complements, and thereby may relate to the forward trade model of price setting duopolists analyzed by Mahenc and Salanié (2004). They found that the firms go long in the futures market (forward buying their own output). This will not result in our model, and in this way, the case of quantity setting duopolists producing com-

[8]The equilibrium outcome corresponding with T^* and $\lambda_i = 1/n \forall i$ equates with the outcome corresponding with $T^* + 1$ and $\lambda_i = 1 \forall i$.

[9]I focus on duopolies for tractability. In case goods are heterogenous, the equation system defined by the first order conditions has to be solved directly, which becomes intractable for $n > 2$ and prevents closed form solutions for general n.

plements differs from price setting duopolists producing substitutes (although both games exhibit strategic complements). Technically, the analysis is very similar to the one made above, and for this reason, the main results are stated immediately.

Proposition 5.1. *Assume $|N| = 2$, $|b_1| > |b_2|$, and $p_i = a - b_1 x_i - b_2 x_j$ for all $i \in N$. For all $T^* \leq T$ and $\lambda_i \in [1 - b_2^2/2b_1^2, 1]$, there exists an equilibrium of the T-round game inducing the prices*

$$p_i = \frac{(1 + b_1\mu_j^1)(a + b_1\mu_i^1\gamma_i + b_2\mu_j^1\gamma_j) - b_2\mu_j^1(a_j + b_1\mu_j^1\gamma_j + b_2\mu_i^1\gamma_i)}{(1 + b_1\mu_i^1)(1 + b_1\mu_j^1) - b_2^2\mu_i^1\mu_j^1} \tag{23}$$

for all $i \in N$ (provided $p_i > c_i$ for all i), using $\mu_i^{T^*} = \lambda_i^{-1} b_1^{-1}$ and

$$\forall t \leq T^* : \ r_i^t = \frac{-b_1(1 + b_1\mu_j^t) + b_2^2\mu_j^t}{(1 + b_1\mu_i^t)(1 + b_1\mu_j^t) - b_2^2\mu_i^t\mu_j^t} \quad \forall t < T^* : \ \mu_i^t = -\mu_i^{t+1} - 1/r_i^{t+1}.$$

The following is implied.

1. *As T^* tends to infinity, μ_i^1 converges to $\bar{\mu} = (b_1^2 - b_2^2)^{-1/2}$ for all i. This limit is strictly below $\mu^B = b_1/(b_1^2 - b_2^2)$ which is implicit in price competition.*

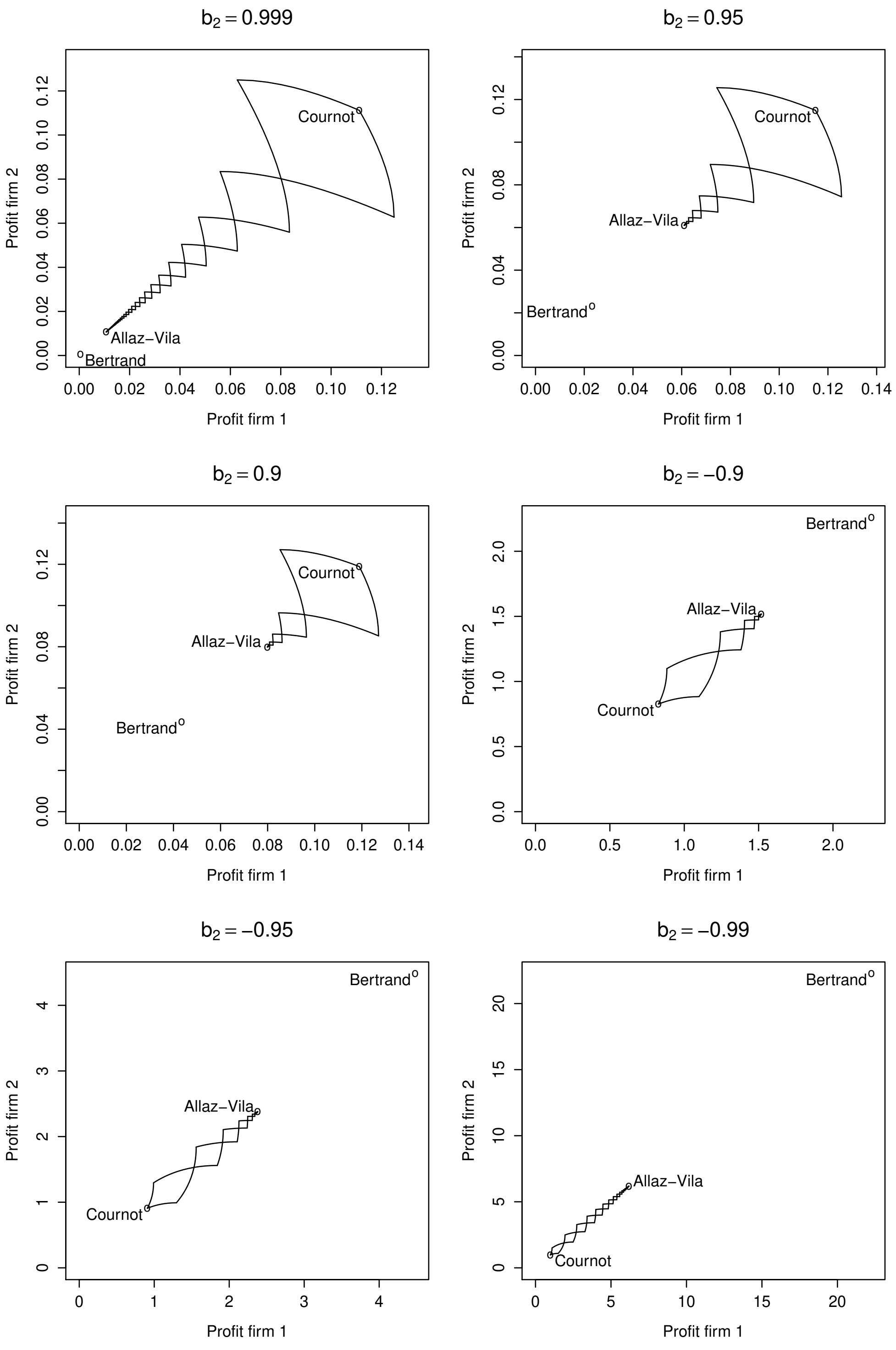
2. *The amount sold forward in planning period t of an equilibrium inducing price p_i equates with $(p_i - \gamma_i) * (\mu_i^t - \mu_i^{t+1})$ and is strictly positive.*

3. *Fix $\lambda_1 = \lambda_2$. The equilibrium prices are decreasing in T^*.*

Figure 3 depicts the equilibrium profits for a variety of cases. The main observation is that the iterated forward trade effect—convergence toward competitive pricing as T approaches infinity—is not robust to relaxing homogeneity of goods. If b_2 drops to $b_2/b_1 = 0.9$, then the profits converge half-way between Cournot and Bertrand profits, and in this case, the Bertrand profits are far from being competitive already. Hence, the observation that forward trading induces competitive pricing if T grows cannot be confirmed in general.

In turn, the results show that forward trading of quantity setting duopolists benefits consumers even when the goods are complements. This contrasts with the results Mahenc and Salanié (2004), who analyzed price setting duopolists producing substitutes and found that the firms go long and tacitly collude in equilibrium. They argued

Figure 3: Equilibrium profits if goods are heterogenous ($a = b_1 = 1$, $\gamma_1 = \gamma_2 = 0$)



Note: The "Allaz-Vila" price refers to the limit as T approaches infinity.

that this observation relates to the fact that price competition exhibits strategic complements, but as $b_2 < 0$ in Proposition 5.1 exhibits strategic complements, too, this relation is weaker than suspected. For, we observe convergence to price competition (and lower prices, see Point 3 of Proposition 5.1) even in this case, while Mahenc and Salanié observe the opposite in their price setting game. Finally, since Bertrand competition benefits both consumers (lower prices) and firms (higher profits) in case the goods are complements, we can conclude that forward trading raises welfare regardless of whether goods are complements or substitutes—provided firms set quantities.

6 Concluding discussion

The paper presents the first analysis of an industry where firms do both, pre-order input and forward contract output, prior to the production period. Pre-ordering input allows firms to pre-build capacity (cumulatively, following e.g. Kreps and Scheinkman, 1983, and Saloner, 1987) and forward contracting with say retailers allows firms to improve their short-term marginal revenue and hence their strategic positioning in the production period (the "forward trade effect" of Allaz and Vila, 1993).

The main contributions can be summarized as follows. The Cournot-Stackelberg endogeneity (Saloner, 1987; Pal, 1991; Romano and Yildirim, 2005) is shown to be robust to capacity accumulation (as opposed to quantity accumulation) and generalized to oligopolistic industries, which shows that the endogeneity is incomplete in oligopolistic games of accumulation. The T-round Allaz-Vila prices are derived for duopolists producing heterogenous goods, which allowed me to show that they do not converge to the Bertrand prices in general, and yet forward trades are welfare improving even in case the goods are complements. Finally, the outcome set of the joined model is characterized using a novel indexation of oligopoly equilibria and it is shown to consist of T hyperrectangles stretching from the Cournot-Stackelberg frontier on one end to the Allaz-Vila price on the other end (which generalizes the Cournot-Stackelberg endogeneity).

Since the Allaz-Vila price converges to the Bertrand price in case the goods are homogenous, the "mode of competition" may therefore be entirely independent of "objective differences" between markets—to the degree that different focal points or historical standards do not constitute objective differences—and different modes are fully self-sustaining in the sense that repeated interaction and complex penal codes are not required.

The assumptions made above are fairly standard. For example, linear demand and constant marginal costs are standard and can be generalized in several ways, although closed-form solutions may then be unavailable (note also the conditions for existence of Cournot equilibria, e.g. Novshek, 1980). Similarly, the assumption that capacity accumulates, i.e. that capacity investments represent sunk costs at later stages, is standard. An issue that deserves more discussion relates to the point raised by Pal (1996) who argues, in the context of two-period Cournot games, that asymmetric equilibria seem implausible in symmetric games. Note that we do not argue that say specific Stackelberg equilibria necessarily result in specific industries, but that asymmetric outcomes may be sustained in stationary equilibrium points of industries with ex-ante symmetric firms. Such industries may well have historically established leadership and follower assignments, even if firms do not move asynchronously, and since the equilibria are shown to be self-sustaining, such role assignments need not disappear over time even if firm owners or managers tend to think myopically (low δ, hence no Folk theorem) or tend to act stationarily or forward-looking (which rules out the possibility of retaliations against firms that deviated from acting as say followers).

To conclude, let me recall that the present paper analyzed a model of competition, and as such, it does not rationalize everything. That is, it generates several falsifiable predictions. As Figures 2 and 3 show, the relative profits of different firms are correlated and bounded in equilibrium (though the correlation weakens as the number of firms grows). Similarly, there is a falsifiable mapping from implicit conjectural derivatives ($= 1 - \lambda_i$ as discussed above) to the set of outcomes. Empirical tests of these predictions may be a topic for further research.

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A Proofs of results in Section 3

Proof of Lemma 3.1 Fix $(\mathbf{z},\mathbf{y})$. For all $i \in N$, define the intervals $X_i' = [y_i, \bar{x}_i]$ such that $a - b\bar{x}_i = 0$. The best response of i to $\mathbf{x}_{-i} \in \mathbf{X}_{-i}$ is unique, continuous in $\mathbf{x}_{-i}$, and necessarily satisfies $x_i \in X_i'$. Since X_i' is compact, closed, and convex for all $i \in N$, existence of a pure strategy equilibrium $(x_i^*)_{i=1,\ldots,n}$ follows from Brouwer's fixed point theorem. Eq. (8) represents the necessary conditions for a profile of mutual best responses, which therefore have to be satisfied in any equilibrium $\mathbf{x}^*$. We have to show that the equilibrium is unique. For any equilibrium $\mathbf{x}^*$ there exist sets $N^-, N^+ \subseteq N$, with $N^- \cap N^+ = \emptyset$, such that

$$x_i^* < z_i \text{ for } i \in N^-, \qquad x_i^* = z_i \text{ for } i \notin N^- \cup N^+, \qquad x_i^* > z_i \text{ for } i \in N^+. \tag{24}$$

Define m and k such that, relabeling the players appropriately, the sets are $N^- = \{1,\ldots,m\}$ and $N^+ = \{k+1,\ldots,n\}$. Using the necessary condition Eq. (8), $\mathbf{x}^*$ solves a linear $(m+n-k)$–dimensional equation system with the solution $(x_1^*,\ldots,x_m^*,x_{k+1}^*,\ldots,x_n^*)$.

$$\begin{pmatrix} x_1^* \\ \cdot \\ x_m^* \\ x_{k+1}^* \\ \cdot \\ x_n^* \end{pmatrix} = \begin{pmatrix} \frac{2s}{s+1} & \cdot & -\frac{2}{s+1} & -\frac{2}{s+1} & \cdot & -\frac{2}{s+1} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -\frac{2}{s+1} & \cdot & \frac{2s}{s+1} & -\frac{2}{s+1} & \cdot & -\frac{2}{s+1} \\ -\frac{2}{s+1} & \cdot & -\frac{2}{s+1} & \frac{2s}{s+1} & \cdot & -\frac{2}{s+1} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -\frac{2}{s+1} & \cdot & -\frac{2}{s+1} & -\frac{2}{s+1} & \cdot & \frac{2s}{s+1} \end{pmatrix} \times \begin{pmatrix} \frac{a+by_1}{2b} \\ \cdot \\ \frac{a+by_m}{2b} \\ \frac{a-c_{k+1}+by_{k+1}}{2b} \\ \cdot \\ \frac{a-c_n+by_n}{2b} \end{pmatrix} \tag{25}$$

where $s := m+n-k$. In turn, any pair N^-, N^+ defines a unique set of necessary conditions Eq. (8) and has a unique solution $\mathbf{x}^*$ through Eq. (25). Hence, the equilibrium is unique if N^- and N^+ are unique. Fix any pair N^-, N^+ such that an equilibrium is induced. By Eq. (8), N^- contains the players $i \in N$ that have the lowest values of $\frac{a+by_i}{2b}$. Hence, if $j \notin N^-$, then he must have a greater value of $\frac{a+by_j}{2b}$.

$$i \in N^- \text{ and } j \notin N^- \qquad \Rightarrow \qquad \frac{a+by_i}{2b} < \frac{a+by_j}{2b}. \tag{26}$$

Also by Eq. (8), N^+ contains the players $i \in N$ with the largest $\frac{a-c_i+by_i}{2b}$, which implies

$$i \in N^+ \text{ and } j \notin N^+ \qquad \Rightarrow \qquad \frac{a-c_i+by_i}{2b} > \frac{a-c_j+by_j}{2b}. \tag{27}$$

If $(\mathbf{z},\mathbf{y})$ and $\mathbf{c}$ are given, then the m players with the lowest values of $\frac{a+by_j}{2b}$ are identified, as are the $n-k$ players with the highest values of $\frac{a-c_i+by_i}{2b}$. Hence, the sets N^-,N^+ are uniquely defined through pairs (m,k) satisfying $m := |N^-|$ and $k := n-|N^+|$, and thus, for any (m,k), there is a unique solution $\mathbf{x}^*$ satisfying Eqs. (8) and (25). Now pick any equilibrium $\mathbf{x}^*$ and for contradiction assume that it is not unique. An equilibrium $\mathbf{x}^{*\prime} \neq \mathbf{x}^*$ exists which is characterized by N'^-,N'^+ where

$$x_i^{*\prime} < z_i \text{ for } i \in N'^-, \qquad x_i^{*\prime} = z_i \text{ for } i \notin N'^- \cup N'^+, \qquad x_i^{*\prime} > z_i \text{ for } i \in N'^+. \tag{28}$$

Further, define $m' := |N'^-|$ and $k' := n-|N'^+|$. If $\mathbf{x}^{*\prime} \neq \mathbf{x}^*$, then $(m',n') \neq (m,n)$ follows from the previous argument. Without loss of generality assume $m' > m$, which implies $N'^- \supset N^-$. We first show that $N'^+ \supset N^+$ follows. Define $\Delta x_i^* := x_i^{*\prime} - x_i^*$ for all i. This implies $\Delta x_i^* < 0$ for all $i \in N'^- \setminus N^-$. Putting the solution Eq. (25) for N'^-,N'^+ in relation to that for N^-,N^+, the condition $\Delta x_i^* < 0$ for all $i \in N'^- \setminus N^-$ implies $\Delta x_i^* < 0$ for all $i \in N^-$. This can be optimal for these firms, by the necessary condition Eq. (8), only if there exists $j \notin N'^-$ such that $\Delta x_j^* > 0$. By Eq. (25), again, this can result only if $N'^+ \supset N^+$. Now, using the necessary conditions Eq. (8), we obtain for all $i \in N'^-$

$$x_i^{*\prime} = \frac{a+by_i}{2b} - \frac{1}{2}\sum_{j\neq i} x_j^{*\prime}, \qquad\qquad x_i^* \leq \frac{a+by_i}{2b} - \frac{1}{2}\sum_{j\neq i} x_j^*$$

(the inequality on the right-hand side is an equality if $i \in N^-$, but not necessarily for $i \in N'^- \setminus N^-$), and similarly for all $i \in N'^+$,

$$x_i^{*\prime} = \frac{a-c_i+by_i}{2b} - \frac{1}{2}\sum_{j\neq i} x_j^{*\prime}, \qquad\qquad x_i^* \geq \frac{a-c_i+by_i}{2b} - \frac{1}{2}\sum_{j\neq i} x_j^*.$$

Hence, for all $i \in N'^{-}$ and $j \in N'^{+}$, we obtain

$$x_i^* + \frac{1}{2}\sum_{k\neq i} x_k^* \leq x_i^{*\prime} + \frac{1}{2}\sum_{k\neq i} x_k^{*\prime}, \qquad x_j^* + \frac{1}{2}\sum_{k\neq j} x_k^* \geq x_j^{*\prime} + \frac{1}{2}\sum_{k\neq j} x_k^{*\prime} \tag{29}$$

and thus

$$x_j^{*\prime} - x_j^* + \frac{1}{2}(x_i^{*\prime} - x_i^*) \leq \frac{1}{2}\sum_{k\neq i,j} (x_k^* - x_k^{*\prime}) \leq x_i^{*\prime} - x_i^* + \frac{1}{2}(x_j^{*\prime} - x_j^*). \tag{30}$$

This can be satisfied only if $x_j^{*\prime} - x_j^* \leq x_i^{*\prime} - x_i^*$, which contradicts $\Delta x_i^* < 0$ and $\Delta x_j^* > 0$. Hence, the equilibrium $\mathbf{x}^*$ is unique. □

Proof of Lemma 3.2 Assume an SPE $(\mathbf{z}^*, \mathbf{y}^*, \mathbf{x}^*)$ exists where $x_i^*(\mathbf{z}^*, \mathbf{y}^*) \neq z_i^*$ for some $i \in N$ (the two cases $x_i^* < z_i^*$ and $x_i^* > z_i^*$ are distinguished below). We show that i benefits by deviating unilaterally to $z_i' = x_i^*(\mathbf{z}^*, \mathbf{y}^*)$ in stage 1. By Lemma 3.1, the choices of $\mathbf{x}^*(z_i', \mathbf{z}_{-i}^*, \mathbf{y}^*)$ following this unilateral deviation are unique, and hence the following quantities, which are mutual best responses, must be played: $x_j^*(z_i', \mathbf{z}_{-i}^*, \mathbf{y}^*) = x_j^*(\mathbf{z}^*, \mathbf{y}^*)$ for all $j \neq i$, and $x_i^*(z_i', \mathbf{z}_{-i}^*, \mathbf{y}^*) = z_i'$. The implied gain of player i is $-(x_i^* - z_i^*)\gamma_i$ if $x_i^*(\mathbf{z}^*, \mathbf{y}^*) < z_i^*$, and $(x_i^* - z_i^*)(c_i - \gamma_i)$ if $x_i^*(\mathbf{z}^*, \mathbf{y}^*) > z_i^*$. Hence, the initially assumed strategy profile is not an SPE under the assumptions of the Lemma. □

Proof of Lemma 3.4 The proof is made by logical induction starting in round T. Assuming $x_{-T} \in \mathbb{R}$ denotes the aggregate quantity of the players acting in previous rounds, the first order condition for all $i \in N_T$ is $\Pi_i' = p - bx_i - \gamma_i = 0$, and hence the aggregate quantity of all $i \in N_T$ is $x_T^a = (a - bx_{-T} - \gamma_i)/b * |N_T|/(1 + |N_T|)$. Fix $t \leq T$. Now assume that the aggregate quantity of all players acting in rounds $t' \geq t$ can be expressed as a function of the aggregate quantity x_{-t} of the players acting in earlier rounds as follows.

$$x_t^a = \frac{\beta_t}{1+\beta_t} \cdot \frac{1}{b}\left(a - bx_{-t} - \gamma_i\right). \tag{31}$$

Using $\beta_T = |N_T|$, this applies for $t = T$. The first order condition for all $i \in N_{t-1}$ is

$$\Pi_i' = p - \frac{1}{\beta_t} \cdot bx_i - \gamma_i = 0, \tag{32}$$

which allows us, in combination with Eq. (31) and $p = a - b\sum_i x_i$, to express the price as a function of $x_{-(t-1)}$ (the aggregate quantity prior to round $t-1$) as follows.

$$p - \gamma_i = \frac{1}{1+\beta_t * (|N_{t-1}|+1)} \cdot \big(a - \gamma_i - bx_{-(t-1)}\big) \tag{33}$$

Substituting this for $p - \gamma_i$ in Eq. (32), again using Eq. (31), yields

$$x^a_{t-1} = \frac{\beta_t * (|N_{t-1}|+1)}{1+\beta_t * (|N_{t-1}|+1)} \cdot \frac{1}{b} \cdot \big(a - \gamma_i - bx_{-(t-1)}\big). \tag{34}$$

Hence, $\beta_{t-1} = \beta_t * (|N_{t-1}|+1) = \prod_{t'=t-1}^{T}(|N_{t'}|+1)$, which thus applies for all $t \leq T$. For all $t \leq T$ and all $i \in N_t$, λ_i in Eq. (13) corresponds with β_{t+1}^{-1} in Eq. (32), and thus it confirms the first part of the lemma. The second part follows, since an equilibrium corresponding with (λ_i) exists under the conditions of Proposition 4.1 if $\lambda_i \geq \frac{1}{n}$ for all $i \in N$. □

Proof of Proposition 3.5 *Point 1.* Fix any SPE $(\mathbf{z}^*, \mathbf{y}^*, \mathbf{x}^*)$, let p denote the corresponding market price, and define $r_i := a - b(z^*_{-i} + 2z^*_i - y^*_i)$ for all i. By Lemma 3.2, $x^*_i(\mathbf{z}^*, \mathbf{y}^*) = z^*_i$ for all $i \in N$ holds true along the path of play in any SPE. Thus, $p = a - b\sum_i x^*_i = a - b\sum_i z^*_i$ and $r_i = a - b(z_{-i} + 2z_i - y_i)$ imply that the following holds true in any SPE.

$$p = b(z^*_i - y^*_i) + r_i \qquad \forall i \in N \tag{35}$$

In any SPE, $\frac{\partial \Pi_i}{\partial z_i} = (p - bz^*_i) * \frac{\partial x_i}{\partial z_i} - \gamma_i \leq p - bz^*_i - \gamma_i \leq 0$ holds true for $\partial z_i > 0$. Due to $y_i = 0$ and Eq. (35), $r_i \leq \gamma_i < c_i$ follows for all $i \in N$. By Lemma 3.1, $r_j < c_j$ implies $\partial x_j(\mathbf{z}^*, \mathbf{y}^*)/\partial z_i = 0$ for $\partial z_i < 0$, and combined, we obtain $\frac{\partial x_j(\mathbf{z}^*, \mathbf{y}^*)}{\partial z_i} = 0$ for all $i, j \in N$. This implies that the Cournot condition

$$\frac{\partial \Pi_i}{\partial z_i} = p - bz^*_i - \gamma_i = 0 \qquad \forall i \in N. \tag{36}$$

is necessary. Since it also is sufficient, the unique solution is $z^*_i = \frac{1}{(n+1)b} * \big(a + \sum_{j \in N} \gamma_j\big) - \frac{\gamma_i}{b}$ for all $i \in N$.

Point 2. Fix a strategy profile $(\mathbf{z}, \mathbf{y}, \mathbf{x})$. Due to the assumption of sufficiently similar (γ_i), we can focus on the case that all capacities are positive, which implies that the induced market price is above marginal costs γ_i for all $i \in N$. Let p denote the induced market price. Hence, there exists a profile $(\lambda_i) \in \mathbb{R}^N_+$ such that $p - \lambda_i b z_i - \gamma_i = 0$ for all $i \in N$. It is easy to verify that $p - \lambda_i b z_i - \gamma_i = 0 \forall i \in N$ induces the equilibrium outcome described in the proposition. Define $k := \#\{j \in N | \lambda_j = 1\}$.

First we show that if $(\mathbf{z}, \mathbf{y}, \mathbf{x})$ is an SPE, then $\lambda_i \in \left[\frac{1}{k+1}, 1\right]$ for all $i \in N$. To begin with, note that $x_i > z_i$ for some $i \in N$ can result in equilibrium only if $k = 1$, $\lambda_i = 1$, and $\lambda_j = \frac{1}{2}$ for all $j \neq i$. Otherwise, some $j \neq i$ would benefit by deviating unilaterally toward a higher z_j in stage 1 (as j's increase in stage 1 would crowd out i's increase in stage 2). This case implies $\lambda_i \in \left[\frac{1}{k+1}, 1\right]$ for all $i \in N$. In all alternative SPEs, capacity must be fully pre-built in stage 1 (along the path of play). The condition that $(\mathbf{z}, \mathbf{y}, \mathbf{x})$ is an SPE implies $\lambda_i \leq 1$ (for all $i \in N$), since i would otherwise benefit by increasing z_i unilaterally in stage 1 (note that no opponent responds to a small increase of z_i by decreasing quantity since $p > \gamma_j \Leftrightarrow \lambda_j > 0$ for all $j \neq i$). It implies $\lambda_i \geq \frac{1}{k+1}$ (for all $i \in N$), since i would otherwise be best off cutting capacity z_i unilaterally in stage 1 (note that k players respond to i's capacity cut by increasing quantity in stage 2).

Second we show that if $\lambda_i \in \left[\frac{1}{k+1}, 1\right]$ for all $i \in N$, then $(\mathbf{z}, \mathbf{y}, \mathbf{x})$ is an SPE. To begin with, $p - \lambda_i b z_i - \gamma_i = 0$ for all i implies (by Lemma 3.1) that quantities equate with capacities in the unique stage 2 equilibrium. This confirms that capacity is fully pre-built in this case. Now, $\lambda_i \in \left[\frac{1}{k+1}, 1\right]$ for all $i \in N$ implies $\exists i \in N : \lambda_i = 1$, i.e. $k \geq 1$. No player may benefit from extending capacity unilaterally in stage 1 because of $\lambda_i \leq 1$ for all $i \in N$. Also, no player may benefit from cutting capacity since $\lambda_i \geq \frac{1}{k+1}$ for all i with $\lambda_i < 1$ and $\lambda_i \geq \frac{1}{k}$ for all i with $\lambda_i = 1$ (note that k and $k-1$ players, respectively, respond to the capacity cut by extending quantity in stage 2). $\square$

B Results on two-stage games with homogeneous goods

Proof of Proposition 4.1 Fix a strategy profile $(\mathbf{z}, \mathbf{y}, \mathbf{x}^*)$ where $\mathbf{x}^*(\mathbf{z}, \mathbf{y})$ is the stage-2 Nash equilibrium for all $(\mathbf{z}, \mathbf{y})$. Let p denote the induced market price. We focus on SPEs where $r_i = c_i$ for all $i \in N$ results along the path of play (it will be shown that such SPEs exist, and it is easy to see that the set of outcomes of SPEs where $r_i \neq c_i$ for at least one $i \in N$ is a subset of the outcomes derived in the following). To abbreviate notation of directional derivatives, let $\nabla_{(\Delta z_i, \Delta y_i)} f(\mathbf{z}, \mathbf{y})$ denote the change of f (which could be x_i, x_j, or Π_i) if i changes (z_i, y_i) along $(\Delta z_i, \Delta y_i)$. By Lemma

3.2, directions $(\Delta z_i, \Delta y_i)$ that induce $\nabla_{(\Delta z_i,\Delta y_i)} x_i^*(\mathbf{z},\mathbf{y}) \neq 0$ are generally sub-optimal. Given the stage 2 solutions $x_i^*(\mathbf{z},\mathbf{y})$ from Eq. (25), it follows that we can focus on directions $(\Delta z_i, \Delta y_i)$ such that either (i) $\Delta z_i > 0$ and $\Delta y_i \leq 2\Delta z_i$, or (ii) $\Delta z_i < 0$ and $\Delta y_i \leq \frac{n+1}{n}\Delta z_i$. It further holds that if a deviation in any direction is profitable, then either of the extreme deviations where Δy_i is bound by an equality must be profitable. Consider first $\Delta z_i > 0$ and $\Delta y_i = 2\Delta z_i$. By Eq. (25), this implies $\nabla_{(\Delta z_i,\Delta y_i)} x_i^*(\mathbf{z},\mathbf{y}) = 0$ and $\nabla_{(\Delta z_i,\Delta y_i)} x_j^*(\mathbf{z},\mathbf{y}) = 0$ for all $j \neq i$, and therefore

$$\nabla_{(\Delta z_i,\Delta y_i)} \Pi_i(\mathbf{z},\mathbf{y}) = p - bz_i - \gamma_i \leq 0 \tag{37}$$

has to be satisfied in equilibrium. Second consider $\Delta z_i < 0$ and $\Delta y_i = \frac{n+1}{n}\Delta z_i$. By Eq. (25), $\nabla_{(\Delta z_i,\Delta y_i)} x_i^*(\mathbf{z},\mathbf{y}) = 0$ and $\nabla_{(\Delta z_i,\Delta y_i)} x_j^*(\mathbf{z},\mathbf{y}) = \frac{1}{n}$ for all $j \neq i$ result, which implies that

$$\nabla_{(\Delta z_i,\Delta y_i)} \Pi_i(\mathbf{z},\mathbf{y}) = -\left(p - \tfrac{1}{n} bz_i - \gamma_i\right) \leq 0 \tag{38}$$

has to be satisfied. In turn, all $(\mathbf{z},\mathbf{y})$ that satisfy both conditions are extended by the stage-2 equilibria $\mathbf{x}^*$ to an SPE. Hence, the necessary and sufficient condition for SPE (conditional on the initial assumption $r_i = c_i$ for all $i \in N$) can be expressed as follows.

$$\forall i \in N \, \exists \lambda_i \in \left[\tfrac{1}{n}, 1\right]: \qquad p - \lambda_i b z_i - \gamma_i = 0 \tag{39}$$

Hence, $bz_i = \lambda_i^{-1} p - \lambda_i^{-1}\gamma_i$ for all i, and since $p = a - b\sum_i z_i$ in equilibrium, this implies $p = \left(a + \sum_i \lambda_i^{-1}\gamma_i\right) / \left(1 + \sum_i \lambda_i^{-1}\right)$. Since $\lambda_i b z_i = p - \gamma_i$, see Eq. (39), it follows that

$$\lambda_i b z_i = \frac{1}{1 + \sum_j \lambda_j^{-1}} \left(a - \gamma_i + \sum_j \lambda_j^{-1}(\gamma_j - \gamma_i)\right) \tag{40}$$

and that $z_i > 0$ are positive for all $i \in N$ and all $(\lambda_i) \in \left[\frac{1}{n}, 1\right]^N$ if the (γ_i) are sufficiently similar. Finally, $r_i = p - b(z_i - y_i)$, see Eq. (35), and $p = \lambda_i b z_i + \gamma_i$, see Eq. (39), imply that the initial condition $r_i = c_i$ is satisfied if $by_i = c_i - \gamma_i + (1 - \lambda_i) b z_i$. Since $\lambda_i \in \left[\frac{1}{n}, 1\right]$, appropriate $y_i \leq z_i$ exist whenever c_i is sufficiently close to γ_i. It is easy to see that these (y_i) do not contradict payoff maximization, since increasing y_i implies $r_i > c_i$, decreasing y_i is payoff irrelevant, and directional variations of (z_i, y_i) are not profitable due to the arguments made above. □

Proof of Lemma 4.2 By a standard argument of upper hemicontinuity it follows that the set of SPEs constructed for the case $\mathbf{c} \approx \boldsymbol{\gamma}$ in Prop. 4.1 remain SPEs when

$\mathbf{c} = \boldsymbol{\gamma}$. Hence, the set of equilibrium outcomes (prices and profits) in case $\mathbf{c} = \boldsymbol{\gamma}$ contains all equilibrium outcomes that may result if $\mathbf{c} \approx \boldsymbol{\gamma}$ (where $\mathbf{c} > \boldsymbol{\gamma}$). It also follows that all outcomes that may result in SPEs in case $\mathbf{c} = \boldsymbol{\gamma}$ but not in case $\mathbf{c} \approx \boldsymbol{\gamma}$ necessitate $x_i > z_i$ for at least one $i \in N$ along the path of play. It has to be shown that the outcomes associated with such equilibria are in the set of equilibrium outcomes even if $\mathbf{c} \approx \boldsymbol{\gamma}$. This follows, as it can be shown that all SPEs where $x_i > z_i$ for at least one $i \in N$ along the path of play induce the Allaz-Vila outcome (price and profits). The details are skipped. □

Proof of Lemma 4.3 We show first that all payoff profiles Eq. (13) associated with some $(\lambda_i) \in [0,1]^N$ where $\min_{i \in N} \lambda_i < \frac{1}{n}$ are Pareto dominated by some $(\lambda_i') \in [0,1]^N$ satisfying $\lambda_i' \geq \lambda_i$ for all $i \in N$ and $\lambda_i' > \lambda_i$ for at least one $i \in N$. Using $r = 0$, the payoff of $i \in N$ at (λ_i) can be expressed as, using $h_i(r) = (r + \lambda_i^{-1})/\lambda_i^{-1}$,

$$\Pi_i(r) = \frac{1}{\lambda_i^{h_i(r)} b} \cdot \left(\frac{a - \gamma_i}{1 + \sum_j \lambda_j^{-h_j(r)}} \right)^2 . \tag{41}$$

The first derivative of $\Pi_i(r)$ with respect to r is proportional to

$$\frac{d\Pi_i(r)}{dr} \propto -\lambda_i \cdot \ln \lambda_i + 2 \cdot \frac{\sum_j \ln \lambda_j}{1 + \sum_j \lambda_j^{-1}} \tag{42}$$

and hence negative if $\lambda_i = 1$ (in this case, some $j \neq i$ exists such that $\lambda_j < 1/n < 1$). Considering the case $\lambda_i < 1$, the aforementioned derivative of Π_i is negative if

$$\lambda_i \cdot \left(1 + \sum_j \lambda_j^{-1}\right) < 2 \cdot \sum_j \log_{\lambda_i} \lambda_j, \tag{43}$$

which is generally satisfied if $\min_i \lambda_i < \frac{1}{n}$. As a result of $d\Pi_i/dr < 0$ for all $i \in N$ if $\min_i \lambda_i < \frac{1}{n}$, for any $(\lambda_i) \in [0,1]^N$ where $\min \lambda_i < \frac{1}{n}$ there exists $(\lambda_i') \in \left[\frac{1}{n}, 1\right]^N$ such that the payoff profile associated with (λ_i) is Pareto dominated by the one associated with (λ_i'). By Lemma 3.4 it thus follows that all outcomes of Stackelberg games are either in the set of outcomes compatible with Prop. 4.1 or Pareto dominated by one of those. □

C Results on T-stage games with homogeneous goods

The proofs in this section use the following extended notation. The set of "states" is denoted by $\overline{T} \times \mathbf{H}$ with $\overline{T} = \{1, \ldots, T\}$ and $\mathbf{H} = \mathbf{Z} \times \mathbf{Y}$. Given any state (t,h), the accumulated capacity is denoted as $\bar{z}_i(h)$, and following Romano and Yildirim (2005) we assume prior capacity investments are sunk. The capacity choices are therefore strategies satisfying

$$z_i : \overline{T} \times H \to Z_i \qquad \text{s.t.} \qquad z_i(t,h) \geq \bar{z}_i(h) \quad \forall (t,h). \tag{44}$$

Similarly, the accumulated amount of forward trades is denoted as $\bar{y}_i(h)$ for $i \in N$, and following Allaz and Vila (1993), forward trades are cumulative, too.

$$y_i : \overline{T} \times H \to Y_i \qquad \text{s.t.} \qquad y_i(t,h) \geq \bar{y}_i(h) \quad \forall (t,h) \tag{45}$$

Finally, the quantity choice has to match the forward trades.

$$x_i : H \to \mathbb{R} \qquad \text{s.t.} \qquad x_i(h) \geq \bar{y}_i(h) \quad \forall h \tag{46}$$

Strategies are tuples (z_i, y_i, x_i) for all $i \in N$.

Proof of Lemma 4.4 Fix $T \geq 1$ and any MPE $(\mathbf{z}, \mathbf{y}, \mathbf{x})$ of $\Gamma(T)$. Construct a strategy profile $(\mathbf{z}', \mathbf{y}', \mathbf{x}')$ of $\Gamma(T+1)$ as follows. (i) For all states (t,h) associated with some $t \leq T$ maintain the strategies from $\Gamma(T)$, i.e. $z_i'(t,h) = z_i(t,h)$ and $y_i'(t,h) = y_i(t,h)$ for all $t \leq T$ and $h \in H$. (ii) In the production period, set x_i according to the unique equilibrium $\mathbf{x}^*(h)$, for all h, derived in Lemma 3.1. (iii) For all states (t,h) associated with $t = T+1$, set z_i equal to the greater of $\bar{z}_i(h)$ and $x_i(h)$, i.e. $z_i'(T+1,h) = \max\{\bar{z}_i(h), x_i(h)\}$, and set $y_i'(T+1,h)$ such that (for all i and h) $p^*(h) - b(x_i(h) - y_i'(T+1,h)) \leq \gamma_i$ where $p^*(h) := a - b\sum_j x_j(h)$. Appropriate $y_i'(T+1,h) \geq \bar{y}_i(h)$ exist for all h since, by Lemma 3.1, the $x_i(h)$ chosen in any SPE imply $p^*(h) - b(x_i(h) - \bar{y}_i(h)) \leq c_i = \gamma_i$ for all i.

Note that $(\mathbf{z}', \mathbf{y}', \mathbf{x}')$ is outcome equivalent to $(\mathbf{z}, \mathbf{y}, \mathbf{x})$. It remains to be shown that it is an MPE of $\Gamma(T+1)$. By construction the latter is satisfied for the production period and also with respect to the y_i' chosen in states (t,h) associated with round $t = T+1$ (they are payoff irrelevant). By Lemma 3.1, the fact that $(\mathbf{z}, \mathbf{y}, \mathbf{x})$ is an

MPE of $\Gamma(T)$ implies $p^* - b(x_i(h) - \bar{y}_i(h)) \in [0, \gamma_i]$ for all i and h, and this in turn implies that $z'_i(T+1,h) = \max\{\bar{z}_i(h), x_i(h)\}$ are mutual best responses in the states associated with period $T+1$. Finally, note that the construction of $(\mathbf{z}', \mathbf{y}', \mathbf{x}')$ implies that for all states (t,h) with $t = T$ and all action profiles viable in this state, the profiles of continuation payoffs are identical under $(\mathbf{z}', \mathbf{y}', \mathbf{x}')$ and $(\mathbf{z}, \mathbf{y}, \mathbf{x})$. Hence, action profiles that constitute mutual best responses in state (T,h) of $\Gamma(T)$ must also be best responses in state (T,h) of $\Gamma(T+1)$, and by backward induction, this applies in all states (t,h) with $t \leq T$. □

Proof of Proposition 4.5 Fix any $T^* \leq T$. The following derives the conditions under which a given outcome can result in an MPE of $\Gamma(T^*)$ subject to the constraint that the quantity sold forward is increased in every planning period of $\Gamma(T^*)$. By Lemma 4.4, an outcome equivalent MPE of $\Gamma(T)$ exists. Hence, the set of outcomes that can be sustained in MPEs of $\Gamma(T)$ is the union of all outcomes as derived next over all $T^* \leq T$. Considering $\Gamma(T^*)$, fix any state (t,h) where $t = T^*$. Similarly to the argument leading to Eq. (39), it can be shown that the necessary and sufficient condition for MPE is (along the equilibrium path, where $z_i(T^*,h) > \bar{z}_i(h)$ can be assumed w.l.o.g.)

$$\forall i \in N \, \exists \lambda_i \in \left[\tfrac{1}{n}, 1\right] : \qquad p^*(T^*,h) - \lambda_i b(z_i(T^*,h) - \bar{y}_i(h)) - \gamma_i = 0, \tag{47}$$

where $p^*(T^*,h)$ denotes the market price resulting along the equilibrium path conditional on state (T^*,h). Using $p = a - b\sum_i z_i$ it follows that

$$p^*(T^*,h) = \frac{1}{1 + \sum_i \lambda_i^{-1}} \left(a + \sum_i \lambda_i^{-1} \gamma_i - b \sum_i \bar{y}_i(h) \right) \tag{48}$$

Define $\beta^{T^*} := \sum_i \lambda_i^{-1}$. Thus, using $\gamma = \gamma_1 = \cdots = \gamma_n$ and $\bar{y}(h) = \sum_i \bar{y}_i(h)$,

$$p^*(T^*,h) = \left(a + \beta^{T^*} \gamma - b\bar{y}(h) \right) / (1 + \beta^{T^*}). \tag{49}$$

We now turn to states (t,h) in arbitrary rounds $t \leq T^*$. Define $\bar{y}_i^{T^*}(t,h)$ as the quantity that is going to be sold forward, prior to round T^* and conditional on the current state (t,h), along the equilibrium path. The induction assumptions are (i) $p^*(t,h) = (a + \beta^t \gamma - b\bar{y}(h))/(1+\beta^t)$, which is satisfied for $t = T^*$ using β^{T^*} as defined above, and (ii) $\bar{y}_i^{T^*}(t,h) = \bar{y}_i(h) + \frac{p^* - \gamma}{b} \cdot \alpha_i^t$, which is satisfied for $t = T^*$ if $\alpha_i^{T^*} = 0$ for all $i \in N$. By definition, the profit of i in state (t,h) is $\Pi_i(t,h) = (z_i^* - \bar{y}_i(h)) * (p - \gamma_i) + p^f * \bar{y}_i(h)$, for some constant p^f and using z_i^* as the capacity that is going to be built eventually conditional on (t,h). Eq. (47) allows us to express z_i^* as a function of $\bar{y}_i^{T^*}(\cdot)$, and the latter can be expressed as $\bar{y}_i^{T^*}(t+1,\cdot) = y_i(t,h) + \frac{p^*-\gamma}{b} \cdot \alpha_i^{t+1}$ by the induction assumption. The following expression of Π_i follows, neglecting the constant term $p^f * \bar{y}_i(h)$.

$$\Pi_i(t,h) = \frac{1}{\lambda_i b}(p^* - \gamma)^2 + \left(y_i(t,h) + \frac{p^* - \gamma}{b} \cdot \alpha_i^{t+1} - \bar{y}_i(h)\right) \cdot (p^* - \gamma) \tag{50}$$

The first order conditions of maximizing $\Pi_i(t,h)$ over $y_i(t,h)$ yield, for all $i \in N$,

$$y_i(t,h) = \bar{y}_i(h) + \frac{p^* - \gamma}{b} \cdot \left[1 + \beta^{t+1} - 2(\alpha_i^{t+1} + \lambda_i^{-1})\right]. \tag{51}$$

Hence, $\alpha_i^t = \alpha_i^{t+1} + \left[1 + \beta^{t+1} - 2(\alpha_i^{t+1} + \lambda_i^{-1})\right] = 1 + \beta^{t+1} - \alpha_i^{t+1} - 2\lambda_i^{-1}$, and

$$\sum_{i \in N} y_i(t,h) = \sum_{i \in N} \bar{y}_i(h) + \frac{p^* - \gamma}{b} \cdot \left[n * (1 + \beta^{t+1}) - 2\textstyle\sum_i (\alpha_i^{t+1} + \lambda_i^{-1})\right]. \tag{52}$$

Using the induction assumption (i) $p^*(t+1,h) = (a + \beta^{t+1}\gamma - b\sum_i y_i(t,h))/(1+\beta^{t+1})$,

$$p^*(t,h) = \frac{a - b\bar{y}(h) + \gamma \cdot \left[n + (n+1)\beta^{t+1} - 2\sum_i(\alpha_i^{t+1} + \lambda_i^{-1})\right]}{(n+1) * (1 + \beta^{t+1}) - 2\sum_i(\alpha_i^{t+1} + \lambda_i^{-1})}$$

It follows that $\beta^t = n + (n+1)\beta^{t+1} - 2\sum_i(\alpha_i^{t+1} + \lambda_i^{-1})$, and recursively both (α_i^t) and β^t are thus well-defined for all $t \leq T^*$. Since $\bar{y}_i(h) = 0$ for all $i \in N$ in $t = 1$ (no output is sold forward prior to round 1), Eq. (48) thus yields the equilibrium price, Eq. (47) yields the equilibrium capacity/quantity for all $i \in N$, and $\bar{y}_i^{T^*}(1,h) = 0 + \frac{p^*-\gamma}{b} \cdot \alpha_i^1$ for all $i \in N$. The equilibrium profit Eq. (19) follows from Eq. (50), using $t = 1$ and $\bar{y}_i^1 = 0$ for all i. To see that β^t is increasing in T^*, resolve the recursive definition of β^t. If $T^* - t$ is even,

$$\beta^t = \beta^{t+1} + n(\beta^{t+1} - 2\beta^{t+2} + 2\beta^{t+3} - \cdots + \cdots - 2\beta^{T^*} - 1) + 2\beta^{T^*} \tag{53}$$

$$\beta^{t+1} = \beta^{t+2} + n(\beta^{t+2} - 2\beta^{t+3} + 2\beta^{t+4} - \cdots + \cdots + 2\beta^{T^*} + 1) - 2\beta^{T^*} \tag{54}$$

and (partially) substituting for β^{t+1}, we obtain $\beta^t = \beta^{t+1} + (n-1)(\beta^{t+1} - \beta^{t+2})$ and the expression provided in the proposition. The same applies if $T^* - t$ is odd. Note $\beta^{T^*-1} - \beta^{T^*} = n + (n-2)\beta^{T^*}$ and $\beta^{T^*} = \sum_i \lambda_i^{-1}$. Hence $\beta^t \to \infty$ as well as $p \to \gamma$ when $T^* \to \infty$. Resolving the recursive definition of α_i^t yields, for all $i \in N$, $\alpha_i^t = \sum_{\tau=t+1}^{T^*}(1 + \beta^\tau - 2\lambda_i^{-1}) * (-1)^{T^*-\tau+1}$. □

D Proof of Proposition 5.1

Lemma D.1. *Assume $|N| = 2$ and $p_i = a - b_1 z_i - b_2 z_j$ for all $i \in N$. If $p_i - (z_i - \overline{y}_i)/\mu_i - \gamma_i = 0$ for all $i \in N$, then*

$$p_i = \frac{(1+b_1\mu_j)A_i - b_2\mu_j A_j}{(1+b_1\mu_i)(1+b_1\mu_j) - b_2^2\mu_i\mu_j} \tag{55}$$

for all $i \in N$, using $A_i := a - b_1\left[\overline{y}_i - \mu_i\gamma_i\right] - b_2\left[\overline{y}_j - \mu_j\gamma_j\right]$ for all i.

Proof. Using $z_i = \overline{y}_i + \mu_i(p_i - \gamma_i)$ and $p_i = a - b_1 z_i - b_2 z_j$, we obtain

$$p_i = a - b_1\left[\overline{y}_i + \mu_i(p_i - \gamma_i)\right] - b_2\left[\overline{y}_j + \mu_j(p_j - \gamma_j)\right] \tag{56}$$

$$(1+b_1\mu_i)p_i + b_2\mu_j p_j = a - b_1\left[\overline{y}_i - \mu_i\gamma_i\right] - b_2\left[\overline{y}_j - \mu_j\gamma_j\right] =: A_i, \tag{57}$$

i.e. an equation system implying

$$\left(\frac{1+b_1\mu_i}{b_2\mu_j} - \frac{b_2\mu_i}{1+b_1\mu_j}\right)p_i = \frac{1}{b_2\mu_j}A_i - \frac{1}{1+b_1\mu_j}A_j \tag{58}$$

which yields the result. □

Lemma D.2. *If $p_i - (z_i^T - \overline{y}_i^{t+1})/\mu_i - \gamma_i = 0$ for all $i \in N$, then $p_i - (z_i^T - \overline{y}_i^t)/(-\mu_i - 1/r_i) - \gamma_i = 0$ for all $i \in N$, using $r_i := dp_i^T/d\overline{y}_i^{t+1}$.*

Proof. The expected profit in round t is

$$\Pi_i(t,h) = \left(z_i^T - \overline{y}_i^t\right) * (p_i - \gamma_i) + p_i^f * \overline{y}_i^t \tag{59}$$

using $p_i - (z_i^T - \overline{y}_i^{t+1})/\mu_i - \gamma_i = 0$ and dropping the constant term $p_i^f * \overline{y}_i^t$ yields

$$\Pi_i^{\text{net}}(t,h) = \mu_i * (p_i - \gamma_i)^2 + (y_i^{t+1} - \overline{y}_i^t) * (p_i - \gamma_i) \tag{60}$$

Now, using $r_i := dp_i^T/d\overline{y}_i^{t+1}$, the first order conditions $d\Pi_i^{\text{net}}/d\overline{y}_i^{t+1} = 0$ for both i become

$$\mu_i * 2r_i(p_i - \gamma_i) + (p_i - \gamma_i) + r_i(y_i^{t+1} - \overline{y}_i^t) = 0 \tag{61}$$

and thus

$$y_i^{t+1} = \overline{y}_i^t - (p_i - \gamma_i) * (2\mu_i + 1/r_i) \tag{62}$$

Substituting this for $\bar{y}_i^{t+1}$ in the initially assumed equilibrium condition $p_i - (z_i^T - y_i^T)/\mu_i - \gamma_i = 0$, we obtain

$$p_i - \left[z_i^T - \bar{y}_i^t + (p_i - \gamma_i) * (2\mu_i + 1/r_i)\right]/\mu_i - \gamma_i = 0 \tag{63}$$

and the result follows. □

Lemma D.3. *The limit of (μ_i^t) as $t \to -\infty$ is $\mu = (b_1^2 - b_2^2)^{-1/2}$.*

Proof. Using the definition of (μ_i^t, μ_j^t),

$$\mu_i^t = \frac{(1 + b_1\mu_i^{t+1})(1 + b_1\mu_j^{t+1}) - b_2^2\mu_i^{t+1}\mu_j^{t+1}}{b_1(1 + b_1\mu_j^{t+1}) - b_2^2\mu_j^{t+1}} - \mu_i^{t+1} \tag{64}$$

its fixed points $\mu^t = \mu^{t+1}$ satisfy

$$\mu_i = \frac{(1 + b_1\mu_i)(1 + b_1\mu_j) - b_2^2\mu_i\mu_j}{b_1(1 + b_1\mu_j) - b_2^2\mu_j} - \mu_i \tag{65}$$

which yields $(b_1^2 - b_2^2)\mu_i\mu_j + b_1(\mu_i - \mu_j) = 1$. Since this is true for all $i \neq j$, it implies $\mu_i = \mu_j$ in the limit, i.e. $(b_1^2 - b_2^2)\mu^2 = 1$ as claimed. Since this fixed point is unique, it must also be the limit of the sequence as $t \to -\infty$. □

Lemma D.4. *Assume $|N| = 2$ and $p_i = a - b_1 z_i - b_2 z_j$ for all $i \in N$. The Bertrand profits are characterized as*

$$p_i - z_i / \left(\tfrac{b_1}{b_1^2 - b_2^2}\right) - \gamma_i = 0 \qquad \forall i \in N. \tag{66}$$

Proof. The demand functions are

$$z_i = \frac{b_1(a - p_i) - b_2(a - p_j)}{b_1^2 - b_2^2} \tag{67}$$

for all $i \in N$, and hence, profit and first order conditions are

$$\Pi_i = (p_i - \gamma_i) * \frac{b_1(a - p_i) - b_2(a - p_j)}{b_1^2 - b_2^2} \tag{68}$$

$$\Pi_i' = \frac{b_1(a - p_i) - b_2(a - p_j)}{b_1^2 - b_2^2} + (p_i - \gamma_i) * \frac{-b_1}{b_1^2 - b_2^2} = 0 \tag{69}$$

$$\Rightarrow \quad p_i - z_i / \left(\tfrac{b_1}{b_1^2 - b_2^2}\right) - \gamma_i = 0. \tag{70}$$

□

Lemma D.5. *Fix $\lambda_1 = \lambda_2$. The equilibrium price (Prop. 5.1) is decreasing in T^*.*

Proof. By definition, $\lambda_1 = \lambda_2$ implies $\mu_1^t = \mu_2^t =: \mu^t$ for all t and μ^1 is increasing in T^*. By Eq. (72), the equilibrium price satisfies $p_i - z_i^T/\mu_i^1 - \gamma_i = 0$ for all $i \in N$. First, consider the case $b_2 > 0$ and assume (for contradiction) that the price is not decreasing in μ. Thus, there exist $\mu' > \mu''$ and $p_i' > p_i''$, $p_j' \gtreqless p_j''$ such that $p_i' - z_i^T/\mu' - \gamma_i = 0$ and $p_i'' - z_i^T/\mu'' - \gamma_i = 0$.

$$p_i = \frac{(1+b_1\mu)(a+b_1\mu\gamma_i+b_2\mu\gamma_j) - b_2\mu(a+b_1\mu\gamma_j+b_2\mu\gamma_i)}{(1+b_1\mu)^2 - b_2^2\mu^2} \tag{71}$$

and its derivative with respect to μ is negative if

$$\begin{aligned}&\left[(1+b_1\mu)(b_1\gamma_i+b_2\gamma_j) - b_2\mu(b_1\gamma_j+b_2\gamma_i)\right] * \left[(1+b_1\mu)^2 - b_2^2\mu^2\right]\\ &< \left[(1+b_1\mu)(a+b_1\mu\gamma_i+b_2\mu\gamma_j) - b_2\mu(a+b_1\mu\gamma_j+b_2\mu\gamma_i)\right] * \left[b_1(1+b_1\mu) - b_2^2\mu\right]\end{aligned}$$

Case 1: $b_2 > 0$. The negativity condition is equivalent to

$$\begin{aligned}&(b_1\gamma_i + b_2\gamma_j) - b_2(b_1\mu\gamma_j + b_2\mu\gamma_i)\\ &< b_1(1+b_1\mu)^2 a - b_2\mu b_1(1+b_1\mu)a - b_2^2\mu(1+b_1\mu)a + b_2^3\mu^2 a\end{aligned}$$

and in case $0 < b_2 < b_1$, this is satisfied, since (using $\mu > 1/b_1$)

$$b_1\gamma_i(1-b_1\mu) + b_2\gamma_j(1-b_1\mu) < b_1 a + b_1^2\mu(a-a)$$

Case 2: $b_2 < 0$. The negativity condition is equivalent to

$$\begin{aligned}&b_1(\gamma_i - b_1\mu a) + b_2\gamma_j + b_2 b_1\mu(a-\gamma_j) - b_2^2\mu\gamma_i\\ &< ab_1 + \mu a(b_1^2 - b_2^2) + b_1\mu^2 a(b_1^2 - b_2^2) - b_2\mu^2 a(b_1^2 - b_2^2)\end{aligned}$$

and since $a > \gamma_i$, $a > \gamma_j$, $\mu > 1/b_1$, and $|b_1| > |b_2|$, this is generally true if $b_2 < 0$. □

Proof of Proposition 5.1 Similar to above, we can show that a necessary and sufficient condition for equilibrium in round T is $p_i - (z_i^T - \bar{y}_i^T)/\lambda_i^{-1}b_1^{-1} - \gamma_i = 0$ for all $i \in N$. Using $\mu_i^T = \lambda_i^{-1}b_1^{-1}$ and $t = T$, this condition can be expressed as follows.

$$p_i - (z_i^T - \bar{y}_i^t)/\mu_i^t - \gamma_i = 0 \qquad \forall i \in N. \tag{72}$$

By Lemma D.1, this implies

$$r_i^t := dp_i^t/d\bar{y}_i^t = \frac{-b_1(1+b_1\mu_j^t)+b_2^2\mu_j^t}{(1+b_1\mu_i^t)(1+b_1\mu_j^t)-b_2^2\mu_i^t\mu_j^t} \tag{73}$$

for all t. In addition, by Lemma D.2, for all $t < T$,

$$\mu_i^t = -\mu_i^{t+1} - 1/r_i^{t+1}. \tag{74}$$

The equilibrium price follows from Lemma D.1 for $t = 1$ and $\bar{y}_i^1 = 0$ for all $i \in N$. The convergence of μ is established in Lemma D.3 and the μ^B characterizing price competition follows from Lemma D.4. By Eq. (62), the amount sold forward in round t is

$$y_i^{t+1} - \bar{y}_i^t = -(p_i - \gamma_i) * (2\mu_i^{t+1} + 1/r_i^{t+1}) \equiv (p_i - \gamma_i) * (\mu_i^t - \mu_i^{t+1}), \tag{75}$$

where $\mu_i^t > \mu_i^{t+1}$ follows from its definition. Finally, the fact that p_i is decreasing in T^* follows from Lemma D.5. □