The Demand for Tailored Goods and the Theory of the Firm

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Abstract

The transaction cost theory predicts that firms are inclined to vertically integrate transactions in response to the specificity of their required inputs. Yet, reality proves that some firms engage in repeated transactions with external suppliers aimed at procuring highly specific inputs. To explain this phenomenon, this paper elaborates on a firm’s make-or-buy decision in a context with relational contracts in order to investigate how this decision is affected by the required input specificity. This paper demonstrates that a high degree of input specificity can lead to repeated market transactions being favored over vertical integration because demanding more specific inputs (i) impose lower costs to maintain repeated market transactions founded on relational contracts; and (ii), facilitate the self-enforcement of these relational contracts.

Keywords: Input specificity, vertical integration, market transactions, relational contracts, transaction cost theory.

JEL classification: D23, L22, L23

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1 Introduction

Since the seminal work of Coase [1937], a stream of economic literature has emerged to explain why firms exist and utilize non-market in place of market transactions.\(^1\) However, a fundamental question remains relatively unanswered: What are the conditions driving firms to integrate transactions? As Williamson [1971, 1979, 1985] and Klein, Crawford, and Alchian [1978] argued, high levels of quasi-rents due to relation-specific investments increase the likelihood of vertical integration.\(^2\) In the same vein, the procurement of highly specific inputs requires relation-specific investments such that input specificity can be considered as an important driver for vertical integration. Market transactions in competitive markets thus appear to be beneficial so long as buyers do not rely on relatively specific inputs. However, if firms were to require highly specific inputs, the value of market transactions appears to be limited because supplying firms may not make the necessary relation-specific investments in anticipation of hold-up.\(^3\) This suggests a strong relationship between the degree of input specificity and firms’ tendency to vertically integrate transactions. For instance, as reported by Monteverde and Teece [1982], car manufacturers like Ford and General Motors procure standardized inputs like mirrors, carpeting, safety belts, and wires from external suppliers. In contrast, engines and automatic transmissions—which are highly specific inputs that must be tailored to car manufacturers’ particular technical requirements—are internally produced.

Contrary to this prediction however, the dependency on highly specific inputs does not always lead to vertical integration. For example, Boeing and Airbus rely on highly specific turbosfan engines for producing commercial airplanes. According to premise of the transaction cost theory, we would expect both firms to vertically integrate the production of aircraft en-


\(^{2}\)For more thoroughly discussions refer to Riordan and Williamson [1985], Joskow [1988], and Demsetz [1988].

\(^{3}\)The Fisher Body - General Motors case reported in Klein et al. [1978] and further analyzed by Klein [1988, 2000] is a widely used example for illustrating hold-up. For a summary of extreme examples of hold-up see Shavell [2007].
engines. Instead, Boeing and Airbus acquire custom-tailored engines from external producers: Rolls Royce or CFM International (a joint venture between General Electric and the French company Snecma). The relationships between the suppliers of aircraft engines—Rolls Royce and CFM—and their customers—Boeing and Airbus—are characterized by long-term contracts imposing significant relation-specific investments on the supplying side. For example, Rolls Royce tailored the turbofan engine Trent XWB to the specific requirements of the Airbus A350 XWB family, whereas CFM developed the turbofan engine CFM56-3 exclusively for Boeing aircrafts. Other engines produced by CFM, like the CFM56-5 series, are only used by Airbus for its commercial airplanes. Chiu [1998] makes a similar observation and argues that the correlation between relation-specific investments and vertical integration is not as strong as the theory predicts.4 Two questions therefore emerge: First, how does the degree of input specificity affect a firm’s input procurement? Second, can input specificity serve as a rationale for the aforementioned counterintuitive phenomenon? The objective of this paper thus endeavors to answer these questions by elaborating on a firm’s make-or-buy decision in light of the specificity of required inputs.

Extant literature on the theory of the firm focused on two important drivers for vertical integration. Firstly, the property rights approach as devised by Grossman and Hart [1986], Hart and Moore [1990], and Bolton and Whinston [1993], paid attention to the efficient allocation of asset ownership as a mean to induce sufficient relation-specific investments. The efficient ownership structure of assets in a context with relational contracts is further investigated by Garvey [1995], Baker, Gibbons, and Murphy [2001], Bragelien [2001], Baker, Gibbons, and Murphy [2002], and Halonen [2002]. Second, asset specificity as a potential explanatory contribution to the theory of the firm—originated in the modern transaction cost theory a lá Williamson [1971, 1979, 1985]—is thoroughly analyzed by Riordan and Williamson [1985], Suzuki [2005], Kvaloy [2005], and Ruzzier [2007].5 Asset specificity governs incentives

4Whinston [2003] points to the same observation by noting that some firms even increase their mutual dependency e.g. by agreeing upon exclusive contracts.

5Input specificity differs from asset specificity in two principal aspects. First, using specific assets is not a necessity for custom-tailoring intermediate products. Even a generic asset can be utilized to produce highly
for the contracting parties to behave opportunistically by taking advantage of the fact that investments in relation-specific assets are not entirely reversible. Conceivably, the need for significant investments in relation-specific assets is deemed as a valid argument for vertical integration.6

Absent from these aforementioned studies is the investigation of the relationship between the dependency on custom-tailored inputs and a firm’s make-or-buy decision. To understand the nature of the firm, however, it is imperative to shed light on how a firm’s input procurement is influenced by the specificity of required inputs. This paper closes a prevailing gap in the literature on the theory of the firm by enhancing our understanding of a firm’s choice for inter- vs. intra-firm trade.

This paper demonstrates that a firm relying on highly specific inputs can indeed favor repeated market transactions over vertical integration. The rationale for this observation is as follows. First, demanding more specific inputs impairs supplying firms’ bargaining positions in repeated trading relationships. This eventually imposes lower costs on demanding firms to maintain repeated market transactions founded on relational contracts. Second, a higher degree of input specificity facilitates the self-enforcement of relational contracts between firms, thereby inducing the efficient level of relation-specific investments. In so doing, this paper offers a theoretical underpinning of the seemingly contradictory phenomenon that some firms utilize repeated market transactions for procuring highly specific inputs.

One important characteristic of the framework analyzed in this paper is the consideration of a firm’s make-or-buy decision in a context with relational contracts. Generally, relational (or implicit) contracts refer to contracts, for which some elements are not enforceable by specific intermediate products. The key is how this asset is being utilized in the production process. Consider for instance the customization of marble plates for kitchen counters or window sills. The asset required to cut marble plates—a diamond saw—can be utilized to produce plates ranging from standard to highly specific sizes. In this sense, firms can produce intermediate products characterized by different degrees of specificity without necessarily investing in specific assets. Second, in contrast to the acquisition of specific assets, producing specific intermediate products is a repeated investment decision, implying that a firm is not entirely tied to these (relation-specific) investments in the future.

6See also the discussion by Masten [1984] and Whinston [2003].
third parties [MacLeod and Malcomson, 1989], or are prohibitively costly to specify *ex ante* [Baker et al., 2002]. As documented in contemporary literature, relational contracts need to be self-enforcing in repeated games in order to eliminate opportunistic behavior. In particular, the framework in this paper comprises relational contracts within firms as analyzed by Bull [1987], Pearce and Stacchetti [1998], and Levin [2003]; and between firms as considered by Telser [1980], Klein and Leffler [1981], and Itoh and Morita [2005].

To model varying degrees of input specificity, the productive party is assumed to be in charge of implementing multidimensional effort. The underlying idea is that the implementation of differential effort allocations leads to intermediate products with distinguishable characteristics. A simple example is the emphasis on producing a high quality at the expense of a high quantity ensuring certain quality characteristics of intermediate products, which are potentially desired by a particular market participant.

The framework in this paper builds partially on the model devised by Baker et al. [2002]. They investigated how asset ownership facilitates the achievement of superior relational contracts within and between firms. Bragelien [2001] analyzed a more general version of their framework by endogenizing a firm’s production technology. He demonstrated that the self-enforcement of relational contracts between two firms might be facilitated by choosing a production technology that requires highly relation-specific investments.

Furthermore, this paper is synonymous with the work of Kvaloy [2005] and Ruzzier [2007], who illustrated that asset specificity can lead to non-integration. However, the grounds for their observations are notably different. Kvaloy’s [2005] result is founded on the fact that a higher degree of asset specificity can support the self-enforcement of relational contracts between firms. In contrast, Ruzzier’s [2007] conclusion follows directly from the application of the "deal-me-out" as the bargaining benchmark for market transactions [see Binmore, Shaked, and Sutton, 1989].

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7Though Baker et al. [2002] also analyzed a firm’s make-or-buy decision within a context of relational contracts, they assumed—in contrast to the framework in this paper—that the agent is not financially constrained such that the asset ownership can be transferred to the agent.
This paper is organized as follows. Section 2 provides an overview of the analyzed economic environment. The optimal contracts under a firm’s alternative input procurements are derived in section 3. In section 4, a firm’s optimal input procurement is identified; and investigated in light of how it is affected by different required degrees of input specificity. Section 5 summarizes the main results and concludes.

2 The Model

Consider a risk-neutral market participant (firm) henceforth referred to as the downstream party. In every period, the downstream party needs an intermediate product to sustain her production. The downstream party owns an asset which can be utilized to produce this input. However, the downstream party lacks either the ability or the time to produce the input by herself.

The downstream party can choose among two alternatives for procuring the required input: \((i)\) she can purchase the input from another risk-neutral market participant (firm) henceforth referred to as the upstream party \((market\ transaction)\); or \((ii)\), she can vertically integrate its production \((employment)\). The upstream party owns an identical asset as the downstream party such that their production technologies are comparable. If the downstream party decides to integrate the production of the required input, she depends on a worker as productive party. The worker is risk-neutral and financially constrained. For parsimony, his reservation utility is zero.

All parties interact for an infinite number of periods and share the same interest rate \(r\).

\(^8\) Infinitely living parties can be obtained by assuming overlapping generations of individuals who in turn live only a certain number of periods [Thomas and Worrall, 1988].

\(^9\) All vectors are column vectors, where ‘\(T\)’ denotes the transpose.
\[ c_i e_i^T e_i / 2, \ i = W, U, \] where for parsimony \( c_W = 1 \). Moreover, to reflect potential agency costs of production for the upstream party, let \( c_U \geq 1 \).

The characteristics of the input are determined by the implemented effort allocation. Let \( \mu = (\mu_1, \mu_2)^T \in \mathbb{R}^2^+ \) and \( \omega = (\omega_1, \omega_2)^T \in \mathbb{R}^2^+ \) represent the relative effort allocation—and therefore the attributes of the input—desired by the downstream party and other market participants, respectively. As illustrated in figure 1, the degree of input specificity required by the downstream party can then be measured by the geometric relation of \( \mu \) and \( \omega \): the angle \( \varphi \). The greater \( \varphi \) is, the more specific is the intermediate product the downstream party requires for further processing. The degree of input specificity—measured by \( \varphi \)—is exogenously determined by the downstream’s production technology which the input is required for.\(^{10}\)

To reduce the notational burden, it is assumed that \( \| \mu \| = \| \omega \| = 1 \), i.e. the lengths of \( \mu \) and \( \omega \) are normalized to one.

\[ \text{Figure 1: Desired Effort Allocations and Input Specificity} \]

As mentioned above, the implemented effort allocation—and hence the properties of the input—determines its value for the downstream party and for the other market participants. In particular, the downstream’s (internal) value \( I \) and the market (external) value \( E \) can be either high (indexed by \( H \)) or low (indexed by \( L \)), where \( \Delta I \equiv I_H - I_L \) and \( \Delta E \equiv E_H - E_L \). The

\(^{10}\)I discuss in section 5 a firm’s choice with respect to the specificity of required inputs if the production technology gives some scope for processing inputs with different characteristics.
Table 1: Combinations of Transactions and Contracts

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot</td>
<td>Spot Employment (SE)</td>
</tr>
<tr>
<td>Relational</td>
<td>Relational Employment (RE)</td>
</tr>
<tr>
<td>Spot</td>
<td>Spot Market (SM)</td>
</tr>
<tr>
<td>Relational</td>
<td>Relational Market (RM)</td>
</tr>
</tbody>
</table>

Internal and external input values are observable by all involved entities, but non-verifiable by third parties. Let

\[
\text{Prob}\{I = I_H | e_i\} = \min\{\mu^T e_i, 1\},
\]

\[
\text{Prob}\{E = E_H | e_i\} = \min\{\omega^T e_i, 1\}
\]

be the conditionally independent probabilities that the high internal and high external input value will be realized. Observe that maximizing the expected internal value requires a different effort allocation than maximizing the expected external value if \(\mu\) and \(\omega\) are linearly independent. In this case, the downstream party requires a specific input with certain attributes diverging from those desired by other market participants. Independent of the realized input value, however, the downstream party always prefers an internal use to sustain her production. Formally, this requires that \(I_H > I_L \geq E_H > E_L\), where \(E_L\) is normalized to zero. Finally, to ensure interior solutions, it is assumed that \(\Delta I, \Delta E < 1\).\(^{11}\)

For both types of transactions being considered—vertical integration and market exchange—the downstream party can utilize either a spot contract, or a relational contract contingent on the non-verifiable, but observable input value. If the involved parties agree upon a relational contract, they play a grim trigger strategy: Once they detect a violation of implicit obligations, they will never rely on relational contracts with the violator again. Subsequently considered combinations of transactions and contracts are summarized in table 1, using the terminology established by Baker et al. [2002].

\(^{11}\)Alternatively, one can let \(\Delta I, \Delta E > 1\) by assuming that \(\mu^T \mu\) and \(\omega^T \omega\) are sufficiently small. In this case, the lengths of \(\mu\) and \(\omega\) appear in the subsequent solutions. Since this does not provide additional insights, I opted for the first alternative for parsimony purposes.
3 Alternative Input Procurements

In the subsequent sections, I elaborate on the downstream’s alternatives to procure the required input as summarized in table 1. This eventually allows me to identify the optimal input procurement from the downstream’s perspective, and to illustrate the effect of input specificity on her make-or-buy decision.

3.1 Spot Employment

Consider first the case where the downstream party recruits the worker as the productive party for one period. Since verifiable information about the realized input value are not available, the downstream party cannot provide the worker with an enforceable incentive contract. Nevertheless, the downstream party can promise to pay a bonus in the event that the high internal input value is realized. Once this occurs however, the downstream party takes the input without paying the promised bonus since she owns the asset and possesses the related property rights [Grossman and Hart, 1986, Hart and Moore, 1990]. Anticipating this opportunistic behavior, the worker implements $e_W = (0, 0)^T$ such that the downstream party obtains $\Pi^{D|SE} = I_L$ under spot employment.

3.2 Relational Employment

If the downstream party repeatedly interacts with the worker, a relational incentive contract can be self-enforcing, and therefore credible. Particularly, the downstream party can promise to pay the worker a bonus $\beta$ in addition to a fixed transfer $\alpha$ in the event that the high internal input value is realized. The worker’s wage payment $w$ therefore takes the form

$$w = \begin{cases} 
\alpha + \beta, & \text{if } I = I_H \\
\alpha, & \text{if } I = I_L,
\end{cases}$$

(1)

where the worker’s liability limit requires that $w$ is always non-negative.

As a consequence of the implicit nature of this bonus contract, the downstream’s promise to pay $\beta$ needs to be reliable. Let $\bar{\Pi}^{RE} \equiv \max\{\Pi^{D|SE}, \Pi^{D|SM}, \Pi^{D|RM}\}$ denote the down-
stream’s expected profit obtained under her best alternative after violating the relational contract with the worker. After behaving opportunistically by reneging on $\beta$, the downstream party can henceforth either choose spot employment ($SE$), or engage in market transactions with the upstream party based on explicit ($SM$) or implicit ($RM$) contracts. Suppose for a moment that the high internal input value is eventually realized. Then, the downstream party honors her implicit obligation to pay $\beta$ if

$$-\beta + \frac{\Pi^{D|RE}_r}{r} \geq \hat{\Pi}^{RE}_r. \quad (2)$$

The left side of (2) represents the downstream’s expected payoff when she delivers on her promise, i.e. paying the bonus $\beta$ but obtaining the expected profit under relational employment $\Pi^{D|RE}_r$ in the future. To be deterred from reneging, this payoff needs to be greater than the present value of her best fall-back position $\hat{\Pi}^{RE}_r$. If this self-enforcement condition is satisfied, the worker anticipates the downstream party to deliver on her promise to pay $\beta$ whenever $I = I_H$ such that he is motivated to implement effort.

The optimal relational contract maximizes the difference between the expected internal input value and the worker’s expected wage payment. Consequently, the downstream’s problem can be stated as follows:

$$\max_{\alpha, \beta, e_W} \Pi^{D|RE}_r = I_L + \Delta I \mu^T e_W - \alpha - \beta \mu^T e_W \quad (3)$$

s.t.

$$\alpha + \beta \mu^T e_W - \frac{1}{2} e_W^T e_W \geq 0 \quad (4)$$

$$e_W \in \arg \max e_W \alpha + \beta \mu^T e_W - \frac{1}{2} e_W^T e_W \quad (5)$$

$$\alpha + \beta \geq 0 \quad (6)$$

$$\alpha \geq 0 \quad (7)$$

$$I_L + \Delta I \mu^T e_W - \alpha - \beta \mu^T e_W - \hat{\Pi}^{RE}_r \geq \beta r. \quad (8)$$

Condition (4) is the worker’s participation constraint and ensures that it is in his interest to enter this employment relationship. Further, (5) is the worker’s incentive constraint, implying
that he implements $e_W = \beta \mu$. Constraints (6) and (7) guarantee that the relational bonus contract is compatible with the worker’s liability limit. Finally, (8) is the self-enforcement condition ensuring that the relational bonus contract does not motivate the downstream party to renege on $\beta$ ex post.

**Proposition 1** Under relational employment, the optimal relational bonus contract $(\alpha^*, \beta^*)$ leads to the downstream’s expected profit

$$\Pi^{D|RE}(r) = \begin{cases} I_L + \frac{1}{4}(\Delta I)^2, & \text{if } r \leq r^{RE} \\ \frac{r}{2} [\Delta I - r + 2\phi] + \bar{\Pi}^{RE}, & \text{if } r^{RE} < r \leq \hat{r}^{RE} \\ I_L, & \text{if } \hat{r}^{RE} < r, \end{cases}$$

(9)

where

$$r^{RE} \equiv \frac{\Delta I}{2} - \frac{2}{\Delta I} \left[\bar{\Pi}^{RE} - I_L\right], \quad \hat{r}^{RE} \equiv \Delta I - 2 \left[\bar{\Pi}^{RE} - I_L\right]^{\frac{1}{2}},$$

$$\phi \equiv \left[\frac{1}{4} (\Delta I - r)^2 + I_L - \bar{\Pi}^{RE}\right]^{\frac{1}{2}}.$$

**Proof** See appendix.

If $r \leq r^{RE}$, the downstream party sufficiently values a sustained employment relationship with the worker such that her promise to pay the optimal bonus $\beta^* = \Delta I/2$ is credible.\(^\text{12}\) For $r^{RE} < r \leq \hat{r}^{RE}$, however, the downstream party cannot credibly commit to pay $\beta^* = \Delta I/2$. This is because a higher interest rate $r$ imposes a less severe ‘penalty’ on the downstream party for violating the relational contract with the worker. Thus, the downstream party would behave opportunistically by reneging ex post on $\beta^*$. Nevertheless, to motivate the worker to implement effort, she needs to diminish the bonus $\beta$ with the aim of satisfying the self-enforcement condition (8). Technically, (8) becomes binding. However, the more the credible bonus $\beta^*(r)$ deviates from the efficient bonus $\beta^* = \Delta I/2$, the lower is the downstream’s expected profit $\Pi^{D|RE}(r)$. Finally, if $r > \hat{r}^{RE}$, the downstream party cannot find a strictly positive bonus which eliminates her reneging temptation. As a result, $\beta^* = 0$, and the downstream party obtains the same expected profit as under spot employment.

\(^{12}\)See proof of proposition 1 in the appendix for a characterization of the optimal relational bonus contract.
3.3 Spot Market

Instead of utilizing an integrated production, the downstream party can alternatively procure the input from the upstream party through a spot market exchange. Under this arrangement, both parties negotiate in every period about a price \( \Upsilon_{SM} \) in exchange for the input. This price is obtained by applying the Nash-Bargaining solution with equal bargaining power. Accordingly, the downstream party pays the external input value \( E \) plus half of the surplus \( I - E \) for its internal use. Consequently, \( \Upsilon_{SM} = (I + E)/2 \).

The upstream party chooses effort \( e_U \) with the objective of maximizing her expected profit \( \Pi_U|SM = E[\Upsilon_{SM}|e_U] - C(e_U) \). Equivalently,

\[
\max_{e_U} \Pi_U|SM = \frac{1}{2} \left[ I_L + \Delta I \mu^T e_U + \Delta E \omega^T e_U \right] - \frac{1}{2} c_U e_U^T e_U, \tag{10}
\]

which directly implies that the upstream party implements

\[
e^*_U = \frac{1}{2c_U} \left[ \Delta I \mu + \Delta E \omega \right]. \tag{11}
\]

Apparently, the upstream party intends to maximize the internal and external input value with the aim of improving her own expected bargaining position, and hence, the price she expects to obtain. Observe further that the upstream party does not perfectly tailor the input to the downstream’s requirements if \( \mu \neq \lambda \omega, \lambda > 0 \). In this case, adjusting the input to the downstream’s and to other market participants’ needs are two competing objectives.

If the upstream party anticipates a spot market transaction with the downstream party, she implements \( e^*_U \) and obtains:

\[
\Pi^{U|SM} = \frac{1}{2} I_L + \frac{1}{8c_U} \left[ (\Delta I)^2 + (\Delta E)^2 + 2\Delta I \Delta E \cos \varphi \right]. \tag{12}
\]

Notice that the upstream’s expected profit \( \Pi^{U|SM} \) is decreasing in the degree of input specificity \( \varphi \) desired by the downstream party. This can be observed because the upstream’s trade-off between tailoring the input to the downstream’s versus the market’s requirements becomes more severe. This eventually deteriorates the upstream’s expected bargaining position, and as a consequence, leads to a smaller expected premium for selling the input to the downstream party in place of the other market participants.
If the downstream party decides in favor of spot market exchange with the upstream party, her expected profit $\Pi_{D|SM}$ is the difference between the expected internal input value and the price she expects to pay. Formally, $\Pi_{D|SM} = E[I - \Upsilon^{SM}|e^*_U]$, which is equivalent to

$$\Pi_{D|SM} = \frac{1}{2} I_L + \frac{1}{4c_U} \left[ (\Delta I)^2 - (\Delta E)^2 \right]. \quad (13)$$

The downstream’s expected profit $\Pi_{D|SM}$ under spot exchange is apparently independent of the specificity measure $\varphi$. Nevertheless, there are two countervailing effects of $\varphi$ on $\Pi_{D|SM}$. It can be shown that

$$\frac{\partial E[I|e^*_U]}{\partial \varphi} = \frac{\partial E[\Upsilon^{SM}|e^*_U]}{\partial \varphi} = -\frac{1}{2c_U} \Delta I \Delta E \sin \varphi. \quad (14)$$

Accordingly, an increase in $\varphi$ leads to a lower expected internal input value and a lower price the downstream party expects to pay. Since the magnitudes of both effects are identical, they cancel each other out such that $\Pi_{D|SM}$ is eventually not affected by the desired degree of input specificity. Generally speaking, a lower expected price perfectly compensates the downstream party for the expected exchange of an insufficiently tailored input.

### 3.4 Relational Market

As demonstrated in the preceding section, utilizing spot market transactions with the upstream party leads to the procurement of an insufficiently tailored input. To ensure the exchange of a perfectly tailored input however, the downstream party can promise the upstream party to pay a certain amount conditional on the realized internal input value $I$. This is aimed at motivating the upstream party to make relation-specific investments because tailoring the input to the downstream’s requirements irretrievably impairs its expected market value.

Let $P_L$ denote the floor payment both parties consent to in an enforceable contract for exchanging the input, regardless of its final value. In addition, to motivate relation-specific investments in the sense of tailoring the input, the downstream party promises to pay the upstream party a higher price $P_H$ if $I = I_H$. Thus, this contract consists of an explicit component $P_L$ and a non-enforceable premium $\Delta P \equiv P_H - P_L$. However, the upstream party is only motivated to tailor the input if the downstream’s promise to pay the premium
\( \Delta P \) is reliable. Let \( \tilde{\Pi}^{RM} \equiv \max\{ \Pi^{D|SE}, \Pi^{D|RE}, \Pi^{D|SM} \} \) denote the downstream’s expected profit she obtains under her best fall-back position. Suppose that the high internal input value is eventually realized. The downstream party has no incentive to hold up the upstream party by reneging on \( \Delta P \) if
\[
-\Delta P + \frac{\Pi^{D|RM}}{r} \geq \frac{\tilde{\Pi}^{RM}}{r}.
\]
(15)
The downstream party adheres to her promise if paying the premium \( \Delta P \) but perpetuating the long-term trading relationship with the upstream party provides her with a higher expected profit than her best fall-back position.

The payments \( P_L \) and \( P_H \) need to guarantee that it is in the upstream’s interest to enter into a long-term trading relationship with the downstream party. This requires that the upstream party is at least weakly better off under relational market than under her best alternative. It is crucial here to note that her best alternative is directly linked to the downstream’s best fall-back position. If the downstream’s best fall-back is to engage in spot market exchange, the upstream’s reservation profit is \( \Pi^{U|SM} \) as derived in section 3.3. By contrast, if vertical integration is the downstream’s best alternative, the upstream’s reservation profit is the one she obtains by selling the intermediate product on the market at the market price \( \Upsilon^M = E \). In this case, it can be verified that the upstream party implements \( e^*_U = \Delta E \omega / c_U \), providing her with
\[
\Pi^{U|M} = \frac{1}{2c_U}(\Delta E)^2.
\]
(16)
The purpose of the optimal relational contract is to maximize the difference between the expected internal input value and the expected payment to the upstream party. The downstream’s problem is therefore
\[
\max_{P_L, P_H, e_U} \Pi^{D|RM} = I_L + \Delta I \mu^T e_U - P_L - \Delta P \mu^T e_U
\]
(17)
s.t.
\[
P_L + \Delta P \mu^T e_U - \frac{1}{2}c_U e_U^T e_U \geq \Pi^U
\]
(18)
\[
e_U \in \arg\max_{e_U} P_L + \Delta P \mu^T \tilde{e}_U - \frac{1}{2}c_U \tilde{e}_U^T \tilde{e}_U
\]
(19)
\[
I_L + \Delta I \mu^T e_U - P_L - \Delta P \mu^T e_U - \tilde{\Pi}^{RM} \geq \Delta P r,
\]
(20)
13
where

\[
\bar{\Pi}^U = \begin{cases} 
\frac{1}{2}I_L + \frac{1}{8c_U} [\Delta I\mu + \Delta E\omega]^2, & \text{if } \bar{\Pi}^{RM} = \Pi^{D|SM} \\
\frac{1}{2c_U} (\Delta E)^2, & \text{if } \bar{\Pi}^{RM} \neq \Pi^{D|SM},
\end{cases}
\]  

(21)
is the upstream’s reservation profit conditional on the downstream’s best fall-back position. Condition (18) is the upstream’s participation constraint guaranteeing that the proposed relational contract makes her at least weakly better off than her best alternative. Further, (19) is the upstream’s incentive constraint, implying that she implements \(e^*_U = \Delta P\mu/c_U\). Finally, the self-enforcement condition (20) ensures that the downstream party is not tempted to renege \(ex\ post\) on the non-enforceable premium \(\Delta P\).

As demonstrated, the upstream’s reservation profit can take one of two values conditional on the downstream’s best fall-back position. For the sake of lucidity, I subsequently consider both cases separately.

**Proposition 2** If \(\bar{\Pi}^{RM} = \Pi^{D|SM}\) under relational market, the optimal relational contract \((P^*_L, P^*_H)\) leads to the downstream’s expected profit

\[
\Pi^{D|RM}(r) = \begin{cases} 
\frac{1}{2}I_L + \frac{1}{2c_U} (\Delta I)^2 - \frac{1}{8c_U} [\Delta I\mu + \Delta E\omega]^2, & \text{if } r \leq r^{RM} \\
r [\Delta I - rCU + \phi] + \Pi^{D|SM}, & \text{if } r^{RM} < r \leq \hat{r}^{RM} \\
\Pi^{D|SM}, & \text{if } \hat{r}^{RM} < r,
\end{cases}
\]  

(22)

where

\[
r^{RM} \equiv \frac{1}{8c_U\Delta I} [\Delta I\mu - \Delta E\omega]^2 \quad \hat{r}^{RM} \equiv \frac{1}{c_U} [\Delta I - \eta^2] \\
\phi \equiv [(\Delta I - rCU)^2 - \eta]^\frac{1}{2} \quad \eta \equiv \frac{1}{4} (\Delta I\mu + \Delta E\omega)^2 + c_U (2\Pi^{D|SM} - I_L).
\]

**Proof** See appendix.

As long as \(r \leq r^{RM}\), the downstream party honors her non-enforceable obligation to pay the optimal premium \(\Delta P^* = \Delta I\) if \(I = I_H\). In this case, the upstream party anticipates that holdup will not occur, and is therefore motivated to make the desired relation-specific

---

\footnote{See proof of proposition 2 in the appendix for the characterization of the optimal relational contract.}
investment in the sense of tailoring the input to the downstream’s requirements. In contrast, if \( r^{RM} < r \leq \hat{r}^{RM} \), the downstream party needs to diminish the premium \( \Delta P^*(r) \) aimed at eliminating her reneging temptation. Technically, the self-enforcement condition (20) becomes binding. The provision of an inefficient (but credible) premium \( \Delta P^*(r) \) inevitably leads to a lower expected profit for the downstream party. Finally, if \( r > \hat{r}^{RM} \), the downstream party cannot find a strictly positive and reliable premium to provide the upstream party with incentives to tailor the input. If so, the downstream party engages in spot market exchange with the upstream party and obtains \( \Pi^{D|SM} \).

To gain further insights, one can re-write \( \Pi^{D|RM} \) for \( r \leq r^{RM} \) as\(^{14}\)

\[
\Pi^{D|RM} = \frac{1}{2} I_L + \frac{1}{2c_U} (\Delta I)^2 - \frac{1}{8c_U} \left[ (\Delta I)^2 + (\Delta E)^2 + 2\Delta I\Delta E \cos \varphi \right].
\]

(23)

Apparently, if \( \tilde{\Pi}^{RM} = \Pi^{D|SM} \), the downstream’s expected profit \( \Pi^{D|RM} \) depends on the specificity measure \( \varphi \). The rationale for this observation is as follows: Ensuring the upstream’s participation requires that she obtains at least the same expected profit under relational market as under mutual spot exchange. The floor payment \( P_L \) thus reflects the upstream’s reservation profit \( \Pi^{U|SM} \), which in turn is a function of \( \varphi \), see section 3.3. Consequently, the degree of input specificity also affects the downstream’s expected profit under relational market if spot exchange is her best fall-back, i.e. if \( \Pi^{D|SM} = \tilde{\Pi}^{RM} \). Furthermore, as discussed in section 3.3, a higher degree of input specificity (greater \( \varphi \)) impairs the upstream’s expected profit \( \Pi^{U|SM} \) under spot market exchange. This in turn makes it less costly for the downstream party to ensure the upstream’s participation, thereby leading to a higher expected profit \( \Pi^{D|RM} \). The same observation applies for \( \Pi^{D|RM} \) if \( r^{RM} < r \leq \hat{r}^{RM} \).

Finally, re-consider the cut off interest rate \( \hat{r}^{RM} \). Substituting \( \Pi^{D|SM} \) gives

\[
\hat{r}^{RM} = \frac{\Delta I}{c_U} - \frac{1}{2c_U} \left[ 3(\Delta I)^2 - (\Delta E)^2 + 2\Delta I\Delta E \cos \varphi \right]^\frac{1}{2}.
\]

(24)

Apparently, \( \hat{r}^{RM} \) is increasing in the specificity measure \( \varphi \). As emphasized, a higher degree of input specificity enhances the downstream’s expected profit under relational market if \( \Pi^{D|SM} = \tilde{\Pi}^{RM} \). This imposes a more severe ‘penalty’ on the downstream party for violating

\(^{14}\)To see this, note that \( \mu^T \omega = \|\mu\|\|\omega\| \cos \varphi \) and \( \|\mu\| = \|\omega\| = 1 \).
the relational contract with the upstream party by reneging on the non-enforceable premium $\Delta P$. The lower reneging temptation is reflected by a higher cut off interest rate $\hat{r}^{RM}$. It can be further shown that the same observation applies for $r^{RM}$. This result is in line with Halonen [2002] who also recognized that less attractive outside options alleviate incentives to behave opportunistically.

**Proposition 3** If $\tilde{\Pi}^{RM} \neq \Pi^{D|SM}$ under relational market, the optimal relational contract $(P^*_L, P^*_H)$ leads to the downstream’s expected profit

$$\Pi^{D|RM}(r) = \begin{cases} I_L + \frac{1}{2c_U} \left[ (\Delta I)^2 - (\Delta E)^2 \right], & \text{if } r \leq r^{RM} \\ r [\Delta I - rc_U + \phi] + \tilde{\Pi}^{RM}, & \text{if } r^{RM} < r \leq \hat{r}^{RM} \\ \Pi^{D|SM}, & \text{if } \hat{r}^{RM} < r, \end{cases}$$

where

$$r^{RM} \equiv \frac{1}{2c_U \Delta I} \left[ (\Delta I)^2 - (\Delta E)^2 - 2c_U \left( \tilde{\Pi}^{RM} - I_L \right) \right], \quad \hat{r}^{RM} \equiv \frac{1}{c_U} \left[ \Delta I - \eta^2 \right]$$

$$\phi \equiv \left[ (\Delta I - rc_u)^2 - \eta \right]^\frac{1}{2}, \quad \eta \equiv \left[ (\Delta E)^2 + 2c_U \left( \tilde{\Pi}^{RM} - I_L \right) \right].$$

**Proof** See appendix.

In principle, the illustrated relationship between the interest rate $r$ and the downstream’s expected profit for the case $\tilde{\Pi}^{RM} = \Pi^{D|SM}$ also applies in the event that $\tilde{\Pi}^{RM} = \Pi^{U|SM}$. Observe however, that the downstream’s expected profit when $\tilde{\Pi}^{RM} = \Pi^{D|SM}$ is not affected by the specificity measure $\varphi$ because the upstream’s reservation profit $\Pi^{U|SM}$ is now independent of $\varphi$. Therefore, the relational contract with the upstream party does not capture $\varphi$, and so does $\Pi^{D|RM}$.

### 4 Input Procurement and Input Specificity

This section elaborates on the efficient input procurement from the perspective of the downstream party. Further, it demonstrates how the downstream’s make-or-buy decision is affected by her desired degree of input specificity.
To characterize the downstream’s optimal input procurement, let us first identify the optimal spot contract. Suppose for a moment that the interest rate \( r \) is sufficiently high such that relational contracts are not feasible. It is straightforward to see that

\[
(\Delta E)^2 \leq (\Delta I)^2 - 2c_U I_L \equiv \Theta \tag{26}
\]

As long as relational contracts cannot be attained and \((\Delta E)^2 \leq \Theta\), procuring the input through mutual spot exchange with the upstream party dominates an internal production.\(^{15}\)

More precisely, a market transaction is superior to an integrated production, if the market valuation towards the difference between the high and low external value of the input is sufficiently low. In this case, the expected bargaining position of the upstream party, and consequently the expected price for the input, are adequately low. By contrast, if \((\Delta E)^2 > \Theta\), the downstream party would prefer an integrated production since acquiring the input via spot market transactions would impose higher costs.

To identify the downstream’s optimal make-or-buy decision, it is further necessary to compare relational employment and relational market in terms of their profitabilities. First, suppose that the interest rate \( r \) is sufficiently low such that both relational contracts with efficient incentive schemes are self-enforcing. Formally, \( r \leq \min\{r^{RM}, r^{RE}\} \). It is demonstrated in the appendix that for \( \Pi^{D|RM} \geq \Pi^{D|RE} \),

\[
\begin{align*}
(\Delta E)^2 &\leq \Phi, \quad \text{if } \Pi^{D|SM} = \tilde{\Pi}^{RM}, &\quad &\text{(27)} \\
(\Delta E)^2 &\leq \Phi, \quad \text{if } \Pi^{D|SM} \neq \tilde{\Pi}^{RM}, &\quad &\text{(28)}
\end{align*}
\]

where

\[
\Phi \equiv \left[ \kappa^2 + (\Delta I)^2 (3 - 2c_U) - 4c_U I_L \right]^{\frac{1}{2}} - \kappa \}
\Phi \equiv \frac{1}{2}(\Delta I)^2 (2 - c_U) \quad \kappa \equiv \Delta I \cos \varphi.
\]

A long-term trading relationship with the upstream party (relational market) is generally more profitable than integration if \( \Delta E \) is sufficiently low. The rationale for this observation is that

\(^{15}\)This particulary requires that \( c_U < (\Delta I)^2/(2I_L) \), i.e. the upstream’s production of the input is not too inefficient relative to the downstream’s. Otherwise, spot employment would be the preferred spot contract for all values of \((\Delta E)^2\).
a lower $\Delta E$ diminishes the value of the upstream’s outside option, and therefore mitigates the downstream’s costs for ensuring her participation. Observe however, that we have two different threshold levels: $\Phi$ and $\bar{\Phi}$. This is because the downstream’s expected profit for relational market is conditional on whether or not a spot market exchange is her best fallback position.

For $r^i < r \leq \hat{r}^i$, $i = RM, RE$, the downstream party needs to adjust the incentive schemes for both relational contracts aimed at ensuring their self-enforcement. In general, $\Pi^{D|RM}(r) \geq \Pi^{D|RE}(r)$ if

$$\begin{align*}
(\Delta E)^2 \leq \Psi(r), & \quad \text{if } \Pi^{RM} = \Pi^{D|SM}, \\
(\Delta E)^2 \leq \bar{\Psi}(r), & \quad \text{if } \Pi^{RM} \neq \Pi^{D|SM},
\end{align*}$$

where $(\Delta E)^2 = \Psi(r)$ implies $\Pi^{D|RM}(r) = \Pi^{D|RE}(r)$ for $\Pi^{RM} = \Pi^{D|SM}$, and $(\Delta E)^2 = \bar{\Psi}(r)$ for $\Pi^{RM} \neq \Pi^{D|SM}$, respectively.\(^{16}\)

The identified cut offs now allow for characterizing the downstream’s optimal input procurement for different values of $r$ and $\Delta E$.

**Proposition 4** The downstream party chooses relational market and receives $\Pi^{D|RM}(r)$ in the intervals

$$\begin{align*}
0 < (\Delta E)^2 & \leq \min\{\Phi, \Theta\} \quad \text{and} \quad \Theta < (\Delta E)^2 \leq \bar{\Phi}, \quad \text{if } r \leq \hat{r}^{RM}; \\
0 < (\Delta E)^2 & \leq \min\{\Psi(r), \Theta\} \quad \text{and} \quad \Theta < (\Delta E)^2 \leq \bar{\Psi}(r), \quad \text{if } \hat{r}^{RM} < r \leq \hat{\hat{r}}^{RM}.
\end{align*}$$

In contrast, the downstream party prefers relational employment and obtains $\Pi^{D|RE}(r)$ for

$$\begin{align*}
\Phi < (\Delta E)^2 & \leq \Theta \quad \text{and} \quad \bar{\Phi} < (\Delta E)^2, \quad \text{if } r \leq \hat{r}^{RE}; \\
\Psi(r) < (\Delta E)^2 & \leq \Theta \quad \text{and} \quad \bar{\Psi}(r) < (\Delta E)^2, \quad \text{if } \hat{r}^{RE} < r \leq \hat{\hat{r}}^{RE}.
\end{align*}$$

Finally, if $r > \hat{r}^{RM}$ or $r > \hat{r}^{RE}$ in the relevant intervals, the downstream party receives $\Pi^D = \max\{\Pi^{D|SM}, \Pi^{D|SE}\}$.

\(^{16}\)Due to the structure of $\Pi^{D|RM}(r)$ and $\Pi^{D|RE}(r)$ for $r^i < r \leq \hat{r}^i$, $i = RM, RE$, one cannot achieve a tractable closed form solution. Nevertheless, using the implicit characterizations does not derogate the subsequent results.
Proof See appendix.

To shed more light on the downstream’s make-or-buy decision, suppose for a moment that $r \leq \min \{ \hat{r}^{RM}, \hat{r}^{RE} \}$, i.e. both relational contracts are self-enforcing.\textsuperscript{17} In general, relational market is superior for low values of $\Delta E$. Nevertheless, relational employment can be temporarily preferred by the downstream party in the interval $\Psi(r) < (\Delta E)^2 \leq \Theta$, whereas for $\Theta < (\Delta E)^2 \leq \Psi(r)$, relational market is again more profitable. The reason for obtaining spanned intervals is as follows: Different fall-back positions for relational market—spot market exchange or integrated spot production—impose diverse costs to the downstream party for ensuring the upstream’s participation. This further provides the downstream party with differential expected profits, and as a consequence, leads to diverse cut off interest rates for the feasibility of relational contracts. Note however, that the previous argumentation applies only if $\Psi(r) < \Theta < \Psi(r)$, which in turn depends on the specific parameter values. Otherwise, there exists only one threshold level of $\Delta E$ where the downstream party is indifferent between relational market and relational employment.

The downstream’s optimal input procurement is illustrated in figure 2, where the squared spread of the external input value $(\Delta E)^2$ is on the horizontal axis, and the interest rate $r$ on the vertical axis. It is important to note here that figure 2 represents the downstream’s make-or-buy decision for the case $\Phi, \Psi(r) > \Theta$ and $(\Delta I)^2 \cos^2 \varphi > \Theta$.\textsuperscript{18} The subsequent explanations generally apply to the other cases as well.

Consider first the downstream’s preferred spot transactions, which eventually determine her best fall-back for relational contracts. Spot contracts are chosen whenever the interest rate $r$ is sufficiently high such that relational contracts cannot be attained. As previously observed, spot market exchange dominates spot employment if the market valuation towards the difference between the high and low external value of the input $\Delta E$ is sufficiently low. By contrast, if the interest rate $r$ is adequately low, the downstream party can utilize a rela-

\textsuperscript{17}See proof of proposition 4 in the appendix for a characterization of $\hat{r}^{RM}, \hat{r}^{RM}, \hat{r}^{RE}$, and $r^{RE}$.

\textsuperscript{18}The latter condition implies that $\hat{r}^{RM}$ is convex decreasing in $(\Delta E)^2$ for $0 < (\Delta E)^2 \leq \min \{ \Psi(r), \Theta \}$. Further, it can be shown that $\hat{r}^{RE}$ is convex increasing in $(\Delta E)^2$ if spot market is the downstream’s best fall-back. In contrast, if spot employment is her best alternative, $\hat{r}^{RE}$ is constant in $(\Delta E)^2$. 

19
A: Relational Market (Spot Market)
B: Relational Market (Relational Employment)

To illustrate the effect of input specificity on the downstream’s make-or-buy decision, suppose the downstream party requires a more specific intermediate product. Formally, $\varphi$ increases to $\varphi'$. As long as spot market exchange is the downstream’s best fall-back for relational market ($\tilde{\Pi}^{RM} = \Pi^{D|SM}$), demanding a more specific input leads to the effects as discussed in section 3.4. First, the profitability of relational market rises such that $\Psi'(r)$ and $\Phi'$ increase in $\varphi$ for $r \leq \hat{r}^{RM}$. Second, as a direct consequence of the enhanced profitability, the cut off interest rates $r^{RM}$ and $\hat{r}^{RM}$ increase.

The effect of varying degrees of input specificity on the downstream’s make-or-buy decision is also depicted in figure 2, where the dashed lines characterize the new cut-offs for $\varphi'$. Consider first area $A$. Here, the downstream party can engage in a superior long-term trad-
ing relationship with the upstream party (relational market), if she depends on a sufficiently specific input (high $\varphi$). Otherwise, she would be compelled to utilize less profitable spot market exchange. Recall that demanding a more specific input—characterized by a higher $\varphi$—deteriorates the upstream’s benefit of engaging in mutual spot exchange as the best alternative to relational market. This further improves the downstream’s expected profit under relational market, and therefore enhances her prospective ‘penalty’ for violating the relational contract with the upstream party. This in turn is reflected by higher threshold interest rates $r_{RM}$ and $\hat{r}_{RM}$, respectively. Generally speaking, the downstream party can now motivate the upstream party to make relation-specific investments in form of perfectly tailoring the intermediate product, even for higher interest rates.

Next, consider area $B$ in figure 2. Here, the downstream party engages in a long-term trading relationship with the upstream party whenever she demands a sufficiently specific input (high $\varphi$). Otherwise, the downstream party would have procured the input through an integrated production. Accordingly, in this area, demanding a highly specific input leads to repeated market transactions dominating vertical integration. Again, this observation is founded on the impairment of the upstream’s best alternative and hence, her bargaining position. This further results in the enhancement of the profitability of repeated market relative to integrated transactions from the downstream’s perspective.

Summarizing the above observations, a firm depending on sufficiently specific intermediate products might favor repeated market transactions over vertical integration for their procurement. This deduction is rooted in the impairment of the upstream’s bargaining position occurring whenever spot market exchange is the downstream’s best alternative to repeated market transactions. Thus, the proposed framework in this paper yields a potential explanation of the contradictory phenomenon whereby certain firms engage in repeated market transactions for procuring highly specific inputs.
5 Conclusion

Prior literature on the theory of the firm presents a number of reasons why firms prefer integrated instead of market transactions. This paper elaborates on the specificity of required inputs and its effect on a firm’s make-or-buy decision. In doing so, it sheds light on the contradictory phenomenon that certain firms engage in repeated market transactions despite a strong reliance on highly specific inputs.

The analysis in this paper highlights one import conclusion: A firm might favor repeated market transactions if the procured input is sufficiently specific. The rationale for this observation is as follows: First, demanding a more specific input impairs the supplier’s bargaining position for mutual market transactions since tailored intermediate products are less likely to be purchased by other market participants. This in turn imposes lower costs on demanding firms for sustaining repeated market transactions founded on relational contracts. Second, relying on more specific inputs facilitates the self-enforcement of relational contracts between firms. The achievement of these relational contracts in turn induces an efficient level of relation-specific investments. This can be observed because repeated market transactions become sufficiently beneficial for the buyer such that hold up is less likely to occur. This paper thus provides a theoretical underpinning of why some firms are inclined to favor repeated market transactions over vertical integration for procuring highly specific inputs.

Indeed, due to the aforementioned reasons, a firm might be better off by procuring highly specific instead of more standardized intermediate products.\textsuperscript{19} Throughout this paper, the desired specificity of an intermediate product was treated as exogenous. This is a reasonable assumption whenever the technology for processing the input does not allow for variations in its characteristics. Alternatively, firms can invest in production technologies that give them some scope to process inputs with differential properties. According to the analysis in this paper, firms might then prefer to procure the most specific inputs that can be still processed by their present technologies. In this case, a firm’s choice in terms of the specificity of the

\textsuperscript{19}It is important to note that the subsequent argumentation implicitly assumes that additional costs imposed by tailoring intermediate products are sufficiently low.
required inputs is made strategically with the aim of deteriorating supplying firm’s bargaining position in repeated inter-firm trade.

From a pragmatic standpoint, a strong dependency on highly specific inputs may not necessarily imply that an internal production is optimal for a firm. Rather, as suggested in this paper, if inputs can be custom-tailored to the extent to which they cannot be processed by any other firms, a long-term trading arrangement founded on relational contracts with an external supplier is the superior alternative for their procurement. In situations of relatively exclusive inputs, a firm can therefore exploit the low bargaining position of its supplier to induce a cost advantage which cannot be replicated within the organization.
Appendix

Proof of Proposition 1.

It is necessary that $\beta > 0$ in order to ensure $e_{W_i} > 0$ for at least one $i \in \{1, 2\}$. Consequently, (6) is satisfied as long as (7) holds, and can therefore be omitted. Assume first that (8) is satisfied for the optimal bonus contract. Since $\mu^T \mu = \|\mu\|^2 = 1$, the Lagrangian is

$$L(\alpha, \beta) = I_L + \Delta I \beta - \alpha - \beta^2 + \lambda \left[ \alpha + \frac{1}{2} \beta^2 \right] + \xi \alpha. \quad (31)$$

The first-order conditions are

$$-1 + \lambda + \xi = 0, \quad (32)$$

$$\Delta I + \beta (\lambda - 2) = 0. \quad (33)$$

To find a solution of this problem, suppose for a moment that $\lambda > 0$. Accordingly, $\alpha + \beta^2/2 = 0$ due to complementary slackness. Since $\alpha \geq 0$, this would imply that $\alpha^* = 0$ and $\beta^* = 0$, and consequently, $e^* = (0, 0)^T$. Hence, $\lambda > 0$ cannot be a solution of this problem. Consequently, $\lambda = 0$, i.e. the worker’s participation constraint is not binding. Furthermore, we can infer from (32) that $\xi = 1$. Then, complementary slackness implies that $\alpha^* = 0$. Re-arranging (33) with $\lambda = 0$ gives $\beta^* = \Delta I/2$. This leads to $\Pi^{D|RE} = I_L + (\Delta I)^2/4$.

Substituting $\Pi^{D|RE}$ and $\beta^* = \Delta I/2$ in (8) gives the cut off interest rate

$$r \leq \frac{\Delta I}{2} - \frac{2}{\Delta I} \left[ \Pi^{RE} - I_L \right] \equiv r^{RE}. \quad (34)$$

If $r > r^{RE}$, $\beta^* = \Delta I/2$ would violate (8). Hence, the downstream party chooses the maximum feasible $\beta$ for $r > r^{RE}$ such that (8) becomes binding. Nevertheless, it is still optimal to set $\alpha^* = 0$. From (8), $\beta^*(r)$ solves

$$\beta^2 - (\Delta I - r) \beta - I_L + \tilde{\Pi}^{RE} = 0, \quad (35)$$

$$\Leftrightarrow \quad \beta^*(r) = \frac{1}{2} (\Delta I - r) \pm \left[ \frac{1}{4} (\Delta I - r)^2 + I_L - \tilde{\Pi}^{RE} \right]^\frac{1}{2}. \quad (36)$$

Since it is optimal to choose the highest feasible $\beta$, the upper bound is relevant. Notice that there exits only a solution if $(\Delta I - r)^2/4 + I_L - \tilde{\Pi}^{RE} \geq 0$, which is equivalent to

$$r \leq \Delta I \pm 2 \left[ \Pi^{RE} - I_L \right]^\frac{1}{2} \equiv \hat{r}^{RE}. \quad (37)$$

24
The upper bound of $\hat{r}^{RE}$ leads to $\beta^*(r) < 0$, which cannot be a solution. Accordingly, the lower bound of $\hat{r}^{RE}$ is relevant. Substituting $\beta^*(r)$ for $r^{RE} < r \leq \hat{r}^{RE}$ in the downstream’s objective function leads to

$$
\Pi^{D|RE}(r) = \frac{T}{2} \left[ \Delta I - r + 2 \left[ \frac{1}{4} (\Delta I - r)^2 + I_L - \tilde{\Pi}^{RE} \right]^{\frac{1}{2}} \right] + \tilde{\Pi}^{RE}.
$$

(38)

Finally, $\beta^*(r) = 0$ for $r > \hat{r}^{RE}$. This leads to $e^*_W = (0, 0)^T$, and hence, $\Pi^{D|RE}(r) = I_L$. □

**Proof of Proposition 2.**

Since $\tilde{\Pi}^{RM} = \Pi^{D|SM}$, it follows $\tilde{\Pi}^U = I_L/2 + [\Delta I\mu + \Delta E\omega]^2/(8c_U)$. The downstream party sets $P_L$ such that (18) becomes binding. By substituting $e^*_U = \Delta P\mu/c_U$, one get

$$
P_L = \frac{1}{2} I_L + \frac{1}{8c_U} (\Delta I\mu + \Delta E\omega)^2 - \frac{1}{2c_U} (\Delta P)^2.
$$

(39)

Suppose for a moment that (20) is satisfied for the optimal premium $\Delta P^*$. Substituting $P_L$ and $e^*_U$ in the downstream’s objective function yields the simplified problem:

$$
\max_{\Delta P} \Pi^{D|RM} = \frac{1}{2} I_L + \frac{1}{2c_U} (\Delta I)^2 - \frac{1}{8c_U} [\Delta I\mu + \Delta E\omega]^2.
$$

(40)

The first derivative leads to $\Delta P^* = \Delta I$. Hence, the downstream party obtains

$$
\Pi^{D|RM} = \frac{1}{2} I_L + \frac{1}{2c_U} (\Delta I)^2 - \frac{1}{8c_U} [\Delta I\mu + \Delta E\omega]^2.
$$

(41)

Substituting $\Pi^{D|RM}$, $\Pi^{D|SM}$, and $\Delta P^* = \Delta I$ in (20) gives

$$
r \leq \frac{1}{8c_U \Delta I} [\Delta I\mu - \Delta E\omega]^2 \equiv r^{RM}.
$$

(42)

If $r > r^{RM}$, the optimal premium $\Delta P^* = \Delta I$ violates (20). Thus, the downstream party chooses the highest feasible $\Delta P$ for $r > r^{RM}$, which satisfies (20). Technically, (20) becomes binding such that $\Delta P^*$ solves

$$
(\Delta P)^2 - 2(\Delta I - rc_U)\Delta P + \frac{1}{4} (\Delta I\mu + \Delta E\omega)^2 + c_U (2\Pi^{D|SM} - I_L) = 0,
$$

(43)

$$
\Leftrightarrow \Delta P^*(r) = \Delta I - rc_U \pm \left[ (\Delta I - rc_U)^2 - \frac{1}{4} (\Delta I\mu + \Delta E\omega)^2 - c_U (2\Pi^{D|SM} - I_L) \right]^{\frac{1}{2}}.
$$

(44)
Since it is optimal to choose the highest feasible \( \Delta P(r) \), the upper bound is relevant. Moreover, there exists only a solution for \( \Delta P(r) \) if

\[
(\Delta I - r_cU)^2 - \frac{1}{4} (\Delta I \mu + \Delta E \omega)^2 - c_U (2\Pi_{D|SM} - I_L) \geq 0, \tag{45}
\]

\[
\iff r \leq \frac{\Delta I}{c_U} \pm \frac{1}{c_U} \left[ \frac{1}{4} (\Delta I \mu + \Delta E \omega)^2 + c_U (2\Pi_{D|SM} - I_L) \right]^{\frac{1}{2}} \equiv \hat{r}_{RM}. \tag{46}
\]

The upper bound of \( \hat{r}_{RM} \) leads to \( \Delta P < 0 \), which cannot be a solution. Thus, the lower bound of \( \hat{r}_{RM} \) is relevant. Substituting \( \Delta P^*(r) \) for \( r_{RM} < r \leq \hat{r}_{RM} \) in the downstream’s objective function yields

\[
\Pi_{D|RM}(r) = r \left[ \Delta I - r_cU + \left( (\Delta I - r_cU)^2 - \frac{1}{4} (\Delta I \mu + \Delta E \omega)^2 - c_U (2\Pi_{D|SM} - I_L) \right)^{\frac{1}{2}} \right] + \Pi_{D|SM}.
\]

Finally, \( \Delta P^* = 0 \) if \( r > \hat{r}_{RM} \). Thus, it is optimal for the downstream party to engage in spot market exchange such that \( \Pi_{D|RM}(r) = \Pi_{D|SM} \) for \( r > \hat{r}_{RM} \).

**Proof of Proposition 3.**

Since \( \Pi^{RM} \neq \Pi^{D|SM} \), it follows \( \Pi^U = (\Delta E)^2/(2c_U) \). The downstream party sets \( P_L \) such that the upstream’s participation constraint becomes binding. By substituting \( e^*_U = \Delta P \mu/c_U \) in (18) and solving for \( P_L \), one get

\[
P_L = \frac{1}{2c_U}(\Delta E)^2 - \frac{1}{2c_U}(\Delta P)^2. \tag{47}
\]

Now suppose that (20) is satisfied for the optimal premium \( \Delta P \). Hence, we can substitute \( P_L \) and \( e^*_U \) in the downstream’s objective function and achieve the simplified problem

\[
\max_{\Delta P} \Pi_{D|RM} = I_L + \frac{1}{c_U} \Delta I \Delta P - \frac{1}{2c_U} (\Delta E)^2 - \frac{1}{2c_U} (\Delta P)^2. \tag{48}
\]

The first-order condition gives to \( \Delta P^* = \Delta I \). Consequently, the downstream party obtains

\[
\Pi_{D|RM} = I_L + \frac{1}{2c_U} \left[ (\Delta I)^2 - (\Delta E)^2 \right]. \tag{49}
\]

Substituting \( \Pi_{D|RM} \) and \( \Delta P^* = \Delta I \) in (20) gives

\[
r \leq \frac{1}{2c_U \Delta I} \left[ (\Delta I)^2 - (\Delta E)^2 - 2c_U \left( \Pi^{RM} - I_L \right) \right] \equiv \tilde{r}_{RM}. \tag{50}
\]
If \( r > r_{RM} \), \( \Delta P^* = \Delta I \) violates (20). Hence, the downstream party chooses the highest feasible \( \Delta P \) for \( r > r_{RM} \), which still satisfies (20). Thus, (20) becomes binding such that \( \Delta P^* \) solves

\[
(\Delta P)^2 - 2(\Delta I - rc_U)\Delta P + (\Delta E)^2 + 2c_U \left( \Pi_{RM} - I_L \right) = 0, \quad (51)
\]

\[
\Leftrightarrow \quad \Delta P^*(r) = \Delta I - rc_U \pm \left[ (\Delta I - rc_U)^2 - (\Delta E)^2 - 2c_U \left( \Pi_{RM} - I_L \right) \right]^\frac{1}{2}. \quad (52)
\]

Again, it is optimal to choose the highest feasible \( \Delta P(r) \) implying that the upper bound is relevant. Furthermore, there exits only a solution for \( \Delta P(r) \) if

\[
(\Delta I - rc_U)^2 - (\Delta E)^2 - 2c_U \left( \Pi_{RM} - I_L \right) \geq 0, \quad (53)
\]

\[
\Leftrightarrow \quad r \leq \frac{\Delta I}{c_U} \pm \frac{1}{c_U} \left[ (\Delta E)^2 + 2c_U \left( \Pi_{RM} - I_L \right) \right]^\frac{1}{2} \equiv \hat{r}_{RM}. \quad (54)
\]

The upper bound of \( \hat{r}_{RM} \) implies \( \Delta P < 0 \), which cannot be a solution. Hence, the lower bound of \( \hat{r}_{RM} \) is relevant. Substituting \( \Delta P^*(r) \) for \( r_{RM} < r \leq \hat{r}_{RM} \) in the downstream’s objective function gives

\[
\Pi_{D|RM}(r) = r \left[ \Delta I - rc_U + \left[ (\Delta I - rc_U)^2 - (\Delta E)^2 - 2c_U \left( \Pi_{RM} - I_L \right) \right]^\frac{1}{2} \right] + \Pi_{RM}. \quad (55)
\]

Finally, \( \Delta P^* = 0 \) if \( r > \hat{r}_{RM} \). Then, it is optimal for the downstream party to engage in spot market exchange such that \( \Pi_{D|RM}(r) = \Pi_{D|SM} \) for \( r > \hat{r}_{RM} \).

\[\square\]

**Comparison of Relational Market and Relational Employment.**

Consider first the case \( \Pi_{RM} = \Pi_{D|SM} \). Then, \( \Pi_{D|RM} \geq \Pi_{D|RE} \) is equivalent to

\[
(\Delta E)^2 + 2\Delta I \cos \varphi \Delta E + 4c_UI_L - (\Delta I)^2 (3 - 2c_U) \leq 0. \quad (56)
\]

First, we can treat (56) as an equality. By applying the quadratic formula, one get

\[
\Delta E = -\Delta I \cos \varphi \pm \sqrt{(\Delta I)^2 \cos^2 \varphi - 4c_UI_L + (\Delta I)^2 (3 - 2c_U)}. \quad (57)
\]

The upper bound is relevant since \( \Delta E > 0 \). Thus, \( \Pi_{D|RM} \geq \Pi_{D|RE} \) requires

\[
(\Delta E)^2 \leq \left[ (\Delta I)^2 \cos^2 \varphi + (\Delta I)^2 (3 - 2c_U) - 4c_UI_L \right]^\frac{1}{2} - \Delta I \cos \varphi \right]^2 \equiv \Phi. \quad (58)
\]
In case \( \tilde{\Pi}^{RM} \neq \Pi^{D|SM} \), it can be shown that \( \Pi^{D|RM} \geq \Pi^{D|RE} \) is equivalent to \((\Delta E)^2 \leq (\Delta I)^2(2 - c_U) / 2 \equiv \Phi \). □

**Proof of Proposition 4.**

Suppose first that \( r \leq r^i, i = RM, RE \). Then, it is necessary to identify whether the downstream party prefers different relational contracts for the same value of \((\Delta E)^2\), but different values of \( r \). First, consider the intervals \( 0 < (\Delta E)^2 \leq \Phi \) and \( \Theta < (\Delta E)^2 \leq \Phi \), where \( \Pi^{D|RM} \geq \Pi^{D|RE} \). Suppose for a moment that \( r^{RM} \geq r^{RE} \), which is equivalent to

\[
\frac{\Pi^{D|RM} - \bar{\Pi}^{RM}}{\Delta I} \geq \frac{2[\Pi^{D|RE} - \bar{\Pi}^{RE}]}{\Delta I}.
\]

If \( r^{RM} \geq r^{RE} \), we have \( \bar{\Pi}^{RM} = \Pi^{D|SM} \) and \( \bar{\Pi}^{RE} = \Pi^{D|RM} \), leading to

\[
3\Pi^{D|RM} \geq 2\Pi^{D|RE} + \Pi^{D|SM},
\]

which is always true in the intervals \( 0 < (\Delta E)^2 \leq \Phi \) and \( \Theta < (\Delta E)^2 \leq \Phi \). Thus, \( r^{RM} \geq r^{RE} \). Since \( \Pi^{D|RM} \geq \Pi^{D|RE} \) in these intervals, the downstream party chooses relational market if \( r \leq r^{RM} \), and a spot contract, otherwise.

Next, consider the intervals \( \Phi < (\Delta E)^2 \leq \Theta \) and \( \Phi < (\Delta E)^2 \leq \Phi \), where \( \Pi^{D|RE} > \Pi^{D|RM} \). Suppose for a moment that \( r^{RE} \geq r^{RM} \), implying \( \bar{\Pi}^{RE} = \Pi^{D|SM} \) and \( \bar{\Pi}^{RM} = \Pi^{D|RE} \). Hence, \( r^{RE} \geq r^{RM} \) is equivalent to

\[
3\Pi^{D|RE} \geq \Pi^{D|RM} + 2\Pi^{D|SM},
\]

which is satisfied for \( \Phi < (\Delta E)^2 \leq \Theta \) and \( \Phi < (\Delta E)^2 \) since \( \Pi^{D|RE} \geq \Pi^{D|RM} \). Thus, \( r^{RE} \geq r^{RM} \) in these two intervals. Since \( \Pi^{D|RE} \geq \Pi^{D|RM} \) in these intervals, the downstream party chooses relational employment if \( r \leq r^{RE} \). Finally, by substituting the respective fallback spot profits in \( r^{RM} \) and \( r^{RE} \), one get

\[
r^{RM} = \begin{cases} 
\frac{1}{8c_U\Delta I} [\Delta I \mu - \Delta E \omega]^2, & \text{if } \bar{\Pi}^{RM} = \Pi^{D|SM} \\
\frac{1}{2c_U\Delta I} [\Delta I - (\Delta E)^2], & \text{if } \bar{\Pi}^{RM} = \Pi^{D|SE},
\end{cases}
\]

\[
r^{RE} = \frac{\Delta I}{2} - \frac{2}{\Delta I} \left[ \max\{\Pi^{D|SM}, \Pi^{D|SE}\} - I_L \right].
\]
Next, consider the case $r^i < r \leq \hat{r}^i$, $i = RM, RE$. Recall that $\Pi^{D|RM}(r) \geq \Pi^{D|RE}(r)$ in the intervals $0 < (\Delta E)^2 \leq \Psi(r)$ and $\Theta < (\Delta E)^2 \leq \overline{\Psi}(r)$, whereas the cut offs $\Psi(r)$ and $\overline{\Psi}(r)$ are valid for all $r \in (r^i, \hat{r}^i]$, $i = RM, RE$.

Finally, the downstream’s expected profits under relational employment and relational market are decreasing in $r$ for $r^i < r \leq \hat{r}^i$, $i = RM, RE$. Thus, it is necessary to identify the cut off interest rate $\bar{r}^i$ where the downstream party is indifferent between the respective relational contract and the best spot alternative. Consider first relational employment. Hence, $\bar{r}^{RE}$ implies

$$
\Pi^{D|RE}(\bar{r}^{RE}) = \tilde{\Pi}^{RE} \iff \bar{r}^{RE} = \frac{\Delta I - \bar{r}^{RE} + 2\left[\frac{1}{4}(\Delta I - \bar{r}^{RE})^2 + I_L - \bar{\Pi}^{RE}\right]^\frac{1}{2}}{2} = 0,
$$

$$
\iff \frac{1}{4}(\Delta I - \bar{r}^{RE})^2 + I_L - \bar{\Pi}^{RE} = \frac{1}{4}(\Delta I - \bar{r}^{RE})^2. \tag{64}
$$

Because $\bar{\Pi}^{RE} = \Pi^{D|SM} > I_L$, (64) cannot be satisfied. Thus, $\hat{r}^{RE}$ is the relevant cut off such that the downstream party obtains $\Pi^{D|SM}$ if $r > \hat{r}^{RE}$. Consider now relational market, where $\bar{r}^{RM}$ implies $\Pi^{D|RM}(\bar{r}^{RM}) = \tilde{\Pi}^{RM}$. By applying the same approach as for relational employment, one can show that the downstream party obtains $\Pi^{D|SM}$ if $r > \hat{r}^{RM}$. By substituting the respective fall-back spot profits in $\hat{r}^{RM}$ and $\hat{r}^{RE}$, one get

$$
\hat{r}^{RM} = \begin{cases} 
\frac{\Delta I}{c_U} - \frac{1}{c_U} \left[\frac{1}{4}(\Delta I \mu + \Delta E \omega)^2 + \frac{1}{2} (\Delta I)^2 - (\Delta E)^2\right]^\frac{1}{2}, & \text{if } \tilde{\Pi}^{RM} = \Pi^{D|SM} \\
\frac{1}{c_U} [\Delta I - \Delta E], & \text{if } \tilde{\Pi}^{RM} = \Pi^{D|SE}, 
\end{cases}
$$

$$
\hat{r}^{RE} = \Delta I - 2 \left[\max\{\Pi^{D|SM}, \Pi^{D|SE}\} - I_L\right]^\frac{1}{2}.
$$

\qed
References


