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Universitat Jaume I de Castellò

2010

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Abstract

Several agent-based models have been proposed in the economic literature to explain the key stylized facts of financial data: heteroscedasticity, fat tails of returns and long-range dependence of volatility. Agent-based models view these empirical regularities as emerging properties of interacting groups of boundedly rational agents in financial markets. The complexity of these interacting agent models has largely constrained their analytical treatment, limiting their analysis mainly to Monte Carlo simulations. In order to overcome this limitation, we introduce a 'minimalist' model of an artificial financial market, along the lines of our previous contributions, based on herding behavior among two types of traders. The simplicity of the model allows for an almost complete analytical characterization of both conditional and unconditional statistical properties of prices and returns. Moreover, the underlying parameters of the model can be estimated directly, which permits an assessment of its goodness-of-fit for empirical data. While the performance of the model for domestic stock markets has been the focus of a previous contribution, in this paper we report results for selected exchange rates against the US dollar.

Keywords: Herd Behavior; Speculative Dynamics; Fat Tails; Volatility Clustering.
JEL Classification: G12; C61.

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1 Introduction

The Efficient Market Hypothesis (EMH hereafter) considers security markets as efficient mechanism for immediate and unbiased incorporation of new information into prices. Within the EMH, as argued by Friedman [14] and Fama [11], the presence of non-rational traders can be neglected, since their idiosyncratic errors would be averaged out in the aggregate so that they could not significantly affect the market price. Rather, they would progressively lose money in favor of arbitrageurs, betting against them, so that the less rational agents would eventually disappear at the end from the market.

Recent empirical and theoretical investigations have attacked the EMH and its implications in various ways. From a theoretical viewpoint, it has been shown that arbitrageurs may have limited capacity to drive the prices close enough to the fundamental value, if they have a finite time horizon, or in the presence of fundamental risks - see Figlewski [12] and Shiller [24]. The seminal paper by De Long *et al.* [10] has demonstrated that noise traders can create "their own space" in the market and that they might even earn higher returns than sophisticated investors. From an empirical point of view, the most relevant piece of evidence against the EMH is excess volatility of prices when compared to the underlying fundamentals, as pointed out by West [26] among others. One might also ask whether it is plausible that informationally efficient prices would give rise to the long list of extremely robust statistical findings such as the conditional structure of the volatility itself - from the ARCH effect, to the multi-scaling of the level of fluctuations of returns - or the fatness of the tails of the distribution of returns (for an authoritative survey see for instance Pagan [23]). The presence of those complex empirical regularities embedded in the time series of prices may also cast some doubts on the simple one-to-one relationship between price changes and information as implied by the EMH. If we assume that the "relevant" information is made up of a collection of non-correlated news, economical, political and even meteorological, it is hard to justify that such 'a composite 'assortment" of news possesses the complex temporal structure observed for volatility. Anyhow, a strict empirical validation of such a relationship is practically impossible, since the information arrival process is not directly observable.

From the viewpoint of agent-based models, these empirical findings might alternatively be viewed as the imprint of an endogenous dynamics of the market which might be partially decoupled from fundamental factors. Several authors have attempted to model financial markets as a system of heterogeneous interacting agents, whose activities might be responsible for this intrinsic force. A long, however partial, list of contributions in this vein ranges from the (very) early papers of Baumol [6] and Zeeman [27], to recent research on noise traders, fundamentalist/chartist interaction and 'artificial' financial markets (Arthur *et al.* [5], De Long *et al.* [10], Kirman [18] and Beja and Goldman [7] being some prominent examples). Much of this literature on financial markets from a dynamical system perspective has developed in parallel with the behavioral finance literature and choice-theoretical works on financial 'anomalies', explaining the rational behavioral roots of overcorrection, herding behavior and other formerly puzzling observations (cf. [8]). Available dynamic market models differ in the degree of heterogeneity of traders or in the way they interact. Despite many differences, many of them can successfully replicate the key stylized facts and explain their universality as an emergent property of the interactions among traders.

One of the main drawbacks of the agent-based models is the complexity of their interactions, which typically prevents an analytical solution, leaving only the possibility for Monte Carlo simulations based on a rough calibration of the underlying parameters (see e.g. [20]). In addition, it is so far hardly possible to directly compare different models, or to asses their goodness-of-fit. As far as we know the only exception is a recent contribution by Gilli and Winker [15], who estimate some of the parameters of Kirman's seminal herding model [17, 18] via an *indirect* simulated method of moments approach. The main contribution of their exercise is that they show that estimated parameters give rise to a bimodal distribution of the population dynamics, i.e. majorities would emerge in the herding process, instead of a balanced distribution of agents on the two groups of chartists and fundamentalists.

A *direct* estimation of the parameters of a related agent-based model, based on a closed-form solution of the unconditional distribution of returns, has been proposed by Alfarano *et al.* [3], who used a modified and generalized version of the stochastic chartist-fundamentalist approach proposed by Kirman. In their approach, the pool of agents is also divided into two distinct categories or types: fundamentalists and noise traders. The traders interact via a similar recruitment mechanism as in Kirman [18], but the second group is assumed to follow changing fads and moods rather than technical receipts. The interactions among the agents is embedded in an extremely simple market structure, characterized by two behavioral rules for excess demand by the two groups the traders belong to. The dynamics governing the switches between the two groups - namely fundamentalists and noise traders - detailed in section 2, together with the market mechanism, described in section 3, allows for an analytical characterization of both conditional and unconditional properties of returns. This enables us to provide a more thorough characterization of the outcome of the model than purely numerical approach of conducting Monte Carlo simulations. On a theoretical level, we can investigate to which extent the pairwise interaction among the traders gives rise to a genuine Pareto behavior of the extreme returns and hyperbolic decline of volatility autocorrelation, rather than reproduce them as pseudo-scaling laws of the synthetic data.[1] Additionally, the theoretical results enable us to estimate the underlying parameters and to evaluate the goodness-of-fit of the model. In particular, the unconditional distribution of log-returns can be derived in closed form, described in section 3, which allows to estimate the equilibrium parameters via Maximum Likelihood, as will be shown in section 4. We provide an illustration of this procedure for the exchange rates of the main currencies against US dollar. Some final remarks conclude the paper.

2 The herding mechanism

In a long series of papers ([17, 18, 19] among others), Kirman employed a simple model of information transmission to describe the behavior of a multitude of heterogeneous interacting agents in a foreign exchange market. He draws his inspiration from work on recruitment in ant colonies. Experimentally, it had been observed that the majority of an ant population, feeding from one of two identical sources, eventually switched to the other. Kirman, adopted this entomologic framework as a model of an artificial foreign exchange market, replacing the ants by financial agents and the two sources of food by two different forecasting rules for exchange rate changes, within the well-established framework of a monetary model with fundamentalist-chartist interaction, as developed by Frankel and Froot [13]. As it turned out, the exchange rate would stay close to its underlying fundamental value during periods of fundamentalists dominance, while speculative bubbles would emerge in this model if chartists take over. The foreign exchange market would, therefore, be characterized by repeated periods of price dynamics decoupled from fundamentals (thus, explaining the 'exchange rate disconnected' puzzle), which, however, come to end when agents switch back to fundamentalist behavior.

The core of his model is the pairwise interaction governing the transition of the agents between the two states, denoted as state 1 and state 2. The system can be conveniently described by the integer number of agents n in the state 1, where $n \in \{1, 2, ..., N\}$. N represents the total number of agents, assumed to be constant over time[2]. To set the stage for the model, we specify the conditional transition probabilities to switch from one state to the other:

$$\begin{aligned} &\rho(n+1, t+\Delta\tau | n, t) = (N-n)(a_1 + bn)\Delta\tau \ , \\ &\rho(n-1, t+\Delta\tau | n, t) = n\big(a_2 + b(N-n)\big)\Delta\tau \ . \end{aligned} \tag{1}$$

where a_1, a_2 and b are constant parameters[3] and $\Delta\tau$ a unit micro-time step. The above probabilities define a finite Markov chain, i.e. a Markovian stochastic process defined on a finite set

[1] Alfarano and Lux [1] have shown that a related version of this herding agent-based model can just 'mimic' the scaling laws of extreme returns and temporal dependence in volatility -for a description of the problem see [22] and references therein.

[2] For a generalization to a variable number of agents see [4].

[3] In Kirman's paper the two constants a_1 and a_2 are assumed to be equal, while, in the generalization introduced by Alfarano *et al.*[3], they might take different values.

of states, with a discrete time variable and stationary transition probabilities. More precisely, the process belongs to the general types of "birth-and-death" or "one-step" stochastic processes, using the terminology of van Kampen [25]. The conditional probability $\rho(n, t+\Delta\tau|n, t)$ to remain in the same state follows from the normalization condition $\sum_{n'} \rho(n', t+\Delta\tau|n, t) = 1$.

The transition probabilities, introduced in eq. (1), consist of two terms: the first term, proportional to a_1 and a_2, which is linear in n, formalizes the idiosyncratic propensity to switch to the other strategy; the second term, quadratic in n and proportional to b, encapsulates the herding tendency, since it is proportional to the product of the number of agents in the two states, $(N-n)n$. The non-linearity in the transition probabilities (1) constitutes a crucial ingredient of the model: the presence of non-linear terms, in fact, is the imprint of interactions among agents, while the occurrence of linearity only would imply independence of the behavior of individuals (for more details see [25]).

From the transition probabilities (1) we can derive the so-called Master equation for the probability[4] $\bar{\omega}_n(t)$ to find n agents in state 1 at time t

$$\frac{\Delta\bar{\omega}_n(t)}{\Delta\tau} = \sum_{n'} \left(\bar{\omega}_{n'}\ \pi(n' \to n)\ -\ \bar{\omega}_n\ \pi(n \to n')\right), \tag{2}$$

where $\pi(n' \to n)$ are the transition probabilities per unit-time. The Master equation governs the time evolution of the probability $\bar{\omega}_n(t)$ as a competition between the outflow and inflow probabilities of the agents from and to a particular state. For large enough N we can represent the group dynamics by a continuous variable $z = n/N$. As derived in [3], the Master equation (2) can be approximated by a Fokker-Planck equation[5]:

$$\frac{\partial\omega(z,t)}{\partial t} = -\frac{\partial}{\partial z}\big(A(z)\omega(z,t)\big) + \frac{1}{2}\frac{\partial^2}{\partial z^2}\big(D(z)\omega(z,t)\big)\ . \tag{3}$$

The function $A(z)$ represents the drift term, while $D(z)$ is the diffusion function, given by:

$$A(z) = a_1 - (a_1 + a_2)z \qquad \text{and} \qquad D(z) = 2b(1-z)z\ . \tag{4}$$

Eq. (3) turns out to be analytically tractable, providing us with closed-form solutions of a wide range of conditional and unconditional properties of the system (equilibrium distribution and autocorrelation functions, for instance). Focusing on the equilibrium properties, it has been shown by Alfarano *et. al* [3], that the equilibrium distribution $\omega_0(z)$ depends only on the ratios $\varepsilon_1 = \frac{a_1}{b}$ and $\varepsilon_2 = \frac{a_2}{b}$, but not on the size of the constants a_1, a_2 and b:

$$\omega_0(z) = \frac{1}{B(\varepsilon_1, \varepsilon_2)}\ z^{\varepsilon_1 - 1}\,(1-z)^{\varepsilon_2 - 1}\ , \tag{5}$$

where

$$B(\varepsilon_1, \varepsilon_2) = \frac{\Gamma(\varepsilon_1)\Gamma(\varepsilon_2)}{\Gamma(\varepsilon_1 + \varepsilon_2)}\ , \tag{6}$$

with $\Gamma(.)$ denoting the Gamma function. Despite the dependence on just two parameters, eq. (5) is extremely flexible in describing different scenarios: both uni- or bimodal asymmetric equilibrium distributions, or monotonic increasing or deceasing distributions are possible depending on the choice of the parameters ε_1 and ε_2.

[4] We denote probabilities referring to n by $\bar{\omega}$ to distinguish them from the probability densities $\omega(z)$ for the continuous variable z introduced below. Both are related by $\omega(n/N) = N\bar{\omega}_n$.

[5] For all the details of the derivation and the underlying approximations we refer the reader to [3].

To summarize, the Markov chain defined by the transition probabilities (1), for large but finite system size N, can be approximated by a continuous diffusion process, governed by the Fokker-Plank equation (3). The asymptotic approximation given by eq. (3) provides an entire pool of analytical results, which can be exploited for estimation of the underlying parameters. What is more, the solutions of the relatively general process of interactions of individuals can be used to also arrive at analytical insights into the dynamics of markets in which this process is combined with behavioral relationships as well as a standard price formation rule. We now turn to this model of a simple artificial market and its dynamical properties.

3 The artificial market

3.1 Description of the market structure

Our market is populated by a fixed number of traders N, falling into two categories or types:

- N_F *fundamentalists*, who buy or sell according to the deviation between the actual price p and the fundamental value p_F;
- N_C *noise traders* who are subject to “irrational” fads or moods as introduced in the seminal paper by De Long *et al* [10].

For simplicity, the fundamental value p_F is assumed to be constant over time. The former state 1 now stands for fundamentalist disposition, while the second state stands for noise traders. The number of agents in each group varies over time according to the stochastic process detailed in section 2. The trading attitudes of the agents translate into a changes of the market price via two behavioral rules for demand and supply. Fundamentalists' excess demand is given by:

$$ED_F = N_F \ln \frac{p_F}{p} \; . \tag{7}$$

We assume that each fundamentalist is characterized by the same reaction to deviations from the fundamental value, buying or selling whenever he perceives an undervaluation or overvaluation of the stock price. The aggregate excess demand of this group is, then, the sum of the demand of a ‘representative’ fundamentalist times the number of fundamentalists, N_F. The noise traders' aggregate excess demand takes the form:

$$ED_C = -r_0 N_C \xi \; , \tag{8}$$

where ξ represents the actual average ‘mood’ of the noise traders. The constant r_0 is a scale factor for their impact on the price formation, and the expression is multiplied by -1 for notational convenience. It is important to highlight that we model the aggregate excess demand of the noise traders' group without accounting for specific technical trading rules, typically found in the literature (moving average, trend extrapolation or pattern recognition). We rather model the aggregate impact of many heterogeneous chartist techniques as a pure stochastic term of random sign and magnitude, whose properties will be described later.

Within a Walrasian scenario, we can compute the equilibrium price by simply setting the total excess demand equal to zero:

$$ED_F + ED_C = 0 \; . \tag{9}$$

We, then, end up with the following formula for the market price:

$$p = p_F \exp\left(r_0 \frac{z}{1-z} \xi \right) \; , \tag{10}$$

where z and $1-z$ are the fractions of the noise traders and fundamentalists among agents, respectively. The returns over a time interval Δt are given by[6]:

$$r(t,\Delta t) = r_0 \left[\frac{z(t+\Delta t)}{1-z(t+\Delta t)}\,\xi(t+\Delta t) \;-\; \frac{z(t)}{1-z(t)}\,\xi(t)\right]. \tag{11}$$

A full analytical solution for eq. (11) turns out to be cumbersome, taking into account the positive correlation of the variable $z/(1-z)$ over time and the presence of two sources of randomness, namely z and ξ. However, we can approximate eq. (11) by assuming a 'faster' dynamics for ξ compared to that of the variable $z/(1-z)$, which can be considered to be constant during a small time interval Δt. This approximation amounts to separating the time scales governing the switching process among attitudes and the underlying dynamics of the 'mood' of the noise traders. Under this assumption, eq. (11) can be approximated by:

$$r(t,\Delta t) = r_0\,\frac{z(t)}{1-z(t)}\,\eta(t,\Delta t) \tag{12}$$

where we define $\eta(t,\Delta t) \equiv \xi(t+\Delta t) - \xi(t)$. Eq. (12) can accordingly be rewritten as:

$$r(t,\Delta t) = \sigma(t)\,\eta(t,\Delta t)\;, \tag{13}$$

where we assume that η is iid with a given distribution $p(\eta)$, and $\sigma(t) = r_0 z/(1-z)$. Eq. (13) possesses a so-called *stochastic volatility* structure, i.e. is given by the product of a white noise, η, and a conditional volatility factor, σ, which describes the empirically observed time-dependencies. The iid-ness of the multiplicative noise η guarantees the absence of linear correlation of returns, in accordance with empirical facts (see for example [23]). The positive correlations of non-linear transformation of returns, squared or absolute values, are then governed by the correlations in the volatility $\sigma(t)$, which originate from and are related to the dynamical properties of $z(t)$.

The average noise traders' mood is a random walk process with increments given by η. Following the set-up in the model by De Long *et al.* [10], the stochastic variable ξ, rather than η, would be iid. However their model is based on an overlapping generation framework, so that the underlying time horizon should be larger than in our approach, which we rather consider as a model for high-frequency data, i.e. daily or even intra-daily price movements. Moreover, the random walk implementation avoids the abrupt variations of the market price implied by the formalism of De Long *et al.*

Figure 1 shows a typical price pattern from a simulation of eq. (10). The market price fluctuates around the fundamental value $p_F = 1$, with both periods of positive and negative deviations from it, which can be interpreted as bubbles, and subsequent returns to the fundamental value. The corresponding time series of returns exhibits volatility clusters, which arise in close correspondence to deviations from the fundamental value, see panel (b) in Figure 1. This intermittent behavior of the returns is related to the change in the market attitude of the traders. Periods of high volatility correspond to time periods with a large fraction of noise traders. *Vice versa*, only minor fluctuations occurs when the market is dominated by fundamentalists. The market as a whole exhibits excess volatility. In our simulation, all the fluctuations of returns are, in fact, generated by the speculative activities of traders, and are disconnected from the fundamental price, here assumed to be constant [7]. The herd behavior among traders, then, provides the ultimate "engine" for this complex market

[6] We define continuously compounded returns as $r(t,\Delta t) = \ln(p_{t+\Delta t}/p_t)$. Note that the time-unit Δt of the returns process is different from the elementary time-unit of the population dynamics $\Delta\tau$. We, therefore, refer to the former as micro-time and the latter as macro-time. Essentially, during a macro-time Δt, z is averaged over the movement of many agents between the two states (see [2] for more details).

[7] We could, of course, add stochastic changes of the fundamental value without changing the overall appearance of the time series.

dynamics which shares the basic stylized facts of high-frequency financial data. The behavior of the autocorrelation of raw returns and their simple non-linear transformations reflect this particular intermittent dynamics; absence of linear correlation in returns and positive significant correlation in absolute and squared returns (as measure of volatility) are robust features of the model, as illustrated in the bottom panel of Figure 1.

3.2 Unconditional distribution of returns

The simple structure of eq. (13) allows to derive a closed-form solution of the unconditional distribution of returns. The equilibrium distribution of the variable $\sigma(t)$ has been derived in [3], and is given by:

$$p(\sigma) = \frac{1}{r_0} \frac{1}{B(\varepsilon_1, \varepsilon_2)} \left(\frac{\sigma}{r_0}\right)^{\varepsilon_1 - 1} \left(\frac{r_0}{\sigma + r_0}\right)^{\varepsilon_1 + \varepsilon_2} . \tag{14}$$

Interestingly, this distribution exhibits a power law behavior in its outer part, with a decay parameter $\mu = \varepsilon_2 + 1$. Under the condition that $E[|\eta|^{\varepsilon_2}] < \infty$, this power law decay also carries over the distribution of r. Two important aspects of eq. (14) are worth mentioning: the first is the *endogenous* generation of power law behavior of extreme returns as a result of the structural properties of our model, which is compatible with empirical evidence; the second is the characterization of its exponent by behavioral parameters governing the speculative dynamics, namely the ratio between the tendency of autonomous switches from fundamentalist to noise trader behavior a_2, and the herding parameter b.

While the power law decay of the tail is very robust with respect to the choice of $p(\eta)$ [8], a parametric choice of this distribution is necessary in order to end up with a closed-form solution for the returns distribution. Since η is not directly observable, our choice here is rather arbitrary: it is mainly driven by the convenience of the explicit solution than by economic or statistical justifications. Thus, we assume a uniform distributed random variable over the interval $[-1, 1]$ for the distribution of η. However different specifications for $p(\eta)$ could be tried and their explanatory power compared.

One can show that, for this parametric choice of $p(\eta)$, the unconditional distribution of absolute returns is given by:

$$p_u(v) = \frac{1}{r_0} \frac{\varepsilon_2}{\varepsilon_1 - 1} \left[1 - \beta\left(\frac{v}{v + r_0}\, ; \varepsilon_1 - 1, \varepsilon_2 + 1\right)\right] , \tag{15}$$

where we have used the underlying symmetry around the mean of eq. (13). $\beta(\cdot; \cdot, \cdot)$ is the incomplete beta function. The subscript u indicates that we have used the uniform distributed multiplicative noise. For more details we refer the reader again to [3].

4 Estimation of the Parameters

Equipped with the above results, we can proceed to estimation of the three parameters of the model, namely r_0, ε_1 and ε_2, via Maximum Likelihood. We should stress, however, that our likelihood, based on eq. (15), is an approximation of the 'true' likelihood associated with the stochastic process (13). We pretend, in fact, that the realizations from this Markovian process are independent and identically distributed, according to the common distribution given by eq. (15), for

[8] The only required condition for the emergence of power law decay in returns is the boundedness of the ε_2-th absolute moment of $p(\eta)$. For a uniformly or normally distributed random variable, for example, all the absolute moments are finite.

more details see [3] and references therein. The estimation is computed under the normalization $E[v] = 1$, which allows to express r_0 as a function of the other two parameters: $r_0 = 2\frac{\varepsilon_2 - 1}{\varepsilon_1}$. Table 1 exhibits the results of the estimation procedure for five time series of major currencies against the US \$. The following currencies have been used: Canadian Dollar (CD), Japanese Yen (JP), Deutsche Mark (DM), British Pound (BP) and Swiss Franc (SF). The samples all consist of a total of 3 913 daily observations, ranging from December 15, 1989. to December 15, 2004. As an

Data Set	$\hat{\varepsilon}_1$	$\hat{\varepsilon}_2$	$-\ln L_{\varepsilon_1,\varepsilon_2}$	$\hat{\alpha}_H$
CD	14 ± 5	5.9 ± 0.9	3291.0	4.1
				$(3.2, 4.9)$
JY	5.2 ± 1.0	7.0 ± 1.0	3707.2	3.7
				$(3.0, 4.5)$
DM	5.2 ± 0.9	14.0 ± 4.0	3517.4	4.5
				$(3.6, 5.5)$
BP	4.9 ± 0.8	9.0 ± 3.0	3478.8	4.4
				$(3.5, 5.3)$
SF	6.0 ± 1.0	12.0 ± 4.0	3336.3	4.5
				$(3.5, 5.5)$

Table 1. *Estimated parameters using the model from eq. (15). The last column shows the tail index estimates computed with the method of Hill [16], for a 2.5% tail size, with their 95% asymptotic confidence interval.*

illustrative example, Figure 2 compares the theoretical and empirical distributuion for the case of DM/USD, which shows the good performance of the model in describing both the probability density and the cumulative distribution.

An important feature of the model is represented by the relationship between the parameters that govern the behavior of the traders, ε_1 and ε_2, and the resulting equilibrium distribution of returns. For example, $\varepsilon_2 > \varepsilon_1$ indicates that, most of the time, the market is dominated by fundamentalists. From Table 1, we may, thus, infer a predominance of fundamentalist attitude of the traders since ε_2 is greater than ε_1 in 4 out of 5 cases. A closer look at Table 1, however, also shows a somewhat disappointing behavior of the estimated values of the parameter ε_2. This parameter represents the index of the tail of the unconditional distribution, i.e. the rate of the approximatively linear decay of the outer part of the empirical distribution, see the inlet in Figure 2. The last column of Table 1 exhibits the estimated empirical tail indices computed using the semi-parametric approach proposed by Hill [16], which are in good agreement with the results found in the empirical literature (see for instance [21]), namely a narrow interval of variability, centered at some value slightly higher than 3 and ranging from 2.5 to 4.5. On the contrary, our parametric estimates are very heterogeneous, from a minimum value of 5.9 for the Canadian Dollar to a maximum value of 14 for the Deutsche Mark, and far from the empirically identified ‘typical’ value of 3.

The two main results of the estimation procedure, namely the dominance of the fundamentalists and the discrepancy between the values of the parametric estimation as compared to the Hill estimates, are not in harmony with our previous results, reported in [3]. In this earlier contribution, considering stock market and precious metal data, we ended up with rather different conclusions: strong evidence on the dominance of noise traders was found for all the time series, and the parametrically estimated values of ε_2 were well aligned to those computed with the Hill estimator.

What might be the reason for this contradictory behavior? A simple comparison of the DM/USD time series and the German stock market index DAX, both shown in Figure 3, might give a hint at qualitatively different behavior. We observe, in fact, several alternating periods of large and small market movements (volatility clustering) in the case of the DAX, while, for the exchange rate DM/USD, the volatility dynamics appears to exhibits less striking fluctuations. Another noticeable dissimilarity is the much wider interval of variability of the absolute returns for the DAX. Those graphical differences are systematically observed in all the analyzed time series (details upon request). It seems plausible that such different behavior of the two sets of time series may generate the differences in the estimated parameters ε_1 and ε_2. A higher value of ε_2, as compared to the typical value of the tail index is the necessary compromise that the ML procedure takes in order to simultaneously fit the small interval of variability of the data and the empirical value of the distribution at the origin (i.e. $p_u(0) = \frac{1}{2}\frac{\varepsilon_2}{\varepsilon_2-1}\frac{\varepsilon_1-1}{\varepsilon_1}$, approximatively $\hat{p}_u(0) \approx 0.7$ for all the analyzed time series).[9] Such large values of ε_2 generate a very rapid decay of the distribution (15), which implies the absence of extreme events and a diminishing interval of variability of returns. For example, the probability to observe a large price change, say $v_n > 10$, with a parameter value $\varepsilon_2 = 10$ is practically negligible.

Finally, these results suggest some words of caution. It has been repeatedly claimed that security prices and floating exchange rates share the same statistical regularities (see [9]). However, for the currencies listed in Table 1, we do not observed the strikingly large daily movements that are regularly observed in stock market indices. Therefore, further research would be necessary to fully understand whether this contradictory behavior of the model, when applied to stock market data or FX rates, is an imprint of real differences in the two markets, or simply an artefact of the estimation procedure. A further interesting addition to the research reported in this paper, would consist in considering another important category of traders in the FX markets, central banks, whose role is not taken into account in the present version of the model, and who might contribute to the stronger fundamentalist tendency in foreign exchange as compared to stock markets.

[9] To confirm our hypothesis, we have performed a numerical experiment: we artificially eliminated from the time series of the DAX all absolute returns larger than 7, which approximatively is the maximum absolute change of the USD/DM time series. The estimated values of $\hat{\varepsilon}_{1,2}$ for this modified sample are in line with those obtained for the FX time series, namely $\varepsilon_2 > \varepsilon_1$ and a large value for ε_2.

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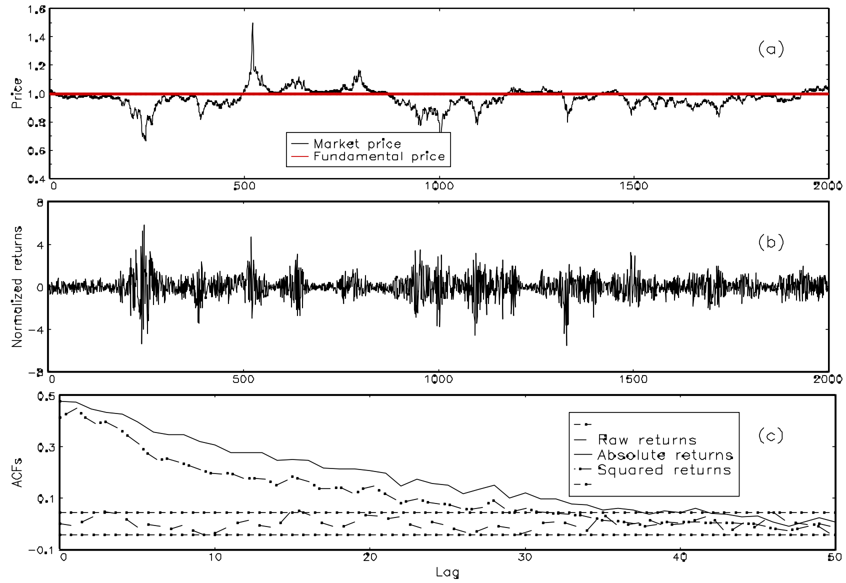

Fig. 1. *Panel **(a)** shows a simulation of the price derived from equation (10) using a uniform distribution for* η*. Panel **(b)** shows the returns obtained by using eq. (12). Panel **(c)** shows the autocorrelation function of raw, absolute and squared returns. As parameters we have chosen* $p_f = 1$, $r_0 = 0.1$ *and* $\Delta t = 1$*. The herding parameters are* $\varepsilon_1 = 3$, $\varepsilon_2 = 4$ *and* $b = 0.003$.

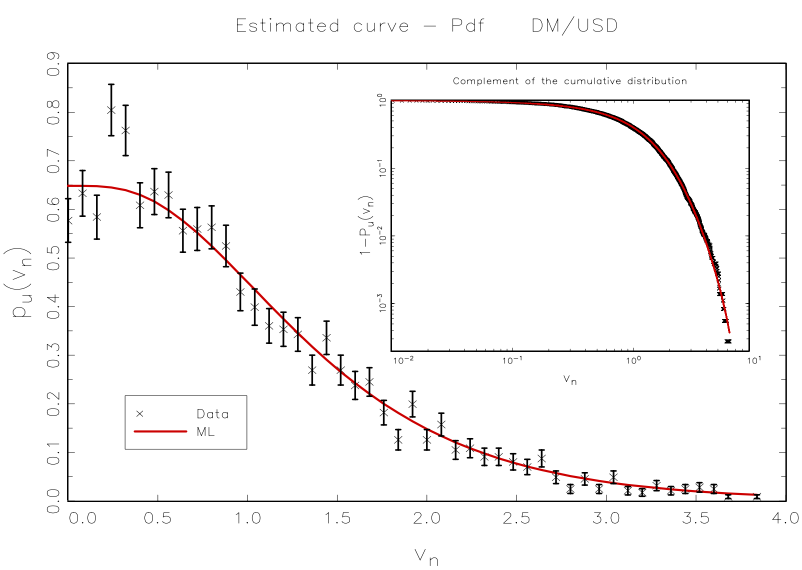

Fig. 2. *The empirical distribution of normalized volatility* $v_n = v/E[v]$ *of DM against USD is compared to the distribution (15), with estimated parameters given in Table 1. The inlet shows the complement of the cumulative distribution* $P(|r| > v_n)$ *in a log-log plot. The graph also shows intervals of* $\pm$ *one standard deviation, which are computed assuming a Normal distribution for the entries in every bin of the histogram.*

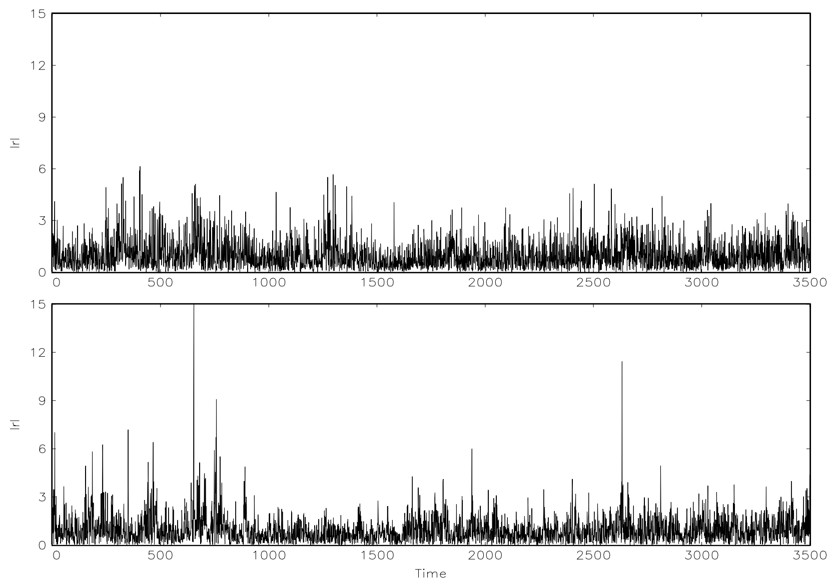

Fig. 3. *The upper panel **(a)** shows the time series of normalized absolute returns for DM/USD. The bottom panel **(b)** shows normalized absolute returns for DAX (1959 to 1969). Note that, due to the normalization, the scales are equal for both time series, but the stock market exhibits much larger daily fluctuation than the foreign exchange market.*