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# Subjective Performance Evaluation and Collusion\*

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## Abstract

This paper considers a principal-agent relationship and explores the incentive provision when the agent's performance cannot be verified. It contrasts two alternatives for the principal to provide incentives: (i) to subjectively evaluate the agent's performance; and (ii), to delegate this task to a supervisor. Supervision induces contractible information about the agent's performance, but could result in vertical collusion. This paper demonstrates that collusion-proofness can require an inefficiently high payment to the supervisor, and too low powered incentives for the agent. The eventuality of collusion is further found to potentially (i), improve the profitability; and (ii), facilitate the achievement of relational contracts based upon subjective performance evaluations.

Keywords: Subjective performance measurement, collusion, relational contracts, limited liability, incentives.

JEL classification: D82, D86

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# 1 Introduction

In many employment relationships, employees' contribution to firm value is non-contractible. Firms therefore need to apply alternative mechanisms to provide their employees with appropriate incentives. One alternative is to draw on subjective performance evaluations which constitute a crucial component of incentives schemes in firms [Gibbons, 2005]. For instance, Gibbs [1995] reported that 25 percent of employees in middle management positions receive bonus payments on the basis of subjective evaluations of their individual performance.<sup>1</sup> Such incentive payments however, cannot be legally enforced, and their reliability is perceived as questionable whenever firms lack sufficient reputations for honoring their non-enforceable obligations. Nevertheless, the credibility of these incentive payments can be augmented by involving a supervisor as supposedly neutral party in the evaluation process. Yet, delegating the subjective performance evaluation to a supervisor can impose an additional inefficiency: The involved parties might be tempted to engage in vertical collusion with the aim of swaying the supervisor's evaluation to their own favor.

The consideration of supervision as a potential device to strengthen the credibility of incentive payments based on subjective evaluations raises two important questions: First, how does potential collusion influence the incentive provision in the absence of verifiable performance measures? Secondly, how do incentive contracts have to be adjusted in order to prevent vertical side-contracting? This paper aims to answer these questions by shedding light on the design of incentive contracts in a principal-agent relationship, when verifiable performance measures are unavailable. This paper elaborates on two alternatives for the principal to provide the agent with incentives: (*i*) to subjectively evaluate his performance; and (*ii*), to delegate the performance evaluation to a supervisor. As a supposedly neutral party, the supervisor can credibly confirm the agent's achieved performance such that it can be applied in an enforceable incentive contract. As mentioned above however, empowering the supervisor to subjectively evaluate the agent's performance might create incentives for the involved parties to engage in collusion.

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<sup>1</sup>Similarly, Gibbs, Merchant, Van der Stede, and Vargus [2004] found that incentive payments of 23 percent of managers in car dealerships are tied to a subjective appraisal of their performance.

This paper points out that the effect of potential collusion on the provision of incentives is twofold. First, it may force firms to adopt less efficient spot contracts, whereas supervision would have been more profitable otherwise. This imposes an inefficiency on organizations whenever superior relational contracts based upon the principal's subjective evaluation are not feasible. Second, potential collusion can be beneficial for organizations since it could facilitate the achievement, or improve the profitability of superior relational contracts contingent on subjective performance evaluations. The latter implication follows from the fact that potential collusion strengthens the principal's credibility for paying non-enforceable incentive payments due to a less profitable contractual alternative. This paper further characterizes the adjustment of incentive contracts necessary to achieve collusion proofness; and exposes two important observations. First, collusion-proofness can require the provision of inefficiently low powered incentives to the agent. Second, the supervisor obtains a compensation, potentially exceeding the efficient level that is necessary to ensure his participation.

There is a growing body of literature investigating the application of subjective performance measures in incentive contracts. One stream, notably Bull [1987], MacLeod and Malcolmson [1989], Levin [2003], and MacLeod [2003], considered the incentive provision when the principal depends solely on subjective performance evaluations as objective, i.e. verifiable measures are unavailable. Although the corresponding relational contracts cannot be legally enforced, they can be self-enforcing in repeated games. This occurs whenever the involved parties have no incentives to deviate from their stipulated behavior. By contrast, Baker, Gibbons, and Murphy [1994], Pearce and Stacchetti [1998], and Demougin and Fabel [2004] focused on the optimal combination of subjective and objective performance measures in incentive contracts. Subjective measures are thereby found to be an integral part of incentive schemes in agency relationships. The aforementioned studies however, restricted their attention to the subjective performance evaluation conducted by the principal. Since it is of particular practical relevance, the next logical step is to incorporate a supervisor as a potential device to augment the credibility of incentive payments contingent on subjective evaluations.

Previous literature concerned with vertical side-contracting in agency relationships focused on the case where the supervisor has an information advantage

compared to the principal. In particular, the supervisor is assumed to privately observe either random productivity shocks [Tirole, 1986, Kofman and Lawarree, 1993, Villadsen, 1995], the true cost of an implemented project [Strausz, 1997], the agent's type [Faure-Grimaud, Laffont, and Martimort, 2003], or the agent's effort [Kessler, 2000]. These information asymmetries could induce the agent to collude with the supervisor aimed at ensuring he withholds or misrepresents his private information. The approach pursued in this paper however, differs in one main aspect. Here, the supervisor is in charge of confirming the agent's performance, which is observable by all involved parties. Since this confirmation eventually determines the agent's incentive payment, the agent as well as the principal could be tempted to collude with the supervisor in order to sway his assessment to their own advantage.

This paper proceeds as follows. The subsequent section introduces the basic model. Section 3 derives the optimal contracts for the principal's respective alternative for providing the agent with incentives. In section 4, the optimal incentive provision is identified; and investigated in light of how it is affected by potential collusion. Section 5 summarizes the main results and concludes.

## 2 The Model

Consider an infinitely repeated employment relationship between a principal and an agent. Both parties are risk-neutral and share the same interest rate  $r$ . The agent is in addition financially constrained and his reservation utility is normalized to zero. In every period, the agent is in charge of producing a good. The value of this good  $V$  can be either high ( $V_H$ ) or low ( $V_L$ ), where  $\Delta V \equiv V_H - V_L$ . The good value  $V$  is observable by all involved entities, but non-verifiable by third parties.<sup>2</sup> By implementing effort  $e \in \mathbb{R}^+$ , the agent determines the likelihood of whether the good value will be high or low. More precisely, let

$$\text{Prob}\{V = V_H|e\} = \rho(e) \in [0, 1) \tag{1}$$

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<sup>2</sup>This occurs for instance when the attainment of a specified quality standard is predominant in the valuation of a good. Quality can be observed and involved parties are able to assess whether a previously defined quality standard is achieved. Nonetheless, it is sometimes either impossible to verify the achieved quality, or the associated costs are prohibitively high.

be the twice-continuously differentiable probability that the high good value will be realized, where  $\rho(0) = 0$ ,  $\rho'(e) > 0$ , and  $\rho''(e) < 0$ . Effort is non-observable and imposes strictly convex increasing costs  $c(e)$  with  $c(0) = c'(0) = 0$ .

Since the realized good value  $V$  is non-verifiable, the principal cannot use this information in an enforceable incentive contract. The principal however, can provide the agent with a relational incentive contract based upon her subjective evaluation of the realized good value  $V$ . In particular, the principal promises to pay a bonus  $\beta$  in addition to a fixed transfer  $\alpha$  in the event that the high good value is realized. The agent's wage  $w^A$  therefore is

$$w^A = \begin{cases} \alpha + \beta, & \text{if } V = V_H \\ \alpha, & \text{if } V = V_L. \end{cases} \quad (2)$$

The principal's promise to pay  $\beta$  needs to be reliable from the agents' perspective since its payment cannot be legally enforced. The agent initially trusts the principal but plays a grim trigger strategy: If  $V = V_H$  and the principal violates her implicit obligation to pay  $\beta$ , the agent will never rely on non-enforceable agreements with the principal again.

As an alternative to subjectively evaluate the agent's performance, the principal can employ a supervisor who is in charge of confirming the manifested good value (*supervision*). The supervisor is risk-neutral and obtains  $\bar{U}^S \geq 0$  under his best alternative. By virtue of his supposedly neutral position, the supervisor's confirmation potentiates that the realized good value is 'quasi-verifiable' from the courts' perspective such that it can be applied in an enforceable incentive contract. In exchange for his service, the principal offers the supervisor the payment  $w^S$ .

### 3 Alternative Provisions of Incentives

The subsequent sections elaborate on the principal's alternatives for providing the agent with incentives to implement effort. Particular, the next section considers the incentive provision by utilizing a spot contracts. The following section investigates the subjective evaluation of the agent's performance in a repeated game. Finally, the case is analyzed where the principal delegates the subjective perfor-

mance evaluation to a supervisor, which potentially creates incentives to engage in vertical side-contracting.

### 3.1 Spot Contract

Suppose the principal offers the agent a contract in a one-shot game. To motivate effort, the principal can promise the agent to pay the bonus  $\beta$  in case the high good value  $V_H$  is realized. Once this occurs however, the principal reneges on  $\beta$  since its payment cannot be enforced. The agent anticipates this opportunistic behavior and refuses to implement effort. It is therefore optimal for the principal to set  $w^{A*} = 0$  such that her profit for utilizing a spot contract is  $\Pi^{SC} = V_L$ .

### 3.2 Subjective Performance Evaluation

If the principal and the agent interact for an infinite number of periods, the principal's promise to pay the bonus  $\beta$  can be credible. To derive the self-enforcement condition for this relational contract, let  $\tilde{\Pi}^{SE} \equiv \max\{\Pi^{SC}, \Pi^S\}$  denote the principal's expected profit she could obtain after reneging on  $\beta$ . Since the agent plays a grim trigger strategy, the principal's best alternative can be either to utilize a spot contract ( $SC$ ), or to employ the supervisor ( $S$ ) for evaluating the agent's performance.

Suppose for a moment that the high good value is realized. Then, the principal is not tempted to renege on  $\beta$  if

$$-\beta + \frac{\Pi^{SE}}{r} \geq \frac{\tilde{\Pi}^{SE}}{r}. \quad (3)$$

The principal adheres to her promise if paying the bonus  $\beta$  but sustaining the employment relationship based upon subjective evaluations leads to a higher present expected profit than her best fall-back  $\tilde{\Pi}^{SE}$ .

The principal's problem is to find the bonus contract  $(\alpha^*, \beta^*)$  which maximizes the difference between the expected good value and the agent's expected wage, while ensuring his participation:

$$\max_{\alpha, \beta, e} \Pi^{SE} = V_L + \Delta V \rho(e) - \alpha - \beta \rho(e) \quad (4)$$

$$\text{s.t.} \quad \alpha + \beta\rho(e) - c(e) \geq 0 \quad (5)$$

$$e \in \arg \max_{\tilde{e}} \alpha + \beta\rho(\tilde{e}) - c(\tilde{e}) \quad (6)$$

$$\alpha + \beta \geq 0 \quad (7)$$

$$\alpha \geq 0 \quad (8)$$

$$V_L + \Delta V\rho(e) - \alpha - \beta\rho(e) - \tilde{\Pi}^{SE} \geq r\beta. \quad (9)$$

Condition (5) is the agent's participation, and (6) his incentive constraint. Moreover, (7) and (8) are the liability limit constraints guaranteeing that all payments to the agent are non-negative. Finally, (9) is the self-enforcement condition ensuring that the principal's promise to pay  $\beta$  is credible.

Observe that (6) is equivalent to  $\beta(e) = c'(e)/\rho'(e)$ , with  $\beta(e)$  as the required bonus to induce an arbitrary effort level  $e$ . Thus, the expected bonus  $B(e) = \beta(e)\rho(e)$  to induce  $e$  is

$$B(e) = \frac{c'(e)\rho(e)}{\rho'(e)}, \quad (10)$$

which is assumed to be convex.<sup>3</sup> The expected bonus  $B(e)$  is characterized by the likelihood ratio  $\rho'(e)/\rho(e)$  which, according to Holmström [1979], measures the principal's propensity to expect that the agent has not implemented the anticipated effort level  $e$  when  $V = V_H$ .

**Proposition 1** *For a subjective performance evaluation, the optimal fixed transfer is  $\alpha^* = 0$ . Moreover, the optimal bonus  $\beta^*$  is characterized as follows:*

- (i): *If  $r \leq r^{SE}$ , the optimal bonus is  $\beta^* = c'(e^*)/\rho'(e^*)$ , where  $e^*$  solves  $\Delta V\rho'(e) = B'(e)$ . Thus, the principal obtains*

$$\Pi^{SE}(e^*) = V_L + \Delta V\rho(e^*) - B(e^*). \quad (11)$$

- (ii): *If  $r^{SE} < r \leq \hat{r}^{SE}$ , the optimal bonus  $\beta^*(r)$  is the highest value of  $\beta$  which implicitly solves*

$$\beta = \frac{1}{r + \rho(e(\beta))} \left[ V_L + \Delta V\rho(e(\beta)) - \tilde{\Pi}^{SE} \right]. \quad (12)$$

Consequently,

$$\Pi^{SE}(\beta^*(r)) = V_L + [\Delta V - \beta^*(r)]\rho(e(\beta^*(r))). \quad (13)$$

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<sup>3</sup>It can be shown that assuming  $c'''(e) \geq 0$  suffices to ensure convexity of  $B(e)$  for all  $e$ .



(iii): If  $r > \hat{r}^{SE}$ , the optimal bonus is  $\beta^*(r) = 0$  such that  $\Pi^{SE}(r) = V_L$ .

**Proof** See appendix.

The optimal bonus contract and the principal's expected profit for different interest rates  $r$  are illustrated in figure 1. The straight line  $r\beta$  thereby represents the right side of the self-enforcement condition (9). Accordingly, the principal can find a credible bonus  $\beta > 0$  whenever  $r\beta$  either tangents or intersects the adjusted profit curve  $\Pi^{SE}(\beta) - \tilde{\Pi}^{SE}$ . The principal can credibly commit to pay the efficient bonus  $\beta^*$  if  $r \leq r^{SE}$ .<sup>4</sup> In this case, the value of a sustained employment relationship based on a subjective performance evaluation eliminates the principal's temptation to renege on  $\beta^*$ . The agent anticipates that the principal would deliver on her promise to pay  $\beta^*$ , and is therefore motivated to implement the efficient (second-best) effort level  $e^*(\beta^*)$ . For  $r^{SE} < r \leq \hat{r}^{SE}$  however, the principal is compelled to adjust  $\beta$  in order to ensure it satisfies the self-enforcement condition (9). This follows from the fact that a higher interest rate  $r$  imposes a less severe 'penalty' on the principal for violating the relational contract with the agent. Indeed, the more  $\beta^*(r)$  deviates from the efficient bonus  $\beta^* = c'(e^*)/\rho'(e^*)$ , the lower is the principal's expected profit  $\Pi^{SE}(r)$ . Finally, if  $r > \hat{r}^{SE}$ , the principal cannot find a strictly positive bonus which satisfies (9) such that  $\beta^*(r) = 0$ . Due to the lack of credible incentives, the agent implements  $e^*(0) = 0$  which implies  $\Pi^{SE}(r) = V_L$ . Thus, the principal obtains the same profit as for utilizing a spot contract.

### 3.3 Supervision

As shown in the preceding section, the agent cannot be motivated to implement the efficient (second-best) effort level if the principal cannot credibly commit herself to pay the efficient bonus. Alternatively, the principal can employ a supervisor who is charged with affirming the realized good value such that it can be applied in an enforceable incentive contract. However, empowering the supervisor to subjectively evaluate the agent's performance could create incentives for the involved parties to engage in vertical side-contracting. To exemplify the effect of potential

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<sup>4</sup>For parsimony, the characterizations of the cut off interest rates for this and the subsequent propositions are relegated to the respective proof in the appendix.

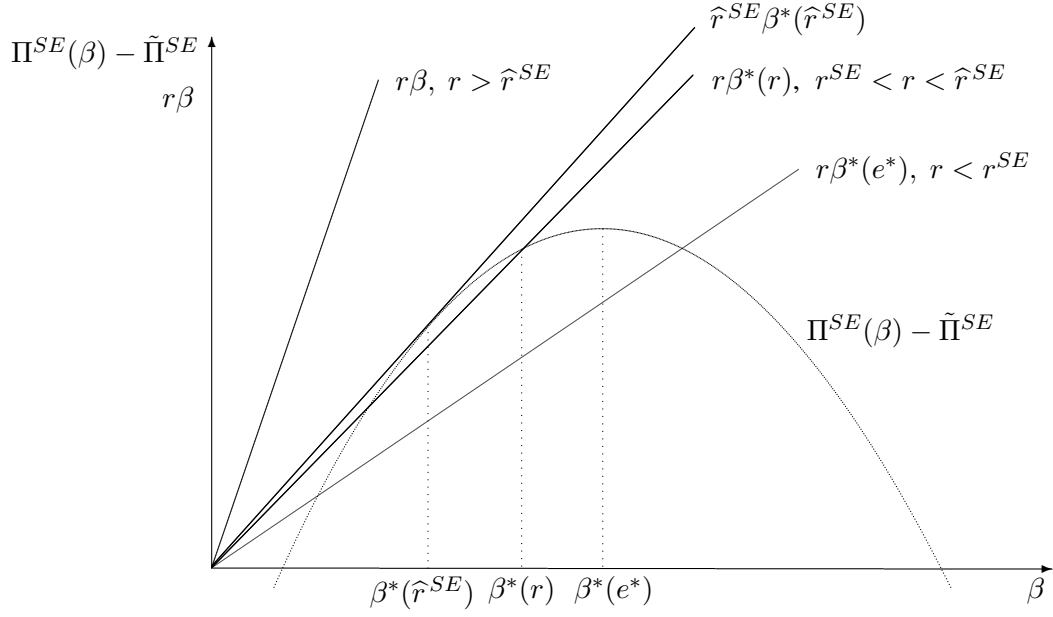


Figure 1: Optimal Bonus Contract for a Subjective Performance Evaluation

collusion on the provision of incentives, I consider the contractual arrangements under supervision with and without potential side-contracting separately.

### 3.3.1 Supervision without Collusion

Under the exclusion of side-contracting by assumption, the supervisor reveals his observation about the realized good value  $V$  always truthfully. Consequently, the principal is forced to pay  $\beta$  in the event that  $V = V_H$ .

The principal's objective is to maximize the difference between the expected good value and the expected compensations for the agent and for the supervisor. Their optimal contracts under collusion-free supervision thus solve

$$\max_{\alpha, \beta, e, w^S} \Pi^{S|NC} = V_L + \Delta V \rho(e) - \alpha - \beta \rho(e) - w^S \quad (14)$$

$$\text{s.t.} \quad w^S \geq \bar{U}^S \quad (15)$$

$$(5), (6), (7), (8),$$

where (15) is the supervisor's participation constraint. This maximization problem is essentially identical to the one considered in section 3.2 for the subjective performance evaluation by the principal. The only difference here is that the self-enforcement condition (9) is not relevant, and  $w^S \geq \bar{U}^S$  appears as an additional

side condition. Thus, we obtain the same optimal bonus contract for the agent as under the principal's evaluation of his performance for  $r \leq r^{SE}$ . Moreover, to minimize costs, the principal sets  $w^{S*} = \bar{U}^S$ . The principal's expected profit therefore is

$$\Pi^{S|NC}(e^*) = V_L + \Delta V \rho(e^*) - B(e^*) - \bar{U}^S, \quad (16)$$

where  $e^*$  solves  $\Delta V \rho'(e) = B'(e)$ . Apparently, employing the supervisor can be only beneficial if his contribution exceeds his compensation, or formally, if

$$w^{S*} = \bar{U}^S \leq \Delta V \rho(e^*) - B(e^*). \quad (17)$$

To expose the effect of potential side-contracting with the supervisor on the incentive provision, I shall assume that  $\bar{U}^S$  is sufficiently small such that (17) holds.

### 3.3.2 Supervision with Potential Collusion

This section investigates the case where collusion among the supervisor and either the agent or the principal is potentially a dominant strategy. All involved parties can perfectly infer whether collusion occurred once the confirmed good value deviates from the one observed. If the agent and the supervisor engaged in side-contracting, the principal can replace them by hiring new employees from the labor market. By contrast, after the principal colluded with the supervisor, the agent does not rely on incentive payments contingent on subjective performance evaluations neither by the supervisor nor by the principal. Consequently, the principal's fall-back is the application of a spot contract as considered in section 3.1.

From the perspective of the external observers such as the courts, the agent and the principal are the immediate parties involved in the dispute over the value of the good, whereas the supervisor is the supposedly neutral entity in this conflict. Therefore, the statement from the supervisor will have a greater weightage in swaying the court's decision. Collusion will thus work in favor of the colluding parties because proving the good's value is prohibitively expensive.

The principal's objective is to find contracts for the agent and supervisor ensuring that no party can be better off by side-contracting. Let  $T^i$  denote the bribe the agent ( $i = A$ ) or the principal ( $i = P$ ) offers the supervisor in exchange for

affirming the desired good value. If the supervisor accepts  $T^i$ , he does not deviate from the stipulated behavior and confirms the requested value. However, if

$$T^i \leq \frac{w^S - \bar{U}^S}{r}, \quad (18)$$

he refuses to engage in side-contracting since the transfer  $T^i$  does not compensate for his prospective loss in utility.

Suppose the low good value is realized such that the agent could be better off by bribing the supervisor with the aim of obtaining his bonus  $\beta$ . The maximum bribe  $\bar{T}^A$  the agent is willing to pay equals his one-time gain  $\beta$  and his discounted loss of expected utility after collusion took place:

$$\bar{T}^A = \beta - \frac{1}{r} [\alpha + \beta\rho(e) - c(e)]. \quad (19)$$

In contrast, if the high good value is eventually realized, the principal could try to avoid the payment of  $\beta$  by bribing the supervisor into confirming a low value. The maximum bribe  $\bar{T}^P$  the principal is willing to pay equals her one-time gain  $\beta$  and her discounted loss of expected profit after she engaged in side-contracting:

$$\bar{T}^P = \beta - \frac{\Pi^{S|C} - \Pi^{SC}}{r}. \quad (20)$$

Whenever  $\bar{T}^P > [w^S - \bar{U}^S] / r$ , the agent anticipates collusion among the principal and the supervisor and is therefore better off by implementing  $e^*(0) = 0$ .

**Proposition 2** *The collusion-proof contracts are characterized as follows:*

- (i): *If  $r \leq r^S \equiv \min\{r^A, r^P\}$ , the efficient contracts with  $\alpha^* = 0$ ,  $w^{S*} = \bar{U}^S$ , and  $\beta^* = c'(e^*)/\rho'(e^*)$  are collusion-proof, where*

$$r^A \equiv \rho(e^*) - \frac{\rho'(e^*)c(e^*)}{c'(e^*)}, \quad r^P \equiv \frac{\rho'(e^*)}{c'(e^*)} [\Delta V \rho(e^*) - \bar{U}^S] - \rho(e^*).$$

- (ii): *If  $r^S < r \leq \hat{r}^S$ , the collusion-proof contracts are characterized by*

$$\left\{ \begin{array}{ll} \alpha^* + w^{S*} = \bar{U}^S + \beta^*(r) [r - \rho(\cdot)] + c(\cdot), & \text{if } r^A < r \leq r^P, \\ \text{where } \alpha^* \geq 0, \quad w^{S*} \geq \bar{U}^S & \text{and } r^P, r^A < r \\ \alpha^* = 0, \quad w^{S*} = \bar{U}^S & \text{if } r^P < r \leq r^A, \end{array} \right.$$

and  $\beta^*(r)$  as the highest value of  $\beta$  which implicitly solves

$$\left\{ \begin{array}{ll} \beta^A(r) \equiv \beta^*(r) : r = [\Delta V \rho'(e(\beta)) - c'(e(\beta))] \frac{\partial e}{\partial \beta}, & \text{if } r^A < r \leq r^P \\ \beta^P(r) \equiv \beta^*(r) : \beta = \frac{\Delta V \rho(e(\beta)) - \bar{U}^S}{r + \rho(e(\beta))}, & \text{if } r^P < r \leq r^A \\ \beta^*(r) = \min\{\beta^A(r), \beta^P(r)\}, & \text{if } r^P, r^A < r. \end{array} \right.$$

(iii): If  $r > \hat{r}^S$ , the principal sets  $\alpha^* = \beta^*(r) = w^{S*} = 0$ .

**Proof** See appendix.

If  $r \leq r^S$ , all parties sufficiently value a sustained employment relationship under supervision such that no one is tempted to collude. The principal therefore provides the agent with the same bonus contract as for the collusion-free case and hence, obtains the same expected profit. In contrast, if  $r^S < r \leq \hat{r}^S$ , the principal is forced to adjust the agent's—and possibly the supervisor's—contract in order to prevent side-contracting. Given the specific contract adjustments, she obtains

$$\Pi^{S|C}(r) = V_L + [\Delta V - \beta^*(r)] \rho(e(\beta^*(r))) - \alpha^* - w^{S*}. \quad (21)$$

For a brief discussion of the potential cases exposed by proposition 2, keep in mind that  $r^P$  refers to the principal's temptation to collude for the efficient contracts  $(\alpha^*, \beta^*)$  and  $w^{S*}$ , and  $r^A$  to the agent's temptation, respectively. If  $r^A < r \leq r^P$ , the agent but not the principal is tempted to collude. Then, the principal needs to enhance either the supervisor's payment  $w^{S*}$ , or the agent's fixed compensation  $\alpha^*$  above their efficient levels to deter both from side-contracting. Contemporaneously, the principal is compelled to offer the agent a bonus  $\beta^*(r)$  below the efficient level  $\beta^*$  such that he is provided with too low powered incentives. The latter adjustment is also required in the event that  $r^P < r \leq r^A$ . For  $r > \hat{r}^S$  however, the principal cannot find a strictly positive bonus which eliminates her temptation to collude.<sup>5</sup> The principal is therefore forced to set  $\beta^*(r) = 0$  which further implies  $e^*(0) = 0$ . Moreover, there is no value in employing the supervisor such that it is optimal to set  $w^{S*} = 0$ . Accordingly,  $\Pi^{S|C}(r) = V_L$  for  $r > \hat{r}^S$ .

<sup>5</sup>The principal can nevertheless find a strictly positive bonus for all  $r$  which deters the agent from side-contracting, see proof of proposition 2.

## 4 Incentive Provision and Collusion

This section elaborates on the optimal provision of incentives, and illustrates how it is affected by potential collusion. For the sake of lucidity, I identify first the optimal incentive schemes for the collusion-free case, where the supervisor confirms the realized good value  $V$  always truthfully.

**Proposition 3** *For the collusion-free case, there exists a threshold interest rate  $\hat{r}$  with  $r^{SE} \leq \hat{r}$ , such that the principal obtains*

$$\Pi^{NC} = \begin{cases} \Pi^{SE}, & \text{if } r \leq \hat{r} \\ \Pi^{S|NC}, & \text{if } r > \hat{r}. \end{cases}$$

**Proof** See appendix.

The principal prefers to subjectively evaluate the agent's performance as long as she can credibly commit to pay a sufficiently high bonus  $\beta^*(r)$ , i.e.  $r \leq \hat{r}$ . In contrast, if  $r > \hat{r}$ , the credible bonus  $\beta^*(r)$  yields a lower expected profit than the employment of the supervisor as a mean to provide the agent with an enforceable incentive contract.

Subsequently, I abrogate the temporarily preclusion of side-contracting. Thus, the contracts under supervision need to be collusion-proof in order to be effective.

**Proposition 4** *For the case with potential collusion, the principal obtains*

$$\Pi^C = \begin{cases} \Pi^{SE}, & \text{if } r \leq \hat{r}^{SE} \\ \Pi^{SC}, & \text{if } r > \hat{r}^{SE}, \end{cases} \quad (22)$$

where  $\hat{r}^{SE} \geq \hat{r}$ .

**Proof** See appendix.

There are two fundamental implications regarding the effect of potential collusion on the incentive provision. Firstly, a subjective evaluation of the agent's performance apparently remains optimal for  $r \leq \hat{r}$ . In addition, the principal now favors a subjective evaluation over supervision for  $\hat{r} \leq r \leq \hat{r}^{SE}$ . If  $r > \hat{r}^{SE}$  however, utilizing a spot contract is now the principal's superior alternative instead of supervision. The rationale for the latter two observations is as follows.

Collusion-proofness under supervision requires either a sufficiently low interest rate  $r$ , or the provision of an inefficient but credible bonus. For the same interest rates however, the principal can commit herself to pay a more efficient bonus  $\beta^*(r)$ , or for  $r \leq r^{SE}$ , the optimal bonus  $\beta^*(e^*)$ . As a result, a subjective performance evaluation is more profitable than supervision for all  $r \leq \hat{r}^{SE}$ .

Secondly, potential collusion leads to a higher cut off interest rate  $\hat{r}^{SE}$  for a subjective performance evaluation to be superior. This reflects the principal's more severe penalty for violating the relational contract with the agent as a result of a less lucrative fall-back. This further implies that the principal can now credibly commit to pay a more efficient bonus for  $r^{SE} < r \leq \hat{r}^{SE}$ , yielding a higher expected profit under the subjective performance evaluation.

## 5 Conclusion

This paper investigated the optimal provision of incentives in an agency relationship, when verifiable measures about the agent's performance are not available. It elaborated on two alternatives for the principal to provide the agent nonetheless with incentives: (i) to subjectively evaluate his performance; and (ii), to delegate the performance evaluation to a supervisor. Supervision however, might constitute incentives for the involved parties to engage in vertical side-contracting.

The analysis in this paper points to the ambiguous effect of anticipated collusive behavior on the provision of incentives. First, if superior relational contracts based upon the principal's subjective evaluation are not feasible, potential collusion compels the principal to utilize inferior spot contracts, whereas supervision would have been more profitable otherwise. Spot contracts are thereby the principal's best alternative to bypass the costs associated with the prevention of side-contracting. Second, anticipated collusion can be advantageous for organizations since it could facilitate the achievement, or improve the profitability of superior relational contracts based on subjective performance evaluations. This can be observed because a worse fall-back position for the principal augments the credibility of non-enforceable incentive payments depending upon her subjective evaluation.

This paper further considered the efficient adjustment of contracts with the aim of preventing side-contracting under supervision. Suppose that being able to observe the agent's performance imposes significant costs on the principal, potentially due to a long hierarchical or geographical distance to the agent. Employing a supervisor as a mean to receive contractible measures about the agent's performance might therefore be the principal's superior alternative. This paper exposed that deterring the involved parties from side-contracting potentially requires the adjustment of contracts in two ways. First, the agent is provided with too low powered incentives inducing him to implement an inefficient effort level. Second, the supervisor obtains a compensation exceeding the efficient level that aims at ensuring his participation.

The latter implication provides a supplementary rationale for high executive compensations. Accordingly, high compensations for executives are necessary to maintain their neutral position for assessing their subordinates' performance. This is shown to be essential for assuring the credibility of such incentive device.



## Appendix

### Proof of Proposition 1.

Note first that  $e > 0$  requires  $\beta > 0$ . Thus, (7) is satisfied if (8) holds, and can therefore be omitted. Assume for a moment that (9) is satisfied for the optimal bonus contract. Let  $\lambda$  and  $\kappa$  be Lagrange multipliers. Then, the Lagrangian is

$$\mathcal{L}(\alpha, e) = V_L + \Delta V \rho(e) - \alpha - B(e) + \lambda [\alpha + B(e) - c(e)] + \xi \alpha. \quad (23)$$

The first-order conditions with respect to  $\alpha$  and  $e$  are

$$-1 + \lambda + \xi = 0, \quad (24)$$

$$\Delta V \rho'(e) - B'(e) + \lambda [B'(e) - c'(e)] = 0. \quad (25)$$

Suppose  $\lambda > 0$ . Then,  $\alpha + B(e) - c(e) = 0$  due to complementary slackness. Since  $\alpha \geq 0$  this would imply that  $B(e) \leq c(e)$  and hence,  $e = 0$ . Thus,  $\lambda > 0$  cannot be a solution which implies that  $\lambda = 0$ . We can then infer from (24) that  $\xi = 1$ , which implies  $\alpha^* = 0$  due to complementary slackness. Since  $\lambda = 0$  it follows from (25) that  $e^*$  solves  $\Delta V \rho'(e) = B'(e)$ . Concavity of  $\rho(e)$  and convexity of  $B(e)$  ensures that the first-order approach is sufficient. Substituting  $\alpha^* = 0$  and  $B(e^*)$  in the principal's objective function gives  $\Pi^{SE}(e^*) = I_L + \Delta V \rho(e^*) - B(e^*)$ . Moreover, substituting  $\Pi^{SE}(e^*)$  and  $\beta^*(e^*) = c'(e^*)/\rho'(e^*)$  in (9) leads to

$$r \leq \frac{\rho'(e^*)}{c'(e^*)} \left[ V_L + \Delta V \rho(e^*) - \tilde{\Pi}^{SE} \right] - \rho(e^*) \equiv r^{SE}. \quad (26)$$

If  $r > r^{SE}$ ,  $\beta^*(e^*)$  would violate (9). In this case, the principal chooses the highest feasible  $\beta$  such that (9) binds:

$$\underbrace{V_L + \Delta V \rho(e(\beta)) - \beta \rho(e(\beta))}_{\Pi^{SE}(\beta)} - \tilde{\Pi}^{SE} = r\beta. \quad (27)$$

The left side is concave increasing in  $\beta$  for  $\beta < \beta^*(e^*)$ , whereas the right side is linear increasing with slope  $r$ . Thus, depending on  $r$ , there are potentially two values of  $\beta$  solving (27). Let  $\beta^*(r)$  denote the maximum value of  $\beta$  which implicitly solves (27), or equivalently,

$$\beta = \frac{1}{r + \rho(e(\beta))} \left[ V_L + \Delta V \rho(e(\beta)) - \tilde{\Pi}^{SE} \right]. \quad (28)$$

Furthermore, implicit differentiating (27) gives

$$\frac{\partial \beta}{\partial r} = \frac{\beta}{\frac{\partial \Pi^{SE}(\beta)}{\partial \beta} - r}. \quad (29)$$

Recall that  $\Pi^{SE}(\beta)$  is concave increasing in  $\beta$  as long as  $\beta < \beta^*(e^*)$ , and the right side of (27) is linear increasing in  $\beta$  with slope  $r$ . Thus,  $\partial \Pi^{SE}(\beta)/\partial \beta|_{\beta=\beta^*(r)} < r$  for  $r^{SE} < r \leq \hat{r}^{SE}$ , where  $\hat{r}^{SE}$  is characterized below. Hence,  $\partial \beta/\partial r < 0$ .

Finally, there exists a cut off  $\hat{r}^{SE}$  such that the principal cannot find a  $\beta > 0$  for  $r > \hat{r}^{SE}$  which satisfies (27). Thus,  $\beta^*(r) = 0$  for  $r > \hat{r}^{SE}$  such that  $e^*(0) = 0$ , and consequently,  $\Pi^{SE}(r) = V_L$ . Since the left side of (27) is concave increasing in  $\beta$  as long as  $\beta < \beta^*(e^*)$ , and the right side is linear increasing,  $\hat{r}^{SE}$  implies that  $r\beta$  tangents  $\Pi^{SE}(\beta) - \tilde{\Pi}^{SE}$ . Thus,  $\hat{r}^{SE}$  is implicitly characterized by the tangent condition  $r = \partial \Pi^{SE}(\beta)/\partial \beta|_{\beta=\beta^*(r)}$ , or equivalently,

$$r = [\Delta V - \beta^*(r)] \rho'(e(\beta^*(r))) \frac{\partial e}{\partial \beta} \Big|_{\beta=\beta^*(r)} - \rho(e(\beta^*(r))). \quad (30)$$

□

## Proof of Proposition 2.

Consider first the optimal contracts  $w^{S*} = \bar{U}^S$  and  $(\alpha^*, \beta^*)$ . According to (18), the supervisor would collude if  $T^i > 0$ ,  $i = A, P$ . Hence, collusion-proofness requires  $\bar{T}^i = 0$ . Substituting  $\alpha^* = 0$  and  $\beta^*(e^*) = c'(e^*)/\rho'(e^*)$  in (19) yields the condition ensuring the agent refrains from side-contracting:

$$r \leq \rho(e^*) - \frac{\rho'(e^*)c(e^*)}{c'(e^*)} \equiv r^A. \quad (31)$$

Likewise, substituting  $\Pi^{S|C}(e^*)$ ,  $\beta^*(e^*)$ , and  $\Pi^{SC} = V_L$  in (20) leads to the condition guaranteeing that the principal has no incentive to collude:

$$r \leq \frac{\rho'(e^*)}{c'(e^*)} [\Delta V \rho(e^*) - \bar{U}^S] - \rho(e^*) \equiv r^P. \quad (32)$$

Thus, the optimal contracts are collusion-proof if  $r \leq r^S \equiv \min\{r^A, r^P\}$ .

For  $r > r^S$ , there are three cases to discuss:  $r^A < r \leq r^P$ ,  $r^P < r \leq r^A$ , and  $r^P, r^A < r$ . Consider first the case  $r^A < r \leq r^P$ . Combining (18) and (19) gives

$$\begin{aligned} \frac{w^S - \bar{U}^S}{r} &\geq \beta - \frac{\alpha + \beta \rho(\cdot) - c(\cdot)}{r}, \\ \Leftrightarrow \alpha + w^S &\geq \bar{U}^S + \beta [r - \rho(\cdot)] + c(\cdot). \end{aligned} \quad (33)$$

To minimize costs, the principal sets  $\alpha$  and  $w^S$  such that (33) binds, provided that  $\alpha^* \geq 0$  and  $w^{S*} \geq \bar{U}^S$ . Substituting  $(\alpha + w^S)$  in the principal's objective functions leads to the simplified problem for  $r^A < r \leq r^P$ :

$$\max_{\beta} \Pi^{S|C} = V_L + \Delta V \rho(e(\beta)) - r\beta - c(e(\beta)) - \bar{U}^S. \quad (34)$$

The first-order condition indicates that  $\beta^*(r)$  implicitly solves

$$r = [\Delta V \rho'(e(\beta)) - c'(e(\beta))] \frac{\partial e}{\partial \beta}. \quad (35)$$

Moreover, implicit differentiation gives

$$\frac{\partial \beta}{\partial r} = \frac{1}{\frac{\partial}{\partial \beta} \left[ [\Delta V \rho'(e(\beta)) - c'(e(\beta))] \frac{\partial e}{\partial \beta} \right]}, \quad (36)$$

where the denominator is strictly negative due to the second-order condition. Thus,  $\partial \beta / \partial r < 0$ . Next, consider the case  $r^P < r \leq r^A$ . Observe from (20) that enhancing  $\alpha$  would reduce  $\Pi^{S|C}$  and thus, raise  $\bar{T}^P$ . Hence,  $\alpha^* = 0$ . Moreover, the marginal effect of raising  $w^S$  on the collusion-proofness condition (18) is  $1/r$ , and on (20) is  $-1/r$ , i.e. changing  $w^S$  does not support collusion-proofness. Thus, to minimize costs,  $w^{S*} = \bar{U}^S$ . Moreover, the principal chooses the highest feasible  $\beta$  such that  $\bar{T}^P = 0$ , which is equivalent to

$$\underbrace{V_L + \Delta V \rho(e(\beta)) - \beta \rho(e(\beta)) - \bar{U}^S}_{\Pi^{S|C}(\beta)} - \Pi^{SC} = r\beta. \quad (37)$$

The left side is concave increasing in  $\beta$  as long as  $\beta < \beta^*(e^*)$ , whereas the right side is linear increasing in  $\beta$ . Consequently, depending on  $r$ , there are potentially two values of  $\beta$  solving (37). Let  $\beta^*(r)$  denote the maximum value of  $\beta$  which implicitly solves (37), or equivalently,

$$\beta = \frac{1}{r + \rho(e(\beta))} [\Delta V \rho(e(\beta)) - \bar{U}^S]. \quad (38)$$

Implicit differentiating (37) gives

$$\frac{\partial \beta}{\partial r} = \frac{\beta}{\frac{\partial \Pi^{S|C}(\beta)}{\partial \beta} - r}. \quad (39)$$

Recall that  $\Pi^{S|C}(\beta)$  is concave increasing in  $\beta$  for  $\beta < \beta^*(e^*)$ , whereas the right side of (37) is linear increasing with slope  $r$ . Hence,  $\partial\Pi^{S|C}(\beta)/\partial\beta|_{\beta=\beta^*(r)} < r$  for  $r > r^P$ . As a result,  $\partial\beta/\partial r < 0$ . In the final case,  $r^P, r^A < r$ , the principal needs to set  $\alpha$  and  $w^S$  as for  $r^A < r \leq r^P$  in order to prevent collusion. Moreover, to ensure that neither the principal nor the agent colludes, it is necessary to choose the lowest  $\beta$  which contemporaneously satisfies (35) and (38).

Finally notice that the principal can always find a  $\beta > 0$  which satisfies (35) for  $r > r^A$ . In contrast, if  $r > r^P$ , there exists a cut off  $\hat{r}^S$  such that the principal cannot find a  $\beta > 0$  for  $r > \hat{r}^S$  which satisfies (37). Thus,  $\beta^*(r) = 0$  for all  $r > \hat{r}^S$ . The cut off  $\hat{r}^S$  thereby implies that  $r\beta$  tangents  $\Pi^{S|C}(\beta) - \Pi^{SC}$ , see (37). As a consequence,  $\hat{r}^S$  is implicitly characterized by the tangent condition  $r = \partial\Pi^{S|C}(\beta)/\partial\beta|_{\beta=\beta^*(r)}$ , or equivalently,

$$r = [\Delta V - \beta^*(r)] \rho'(e(\beta^*(r))) \frac{\partial e}{\partial \beta} \Big|_{\beta=\beta^*(r)} - \rho(e(\beta^*(r))), \quad (40)$$

□

### Proof of Proposition 3.

Note first that  $\Pi^{S|NC} \geq \Pi^{SC} = V_L$  for all values of  $r$ . Moreover,  $\Pi^{SE} \geq \Pi^{S|NC}$  is satisfied for  $r \leq r^{SE}$ . As shown in proof of proposition 1,  $\partial\beta/\partial r < 0$  under a subjective evaluation. Hence,  $\Pi^{SE}(r)$  is decreasing in  $r$  for  $r^{SE} < r \leq \hat{r}^{SE}$  such that there exists a cut off  $\hat{r}$  satisfying  $\Pi^{SE}(\beta^*(r)) = \Pi^{S|NC}$ , or equivalently,

$$\Delta V [\rho(e(\beta^*(r))) - \rho(e^*(\beta^*))] = B(e(\beta^*(r))) - B(e^*(\beta^*)) - \bar{U}^S \quad (41)$$

Finally,  $r^{SE} \leq \hat{r}$  since  $\Pi^{SE} = \Pi^{S|NC}$  for  $r \leq r^{SE}$  and  $\bar{U}^S = 0$ . □

### Proof of Proposition 4.

Observe first that  $\Pi^{SE} \geq \Pi^{S|C}$  for  $r \leq r^{SE}$ . For the subsequent proof, it is necessary to show that  $r^{SE} \geq r^S = \min\{r^A, r^P\}$ . Apparently, it suffices to demonstrate that  $r^{SE} \geq r^P$ . Since the agent plays grim trigger strategy, utilizing a spot contract is the principal's best fall-back after she either reneged on  $\beta$  (subjective

performance evaluation) or colluded with the supervisor (supervision). Hence,  $\tilde{\Pi}^{SE} = \tilde{\Pi}^{S|C} = \Pi^{SC}$ . As a result,  $r^{SE} \geq r^P$  is equivalent to

$$\frac{\Pi^{SE} - \Pi^{SC}}{\beta^*(e^*)} \geq \frac{\Pi^{S|C} - \Pi^{SC}}{\beta^*(e^*)}, \quad (42)$$

which is satisfied since  $\Pi^{SE} \geq \Pi^{S|C}$ . Thus,  $r^{SE} \geq r^S = \min\{r^A, r^P\}$ .

Next, it is necessary to verify that  $\Pi^{SE}(r) \geq \Pi^{S|C}(r)$  for  $r^{SE} < r \leq \hat{r}^{SE}$ . There are two cases to consider: (i)  $r^{SE} \geq \hat{r}^S$ ; and (ii),  $r^{SE} < \hat{r}^S$ . For case (i), it follows directly that  $\Pi^{SE}(r) \geq \Pi^{S|C}(r)$  for  $r^{SE} < r \leq \hat{r}^{SE}$  since  $\Pi^{S|C}(r) = V_L$  for  $r > \hat{r}^S$ . Now consider case (ii). As exposed by proposition 2, there are three potential cases for adjusting the contracts under supervision appropriately: (a)  $r^A < r \leq r^P$ , (b)  $r^P < r \leq r^A$ ; and (c),  $r^P, r^A < r$ . Consider first case (a). As shown,  $r^{SE} \geq r^P$  such that  $\Pi^{SE} \geq \Pi^{S|C}(r)$  for  $r^A < r \leq r^P$ . Next, consider case (b). As (29) in connection with (27), and (39) in connection with (37) indicate,  $\beta^*(r)$  is decreasing in  $r$  with the same rate under a subjective performance evaluation as under supervision. Thus,  $\Pi^{SE}(r)$  is decreasing in  $r$  for  $r^{SE} < r \leq \hat{r}^{SE}$  with the same rate as  $\Pi^{S|C}(r)$  for  $r^P < r \leq r^A$ . Since  $r^{SE} \geq r^P$  it follows that  $\Pi^{SE}(r) \geq \Pi^{S|C}(r)$  for  $r^P < r \leq r^A$ . If (c) applies, the principal chooses the lowest  $\beta$  which contemporaneously satisfies (35) and (38), see proof of proposition 2. Thus,  $\beta^*(r)$  under a subjective performance evaluation is greater than the adjusted  $\beta^*(r)$  under supervision. Hence,  $\Pi^{SE}(r) \geq \Pi^{S|C}(r)$  for  $r^P, r^A < r$ . In sum,  $\Pi^{SE}(r) \geq \Pi^{S|C}(r)$  for  $r^{SE} < r \leq \hat{r}^{SE}$ . For  $r > \hat{r}^{SE}$  however, the principal utilizes a spot contract because a strictly positive bonus  $\beta$  is not credible under a subjective performance evaluation; and induces collusion under supervision.

Finally,  $\tilde{\Pi}^{SE} = \Pi^{S|NC}$  (collusion-free case) changes to  $\tilde{\Pi}^{SE} = \Pi^{SC}$  (collusion case), where  $\Pi^{S|NC} \geq \Pi^{SC}$ . Hence, due to a worse fall-back position for the principal,  $\hat{r}^{SE} \geq \hat{r}$ .  $\square$

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