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Halkos, George

University of Thessaly, Department of Economics

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George E. Halkos and George Papageorgiou

University of Thessaly, Department of Economics,
Korai 43, 38333, Volos, Greece

Abstract

Traditional economic theory, up to the middle of the twentieth century, builds up the production functions regardless the inputs' scarcity. In the last few decades has been clear that both the inputs are depletable quantities and a lot of constraints are imposed in their usage in order to ensure economic sustainability. Furthermore, the management of exploitation and use of natural resources (either exhaustible or renewable) has been discussed by analyzing dynamic models applying methods of Optimal Control Theory. This theory provides solutions that are concerned with a single decision maker who can control the model dynamics facing a certain performance index to be optimized. In fact, market structures or exploitation patterns are often oligopolistic, i.e. there are several decision makers whose policies influence each other. So, game theoretical approaches are introduced into the discussion. According to the theory of continuous time models of Optimal Control, the appropriate analogue of differential games is used. Roughly, this is the extension of Optimal Control, when there is exactly one decision maker, to the case of $N\left(N \geq 2\right)$ decision makers interacting with each other.

Keywords: Nonrenewable resources; dynamic interaction; economic regulation; differential games.

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Address for correspondence

George E. Halkos
Director of Postgraduate Studies
Director of the Operations Research Laboratory
Deputy Head
Department of Economics
University of Thessaly
Korai 43
Volos 38333, Greece
Tel. 0030 24210 74920
FAX 0030 24210 74772
email: halkos@uth.gr
http://www.halkos.gr/

1. Introduction

In the literature of environmental economics, existing models often make an assumption in which the involved agents exploit the resource from a common pool area in a non-cooperative way. This approach yields inefficiency in the well known sense "tragedy of commons" (Benchekroun, 2003). Tragedy of commons refers to a situation in which a producible asset is exploited jointly by several economic agents whose "non-cooperative" behavior results in overexploitation of the asset, i.e., an exploitation of the asset that is not jointly efficient (Pareto optimal).

In fact, market structures of exploitation patterns are often oligopolistic, i.e., there are several decision makers whose policies influence each other. So, game theoretical approaches are introduced into the discussion. According to the theory of continuous time models of Optimal Control, the appropriate analogue of differential games is used. Roughly, this is an extension of Optimal Control, when there is an exactly one decision maker, to the case of N (N>1) decision makers interacting with each other.

Dynamic models of exploitation (or harvesting) and use of natural resources refer to two different systems of property rights: In the case of sole ownership, optimal extraction policies can be obtained by means of Optimal Control Theory (Clark, 1976); in the case of open access or common property exploitation, game theoretical models are applicable in the sense that all decision makers exploit a resource from a common pool without any restriction, looking only at their own profits over some time horizon, and without considering the stock of the resource, which is diminished due to the extraction policies of all the players of the game who share the common pool (Clark, 1980; Dockner *et al*., 1989).

Environmental problems can be understood as the exploitation of a common pool of a natural resource by several players. For example, activities with polluting results have cumulative future consequences. Activities of some nations may affect the interests of other nations, that is a kind of players' interdependence. Sulphur emissions lead to acid rain, which does not respect borders, or pollution of the sea caused by industrial activities in areas located far away but connected with the sea by a river which transports the waste industrial residuals. This pollution may have impacts on other economic sectors of the same nation or of borders.

Whenever decision makers are few, one cannot use models of perfect competition, but the appropriate framework for the discussion of these problems is given by theoretical approaches with special regard to the question of "how to play the game": cooperatively or non–cooperatively?. Moreover if we assume that there is a regulator managing environmental and natural resources problems, caused by natural resource extraction, it is impossible in our opinion to settle the problem because is difficult to find which is the polluter in order to compensate the pollutee for the damage incurred, from the Pigouvian point of view.

In natural resources economics there is a chain of externalities arisen by human activities, known as environmental externalities. Once a natural resource is explored and is ready for exploitation the first externality arises from the fact that the extraction cost raises not only with the current exploitation rate, but with the cumulative amount extracted to date. Consequently, a unit of resource extracted today will inflict an intertemporal externality in the form of pushing up extraction costs at all future dates, assuming a twice continuously differentiable cost function. The cost function, along the extraction path, must be an increasing function not only with respect to the extraction rate but also it must be an increasing function of the remainder stock. In such a way it is possible to assume that the marginal current exploitation cost is higher both at higher exploitation rate and, for a constant rate of exploitation, at higher depletion rates.

The second externality is in association with the use of the extracted resource. The resource use not only damages the environment through the current flow of an externality, but also damages the environment indirectly by adding to the accumulated stock of an externality and pushing it toward to a critical level. A well known example is the externalities associated with fossil fuel use when the flow externality may take one of the forms of air pollution, pollution of the seas by oil spills, land pollution caused by dumping of coal wastes, while the stock externality takes the form of greenhouse warming and acid rain.

From the supply side point of view, resource-extracting oligopolists continually engage in the search for additional stocks or in finding new technologies to transform resources that are economically non-exploitable into resources that can be profitably extracted. If the demand curve facing the industry is elastic, the discovery of additional stocks will raise the industry's profit. It is not clear, however, if all firms will benefit from a windfall "gain" (discovery) that increases the stock of each firm.

When a given number of firms deplete an exhaustible resource with zero extraction costs and iso–elastic demand, it has been argued that the oligopoly and cartel outcomes are efficient and that firms deplete according to the Hotelling's rule (Dasgupta and Heal, 1979). This implies that dynamic oligopolies and cartels cannot be distinguished from perfect competition and that firms act as if there are well defined private property rights. These results are somewhat counter intuitive and cannot explain the phenomena of 'wild – cutting'. One reason for excessive extraction rates in oligopolistic resource markets may be that firms are worried that, if they announce to extract efficiently, one of their rivals with access to current stock levels will have an incentive to deplete more rapidly, therefore yielding inefficiency.

In this paper, we consider oligopolistic equilibrium in subgame-perfect strategies in continuous time, and investigate the effect of stock discovery on the profits of non-identical oligopolists. We show that a uniform addition to all stocks could harm firms that are originally larger than average.

In a static model, this result is not surprising. Starting from a Cournot equilibrium it is well known that a marginal reduction of all firms' production will be beneficial to the firms and will move them closer to the cooperative equilibrium. Conversely, increasing the output of all firms is likely to move them further from the cooperative outcome and will reduce their profits. In a dynamic framework with free time horizon, this reasoning is not necessarily valid. The typical extraction path under non-cooperation is monotonically decreasing over time with production level below the production level of cooperative exploitation for at least some interval of time, which we refer to as a scarcity phase. When a firm receives an additional stock it splits its extra-exploitation between the scarcity phase and the phase where production is above the cooperative level. Increasing exploitation during the phase where production is above the cooperative level decreases instantaneous profits but increasing exploitation in the former phase increases instantaneous profits, resulting in an unclear conclusion for the overall impact in firms profits.

Existing models of natural resource oligopoly that use the concept of Markov perfect Nash equilibria are typically based on the assumption that there is only one stock, to which all firms have equal common access (see for instance Benchekroun, 2003; Benchekroun and Long, 2002; Dockner and Sorger, 1996; Benhabib and Radner, 1992). Our model has N stocks, and we rule out common access.

The rest of the paper is organized as follows. Section 2 describes the model of resource extraction with an isoelastic demand function. Section 3 provides the Markov perfect Nash equilibrium strategies that are time consistent and the resulting value function for the strategies. Section 4 proposes some policy instruments based on changes (marginal of uniform) of the allowed resource stock, while section 5 concludes the paper.

2. The basic model

Let us assume that there are N firms in an oligopoly market. Firm i is endowed with a stock of a resource $S_i(t)$ at time t, with $S_i(0)=S_i^0$. Let $S(t)$ denote the sum of all stocks at time t, that is

$$S(t)=\sum_{i=1}^{N} S_i(t)$$

We define $S_{-i}(t)=S(t)-S_i(t)$. We then also assume that the rate of change of firm's i resource stock is[1]

$$\frac{dS_i(t)}{dt}=\dot{S}_i(t)=-h_i(t)$$

where $h_i(t)$ is firm's i extraction rate at time t. The inverse demand function is given by

$$D(h(t))=(h(t))^{f(a(t))}$$

with $f(a(t))=-a(t)$, $a(t)\in(0,1]$ $\forall t\in(0,+\infty)$ and $h(t)=\sum_{i=1}^{N} h_i(t)$ denotes the overall extracted quantity.

The function $\left[f(a(t))\right]^{(-1)}=1/a(t)\geq 1$ determines, in absolute value, the instantaneous elasticity of demand, i.e. the inverse demand function is always elastic and takes the hyperbolic shape if $a(t)=1$ (i.e. a constant), but is always convex.

Here in order to form the dynamic problem we assume utility derived from revenues, so firms in industry are rather revenues maximizers. Moreover we assume that the resource stock is not restrictive for the firms' decisions (i.e. extraction rate) but the regulator is the decision maker of the state variable, i.e. the remainder resource

[1] A similar adoption in the resource reduction equation is made by Batabyal (1995a, 1995b)

stock as you will see below. One of the results[2] of the paper is that the control trajectory is strictly dependent on the extraction trajectory and on instantaneous elasticity as well. So, the state variable as affected from the control, the problem is an optimal control for every involved firm.

Having these assumptions the dynamic formulation can be presented as follows. Firm's i revenues are given by the expression:

$$R_i\left(h_i, h_{-i}\right) = h_i\left(h_i + h_{-i}\right)^{f(a(t))}$$

where $$h_{-i} = h - h_i$$

The objective function of firm i is to maximize the present value of the stream of cash flow subject to the system dynamics, that is the problem[3]

$$\max \int_0^\infty h_i\left(h_i + h_{-i}\right)^{f(a(t))} e^{-\rho t} dt \tag{1}$$

subject to $$\dot{S}_i(t) = -h_i(t) \tag{1a}$$

with $$S_i(0) = S_i^0$$

The control variable of firm i is its quantity h_i, while the state variable is its remainder resource S_i.

We seek to find a strategy and the value function of the dynamic problem under the Closed Loop[4] or Markovian Nash informational structure equilibrium which is by definition the concept of equilibrium in which the choice of player's i current action is conditioned on current time t and on state vector too.

[2] In another perspective a second result could be the fact that a tightening of the regulation on total allowent resource stock can lead to an increase in firms' NPV of discounted revenues.
[3] In this setting i.e. the state variable doesn't enter into objective function, the induced game seems to be a trivial one.
[4] For more details about the informational structures of the dynamics games, see Olsder and Basar (1998).

Under the closed–loop informational structure and stationarity of the game the player's i strategy space[5] is this of mappings

$$\phi_i : \mathbb{R}_+^n \to \mathbb{R}$$

which associates to a vector of resource stock $\left(S_1, S_2, ..., S_N\right) \in R_+^n$ the quantity $\phi_i\left(S_1, S_2, ..., S_N\right)$ to extract. Each player i of the game has to choose a quantity $h_i\left(t\right) \in R$ of the resource, and the price of that resource is then set according to

$$D\left(h_1, h_2, ..., h_N\right) = \left(\sum_{i=1}^{N} h_i\left(t\right)\right)^{f\left(a(t)\right)}$$

The utility (total revenues) enjoyed by firm i is then given by

$$U_i : \left(\phi_1, \phi_2, ..., \phi_N\right) \to \int_0^{\infty} D\left(\phi_1\left(S\right), \phi_2\left(S\right), ..., \phi_i\left(S\right), ..., \phi_N\left(S\right)\right) e^{-\rho t} dt$$

where $\left(S_k\right)_{k=1,...,N}$ evolve according to the differential equation determined by $\left(1a\right)$. Equilibrium should then be defined as a set of strategies for which no player has a profitable deviation.

Imposing this assumption on informational structure of the game, clearly the history of the game is important and is reflected in the current value of the state vector. Consequently, player's i optimal time paths take into account at any point of time the control variables (quantity extracted) of the other players. This type of equilibrium affects the state variables, requiring a revision of the player's i controls at any time instant. Here we apply the Hamilton – Jacobi – Bellman (HJB) equation in order to prove that the conjectured strategy we propose is a Markovian strategy and

[5] By strategy spaces, we mean the information available to each player together with a set of functions with this information as domain. These functions are actually the permissible ways in which the players are allowed to use that information. Open loop strategies, where at each instant of time t the players have knowledge of the present time instant t and the initial condition S(0) of the state, result in different equilibrium from the strategies where at each instant of time t the players have knowledge of the time t, the initial state S(0) and the current state S(t).

consequently a strongly time consistent one. In contrast to the open loop informational structure the closed loop is a strongly time consistent one, but the open loop is not. Here the time consistent property is in the sense of sub–game perfectness (for more details see Dockner *et al*. 2000).

3. Markov Perfect Nash Equilibrium (MPNE)

We denote by ϕ_i the strategy that specifies firm's i extraction rate as a function of time t and the vector of remainder resource stock at the same time. This is the strategy

$$h_i(t) = \phi_i\left(S(t)\right)$$

Each firm takes competitors strategies as given and determines its optimal strategy that solves problem (1) with constraint $(1a)$.

Proposition 1.

A MPNE exists, where the equilibrium strategy of firm i has the property that its extraction level depends on its own resource stock and on elasticity of demand. That is

$$h_i(t) = \frac{\rho}{a(t)} S_i(t) \qquad i = 1,\dots,N$$

The discounted sum of firm's i revenues $V_i(S)$, when the total resource stock is S, are given by

$$V_i(S) = \left(\frac{a(t)}{\rho}\right)^{a(t)} \frac{S_i(t)}{S^{a(t)}} \tag{2}$$

Proof (is given in the appendix)

Given the discounted revenues expression by (2), we are able to see the impact from a change in elasticity of demand in the discounted revenues. Therefore we take the derivative of the value function expression given by (2) with respect to the demand curvature a, assuming that the initial resource stock of firm's i and the overall resource stock S remains unchanged, that is

$$\frac{dV_i}{da}=\left(\frac{\rho S}{a}\right)^{-a}\left(-\ln\left(\frac{\rho S}{a}\right)+1\right)S_i=V_i\left(S\right)\left(1-\ln\left(\frac{\rho S}{a}\right)\right)$$

which is a negative or positive quantity, meaning that the discounted revenues change will be negative or positive, depending on the sign of the quantity $1-\ln\left(\frac{\rho S}{a}\right)$.

3.1. The unary elasticity demand (A special case)

Consider for a moment that elasticity of demand equals to one independent of time, $a(t)=1$. As it is simply clear in this case the market demand function collapses to a hyperbolic shape, this being a special case of a more general class of models based on isoelastic demand curves. An isoelastic demand function was used to study the stability for a general Cournot oligopoly (Chiarella and Szidarovsky, 2002) and in many variations (Puu, 1991, 1996; Puu and Norin 2003; Puu and Marin, 2006). Furthermore isoelastic demand functions is a result in the case the consumers maximize utility functions of the Cobb – Douglas type in a static environment. The static problem for the i consumer is $\max\left(D_1^i\right)^{a_1^i}\left(D_2^i\right)^{a_2^i}\ldots$, subject to the budget constraint $y^i=p_1D_1^i+p_2D_2^i+\ldots$ with p_k to denote the prices of the commodities and D_k^i denote the quantities demanded. The well known outcome of this static constrained maximization is $p_kD_k^i=a_k^iy^i$ whence a_k^i is the fixed spending share of the i's consumer income y^i on the k-th good.

From the above problem solution the resulting demand for each consumer is reciprocal to price charged that is $D_k^i=\frac{a_k^iy^i}{p_k}$, so dropping commodities indices, aggregate demand obtained (the sum of all consumers) as $D=\sum_i D^i=\frac{\sum_i a^iy^i}{p}=\frac{R}{p}$.

Puu (2008) also uses the following price specification $p = \frac{R}{\sum_{i=1}^{N} q_i}$ where p denotes market price, $\sum_{i=1}^{N} q_i$ is the total quantity produced, while R is the sum of the total budget shares that all consumers spend in the particular good. It is well known from the literature[6] in such a case the maximum problem of a firm choosing the output level is indeterminate if marginal cost is zero, since the revenues generated by a hyperbolic demand are constant, thus economically unacceptable. But even in this special case our model under closed loop informational structure yields linear strategies and value function as well. More precise setting demand elasticity to one, $a = 1$, the model solution yields the following results for strategies and value function respectively:

$$h_i = \rho S_i \qquad i = 1, \dots, N \tag{3}$$

$$V_i(S) = \frac{S_i}{\rho S} \tag{4}$$

The latter reasoning leads us to conclude the following corollary.

Corollary 1

The above proposed model of an exhaustible resource extraction even in the special case of isoelastic demand, so for constant consumers' budget share, yields deterministic Markovian linear strategies and value functions given by (3), (4) *respectively.*

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[6] For an exposition of a differential oligopoly model where firms face implicit menu costs of adjusting output over time due to sticky market price, see Lambertini (2007).

4. Regulating policies in the allowed resource stock

We consider now the impact of a marginal change in the allowed resource stock imposed by an authority into the firms' value function. For this purpose we investigate the total differentiation of the value function

$$V_i(S) = \left(\frac{a}{\rho}\right)^a \frac{S_i(t)}{S^a}$$

with respect to the remainder resource, that is

$$dV_i = \frac{\partial V_i}{\partial S_i} dS_i + \sum_{j \neq i, j=1}^{N} \frac{\partial V_i}{\partial S_j} dS_j \qquad (5)$$

In order to have a unified result into the previous found value function of each firm we record the following proposition.

Proposition 2.

A marginal increase in the total resource stock, affects incrementally the discounted firm's i revenues, if the inequality $\frac{dS_i}{dS} > a\frac{S_i}{S}$ holds, otherwise an increase in the total resource stock reduces the discounted sum of firm's i revenues.

Proof (See Appendix)

In proposition 2 the elasticity of demand plays a crucial role. Inequality in proposition 2 implies that if the marginal changes in resource shares are greater than the marginal resource shares multiplied by the inverse elasticity then every firm has incremental revenues. Assuming for a moment that a firm decides to extract a more inelastic resource with $a_1 > a$ and with the same resource stock S_i. Clearly the extraction rate of the more inelastic resource will be higher as the Markov strategy in proposition 1 reveals. But it is not clear that the raised revenues requirement

$\frac{dS_i}{dS} > a\frac{S_i}{S}$ maintains the same inequality. So the firm decides at the margin which elasticity prefers to supply.

The total derivative of the value function after manipulations (see in the appendix) is given by the expression

$$dV_i = \left(\frac{\rho}{a}S\right)^{-a} dS\left(\frac{dS_i}{aS} - a\frac{S_i}{S}\right) \tag{6}$$

Since the term $\left(\frac{\rho}{a}S\right)^{-a}$ of (6) always measures the aggregate demand, as $h = \sum_{i=1}^{N} h_i = \left(\frac{\rho}{a}\sum_{i=1}^{N} S_i\right)^{-a} = \left(\frac{\rho}{a}S\right)^{-a}$ and we have set $\sum_{i=1}^{N} S_i = S$, the rest of term (6) $dS\left(\frac{dS_i}{dS} - a\frac{S_i}{S}\right)$ measures the amount multiplied with the total demand, giving the total marginal change on the discounted revenues.

Furthermore, we assume that the sign of the expression (6) is positive, the latter assumption implies an increment of the discounted revenues, that is, $\frac{dS_i}{dS} > a\frac{S_i}{S}$ and firms are ranked by an increasing order of the allowed resource stock, $S_1 < S_2 < \ldots < S_i < S_{i+1} < \ldots < S_N$, so the initial resource shares are $\frac{S_i}{S} < \frac{S_{i+1}}{S}$. We have

$$dS_i > adS\frac{S_i}{S}, \qquad dS_{i+1} > adS\frac{S_{i+1}}{S}$$

Subtracting the LHS and RHS of the two relations we have

$$dS_i - dS_{i+1} > a\frac{dS}{S}\left(S_i - S_{i+1}\right) \tag{7}$$

The LHS of (7) is a small negative number. For a perfectly elastic demand $(a \approx 0)$ the LHS of (7) tends to zero, so we have $dS_i > dS_{i+1}$ $(\forall i)$. According to (7) we conclude the following corollary.

Corollary 2

In the case of a large number of substitutes an increment in the discounted revenues caused by a marginal increment of the total allowed resource stock, dS *, the order of marginal increments of individual firms,* dS_i*, is ranked by the reverse order rather than the originally allowed set of resource stocks. That is, if* $S_1 < S_2 < ... < S_N$ *the result in the above marginal increase is* $dS_1 > dS_2 > ... > dS_N$

The impact of an absolute increase to the pollution stocks $dS_i = \varepsilon$ $(\forall i)$ can be expressed as follows.

Corollary 3

A uniform absolute increase in all resource stocks by $dS_i = \varepsilon > 0$ *reduces firm's* i *discounted revenues if and only if* $\frac{S_i}{S} > \frac{1}{aN}$.

Proof

The result is easily obtained since $dS = \sum_{i=1}^{N} dS_i = \varepsilon N \Rightarrow \frac{dS_i}{dS} = \frac{1}{N}$ and $a\frac{S_i}{S} < \frac{dS_i}{dS}$.

Next we consider a new allocation of the allowed stocks. With S_i^O we denote firm's i old allowed stock and with S_i^N the reallocated (new) allowed resource stock. Moreover we assume that the new allowed pollution is less than the original, $S_i^O > S_i^N$ $(i = 1,..,N)$.

The next proposition joins the two pollution stocks assuming the last given order.

Proposition 3.

The discounted revenues of each firm increases while the total resource stock falls, caused by a new allocation, if and only if $\sigma_i > (1-\sigma)^a + 1$, *where*

$$\sigma = 1 - \frac{\left(\sum_{k=1}^{N} S_k^N\right)}{\left(\sum_{k=1}^{N} S_k^O\right)} \text{ and } \sigma_i = 1 - \frac{S_i^N}{S_i^O}.$$

Proof (See Appendix)

Remark

The results of Proposition 3 may be used as follows. Suppose that an authority decides to decrease the total resource stock by an amount ΔS $\left(\sigma = \frac{\Delta S}{\sum_{i=1}^{N} S_i}\right)$, so in order to have each firm higher revenues, its allowed resource stock must be reduced by the amount $S_i^O - S_i^N = \Delta S_i = \left((1-\sigma)^a + 1\right) S_i^O$.

In the same way we consider a uniform decrease η to all firms' resource stock. Then

$$\sigma = 1 - \frac{N\eta + \sum_{i=1}^{N} S_i^O}{\sum_{i=1}^{N} S_i^O} = \frac{N\eta}{\sum_{i=1}^{N} S_i^O}$$

and the raised revenues requirement is $\sigma > 1 - (\sigma_i - 1)^{-a}$

and finally

$$\frac{N\eta}{\sum_{i=1}^{N} S_i^O} > 1 - (\sigma_i - 1)^{-a}$$

where $S^O = \sum_{i=1}^{N} S_i^O$ is the initial allocation of the pollution and $\sigma_i = 1 - \frac{S_i^N}{S_i^O}$ the percentage change on firm's i allowed resource stock.

5. Concluding remarks

In this paper we set up a very simple model of extracting oligopolists where the demand is not linear and the resulting game is not a linear quadratic one. We also make the assumption that each firm is allowed to extract to a variable size depending on the criterion that is given by an authority. The results, in our opinion, are useful for a policy maker to make distributed extraction policies on the industry in total as well as partially on a firm.

One conclusion that could be drawn as a result of the above model is that a new technology that reduces the total amount of the extraction stock is not necessarily welcomed by all firms in the industry. If for example any authority decides to improve the technology that is used by firms previous analysis shows that the bigger, with respect to the allowed resource stock, firm does not always benefit from this decision.

Specifically, our results on a strong time consistent (Markov) equilibrium with conjectured value function and strategies are surprising. Although without exposing the solutions of the problem in full generality, as Tsutsui and Mino (1990) face their linear quadratic differential game in a duopoly with sticky prices, a strong time consistent solution is obtained using the conjectured method.

Moreover testing the above strategies and the value function obtained we are able to conclude some interesting policy implications. In general, we expect that for each firm the higher the extraction rates is the more the utility (discounted revenues) will be. However, the findings of the model are slightly different. It is possible a marginal decrease on the total extraction stock to increase the firms' discounted revenues, provided that the original allowed share multiplied by the elasticity of demand is greater than the marginal change share.

Additionally, a reallocation caused by a uniform decrease into all firms resources, reorders the marginal change of the stocks in reverse to the original order of the allowed stocks and again the reallocation is possible to raise the discounted revenues of each firm.

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Appendix

Proofs of Propositions

Proof of Proposition 1

First we check that if firm's j strategy is $h_j = \frac{\rho}{a} S_j$, then firm's i best response will be $h_i = \frac{\rho}{a} S_i$. The Hamilton-Jacobi-Bellman (hereafter HJB) equation for firm's i maximization problem is the following

$$\rho V_i = h_i \left(h_i + \frac{\rho}{a} S_{-i} \right)^{-a} + \frac{\partial V_i}{\partial S_i} (-h_i) + \sum_{j \neq i, j=1}^{N} \frac{\partial V_i}{\partial S_j} \left(-\frac{\rho}{a} S_j \right)$$

Maximization of the RHS of the HJB equation with respect to h_i gives

$$\left(h_i + \frac{\rho}{a} S_{-i} \right)^{-a} - a \frac{h_i \left(h_i + \frac{\rho}{a} S_{-i} \right)^{-a}}{\left(h_i + \frac{\rho}{a} S_{-i} \right)} - \frac{\partial V_i}{\partial S_i} = 0$$

or equivalently

$$\frac{\partial V_i}{\partial S_i} = \left(h_i + \frac{\rho}{a} S_{-i} \right)^{-a} \left[1 - a \frac{h_i}{\left(h_i + \frac{\rho}{a} S_{-i} \right)} \right] \qquad (A.1)$$

Where S_{-i} represents the sum of all resource stocks except firm's i stock, that is $S_{-i} = S - S_i$ and $S = \sum_{j=1}^{N} S_j$

Now we make use the nonlinear conjectured value function

$$V_i = \left(\frac{a}{\rho} \right)^a S_i \left(\sum_{j=1}^{N} S_j \right)^{-a}$$

Differentiation of the value function with respect to S_i yields

$$\frac{\partial V_i}{\partial S_i} = \left(\frac{\rho}{a} S \right)^{-a} \left(1 - a \frac{S_i}{S} \right) \qquad (A.2)$$

with $S=\sum_{j=1}^{N}S_j$ the same as above.

Equating the terms with the same power of $(A.1)$ and $(A.2)$ we have the resulting system of equations.

$$1-a\frac{h_i}{\left(h_i+\frac{\rho}{a}S_{-i}\right)}=1-a\frac{S_i}{S} \qquad (A.3)$$

and

$$h_i+\frac{\rho}{a}S_{-i}=\frac{\rho}{a}S \qquad (A.4)$$

Both equations $(A.3)$ and $(A.4)$ have the same solution $h_i=\frac{\rho}{a}S_i$.

Now we prove that substituting the above strategies into the RHS of the HJB function we have equality with the LHS of the same equation. The partial derivative of the value function V_i with respect to S_j is

$$\frac{\partial V_i}{\partial S_j}=-a\left(\frac{a}{\rho}\right)^a S_i S^{-(a+1)}<0 \qquad (A.5)$$

so the RHS of the HJB becomes

$$\text{RHS(HJB)}=\frac{\rho}{a}S_i\left(\frac{\rho}{a}S\right)^{-a}-\frac{\rho}{a}S_i\left(1-a\frac{S_i}{S}\right)\left(\frac{\rho}{a}S\right)^{-a}+\sum_{j\neq i,j=1}^{N}-a\left(\frac{a}{\rho}\right)^a S_i S^{-(a+1)}\left(-\frac{\rho}{a}S_j\right)=$$

$$=\frac{\rho}{a}S_i\left(\frac{\rho}{a}S\right)^{-a}\left(1-1+a\frac{S_i}{S}+a\frac{S_{-i}}{S}\right)=\rho S_i\left(\frac{\rho}{a}S\right)^{-a}=\rho V_i(S)=\text{LHS(HJB)}$$

Where as above we have set $S=\sum_{j=1}^{N}S_j$ and $S_{-i}=\sum_{j=1,j\neq i}^{N}S_j$

Proof of Proposition 2

The total derivative of the value function for the moment t is

$$dV_i=\frac{\partial V_i}{\partial S_i}dS_i+\sum_{j\neq i,j=1}^{N}\frac{\partial V_i}{\partial S_j}dS_j \qquad (A.6)$$

$$\frac{\partial V_i}{\partial S_i}=\left(\frac{\rho}{a}S\right)^{-a}\left(1-a\frac{S_i}{S}\right)(A.2)\ \frac{\partial V_i}{\partial S_j}=-a\left(\frac{a}{\rho}\right)^{a}S_iS^{-(a+1)}<0\ (A.5)$$

Substituting the partial derivatives $(A.2)$ and $(A.5)$ previously found, into $(A.6)$ the derivative of the value function takes the form:

$$dV_i=\left(\frac{\rho}{a}S\right)^{-a}\left(1-a\frac{S_i}{S}\right)dS_i+\sum_{j\neq i,j=1}^{N}-a\left(\frac{\rho}{a}\right)^{-a}S_jS^{-(a+1)}dS_j$$

Putting the term $-\left(\frac{\rho}{a}S\right)^{-a}a\frac{S_i}{S}dS_i$ inside the sum, the above expression simplifies to

$$dV_i=\left(\frac{\rho}{a}S\right)^{-a}dS_i+\sum_{j=1}^{N}-a\left(\frac{\rho}{a}\right)^{-a}S_jS^{-(a+1)}dS_j$$

Multiplying and divide the first term the RHS of the latter by $a\left(\frac{\rho}{a}S\right)^{-a}\frac{1}{S}\sum_{j=1}^{N}S_idS_j$

we have

$$dV_i=\left(\frac{\rho}{a}S\right)^{-a}\left(\sum_{j=1}^{N}dS_j\right)\left(\frac{dS_i}{\sum_{j=1}^{N}dS_j}-a\frac{S_j}{S}\right)$$

Setting $dS\equiv\left(\sum_{j=1}^{N}dS_j\right)$ the latter simplifies to

$$dV_i=\left(\frac{\rho}{a}S\right)^{-a}dS\left(\frac{dS_i}{dS}-a\frac{S_i}{S}\right)\qquad(A.7)$$

The meaning of $(A.7)$ is, as we expect that a change in the allowed resource stocks results in the same sign change on firm's i discounted revenues depending on the sign of the term inside the brackets. That is, if the sign of the bracketed term is positive an increase in the total resource stock dS increases the discounted revenues of firm's i, as the term outside brackets reveals and vice versa.

Now consider the term of $(A.7)$ $\left(\frac{dS_i}{dS} - a\frac{S_i}{S}\right)$, which shows how the change on firm's i revenues responds to a marginal change in the resource stock. The term under consideration has positive sign which means that $a\frac{S_i}{S} < \frac{dS_i}{dS}$ $\quad (A.8)$

i.e. the original allowed resource stock share multiplied by the reverse elasticity is less than the marginal change share.

Proof of Proposition 3

From solution of the original value function we have the two value functions of the discounted revenues

$$V_i\left(S^O\right) = \left(\frac{\rho}{a}\right)^{-a} S_i^O \left(\sum_{k=1}^{N} S_k^O\right)^{-a} \qquad (A.9)$$

$$V_i\left(S^N\right) = \left(\frac{\rho}{a}\right)^{-a} S_i^N \left(\sum_{k=1}^{N} S_k^N\right)^{-a} \qquad (A.10)$$

Subtracting $(A.9)$ from $(A.10)$ to have incremental revenues, the positive change in firm's i revenues due to reallocation is

$$\Delta V_i = V_i\left(S^N\right) - V_i\left(S^O\right) = \left(\frac{\rho}{a}\right)^{-a} S_i^N \left(\sum_{k=1}^{N} S_k^N\right)^{-a} - \left(\frac{\rho}{a}\right)^{-a} S_i^O \left(\sum_{k=1}^{N} S_k^O\right)^{-a} =$$

$$= \left(\frac{\rho}{a}\right)^{-a} S_i^N \left(\sum_{k=1}^{N} S_k^O\right)^{-a} \left[\frac{\left(\sum_{k=1}^{N} S_k^N\right)^{-a}}{\left(\sum_{k=1}^{N} S_k^O\right)^{-a}} - \frac{S_i^O}{S_i^N}\right]$$

The latter expression simplifies denoting by $\sigma = 1 - \frac{\left(\sum_{k=1}^{N} S_k^N\right)}{\left(\sum_{k=1}^{N} S_k^O\right)}$ the percentage decrement into the total resource stock and with $\sigma_i = 1 - \frac{S_i^N}{S_i^O}$ the percentage change into firm's i resource stock. In order to have an increment into firms' i discounted revenues it suffices to hold the condition $\quad (1-\sigma)^{-a} > \frac{1}{\sigma_i - 1}$.