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VOTING ON TRAFFIC CONGESTION POLICY WITH TWO LEVELS OF GOVERNMENT

-Preliminary version-

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November 6, 2010

Abstract

I study how the political decision process affects urban traffic congestion policy. First, I look at the case of a single government deciding, through majority voting, on a monetary charge to be paid to drive to a city's Central Business District (CBD): if the majority of individuals prefers to drive more (resp. less) than the average, a voting equilibrium with lower (higher) charge emerges. Next, I consider the case of two government levels involved in traffic policy: parking charges in (resp. cordon tolls around) a city's CBD and capacity investments are chosen by a local (resp. regional) government, through a majority voting process. While tax exporting motives and the imperfect coordination among the two governments may lead to higher overall charges than in the case of a single government, strong preferences for driving across the population can still bring to an equilibiurm with suboptimal total charges.

JEL classification: D78, H23, H77, L98

Keywords: Traffic congestion, tolls, parking, voting, fiscal competition

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1 Introduction

Traffic congestion externalities have been the subject of economic enquiry for a long time: transportation economists generally agree on the merits of pricing measures in reducing inefficient road congestion. Nevertheless, while congestion becomes a more and more significant problem, it is still the case in most cities that traffic policy relies insufficiently on pricing instruments: politicians' reluctance to implement them has been recognized as one of the main factors contributing to their scarce application (Jones, 1998). This appears to be true for such instruments as congestion charges, but also for parking charges, which can represent an interesting alternative to discourage car travel (Calthrop *et al.*, 2000).

It also appears that, although in a generally unfavorable political climate to the introduction of pricing instruments to control congestion, local politicians are less restive to use certain among them than others. For instance, while it has been pointed out that parking places in central cities are still significantly underpriced (Shoup, 2005), parking charges seem to face less reluctance from local decision makers than congestion tolls. To make some examples, plans to introduce cordon tolls have been recently considered in cities such as Edinburgh, Manchester, Dublin, New York , and abandoned due to insufficient political support. Interstingly, in the same cities, plans for congestion tolls have been rejected almost at the same time as plans for raising parking taxes in central areas were being successfully implemented[1]. While various factors may be behind this stylised fact (for instance, differences in the costs of implementation), it seems reasonable (and it has been suggested by previous studies, see Proost and Sen (2006)) to link the relation between different pricing instruments to the institutional setup and the assignment of their control to different governmental institutions. In particular, one may observe that, while parking policy is more likely to be managed locally (by city or district councils, for instance), decisions about the implementation and management of congestion charging (even in the, relatively common, form of a cordon toll around central city areas) appear to be more likely to see the involvement of institutions representing larger portions of the population interested by the scheme. To stick to our previous examples: while New York City's DOT controls parking policy, the proposal for the New York Congestion Pricing scheme, championed by Mayor Bloomberg, was blocked in the New York State Assembly because of "overwhelming opposition". In Manch-

[1]For instance, the city of New york has recently abandoned plans for a congestion toll in Manhattan, while parking meter prices in Greenwhich Village have recently been raised by 50%. Moreover, the city plans to expand the scheme to other areas as well (see Litman, 2010).

ester parking policy is under the responsibility of the Manchester City Council, while the proposed (and recently rejected) congestion toll scheme was to be managed by the Association of Greater Manchester Authorities (representing the whole metropolitan area). Finally, in Edinburgh, the City of Edinburgh Council is in charge of parking policy in central city areas, while the decision proposed cordon toll scheme (although ultimately taken by referendum only by the same constituency) saw the involvement of outer councils (Fife, West Lothian, Midlothian): it seems likely that the latters' strong opposition to the scheme played at least some part in its final rejection[2].

Based on these observations, one may ask some questions about the way governments seem to prefer fighting traffic congestion: why do traffic charges (in general) tend to raise significant political opposition? Why do some of them seem to be even less palatable than others, so that governments tend to avoid them altogether? Does the presence of multiple (and possibly non-coordinating) government levels affect the shape and the welfare gains of congestion policy? How does the scale of implementation of congestion control measures influence their features? The objective of this article is to study how the political decision process may influence the choice of traffic congestion policy, in order to try to answer these questions.

We build a model in which individuals have heterogeneous preferences for driving to the city centre, a costly activity in terms of money and time (time costs being increasing with the amount of congestion). In the first part of the paper, we consider the presence of a single level of government, controlling a generalised charge on car trips (which could be seen as a cordon toll around the city centre, a charge to park the car once there or a combination of the two): we obtain that when the majority of the population has stronger (resp. weaker) preferences for using cars to travel to the city centre than the representative individual in the population, the voting equilibrium policy is characterized by suboptimal (resp. higher than optimal) driving charges. We argue that a distribution of preferences for car driving such that the majority drives more than the average may be consistent with what is often observed in reality, for instance in metropolitan areas that are signifcantly "car dependent" (where the majority of the population considers the car to be the main, if not the only, viable option to meet its daily travel needs).

[2]Similar assignments of responsibilities can be observed also in other cases, where cordon tolls were successfully implemented: in London, borough councils are responsible for parking policy and pricing of public parking places in their own jurisdictions, while the London Congestion Charge is under the responsibility of the Mayor of London (representing the whole urban area). In Stockholm, parking policy is managed by the Stockholm Municipality, while the congestion charge was established by the national government and consultative referendums were held, prior to implementation, in several other municipalities of Stockholm County.

In the second part of the paper, we assume congestion policy to consist of parking charges and a cordon toll around the CBD. The parking charge is under the jurisdiction of a local government (representing only people living inside a certain area, for instance the city's administrative boundaries) and the road toll is under the control of a regional government (e.g. an urban agglomeration authority, representing both people living inside and outside the area's boundaries). The possibility for the local population to exploit outside commuters to generate revenues (assuming the local government is not required to share revenues with the regional one) is likely to determine equilibria with higher parking than congestion charges. Since higher parking charges tend to discourage drivers from accepting higher tolls, it may also contribute to generate hostility to cordon tolls. Moreover, the imperfect coordination among the two governments also turns out to play a role: we find that, in the presence of two government levels, the equilibrium total level of charges is likely to be higher than with a single government level, because of the imperfect coordination between the two (vertical tax competition, as both charge non-cooperatively the same tax base).

The rest of the paper is organized as follows: Section 2 by relates this work to existing literature. Section 3 introduces the model and derives the benchmark vector of policy parameters. Section 4 considers the case of majority voting with a single government level, while Section 5 considers the same problem but with two government layers involved (Proofs of all propositions and lemmas will be provided in the appendix). Section 6 concludes. All proofs are provided in the Appendix.

2 Related literature

As mentioned above, there is a large body of literature studying road congestion policy from a normative perspective: most of these studies focus on road charges (and capacity investments), in different scenarios (see Small and Verhoef, 2007). There is also a strand of the transport economics literature looking at parking policy (see, e.g., Arnott and Inci (2006)). Calthrop *et al.* (2000) look at the optimal policy mix when governments can use both parking and road tolls to control congestion: they find that parking and congestion charges can be seen as substitute instruments for reducing traffic. However, they take a purely normative perspective, also neglecting the presence of multiple governments involved in congestion policy.

Although political acceptability is one of the main issues holding back the implementation of road pricing in the urban context, there are, quite surprisingly, only few studies looking at congestion charges from a positive perspective. To the best of my knowledge, only Marcucci *et al.* (2005) and Glazer and Proost (2007) study (analytically) congestion tolls from a political economy perspective. In the first paper, a citizen-candidate game is used to model the political decision process on congestion tolls, assuming the government uses revenues to finance public transportation. In the second the authors use a majority voting setup and find that when aggregate income is high enough that drivers constitute the majority of the voting population, they will vote for suboptimal road tolls and higher than optimal capacity. While this result is quite related to that of this paper, we consider individuals that are heterogeneous in preferences for using cars and relate their reluctance to accept high charges to car dependence. Moreover, our paper studies the issue of multiple (non coordinating) governments intervening in urban traffic policy with different instruments. This seems important in light of its relatively small spatial scale of implementation.

There is a growing body of literature that focuses on the issue of govenrmental competition in tolling of road networks. De Borger *et al.* (2005) and (2007) study the interacton of different governments in setting traffic policy on parallel and serial networks. They find that imperfect coordination among governments can lead to significant deviations from the optimal pricing and investment scheme. Ubbels and Verhoef (2008), study the choice of pricing and capacity investments by a city and a hinterland government, each controlling one part of a two link road network leading to the city's Central Business District (CBD): the lack of coordination among governments, but also, importantly, tax exporting motives for the city population, who can exploit demand from outside commuters for the use of its own part of the network, leads to too high total charges and higher tolls on the city than on the hinterland section of the network. However, while these forces are surely relevant (and indeed similar phenomena are modelled in the present work), they do not explain why cordon toll schemes to enter cities' CBDs often find strong political opposition and are rarely implemented in reality. Our paper provides a possible explanation.

Another paper closely related to this one is Proost and Sen (2006): they study the interactions between overlapping (city and regional) governments in charge, respectively, of parking charge and cordon toll. Their setup is therefore very similar to ours (although urbanites do not have to pay the toll and its revenues are redistributed only ot external commuters). Indeed, they find a tendency for the local government to overcharge for parking, while the regional government responds by reducing the cordon toll: these results

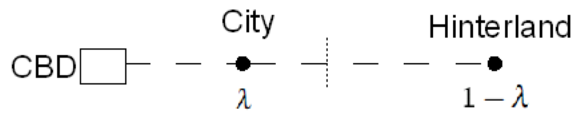


Figure 1:

are consistent with what we find.

3 The model

Spatial structure We consider an economy with the following spatial structure: there is a "large" population of individuals (whose size is normalized to 1) living along a line ending in the CBD. A first group of individuals, comprising a fraction $\lambda \epsilon\ (0, 1]$ of the total population is assumed to live within a certain administrative area (for instance, a city's jurisdiction), while a second group (the remanining $1 - \lambda$ fraction of total population) lives outside them, (for instance, in the city's hinterland). Figure 1 provides a graphical representation of the spatial structure of our economy: we make the important assumption that all car trips (to be introduced later on) are return trips to the CBD.

Individuals Individuals derive utility from consuming two goods: a consumption good c (the numeraire, whose price is fixed and normalized to one) and car trips to the city's CBD, denoted by q (assumed for simplicity to be a continuous variable). Utility also depends on the preference parameter r (non-negative) which increases the marginal utility derived from a car trip. Individuals suffer some disutility $X(q, T)$ for spending time stuck in their cars in traffic: it is proportional to the number of car trips taken, in a given time period, and to the amount of time T required to complete a single trip (see below): therefore $X(q, T) = qT$. We assume the utility function to be[3]

$$U(q, c, X; r) = 2\,(qr)^{\frac{1}{2}} + c - qT$$

Individuals differ only in the parameter r, which is exogenously distributed according to the CdF $F(r)$, with support $[r^l, r^u]$ over the entire population. We denote the average value of r as $\bar{r} = \int_{r^l}^{r^u} r dF(r)$ and its

[3] We use this functional form for simplicity, since it allows to get cleaner results: other functional forms could be used, without changing the qualitative results, while complicating exposition.

median value as $\hat{r}$, so $F(\hat{r}) = \frac{1}{2}$.

We assume that types are distributed according to the same distribution $F(r)$ in both the city and the hinterland: the same distribution of types $F(r)$ characterizes both subgroups and, consequently, the entire population in the region. The case of two different distributions is of course more general but also more complex to treat and left for future work.

Costs of driving and congestion externalities There is a basic resource (monetary) cost for each car trip, denoted by d (in units of the numeraire) and assumed to be the same for all the individuals of the population, regardless of their location[4]. We assume a trip to be composed by two sub-activities: driving to the CBD and parking the car once there. The two are strictly complementary and both are potential targets for governmental levies, for all drivers (in what follows, we will assume that governments can impose a cordon toll to enter the CBD and a parking charge to leave the car once inside it[5]). The direct monetary cost (we will denote it by p) of a car trip for an individual is the sum of d and the eventual charges. We denote the amount of time to drive to the CBD and to park the car, for a single trip, as T, assume it is an increasing function of the ratio of traffic volume Q with the following form:

$$T(Q) = bQ$$

The overall unit "cost" of a car trip, including the marginal disutility of time lost in traffic, is thus $p + T(Q)$[6].

We take the standard assumption that, when deciding on the number of trips to take, the individual will take $T(Q)$ as given, disregarding the effect of her own contribution to total congestion. We also disregard

[4] The underlying assumption is that, even though individuals may have to travel different distances, travel is seamless and takes place in ideal conditions up to the point where they get to the boundary of the CBD, where the only bottleneck is placed. Assuming the costs of a seamless trip to be approximately invariant with distance (and the difference in distances to be travelled sufficiently small anyway), we have that all car trips have the same time and resource costs. This is obviously a simplification but is not without precedents in the literature: for example, it is the typical assumption of the "Bottleneck Model" of traffic congestion (see, e.g., Small and Verhoef, 2007).

[5] We assume that all drivers have to pay for both charges. In most cities many drivers do not pay for parking and payers may even be a minority. This may be due to the limited powers of local governments (unable, for instance, to force employers to make workers pay for parking at the workplace, see Bonsall and Young (2010)), but it may also be due to lack of political will: it seems therefore appropriate to study the behavior of local governments allowing them, a priori, to fully price parking for every trip to the CBD. When studying the interactions of two governments involved in congestion policy, Proost and Sen (2006) take the same assumption.

[6] Notice that we assume that individuals assign the car trips they decide to take randomly along the period considered (a week, for instance). This implies that, even if individuals do not always travel at the same time, since the population is "large", the number of cars found on the road at any time is equal to the time average of the trips taken by all the population. This justifies using an undifferentiated measure of time cost per trip T. A similar assumption is used in Parry (2002).

other forms of externalities, such as air pollution.

Government We will start by assuming the existence of a single government, representing the fraction of the population living within the local boundaries (including the special case in which $\lambda = 1$ and the city government covers the entire urban area). This government controls a monetary charge t to be paid to to drive to the CBD (this could either represent, in our setup, a cordon toll around the CBD or a parking charge, or any combination of the two, assuming all drivers have to pay for them). The government is assumed to fully rebate to each individual in the population it represents an equal share of charge revenues (given by tQ) using an undifferentiated lump sum transfer L. The government's budget constraint is thus

$$\lambda L = tQ \tag{1}$$

with $\lambda = 1$ in the case the administrative boundaries of the government in charge of traffic policy include the entire metropolitan area. We assume L to take the form of a non-distortionary lump-sum tax in case the revenues they generate were insufficient to cover capacity investments. As a result, the individuals' budget constraint is:

$$M + L \geq c + pq$$

Timing We model a two-stage decision process. In the first stage, individuals vote on t. We assume individuals vote perfectly anticipating their welfare at the following stage. In the second stage of the game, once t has been set through the voting process, individuals allocate their resources deciding the amount of car trips to take q and consumption c, maximising their utility $U(.)$.

3.1 Individuals' behaviour after policy parameters have been set

Let us describe the equilibrium allocation $(\{q\}, \{c\}, L)$ in the economy once t is set. Individuals choose the number of times q they want to drive to the CBD (in the given time period considered), their consumption c and receive transfer L from the government. Each individual solves the following utility maximization problem:

$$max\, U(G, c, X)$$

with respect to q and c subject to

$$M + L \geq c + pq$$

We get the following demand function for car trips[7]:

$$q(p; r) = \frac{r}{(p + T(Q))^2} \tag{2}$$

by the linearity of $U(.)$ in c, demand $q(p; r)$ is independent of income and, therefore, also of transfer L. Notice also that monetary and time costs, in our setup, have exactly the same discouraging impact on the demand for car trips q (this is due to the fact that both unit costs enter linearly in the individuals' utility, regardless of their type). Thus, the demand for car trips is decreasing in both p and T and increasing (linearly) in r, the individual's type. Substituting $q(r)$ into $U(.)$ above, and using the individual's budget constraint, we get the indirect utility function

$$V(p, L; r) = \frac{r}{(p + T(Q))} + M + L \tag{3}$$

To obtain the aggregate demand Q, integrate $q(r)$ over the support $[r^l, r^u]$ of $F(r)$ to get[8]:

$$Q(p) = q(p; \bar{r}) = \frac{\bar{r}}{(p + T(Q))^2} \tag{4}$$

(in the following we denote $Q(p)$ simply as Q to save notation) thus, we obtain that the aggregate amount of driving in the economy coincides with that of the "average" individual, for which $r = \bar{r}$. By a similar integration of the indirect utility function $V(p, L; r)$ we obtain the (utilitarian) social welfare function (for

[7] from now on, we will denote, in order to save on notation, $q(p; r)$ simply as $q(r)$.

[8] While it is quite straightforward that a solution to the individual maximization problem exists (the objective function being strictly concave in q and the budget constraint being linear in q), for any value of r, one may wonder whether there will always exist an aggregate equilibrium on the "car trips market": in particular, Q would be the fixed point such that $Q = \frac{\bar{r}}{(p+T(Q))^2}$. Since this fixed point coincides with the q that maximizes $U(.)$ for the individual with $r = \bar{r}$, then it must exist for all values of p and $\bar{r}$.

a government taking care of the *entire* population)

$$W(p) = \int_{r^l}^{r^u} V(p, L; r) dF(r) = \frac{\bar{r}}{(p + T(Q))} + M + \lambda L \tag{5}$$

notice that, by assumption only the fraction λ of the population receives the transfer L[9].

We now study the behavior of aggregate demand for car trips with respect to changes in the unit cost (including the disutility of time lost in traffic) of q, that is $p + T(Q)$. This is important for studying the effects of monetary charges on car driving, since they translate one-to-one in increases in the total monetary cost of a car trip (that is $\frac{dp}{dt} = 1$). We have, totally differentiating Q in (4) with respect to p:

$$\frac{dQ}{dp} = -\frac{\bar{r}}{\frac{1}{2}(p + T(Q))^3} - \frac{\bar{r}(T_Q \frac{dQ}{dp})}{\frac{1}{2}(p + T(Q))^3} \Rightarrow \frac{dQ}{dp} = -\frac{\bar{r}}{\frac{1}{2}(p + T(Q))^3 + \bar{r} T_Q}$$

(where $T_Q \equiv \frac{\partial T}{\partial Q} = b$). As the above formula shows, $\frac{dQ}{dp}$ is the result of two effects: a direct effect (first term in the sum on the left), which is negative due to the change in the monetary cost of car trips p (for a given amount of traffic Q), and an indirect effect (second term in the sum on the left) accounting for the reduction of $T(Q)$ due to lower Q and congestion on the road and in parking search (a positive feedback effect, which, per se, would stimulate additional demand). Notice that the first effect always dominates on the second, so $\frac{dQ}{dp} < 0$.

3.2 Benchmark: the utilitarian optimum

We study here the optimal charge t^{FB} that would be chosen by a welfare maximizing government representing the entire population, wanting to decentralize the welfare maximising allocation, using the instruments in π and the uniform lump sum transfer L. The government's problem is

$$max \quad W(p)$$

[9]However, this does not affect the welfare maximising choice of t, since a utilitarian government, when individuals have quasilinear preferences, is not concerned with the redistribution of resources.

(with respect to t), subject to (1). Taking partial derivatives of $W(p)$ with respect to t (after substituting for L from (1) in $W(p)$), we obtain the FOC

$$\partial W(p)/\partial t : -Q[1 + T_Q \frac{dQ}{dp}] + Q + t\frac{dQ}{dp} = 0 \Rightarrow \quad t = QT_Q \tag{6}$$

The first equation tells us that the optimal charge t is such that it is equal to QT_Q, the marginal external (time) costs of an additional car trip: it is a Pigouvian tax. Moreover, since $W(p)$ is utilitarian (and preferences are quasilinear), there is no concern for the distributional effects of the charge, neither intra nor inter-jurisdictional. The results are summarised in Proposition 1:

PROPOSITION 1: *First Best charge t^{FB}, defined as the choice of a single government maximizing $W(p)$ (a utilitarian social welfare function), is such that*

$$t^{FB} = Q(p^{FB})(T_Q)$$

and is therefore a Pigouvian tax.

4 Voting on traffic policy with a single government level

We now introduce majority voting as the social choice process that leads to the determination of t.

4.1 Voting over the generalised charge

Individuals' preferences over t In order to describe individuals' preferences over t, we start from the *reduced* indirect utility function $V(p;r)$ (written, that is, after using (1) to substitute for L), for a generic individual of type r

$$V(p;r) = \frac{r}{(p + T(Q))} + M + \frac{tQ}{\lambda} \tag{7}$$

To find the most preferred charge $t^*(r)$ by a type-r individual, we maximize $V(t;r)$ with respect to t. Taking partial derivatives of (7) with respect to t, we obtain that $t^*(r)$ is defined by the following FOC

$$\partial V(p,r)/\partial t : -q(r)[1 + T_Q \frac{dQ}{dp}] + \frac{Q + t\frac{dQ}{dp}}{\lambda} = 0 \tag{8}$$

A marginal increase in the generalised charge affects $V(t;r)$ in two ways: it changes (in particular, it raises, since $0 < 1 + T_Q \frac{dQ}{dp} < 1$) the generalized unit cost of a car trip $p + T$, which affects the individuals to different extents, depending on their amount of driving $q(r)$[10].

Secondly, a marginal increase in t also affects the extent of the transfer L received by each individual, by changing the total amount of charge revenues tQ. The relevance of the latter effect is greater the smaller is λ because, for a given amount of revenues, the share of them to which each voter is entitled to is greater, given that individuals living outside the government's jurisdiction are excluded from their redistribution.

Voting equilibrium We can prove that individuals' preferences on t (described by (7)) satisfy the Single Crossing condition. Therefore, the voting equilibrium, (for any λ), denoted by $t^E(\lambda)$ will, following the result of Gans and Smart (1996), exist and coincide with the most preferred choice of the median individual in the population considered, $t^*(\lambda;\hat{r})$

LEMMA 1: *Under a single level of government, when individuals vote (Shepsle procedure) on the generalised charge t, for any given value of λ, there exists a unique majority voting equilibrium $t^E(\lambda)$, coinciding with the choice of the median individual $t^*(\lambda;\hat{r})$*

Lemma 2 describes the nature of the equilibrium vector obtained and how it changes with respect to marginal changes in the exogenous parameters describing the preferences of the decisive individual $\hat{r}$, the relative size of the city population (compared to the total) λ.

LEMMA 2: *The generalised charge t^E is decreasing in the preference for driving $\hat{r}$ of the pivotal individual and in the relative size of the voting population λ*

[10]This particular feature depends on linearity of individuals' utility in both monetary and time costs of a car trip, so that marginal utility of both money and time are the same across the population. In a more general model of individual utility, it would be reasonable to expect, for instance, certain individuals to have a higher marginal utility of time than others and, therefore, to be less negatively affected (or even benefit) from a marginal increase in monetary charges (after accounting for the equilibrium decrease in congestion).

The intuition is quite simple: the higher $\hat{r}$, the more frequently the individual drives, relative to the others, and, as a consequence, the more he will suffer from an increase in the generalized price of driving due to a raise in p. Therefore, individuals with higher r will prefer lower values of t. A marginally lower value of λ will instead induce a higher total level of charges t^E, because of their increased effectiveness as a device to exploit people commuting from the hinterland in order to generate additional revenues[11].

The results in Lemma 2 are instrumental in establishing Proposition 2, in which we describe conditions for the comparison among the components of t^E and those of the vector t^{FB}

PROPOSITION 2: *When the traffic policy vector t is decided via majority voting, under a Shepsle procedure, by a single government, the equilibrium vector t^E is such that:*

- $t^E = t^{FB}$ *if and only if* $\hat{r} = \frac{\bar{r}}{\lambda}$
- *If* $\hat{r} < \frac{\bar{r}}{\lambda}$*, the generalised charge level is higher than optimal,* $t^E > t^{FB}$
- *If* $\hat{r} > \frac{\bar{r}}{\lambda}$*, the generalised charge level is lower than optimal,* $t^E < t^{FB}$

These results provide us with some ingredients to explain the shape of traffic congestion policy observed in reality: the general prediction is that when the majority of the population has sufficiently stronger preferences for driving than average (so that it also travels to the CBD by car sufficiently more frequently than average), then we should observe a tendency by governments to underprice congestion. On the contrary, when the majority of the population has weaker preferences than average for driving, the opposite should occur. In addition, the relative size of the local population compared to the total is important: the smaller it is, the more inclined, all else being equal, will voters be to choose high charges. Finally, the results suggest that the smaller the size of the population involved in the voting process with respect to the total, the more likely the government is to adopt a policy of high monetary charges. This is due to a (well known) phenomenon of tax exporting behavior (charging non-voting outside commuters for a tax that will be reabted only to voters).

Observation suggests that governments are usually reluctant to curb traffic congestion using pricing instruments, while more willing to invest, when feasible, in additional road capacity. Our results say that

[11] The above results concerning the size of the local population λ, would also hold in a model involving a welfare maximising government (acting in the interest of the population's representative individual): the size of the voting population (with the tax exporting motive) affect the impact of the proposed measures on the welfare of the pivotal individual in the same way as they affect that of every other voter.

a left-skewed shape of preferences for driving characterising the population of a metropolitan area could contribute to generate such behavior. Contemporary urban societies often appear to be characterised by significant "car dependence" (Kenworthy, 1999), with large shares of the population finding the car as a difficult to subsitute means to satisfy their mobility needs[12]. Many factors can be behind such a phenomenon: lifestyles and habits that revolve increasingly around cars, supply of road and parking infrastructure, urban sprawl and progressive suburbanization of the population of large urban areas, combined with insufficient investment in public transportation, can all contribute to make the car hard to substitute with alternative transport modes (including also walking and cycling). While our model clearly does not capture all of them, and relates differences in driving habits across the population only to heterogenity in preferences for using cars, a scenario in which a majority of the population has stronger preferences for driving than average, with only a avoiding car travel, does not appear inconsistent with the widespread car dependence observed in many urban contexts. This, in turn, could contribute to explain, according to our findings, the reluctance by politicians to use pricing instruments to control congestion.

5 Voting on traffic policy with two government levels

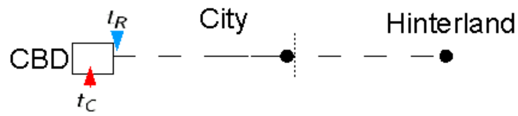
The setup

The setup is the same as that of the previous section, except that now we introduce the presence of a regional and a local (city) government, that represent the respective populations, still deciding on policy through majority voting. So, while the second stage of the game we described is the same as in the previous sections, the first (voting) stage now involves two distinct governments, representing different (though partially overlapping) populations.

Traffic policy is now assumed to consist of two parameters: a parking charge t_C to be paid to park in the CBD and a cordon toll around it t_R. We assume the following institutional setup: the local (city) government has authority over parking charge and capacity t_C, while the regional government decides on the congestion charge t_R. The two governments represent overlapping polities: while the regional government's

[12]While this seems to be a particularly relevant phenomenon in north american and australian cities, it is not of secondary importance in most european and asian cities as well (for some anecdotal evidence, see the case of Dublin, cited in Khan , or the study by Kenworthy, cit.)

Figure 2:



includes all the population considered (that is, both the city's and the hinterland's), only the individuals living within the city's boundaries are represented by the local government. The spatial setup (as well as the allocation of responsibilities for the components of traffic policy among the two governments) is similar to that used in Proost and Sen (2006). Figure 2 provides a graphical representation.

All individuals face the same charges (we assume, therefore, that no one lives inside the CBD and can "escape" any of the levies). The total level of charges (called the "generalised charge" in the previous section) is now $t = t_C + t_R$[13]. Both governments are assumed to fully rebate to each individual *in their respective polities* an equal share of the charge revenues net of capacity investments (given by $t_R Q$ and $t_C Q$ respectively) using undifferentiated lump sum transfers L_R and L_C. The regional government's budget constraint is, thus

$$L_R = t_R Q \tag{9}$$

and the city government's is

$$\lambda L_C = t_C Q \tag{10}$$

(recall that it represents only people living inside the city boundaries, a fraction $\lambda \epsilon (0, 1]$ of the total popualtion, and we assume it has no reason to redistribute revenues to people living in the hinterland, who do not belong to its constituency). As a result, the individuals' budget constraint is:

$$M + L_R + L_C \geq c + pq$$

for individuals living in the city and

$$M + L_R \geq c + pq$$

[13]Notice, therefore, that, in terms of their "contribution" to the monetary price of a car trip p, the two charges are, in our setup, completely equivalent: they are, therefore, equivalent instruments to implement a given allocation on the car transport market. The key difference between the two is in the way the revenues they generate are redistributed to the public, as well as their assignment to the control of different governments.

if the individual lives in the hinterland.

The outcome if only one government were in charge of traffic policy

Before we start introducing the results for the case of two governments, it may be useful to say something about what would happen if traffic policy consisted in the two charges described above (t_C, t_R) but they were all set by a single government, as in the previous sections[14].

In that case, we can expect the choice of the government to coincide with that of Section 2, in the case it simply acted as a welfare maximiser, or with that of the single government of Section 3, in case the choice of policy was determined through majority voting, with the only difference that only the "generalised charge" $t = t_C + t_R$ would be determined and there would be a degree of freedom in setting one of the two charges t_C and t_R. The reason is that, under the stylised setup of our model, driving and parking in the CBD are perfectly complementary activities and all individuals, when taking a car trip, are assumed to have to pay both charges: therefore, the impact any of the two has on the generalised price of a car trip is the same. Moreover, in a single government scenario, the asymmetry between the two charges, in terms of rebated revenues, that is assumed in the two government case is eliminated: every individual in the polity is entitled to an equal share of net revenues. Therefore, in order to implement any allocation of resources , the government will be indifferent among any combination of the two charges that yields the desired level of "generalised charge" t: either the welfare maximising t^{FB} or the most preferred by any of the individuals in the population $t^*(r)$.

5.1 The voting procedure with two governments

Since we want to capture the imperfect interaction among two governments, we still study voting using a Shepsle procedure (Shepsle, 1979): this allows us, importantly, to work under the assumption that voting takes place with governments that do not coordinate their policies. Moreover, it allows us to avoid the difficulties of multidimensional voting. The vector of parameters obtained as the outcome of the voting procedure, denoted by $\Pi^{NE} = (t_C^{NE}, t_R^{NE})$, will be a Nash Equilibrium, in which each of the governments

[14] We would assume that the government cares only for the polity it represents, and would redistribute the (residual) revenues from both charges only to its polity, as in the previous section.

involved chooses (through majority voting) its respective policy variable as the best response to those expressed by the other.

5.1.1 Voting by the local polity

Individuals' preferences We start by looking at preferences for t_C in the local population. Consider an individual of type r: using the local and the regional governments' budget constraints (9) and (10), her (reduced) indirect utility function $V(p,r)$ can be written as

$$V(p,r) = \frac{r}{(p+T(Q))} + M + \frac{t_C Q}{\lambda} + t_R Q \tag{11}$$

As anticipated, we assume that individuals vote separately but simultaneously on the parking charge t_C, taking t_R as given. When choosing the parking charge rate t_C, type-r individual's most preferred charge level $t_C^*(t_R, \lambda; r)$ will have to satisfy the following FOC

$$\partial V(p,r)/\partial t_C : -q(r)[1 + T_Q \frac{dQ}{dp}] + \frac{Q + t_C \frac{dQ}{dp}}{\lambda} + t_R \frac{dQ}{dp} = 0 \tag{12}$$

at the given t_R. Notice that, as anticipated, a maginal increase in t_C translates entirely, given t_R, in a marginal increase in p. Another important thing to notice is that the parking charge t_C's revenues are redistributed by the city government only to its own constituency, while outside commuters pay for it but do not get any of its revenues: this is why the impact of a marginal change in t_C (and therefore, in p) on parking charge revenues (the last term on the right hand side of the equality) is divided by $\lambda \epsilon (0,1)$. Parking charges are, therefore, affected by the tax exporting motive that we identified in the previous sections. Notice, in addition, that city individuals are nonetheless entitled to a share in the revenues generated via t_R (through the transfer L_R) and so they take into account the fact that raising t_C can marginally reduce the tax base for t_R.

Equilibrium choice of the local government, given the choice of the regional government The city population's preferences on t_C satisfy the Single Crossing property. Therefore, for each of the two voting dimensions, a majority voting equilibrium exists, in which the $\hat{r}$ individual (that is, the individual with median value of r for the city population) is pivotal. In turn, the equilibrium of the voting procedure coincides with the most preferred policy $t_C^*(\hat{r}; t_R)$ of the (local) pivotal individual $\hat{r}$ (conditionally on t_R set by the regional government).

LEMMA 3: *When the local population votes on parking charge t_C , under a Shpesle voting procedure, for any given value of t_R, there exists a unique equilibrium $t_C^{BR}(t_R) = t_C^*(t_R, \lambda; \hat{r})$.*

To conclude this part we can derive an (implict) reaction function for the (pivotal) type-$\hat{r}$ individual for the city when voting on π_C, as a best response to t_R set by the regional government:

LEMMA 4: *The local government's best response $t_C^{BR}(t_R)$ to the strategy played by its regional countrerpart is implictly defined by the following condition:*

$$-q(r)[1 + T_Q \tfrac{dQ}{dp}] + \tfrac{Q + t_C \frac{dQ}{dp}}{\lambda} + t_R \tfrac{dQ}{dp} = 0$$

and we have

$$-1 \quad < \tfrac{\partial t_C^{BR}}{\partial t_R} \leq 0 \quad ; \qquad \frac{\partial t_C^{BR}}{\partial \hat{r}}; \frac{\partial t_C^{BR}}{\partial \lambda} < 0$$

Therefore, the local government will respond to a marginal increase in t_R by reducing t_C less than proportionally. The intuition for the comparative statics involving $\hat{r}$ and λ follow the same lines as those of Section 3.

5.1.2 Voting by the regional polity

Individuals' preferences We consider here preferences for t_R in the regional (that is, the entire) population. A crucial distinction among individuals is that between those living in the city and in the hinterland (the latter not being entitled to receive any of the revenues generated by the parking charge t_C): this plays a role in determining their attitudes towards t_R. While for an individual living inside the city the relevant $V(p, r)$ to be maximised with respect to t_R (taking t_C as given) is given by (11), for an individual living in the hinterland it will be

$$V(p, r) = \frac{r}{(p + T(Q))} + M + t_R Q \tag{13}$$

which does not account for the revenues $t_C Q$ coming from the parking charge, as well as capacity investments. When choosing (taking all the other parameters in $\mathbf{\Pi}$ as given) the cordon toll rate t_R, type-r individual's most preferred $t_R^*(t_C; r)$ will have to satisfy the following FOC

$$\partial V(p,r)/\partial t_R : -q(r)[1 + T_Q \frac{dQ}{dp}] + i \left(\frac{t_C \frac{dQ}{dp}}{\lambda} \right) + t_R \frac{dQ}{dp} + Q = 0 \quad i = \{0, 1\} \tag{14}$$

and is equal to zero otherwise. Note the index i, which takes value 1 if the individual lives in the city and 0 if in the hinterland, the key difference in attitudes mentioned above: in the former case, individuals take into account the fact that a higher t_R reduces aggregate demand for car trips Q and, thus, revenues generated by t_C. In the latter, this effect is neglected. Notice that toll revenues are redistributed by the region's government to the entire population: contrary to the case of parking charges, there is no asymmetry between the population who pays t_R and that which receives its revenues.

Equilibrium choice of the regional government, given the choice of the local government The different attitude towards t_R for people living in the city and in the hinterland mentioned above, is the crucial reason behind the fact that, when they vote on t_R (given t_C), the Single Crossing condition fails to hold, as this introduces an additional dimension of heterogeneity among individuals, orthogonal to preferences for driving: this makes it impossible to identify a pivotal individual, which in turns does not allow us to define a proper reaction function $t_R^{BR}(t_C)$ for the (decisive individual in the) regional government.

However, we are able to identify the individuals that would be pivotal if city and hinterland voted separately on t_R (denote them, respectively r_C^d and r_H^d): since, taking each of the subpopulations separately, preferences on t_R do satisfy the Single Crossing property, they are the medians in the distributions of types r for the two subgroups (incidentally, since we assumed that the two subpopulations are characterized by the same distribution of types $F(r)$, then the two individuals both have the same type, so $r_C^d = r_H^d = \hat{r}$). Moreover, it is also the case that we can always rank their most preferred values of t_R, denoted $t_R^*(r_C^d)$ and $t_R^*(r_H^d)$ respectively: for any value of t_C set by the city government, we have $t_R^*(r_C^d, t_C) \leq t_R^*(r_H^d, t_C)$. This is because the objective function individuals r_C^d and r_H^d look to maximise with respect to t_R is not the same, since, as explained above, those living in the city are, *coeteris paribus*, less keen to raise the congestion charge than their counterparts in the hinterland, in order to preserve parking charge revenues. Using these important pieces of information, we can argue that a Condorcet Winner in the voting on t_R exists and, importantly, that it necessarily coincides with the most preferred choice of one of the individual in the population, belonging to the interval $\left[t_R^*(r_C^d), t_R^*(r_H^d)\right]$.

LEMMA 5: *When the region population votes on t_R, taking as given the values of t_C and K, there is no individual who is decisive in every pairwise majority voting contest. Nonetheless, a majority voting equilibrium $t_R^{NE}(t_C)$ exists and is unique for all values of t_C.*

Define $t_R^*(r_C^d)$ the most preferred value of t_R for the type-r individual who would be decisive if the city population were the only one voting on it and $t_R^*(r_H^d)$ the most preferred value of t_R for the type-r individual who would be decisive if the population living outside the city were the only one voting on it. Then, $t_R^{NE}(t_C)$ necessarily belongs to the interval $\left[t_R^*(r_C^d), t_R^*(r_H^d)\right]$.

To prove this, we use an argument proposed by De Donder (2010): he shows that, when a population composed of subgroups that face different governments' budget constraints (in our case, city and hinterland population face different rebating transfers for the same taxes on car trips that they pay) votes on a single policy parameter, as long as the Single Crossing condition holds if each group is taken separately (, then a condorcet winner, when the entire population votes, exists and coincides with the most preferred choice of one of the individuals in the population[15].

We are able to identify a subset of the population among which the condorcet winner has to lie thanks to the fact that preferences on t_R, if the two populations are taken separately, satisfy the single crossing condition and that $r_C^d = r_H^d$. These conditions are sufficient to say that any value outside $\left[t_R^*(r_C^d), t_R^*(r_H^d)\right]$cannot be the condorcet winner (the explanation is left for the appendix).

5.1.3 The Shepsle equilibrium

As mentioned at the beginning of this section, we describe the outcome of the voting procedure as a Nash Equilibrium in which every tax rate is chosen, by the relevant polity, as a best response to the other

[15]In our setup, taking each subgroup separately clearly eliminates the additional element of heterogeneity among individuals mentioned above. Moreover, since, for any individual in our population, the most preferred choice is unique, then we know that also the voting equilibrium on t_R is unique.

two. Unfortunately, the result of the voting procedure on t_R limits the amount of information that can be obtained on Π, as we are unable to identify, as was the case in the voting for the local population, a reaction function characterising the choice of the regional government.

We can, however, derive some important information on the nature of the equilibrium. First, we can draw a comparison between the two taxes on car trips t_C and t_R. We find that, unless λ is sufficiently large, the parking charge will always be the higher tax:

PROPOSITION 3 *: There exist a unique threshold $\tilde{\lambda}$ such that:*

- $\lambda < \tilde{\lambda}$ *is a sufficient condition for* $t_C^{NE} > t_R^{NE}$
- $\lambda \geq \tilde{\lambda}$ *is a necessary, but not sufficient, condition for* $t_R^{NE} \geq t_C^{NE}$

Two forces are behind this result: on the one hand, the tax exporting motive determined by the smaller size of the city population with respect to the total. Such motive is stronger, for each city voter, the smaller is λ. On the other hand, the individuals in the city vote on t_C taking into account the effect it has on the tax base for t_R (recall that people living in the city are entitled to t_R revenues as much as those living outside), while, when voting on t_R, they are the only ones to care for the effect it may have on the tax base for t_C: this is an asymmetry that, in itself, would make voters on t_C more reluctant to support its increase than voters in the hinterland to support a raise in t_R, but, as long as λ is not too big, its effect is surely dominated by the tax exporting motive and we would end up with an equilibrium with $t_C^{NE} > t_R^{NE}$. Instead, if λ is large enough, we may end up in an equilibrium with t_R that is higher than t_C.

These results are consistent with the casual observation that, even if total charges to drive or park cars in central cities are too low, given the associated externalities (congestion being the main one), local politicians seem less reluctant to use parking charges as instruments to curb congestion (and also raise revenues) than cordon tolls (which are rarely implemented at all): our results suggests that an equilibirum policy vector with higher parking than congestion charges would result if the city government can use the parking charge to exploit, in a certain mesure, outside commuters to generate revenues. The key force lying behind this particular finding is a tax exporting motive driving the choices of local voters (and governments), which seems to be less likely for cordon tolls, since they are generally administered by government levels that represent larger portions of the population, including commuters coming from the hinterland. Moreover, due to the substitutability between the two charges, the fact that high parking charges are levied in the CBD would further discourage the regional government from implementing congestion tolls.

To be sure, the above findings (and the driving forces behind them) are not specific to our majority voting setup and would also hold in models with purely welfare maximising governments (they are indeed in line with those of Proost and Sen (2006), who consider a similar insititutional setup to ours, but with no voting). However, they are not the only ones shaping governmental behavior in our setup: unlike previous works in the literature, governments here also respond to voters' heterogeneous preferences for driving. In particular, as we will see later, our findings suggest that the presence of tax exporting motives do not mean that the local government will necessarily set a parking charge that is high neither in absolute terms nor compared to the external costs of car trips: if the population consists of sufficiently frequent drivers, as we argue in the following section, t_C^{NE} while still likely to be higher than t_R^{NE}, may still be quite low (at least compared to the marginal external costs of a car trip).

5.2 Comparison of the equilibrium of the voting game to the first best policy vector

To conclude this part, we compare $t^{NE} = t_C^{NE} + t_R^{NE}$ to the first best policy tax t^{FB}. Once again, the inability to identify the Condorcet Winner on t_R^{NE} does not allow us to give a complete comparison between benchmark and equilibrium vectors. However, we can give sufficient conditions for t^{NE} to lie below t^{FB}, as well as necessary conditions for the opposite to happen:

PROPOSITION 4: *The equilibrium of the voting game* $\Pi^{NE}(t_C^{NE}, t_R^{NE})$ *is such that :*

- $\hat{r} > \bar{r}\left(1 + \frac{1-\lambda}{1+\frac{dQ}{dp}(p^{FB})}\right) > \frac{\bar{r}}{\lambda} \Rightarrow t^{NE} < t^{FB}$
- $t^{NE} \geq t^{FB} \Longrightarrow \hat{r} \leq \bar{r}\left(1 + \frac{1-\lambda}{1+\frac{dQ}{dp}(p^{FB})}\right)$

What this implies is that if the populations considered are such that a majority of individuals has preferences for driving that are sufficiently stronger than average, we may expect to end up with suboptimal level of total traffic charges, and, on the contrary, if the majority has sufficiently weaker preferences than the average, the opposite may happen.

The role of tax exporting and competition among governments

Previous literature has taught us that the imperfect coordination among the two governments in setting traffic policy (particularly in the setting of charges for the use of the road infrastructure) can have important implications for the policy outcome. In a setting like ours, where two overlapping governments can charge for access to the same piece of infrastructure, we could expect, at least to some extent, their imperfect coordination to determine an increase in the total level of charges, with respect to the case of a single government[16]. The above results suggest that such an effect can be relevant in our model. Suppose for a moment that the individual who is pivotal for the city population (r^d_C) were decisive in the voting on the entire π: then we could expect t^{NE}_R to be infinitely close to zero, for the reason that t_R and t_C would be, from her perspective, two equivalent instruments to implement her most preferred allocation of resources, except for the fact that raising the former implies a lower net monetary loss than the latter: therefore,r^d_C would always strictly prefer to use only the former and not the latter. In such a situation, it would be as if the two governments were perfectly coordinating their choices and the sufficient condition to have a suboptimal total charge would, indeed, be the same as the one that would be relevant if only one government were in charge of the whole set of policy parameters (that is $\hat{r} > \frac{\bar{r}}{\lambda}$, cfr. Proposition 2).

However, we cannot be sure that r^d_C will be decisive when voting on t_R. Suppose instead that the Condorcet Winnner in the voting on t_R coincided with the most preferred t_R for an individual living in the hinterland. He would vote on it without taking into account the impact on the revenues from t_C. This, as previously argued, would induce him to prefer a level of t_R higher than that most preferred by an individual of identical type r but living inside the city. We thus have a source of imperfect coordination between the two subpopulations and, ultimately, the two governments. This can lead to an equilibirum entailing higher total taxes than if only one population were voting on the whole policy vector, *coeteris paribus*. Indeed, in the two government case, the sufficient conditon for having a suboptimal t^{NE} (given in Proposition 4), when two different governments are involved, requires the difference between $\hat{r}$ and $\bar{r}$ to be larger than in the case of a single government.

[16] Similarly to the setup of Proost and Sen (2006). The authors find that the imperfect coordination between a city government setting parking charges and a regional one setting a cordon toll leads to vertical tax competition and increases overall charges. The extent of this effect, the authors argue, is however limited, due to the fact that the regional government sets its own charge taking into account the welfare of the city individuals as well.

These results suggest that, even in the presence of imperfect coordination between governments and of tax exporting motives for a local government, the total level of charges on car usage may be too low with respect to the social optimum. While neither tax exporting behavior by local governments nor the effects of imperfect coordination among two taxing governments are new to the literature, this latter finding, determined by voters' preferences for car driving, comes from one of the novelties of our approach: that of studying the behavior of a regional and a local government, imperfectly coordinating in setting traffic congestion policy, while also being deomcratically elected and having to respond to the will of heterogenous voters.

6 Conclusions

We have studied how the political process may determine the shape of traffic congestion policy when two overlapping and democratically elected governments are involved. The non-cooperative interaction between different levels of government (one of which represents only a subset of the total population), may lead to higher charges than with a single government involved and the possibility of exploiting people outside the government's jurisdiction to generate revenues generates an important tax exporting mechanism. However, as long as the majority of individuals in the population has sufficiently strong preferences for driving, total charges will still be suboptimal: a novel result proposed by this paper is that a distribution of preferences for using cars among the population such that the majority of individuals drives more frequently than the average (which may, we have argued, not be inconsistent with a significantly widespread "car dependence" phenomenon observed in many cities), can determine insufficient political support for traffic charges. While the last two driving forces mentioned above are not specific to our setup and would also characterize models of purely welfare maximising governments, an important novelty of our approach is that of combining the three in order to give some insights on the way governments act when setting traffic congestion policy.

Our results suggest that the effects of the lack of coordination among governments and of their divergent objectives may be less harmful to social welfare than suggested in previous literature. A common insight of preceding studies on the effects of governmental competition in traffic policy is that its effects are harmful since it leads to generally to high levels of taxation and charges on the use of road infrastructure, although to different extents depending on the institutional setup. However, these works importantly neglected the fact that governments do not, in general, simply act as welfare maximisers (though not coordinating) for the respective populations, but respond to the preferences of voters. Now one of the main novelties of the paper is to embed the effects of governmental competition in a majority voting framework. Indeed, we find that, in the presence of significant political opposition to the use of pricing instruments (a quite relevant phenomenon in reality), the higher total tax level induced by governmental competition may actually help mitigate the bias from the first best policy.

The results obtained rest on some important assumptions: first of all, our institutional setup is crucially linked to the forces shaping congestion policy in the model. Although relatively common, the setup is clearly not universally valid: for instance, in the city of Milan, both parking and a (recently implemented) cordon toll are under the control of the city government: we provide some insights about such situations in Section 3. Moreover, we assumed that the local government is entitled to keep the revenues from parking charges, while, in reality, the charge revenues may benefit also individuals that do not belong to the city's polity. Nevertheless, it seems fair to say that, while parking policy is more likely to characterise itself as a strictly local policy, debates about (and political consultations on) congestion tolls have often assumed, geographically, a much broader connotation: therefore, even if it is ultimately a local government running all of congestion policy, the use of some of its instruments is more likely to be subject to pressures from outside government levels than others. Secondly, we have assumed that both in the city and in the regional population, all individuals have to pay both charges, which may not always be the case in reality: for instance, we did not consider the possibility of "resident discounts" schemes, nor the issue of privately (or employer) provided parking. Third, we have assumed a monocentric city model, with all traffic flows going in the same direction (towards the CBD), neglecting the phenomenon of "multicentric" cities. Fourth, we have assumed, for simplicity, that revenues from the charges are redistributed using uniform lump sum transfers: this may not necessarily be the case in reality. Finally, we have neglected the possiblity of having different distributions of preferences characterizing the city and the hinterland population. All these would be an interesting questions to extend the study for the future.

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Appendix

Proof of Proposition 1

We have, after substituting for $L = tQ$ from the government's budget constraint,

$$W(p) = \int_{r^l}^{r^u} V(p;r)dF(r) = \frac{\bar{r}}{(p + T(Q))} + M + tQ$$

that the government maximizes with respect to t. Taking first order derivative

$$\partial W/\partial t: \quad -Q\left[1 + T_Q \tfrac{dQ}{dp}\right] + Q + t\tfrac{dQ}{dp} \tag{15}$$

equating the expression to zero one obtains $t = T_Q Q$ as in the text. It is easy to see that, taking the second order derivative of $W(p)$ and imposing $t = T_Q Q$, we obtain a negative value: all points satisfying the FOC above are local maximisers of $W(p)$. Therefore, it has to be the case that there is only one of them: otherwise, given that $W(p)$ is continuously differentiable in all its domain, there would be at least one other point satisfying the FOCs being a local minimizer.

Proof of Lemma 1

To prove that preferences over t satisfy the Single Crossing condition, we use the result of Gans and Smart (1996): they prove that such a condition is satisfied if and only if

$$\partial(MRS_{tL}(r))/\partial r = \partial\left(\frac{\partial V}{\partial t} / \frac{\partial V}{\partial L}\right)/\partial r$$

has a constant sign, for all values of t, L and r. The expression has the following form:

$$\partial(MRS_{tL}(r))/\partial r = -\frac{\partial q(r)}{\partial r}[1 + T_Q \frac{dQ}{dp}]$$

which, since $\frac{\partial q(p;r)}{\partial r} > 0$ and $1 + T_Q \frac{dQ}{dp} > 0$ for all t and r, is strictly negative, for all t, L and r. Therefore, the Single Crossing condition holds. As a consequence, we can claim that a voting equilibrium exists and coincides with $t^*(\lambda; \hat{r})$, as claimed in the text.

Proof of Lemma 2

We use the Implicit Function Theorem. Condition (8) can be seen as a function of t^E, r^d and λ:

$$-q(\hat{r})[1 + T_Q \frac{dQ}{dp}] + \frac{Q + t\frac{dQ}{dp}}{\lambda} = F(t^E, \hat{r}, \lambda) = 0$$

By the IFT, we know that, in a neighbourhood of t^E satisfying the above conditions, we can express t^E as functions of $\hat{r}$ and λ. The IFT tells us that

$$\frac{\partial t}{\partial \hat{r}} = -\frac{\frac{\partial F}{\partial \hat{r}}}{\frac{\partial F}{\partial t}} \qquad \frac{\partial t}{\partial \lambda} = -\frac{\frac{\partial F}{\partial \lambda}}{\frac{\partial F}{\partial t}}$$

it is easy to see that the numerator is negative in both expressions. As for the denominator, its value is negative because of second order conditions, always verified[17]), for all values of $\hat{r}$. Therefore, t^E at

[17]The second order condition writes as

$$\frac{\partial^2 V}{\partial t^2} = -\frac{dq}{dp}\left[1 + T_Q \frac{dQ}{dp}\right] - q\left[T_Q \frac{d^2 Q}{dp^2}\right] + \frac{2\frac{dQ}{dp} + t\frac{d^2 Q}{dp^2}}{\lambda}$$

equilibrium is decreasing in $\hat{r}$ and in λ.

Proof of Proposition 2

To prove the first part of the proposition, simply compare condition (6) defining t^{FB}, to (8) that defining t^E. They coincide, by the linearity of $q(r)$ in r, if and only if $\frac{\bar{r}}{\lambda} = \hat{r}$. It is only in that case, then, that we have that t^E coincides with the first best one t^{FB}. The rest of the claim follows from Lemma 2.

Proof of Lemma 3

When the city population votes on t_C, we have, following the same steps as in the proof for Lemma 1, that a voting equilibrium exists since individuals' preferences for t_C satisfy the Single Crossing condtion. The equilibrium of the voting procedure involving the city government, for any t_R, coincides with the most preferred policy of the median individual in the city $t_C^E = t_C^*(t_R; \lambda, \hat{r})$.

To prove that t_C^E is unique, we simply maximise, for any value of t_R, the $\hat{r}$ individual's reduced indirect utitlity $V(p, \hat{r})$, with respect to t_C. The FOC is

$$-q(\hat{r})[1 + T_Q \tfrac{dQ}{dp}] + \frac{Q(p) + t_C \frac{dQ}{dp}}{\lambda} + t_R \tfrac{dQ}{dp} = 0$$

now, every value t_C satisyfing the FOC above is such that, in its neighbourhood, the second order derivative $\frac{\partial^2 V}{\partial t_C^2}$ is negative:

$$\frac{\partial^2 V}{\partial t_C^2} = -\frac{dq}{dp}\left[1 + T_Q \frac{dQ}{dp}\right] - q\left[T_Q \frac{d^2 Q}{dp^2}\right] + \frac{2\frac{dQ}{dp} + t_C \frac{d^2 Q}{dp^2}}{\lambda} + t_R \frac{d^2 Q}{dp^2}$$

which evaluated at the FOC above, is negative (the condition is the same as that for the footnote above). As a consequence, since the function $V(p, \hat{r})$ is continuously differentiable and concave in the neighbourhood of any point t_C satisfying the FOC, it has to be the case not only that this point is a local maximiser, but

which, evaluated at t satisfying the FOC above is negative as long as

$$\frac{T}{T+p} > \frac{1}{2}\left(\frac{\hat{r}\lambda}{\bar{r}2} - 1\right) - \frac{3}{8\bar{r}}\left(\frac{\lambda\hat{r}}{\bar{r}} - 1\right)$$

which, since necessarily $\hat{r} \leq 2\bar{r}$, is such that the right hand side is always negative and is therefore always verified.

that it is the only one.

Proof of Lemma 4

Write conditions implictly defining the Best Response vector $t_C^{NE}(t_R)$ as

$$-q(\hat{r})[1+T_Q\frac{dQ}{dp}]+\frac{Q+t_C\frac{dQ}{dp}}{\lambda}+t_R\frac{dQ}{dp} \quad = F_1(t_C,t_R,\hat{r},\lambda) = \quad 0$$

by the Implicit Function Theorem, we know that, in a neighbourhood of $t_C^{NE}(t_R)$ satisfying the above conditions, we can express t_C as function of $\hat{r}$, λ and t_R. The IFT tells us that

$$\frac{\partial t_C}{\partial t_R} = -\frac{\frac{\partial F_1}{\partial t_R}}{\frac{\partial F_1}{\partial t_C}}$$

The denominator is negative (by SOC, as discussed in the previous Lemma). As for the numerator, we have

$$\frac{\partial F_1}{\partial t_R} = -\frac{dq}{dp}\left[1+T_Q\frac{dQ}{dp}\right]-q\left[T_Q\frac{d^2Q}{dp^2}\right]+\frac{\frac{dQ}{dp}+t_C\frac{d^2Q}{dp^2}}{\lambda}+t_R\frac{d^2Q}{dp^2}+\frac{dQ}{dp}$$

which is negative as long as

$$\frac{T}{T+p} > \frac{1}{2}\left(\frac{\hat{r}\lambda}{\bar{r}2}-1\right)-\frac{3}{4\left(\lambda+1\right)\bar{r}}\left(\frac{\lambda\hat{r}}{\bar{r}}-1\right)$$

which is always verified as long as $\bar{r} \geq 3/2$: this is simply a scaling condition that we caan always assume to be verified without loss of generality. Moreover, since $\frac{\partial F_1}{\partial t_R}$ is negative but differs from $\frac{\partial F_1}{\partial t_C}$ by $\frac{dQ}{dp}(\frac{1}{\lambda}-1)$, which is negative, it has to be the case that, in absolute value, the numerator is smaller than the numerator. Therefore, impicitly defined derviatives in the neighborhood of the equilibrium vector, satisfy the inequalities claimed in the text. As for $\frac{\partial t_C}{\partial r}$ and $\frac{\partial t_C}{\partial \lambda}$, the proof goes along the same lines.

Proof of Lemma 5

Let us prove, first of all, that the preferences of subgroup C (city) and H (hinterland) in the population, taken separately, satisfy the Single Crossing condition when voting on t_R, while they do not satisfy it when considered jointly. Consider an individual belonging to C: the objective function, for a type-r individual, is

$$V(p;r) = \frac{r}{(p+T(Q))} + M + \frac{t_C}{\lambda}Q + L_R$$

while for an individual belonging to H is

$$V(p;r) = \frac{r}{(p+T(Q))} + M + L_R$$

We have that, for any given t_C

$$MRS_{t_R L}(r) = \tfrac{\partial V}{\partial t_R} / \tfrac{\partial V}{\partial L_R} = -q(r)[1 + T_Q \frac{dQ}{dp}] + i\left(\frac{t_C}{\lambda}\frac{dQ}{dp}\right)$$

with $i = 1$ for the individual living in the city and $i = 0$ otherwise. So

$$\partial\big(MRS_{t_R L}(r)\big)/\partial r = -\frac{\partial q(r)}{\partial r}[1 + T_Q \frac{dQ}{dp}]$$

which is strictly negative, for any initial t_R. Therefore, as argued by Gans and Smart (1996), preferences over t_R in the two subgroups satisfy the SC property. This implies that, in the hypothetical case that the C were the only subgroup involved in the voting on t_R, then a majority voting equilibrium would exist and would coincide with the most preferred t_R of the individual with median r among C, $\hat{r}$: we denote this value by $t_R^*(\hat{r}, C)$. Similarly, if group H was the only one allowed to vote on t_R, a majority voting equilibrium would exist and would coincide with $t_R^*(\hat{r}, H)$.

Looking at the expression above, we see that the $MRS_{t_R L}$ is generally nonmonotonic with respect to r when the whole population is considered, as for any t_C and t_R, since $MRS_{t_R L}(r)^H > MRS_{t_R L}(r)^C$. Therefore, the Single Crossing condition does not hold when both subgroups vote on t_R. Moreover, an individual in the city of given preference parameter r, always prefers a (weakly) lower t_R than an individual with same r living in the hinterland. This also implies that we can rank the two most preferred values for the two individuals that are pivotal in each subgroup, for any given t_C: $t_R^*(\hat{r}, C) \leq t_R^*(\hat{r}, H)$.

Finally, since SC holds in the two subgroups, it is not possible that the CW lies outside $[t_R^*(\hat{r}, C), t_R^*(\hat{r}, H)]$.

Suppose (by absurd) that the CW is on the left of this interval: then surely at least half of the city population would strictly prefer any value inside the interval to it. A fortiori, at least half the population in the hinterland would strictly prefer values in the interval to it. Therefore, the CW cannot lie to the left of the interval. A similar reasoning shows it cannot lie to the right either.

Proof of Proposition 3

We proceed, for the moment, assuming the condorcet winner for t_R coincides with $t_R^*(\hat{r}, H)$. In this case, we provide first of all a lemma stating that the equilibrium vector Π^{NE} would be unique.

> LEMMA 1A: *When Π is chosen by majority voting, under a Shepsle voting procedure, assuming the condorcet winner for t_R coincides with $t_R^*(\hat{r}, H)$*
>
> - *We can (implicitly) define a best response function $t_R^{BR}(t_C)$, such that $\frac{\partial t_R^{BR}}{\partial t_C} < 0$*
> - *There exists a unique equlibrium vector $\Pi^{NE}(t_C^{NE}, t_R^{NE})$*

PROOF: In the case the condorcet winner $t_R^{BR}(t_C)$ in the regional voting coincides with the most preferred $t_R^*(\hat{r}, H)$ for the individual decisive among the commuters (as defined for Lemma 5), his best response t_R^{BR} is unique. This value has to satisfy the FOC

$$-q(\hat{r})[1 + T_Q \frac{dQ}{dp}] + t_R \frac{dQ}{dp} + Q \equiv F(t_R, t_C, \hat{r}, \lambda) = 0$$

and we can easily show that any $t_R^{BR}(t_C)$ satisfying the above, also satisfies the SOC. Uniqueness follows. As for $\frac{\partial t_R^{BR}}{\partial t_C} < 0$, we can use the implicit function theorem:

$$\frac{\partial t_R^{BR}}{\partial t_C} = -\frac{\frac{\partial F}{\partial t_C}}{\frac{\partial F}{\partial t_R}}$$

the denominator is negative, by SOC. As for the numerator, we have

$$\frac{\partial F}{\partial t_C} = -\frac{dq}{dp}\left[1 + T_Q \frac{dQ}{dp}\right] - q\left[T_Q \frac{d^2Q}{dp^2}\right] + t_R \frac{d^2Q}{dp^2} + \frac{dQ}{dp}$$

which, evaluated at t_R satisfying the FOC above (the relvant neighbourhood we need to focus on), is negative if

$$\frac{T}{T+p} > \frac{1}{2}\left(\frac{\hat{r}}{\bar{r}2}-1\right)-\frac{3}{4\bar{r}}\left(\frac{\hat{r}}{\bar{r}}-1\right)$$

which is always verified as long as $\bar{r} \geq 3/2$, a scaling condition that we can always assume to be verified without loss of generality.

Next, we prove that the equilibrium vector Π^{NE} is unique. We know that, if the CW in the regional voting coincides with $t_R^*(\hat{r}, H)$, Π^{NE} would be implicitly defined by the following conditions holding simultaneously

$$\begin{aligned} -q(\hat{r})[1+T_Q\frac{dQ}{dp}]+\frac{Q+t_C\frac{dQ}{dp}}{\lambda}+t_R\frac{dQ}{dp} \qquad &\equiv F_1(t_R,t_C,\hat{r},\lambda)=0 \\ -q(\hat{r})[1+T_Q\frac{dQ}{dp}]+t_R\frac{dQ}{dp} \quad &+Q \equiv F_2(t_R,t_C,\hat{r},\lambda)=0 \end{aligned}$$

Let us look at $F_1(t_R,t_C,\hat{r},\lambda)=0$ and $F_2(t_R,t_C,\hat{r},\lambda)=0$ as functions implictly defining, in a neighbourhood of the equilibrium vector Π^{NE}, for given $\hat{r}$ andλ, t_R as function of t_C. We get, evaluating derivatives at points (t_R,t_C) satisfing F_1 and F_2 above:

$$\frac{\partial t_R^{NE}}{\partial t_C^{NE}}_{F1} = -\frac{\partial F1/\partial t_C}{\partial F1/\partial t_R} = -\frac{-\frac{dq}{dp}\left[1+T_Q\frac{dQ}{dp}\right]-q\left[T_Q\frac{d^2Q}{dp^2}\right]+\frac{2\frac{dQ}{dp}+t_C\frac{d^2Q}{dp^2}}{\lambda}+t_R\frac{d^2Q}{dp^2}}{-\frac{dq}{dp}\left[1+T_Q\frac{dQ}{dp}\right]-q\left[T_Q\frac{d^2Q}{dp^2}\right]+\frac{\frac{dQ}{dp}+t_C\frac{d^2Q}{dp^2}}{\lambda}+t_R\frac{d^2Q}{dp^2}+\frac{dQ}{dp}}\epsilon\,(-\infty;-1)$$

$$\frac{\partial t_R^{NE}}{\partial t_C^{NE}}_{F2} = -\frac{\partial F2/\partial t_C}{\partial F2/\partial t_R} = -\frac{-\frac{dq}{dp}\left[1+T_Q\frac{dQ}{dp}\right]-q\left[T_Q\frac{d^2Q}{dp^2}\right]+t_R\frac{d^2Q}{dp^2}+\frac{dQ}{dp}}{-\frac{dq}{dp}\left[1+T_Q\frac{dQ}{dp}\right]-q\left[T_Q\frac{d^2Q}{dp^2}\right]+t_R\frac{d^2Q}{dp^2}+2\frac{dQ}{dp}}\epsilon\,(-1;0)$$

Both derivatives are strictly negative: this tells us each of these functions define, in a neighbourhood of each point (t_R,t_C) satisfying them, t_R as decreasing function of t_C. Now, all candidate equilibiurm points Π^{NE} are such that both these functions cross on the (t_R,t_C) plane: if we can prove that they cross only once, we can be sure of uniqueness of Π^{NE}. Since we have (at the points satisfying both F_1 and F_2) that

$$\frac{\partial t_R^{NE}}{\partial t_C^{NE}}_{F1} < \frac{\partial t_R^{NE}}{\partial t_C^{NE}}_{F2} < 0$$

at any such point, it has to be the case that the $t_R^{BR}(t_C)$ from the first function crosses that from the second one only from above, at any equilibirum point. Since both functions are continuous, it has to be the case that the crossing is unique and, therefore, the couple Π^{NE} has to be unique.

—END OF PROOF

Let us continue assuming that, in the equilirium the CW for t_R coincides with $t_R^*(\hat{r}, H)$. Now, equilibrium conditions describing Π^{NE} are

$$\begin{aligned} -q(\hat{r})[1 + T_Q \frac{dQ}{dp}] + \frac{Q + t_C \frac{dQ}{dp}}{\lambda} + t_R \frac{dQ}{dp} &= 0 \\ -q(\hat{r})[1 + T_Q \frac{dQ}{dp}] + t_R \frac{dQ}{dp} + Q &= 0 \end{aligned}$$

Let us focus on the limit case in which $\lambda \to 0$. In that case, the first equation gives

$$Q = -t_C \frac{dQ}{dp}$$

which we can plug into the second equation and rewrite it as

$$(t_R - t_C) \frac{dQ}{dp} = q(\boldsymbol{\pi}; \hat{r})[1 + T_Q \frac{dQ}{dp}]$$

now, the right hand side of this expression is positive. Therefore, at equilibrium, when $\lambda \to 0$, we must have $t_C > t_R$. Since, if t_R coincides with $t_R^*(\hat{r}, H)$, it is increasing and t_C decreasing in λ (—proof on paper—), a value $\tilde{\lambda}$ such that $t_C^{NE} = t_R^{NE}$, given the above findings, must be unique, as long as it exists. The relation describing $\tilde{\lambda}$ can be found using the above expressions and imposing the condition $t_C = t_R = t/2$. This leads, isolating λ, to the following expression

$$\lambda = \frac{\hat{r}}{\bar{r}} \left(\frac{p + T}{p + 3T} \right)$$

At equilibrium, the right hand side is a function of λ itself (T depends ultimately on p, and p is a function of λ). However, the function $z(\lambda) = \lambda - \frac{\hat{r}}{\bar{r}} \left(\frac{p+T}{p+3T} \right)$ is strictly increasing in λ (this requires using the fact that, at equilibrium, $\frac{dp}{d\lambda} = \frac{dp}{dt} \frac{dt}{d\lambda} < 0$, which is proved below). We have $z'(\lambda) = 1 - \frac{\hat{r}}{\bar{r}} \left(\frac{2T - 2pT_Q \frac{dQ}{dp}}{(p+3T)^2} \right) \frac{dt}{d\lambda} > 0$.

Therefore, there can be at most one value of $\tilde{\lambda}$ satisfying the above expression. It may be possible that this value is out of the interval $(0,1)$: in that case, we will have either $\tilde{\lambda}=0$ or $\tilde{\lambda}=1$ as corner solutions.

Now, we know that the equilibrim t_R has to lie in the interval $[t_R^*(\hat{r},C),t_R^*(\hat{r},H)]$. Then we can conclude that if $\lambda \leq \tilde{\lambda}$, surely $t_C \geq t_R$: if the CW coincided with any other value among $[t_R^*(\hat{r},C),t_R^*(\hat{r},H)]$, we would have an even lower t_R and a higher t_C, so $t_C \geq t_R$ would hold a fortiori[18].

Finally, when $\lambda > \tilde{\lambda}$, then $t_R > t_C$ if the CW coincides with $t_R^*(\hat{r},H)$. By continuity, the inequality will hold also if the CW coincides with the most preferred values of t_R for individuals living in the hinterland whose preferences are sufficiently close (but stronger) to $\hat{r}$. However, we cannot be sure that this will always be the case: the CW may coincide with the most preferred choice of t_R for individuals, living either in the city or in the hinterland, whose attitudes are sufficiently strong against it to determine a reversal of the inequality (this would be, for example, the case if the CW coincided with $t_R^*(\hat{r},C)$). Therefore, $\lambda > \tilde{\lambda}$ is a necessary, not sufficient, condition, to have $t_R > t_C$ in equilibrium.

Proof of Proposition 4

First of all, we need to establish that, assuming t_R coincides with $t_R^*(\hat{r},H)$, the equilibrium policy would be such that t is strictly decreasing with $\hat{r}$ and λ. The conditions defining the equilibrium in this case are

$$\begin{aligned} -q(\hat{r})[1+T_Q\frac{dQ}{dp}]+\frac{Q+t_C\frac{dQ}{dp}}{\lambda}+t_R\frac{dQ}{dp} &= 0 \\ -q(\hat{r})[1+T_Q\frac{dQ}{dp}]+t_R\frac{dQ}{dp} + Q &= 0 \end{aligned}$$

next, substituting $-Q=-q(\hat{r})[1+T_Q\frac{dQ}{dp}]+t_R\frac{dQ}{dp}$ from equation 2 into 1 and multiplying both sides of the resulting expression by λ, and then adding it to 2 we obtain an expression that can replace equation 1 to

[18] Notice that for any other CW in $\left[t_R^*(\hat{r},C),t_R^*(\hat{r},H)\right]$, we do not draw any conclusion of the unicity of the equilibrium: what is important for us is that, in any event, the (possibly multiple) equilibria will be such that $t_R^{NE} < t_R^*(\hat{r},H)$ and lie on the part of the best response function $t_C^{BR}(t_R)$ that involves $t_C > t_C^{BR}(t_R^*(\hat{r},H))$

obtain the following equivalent system

$$
\begin{aligned}
-q(\hat{r})[1+T_Q\frac{dQ}{dp}]+(t_C+t_R)\frac{dQ}{dp}+(2-\lambda)Q \qquad &= F_1(t_R,t_C,\hat{r},\lambda)=0 \\
-q(\hat{r})[1+T_Q\frac{dQ}{dp}]+t_R\frac{dQ}{dp} \quad +Q &= F_2(t_R,t_C,\hat{r},\lambda)=0
\end{aligned}
$$

Importantly, the first expression contains only terms that are functions only the total charge t, not of its components. Since it must, however, always hold at the equilibrium policy vector, we can use the implicit function theorem to obtain that

$$\frac{\partial t}{\partial \hat{r}} = -\frac{\frac{\partial F_1}{\partial \hat{r}}}{\frac{\partial F_1}{\partial t}}$$

both numerator and denominator are negative: therefore the whole derivative also is. We can repeat the reasoning using λ as the independent variable, instead of $\hat{r}$, and obtain similar results.

Next, we prove that there exists a unique value $r^x \equiv \bar{r}\left(1+\frac{1-\lambda}{1+T_Q\frac{dQ}{dp}(p^{FB})}\right)$ such that if $\hat{r}=r^x$, then, assuming that t_R coincides with $t_R^*(\hat{r},M)$, $t^{NE}=t^{FB}$. Take condition F1 and add and substract the following

$$QT_Q\frac{dQ}{dp}$$

the equality we obtain implies (using the explicit formula for Q) that

$$\frac{[(2-\lambda)\bar{r}-\hat{r}]}{(p+T(Q))}+[Q-q(\hat{r})]\,T_Q\frac{dQ}{dp} \gtreqless 0 \Leftrightarrow [t-T_Q Q]\,\frac{dQ}{dp} \lesseqgtr 0 \Leftrightarrow [t-T_Q Q] \gtreqless 0$$

now since

$$h(t)=t-T_Q Q$$

is a strictly increasing function of t, then it has to be the case that it is equal to zero if and only if $t=T_Q Q$. In other words, we have

$$t=t^{FB} \Leftrightarrow \frac{[(2-\lambda)\bar{r}-\hat{r}]}{(p+T(Q^{FB}))}+\left[Q^{FB}-q(\hat{r})^{FB}\right]T_Q\frac{dQ}{dp}=0$$

which translates in the condition on parameters

$$\hat{r} = \bar{r}\left(1 + \frac{1-\lambda}{1 + T_Q \frac{dQ}{dp}(p^{FB})}\right)$$

Now, combining this result with $\frac{\partial t}{\partial \hat{r}} < 0$, we have that

$$p \gtreqless p^{FB} \Leftrightarrow \hat{r} \lesseqgtr \bar{r}\left(1 + \frac{1-\lambda}{1 + T_Q \frac{dQ}{dp}(p^{FB})}\right)$$

Since $t_R^*(\hat{r}, H)$ is an upper bound on the set of condorcet winners on t_R and since, as proven for Lemma 4

$$-1 \quad < \frac{\partial t_C^{BR}}{\partial t_R} \leq 0$$

we have the first part of the claim: as long a r is large enough, then the equilibrium t lies certainly below the First Best one, since, even if tr were not to coincide with $t_R^*(\hat{r}, M)$, it would be strictly lower than in the equilibrium obtained. But since the reaction function for $t_C^{BR}(t_R)$ has the properties derived above, the sum of the two charges cannot be higher and capacity cannot be lower than those obtained under the assumption.

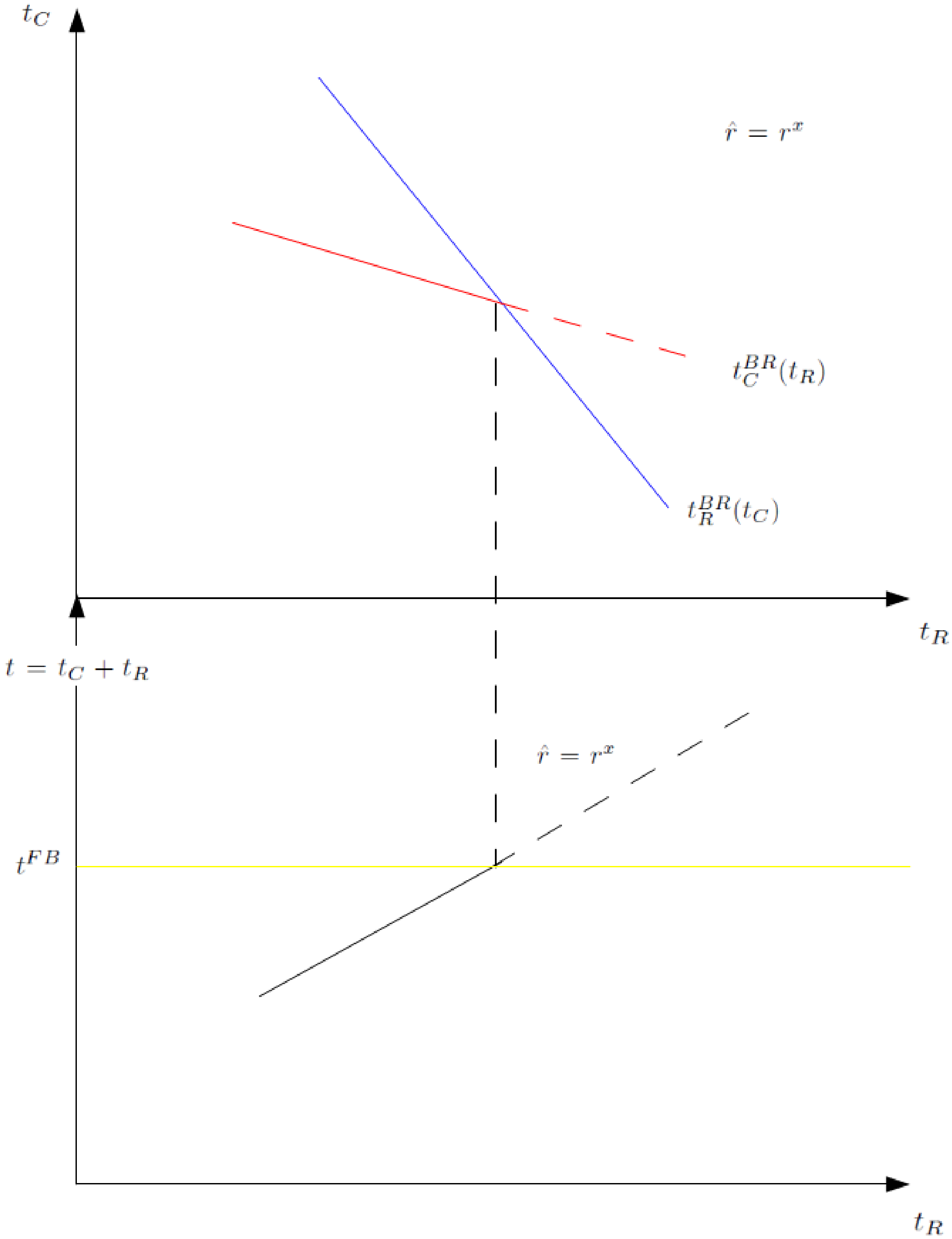

Figure 3:

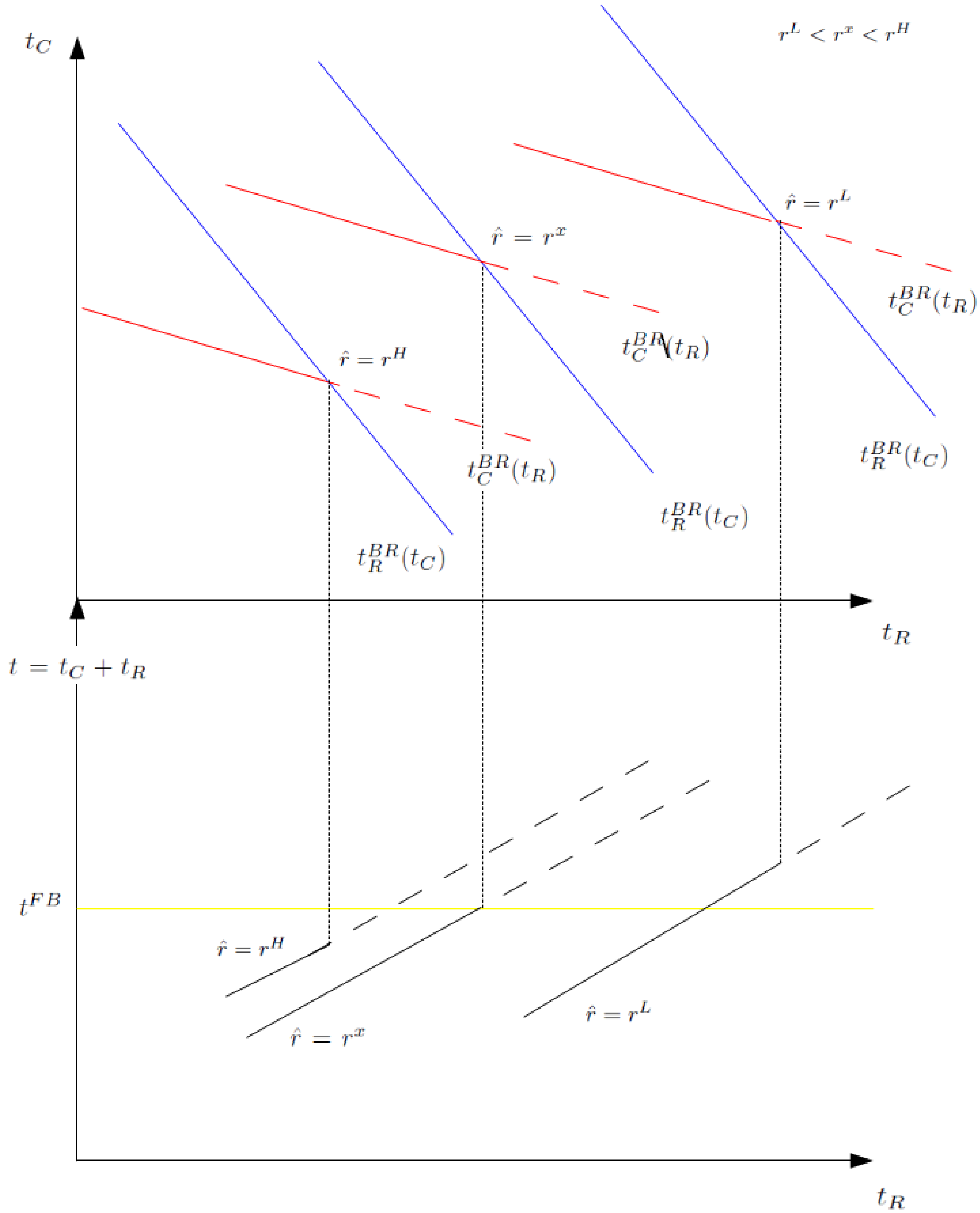
t_C
$r^L < r^x < r^H$
$\hat{r} = r^L$
$\hat{r} = r^x$
$\hat{r} = r^H$
$t_C^{BR}(t_R)$
$t_C^{BR}(t_R)$
$t_C^{BR}(t_R)$
$t_R^{BR}(t_C)$
$t_R^{BR}(t_C)$
$t_R^{BR}(t_C)$
t_R
$t = t_C + t_R$
t^{FB}
$\hat{r} = r^H$
$\hat{r} = r^x$
$\hat{r} = r^L$
t_R

Figure 4: