International Capital Flows: An empirical study of the relationship between equity and debt investments

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INTERNATIONAL CAPITAL FLOWS:
AN EMPIRICAL STUDY OF THE
RELATIONSHIP BETWEEN EQUITY
AND DEBT INVESTMENTS

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Capital flows in today's arena is a hot topic attracting many researchers to do some contributions. International capital flows are rapidly increasing day by day with huge increases in volume of trade in equity and debt markets. This piece of work is an effort and a kind of approach in finding out the cross relationship between equity and debt investments in capital markets and the investor's portfolio decisions between equity and debt investments in international financial markets. In this paper, I have studied intensively world equity market, world debt and bond market. The relationship between equity and debt investment has been computed using statistical methods like covariance and correlation with linear regression analysis with US market data of past 200 years. I examine the relationship between equity and debt and reach with a conclusion that there is an inverse relationship between equity and debt investment.

JEL Classification: F21, G11, G15


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# TABLE OF CONTENTS

1. Abstract ..................................................................................................................A

2. Introduction ........................................................................................................... 1

3. A Primer on World Equity and Debt Markets ....................................................... 2
   3.1 World Equity Markets ....................................................................................... 2
       3.1.1 Public Equity Versus Private Equity .......................................................... 3
   3.2 World Debt Markets ....................................................................................... 4
       3.2.1 Public Debts & External Debts ................................................................. 5
       3.2.2 Private Debts ......................................................................................... 7
       3.2.3 World Bond Markets ............................................................................. 8

   4.1 A brief literature review ................................................................................... 9
       4.1.1 Financing choice of firms ......................................................................... 9
       4.1.2 Investment choice of investors ................................................................. 9
       4.1.3 Risk - return trade off and debt –equity substitution & swap ................. 10
   4.2 Equity and debt: Risk and returns .................................................................. 10
       4.2.1 Equity market returns ............................................................................ 10
       4.2.2 Government bond and debt market returns ............................................. 11
       4.2.3 Market Risk ........................................................................................... 12
       4.2.4 Costs, capital structures and financing choices ....................................... 13
       4.2.5 Analysis of correlation between equity and debt investments ............... 13

5. Conclusion .............................................................................................................. 15

Reference ..................................................................................................................I - II

Appendix ..................................................................................................................III – XII

Additional data .......................................................................................................XIII - XIV
1. Introduction

The international capital flow has been a very interesting research topic over a couple of decades. The capital structure (equity, debt and FDI) of capital flow plays a major role both in primary and secondary markets. The relationship between debt investment and equity investment is quite important to find out its major impact on global economy and it is the main objective of this paper. My thesis makes an in-depth empirical investigation on the relationship existing between equity and debt investment. I analyze major economies, particularly international capital markets and look into the data of past 200 years of US market to calculate the coherent correlation between equity and debt investment concerned with their risks and returns.

First, my paper pays a depth study on world equity and debt market in order to study its trend and magnitude. Then, I focus on the equity and debt relation and find that their relationship is inverse. I use correlation and linear regression model to proof that the existing relationship between debt and equity investment is negative. Though, there are many factors responsible behind this relationship but risk is the most important factor that shapes the returns and so is their relationship.

The first part of the study is mainly focused on world debt and equity market. In equity market, I focus on private equity and public equity and their trends in current market economy, whereas, in debt market, I analyze public debt, private debt and external debt, and finally on the world bond market. In equity market, besides the study of the high equity and high return, I further look into private equity as it is tremendously increasing and the total net profit since inception through 2007 inception through 2007 were $1.12 trillion(Source: Perqine). In debt market, I examine on how too much of public debts burden lead to ‘more risks appetite’ and ‘bankruptcy’. I study the debit flows of the developing countries. Finally, I focus on bond market of emerging economies and illustrated briefly on world bond market. It is noted that the world bond market has increased 31% in 2008 and has decreased 43% in financial crisis between 2008 to 2009 subsequently.

The second part of this paper examines the relationship which is the main theme of this paper. I describe the debt and equity investment with market risk, return and cost, which play a major role in shaping their relationship. I develop correlation analysis along with linear regression analysis as the model and methodology of my research. I calculate the US market data of pasta 200 years. My study focus on equity return and treasury bill return and the risk premium. I finally find the correlation between common stock and risk premium (typically risk : $R_m-R_f$) is 0.97 shows a stronger positive relation (risk is inevitable and always consists with equity), whereas, treasury bill and risk premium see a stronger negative relationship resulting -0.74, as, there is almost little market risk (guaranteed by the US government) besides the default credit risk. But, some debt stocks (large share of debt stock invests in capital market) have also risks, provided that, their returns are co-parallel with the calculated risks and, in that case, the correlation will be positive. Throughout the paper, my main focus is to find out the relationship between equity and debt investment and it is inverse.
2. A Primer on World Equity and Debt Markets

The recent movement towards globalization of equity market adopted by rapid privatization of state-owned (public) enterprises of Europe, Latin America and Asia, and unprecedented explosion of free trade and exchange (liberalization) of markets among developed countries followed by other industrialized countries for understanding the significance of international diversification of funds and cross-border equity investment that gave rise the flow of funds from developed countries to developing countries. The benefits of global diversification has surely provided an incentive to utilize the capital for investment across national borders and outside national borders whereas the capital requirement by developing countries have the same effects in helping the growth of globalized capital markets. The barriers between investing nations and fund seeking nations have been decreased over time as information and communication technology (ICT) develops and the capital controls draw away due to well understanding and improvement in trade and diplomatic relationships among countries. This part of my paper highlights the significance of both equity and debt markets for the growth of an economy and examines some important aspects (types) of equity including bonds in the investment world.

2.1 World Equity Markets

In today’s world, equity markets play a prominent role in driving the growth of economy. The need of an efficient equity market is being felt by an emerging economy. The main reason behind the huge capital flows is the significance rise in trade volume of both equity and debt investments in the current world capital markets. Both equity and debt investments take their own shapes in capital markets as a result of investment decision and issuance decision of investors and companies respectively. Investors always want to diversify its investment portfolio (both equities and debts) in order to gain substantial profits out of its investments. The rise in capital movements both in short run and long run have significantly changed the growth of market and economy as a whole. IMF data indicates that the gross capital flow between industrialized countries has expanded to 300 percent between 1991 to 2000. However, Equity always contributes a major portions to the gross capital flows while debt has a lower contributions.

The increasing trade openness among countries is allowing the world equity markets to integrate financially with the world capital markets. Stock markets across the world are becoming larger both in volumes and sizes and the world economy is getting more liquidity. Though the equity returns are more volatile in compare to debt returns but we can see a strong correlation among capital markets across the world. Equity markets aim at long-term growth support to both public and private enterprises consisting of both high risks and high returns. It also provides supports in economic reforms as such corporate governance.

With the current economic crisis, World global equity market fell to 47% in 2008 ($32.6 trillion) and IPOs fell by more than a half and global M&A volumes declined 30% (Source: IFL Equity Research 2009).

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2. Discussed in latter part of this paper 'Relationship between Equity and Debt: Is it Inverse? Why??'.
3. IMF World Economic and Financial Survey.
2.1.1 Public Equity Versus Private Equity

We can classify equity in two types, i.e., public equity and private equity. Public equity has been a major source of capital supply to the public institutions that they have been raising a sizable portions of their funding requirements through this equity markets since a long time. In brief, we can say, that public equity being a major portion of their of funding, it helps to grow their business as well as business expansions from one part of the world to other parts. Private equity has also its own recognitions among private sector companies, institutional investors and capital markets and it has been proven as a powerful investment engine in creating economic value to the economic society as a whole.

Public equity, as we can define, the equity capital that is generally raised through Initial Public Offerings(IPOs) with a load of corporate governances, rules and regulations of the concerned stock exchanges, corporate institutions and government body. Whereas private equity can involve the mergers and acquisitions(M & As) of private entities with an intention to grow the company to the higher level that it could not achieve at the present level due to financial sickness or/and lack of corporate governances. It may also involve the private investments or funding through venture capitals that the founders of the company need in order to expand the business or unlock the illness of financial instability. A privatizing of public company in order to achieve its short term goals may also be called private equity investment. Private equity is often called ‘Active Ownership’ as it does not trade publicly on stock exchange and more evident lies with intangible assets. A recent study by Private Equity Council found that there is a growing demand of public to private transactions and private equity firms are looking out companies in which they can bring out a significant value of change by investing new capital(private), business strategy or introducing a new leadership talent. As costs and returns are concerned, we can better compare between the both two. Cost is something that company has to bear in order to attract investors and initiate to stay with them and invest with while returns come with, in form of dividends on shares and change in market value of shares. As always high returns are associated with high risks, private equities have always have higher returns and long run prospective. The total net profits distributed to investors worldwide by private equity funds raised since inception through 2007 were $1.12 trillion(Source: Perqine).

Figure 1.1, Source: Private Equity Council, 2009.
Private equity funds have little debts or no debts. The systematic risk is generally higher compare to public equity as it is not deeply interconnected with other financial markets that can help to trigger the financial losses or can veil to cover any systematic risks.

The above chart shows that private equity fund raised $341bn compared to $129bn in 1998. It also shows that equity capital is continuously growing up every year without any economic policies intervention or market pressure.

![Pie chart showing sources of cash flow growth for global private equity investment.](image)

**Figure 1.2. Source: Private Equity Council, 2009.**

Many corporate organizations unveil that the ownership changes from public to private have been a great benefit, favorable and growth initiative. Jon Luther, CEO of Dunkin’ Brands recently told the U.S. House of Representatives Financial Services Committee, “The benefits of our new ownership to our company have been enormous. Their financial expertise led to a ground-breaking securitization deal that resulted in very favorable financing at favorable interest rates. This has enabled us to make significant investments in our infrastructure and our growth initiatives. They have opened the door to opportunities that were previously beyond our reach.”

Private Investment in Public Equity(PIPE) is that investors purchase shares through private placements. This type of private equity investment is expanding and represents approximately eight per cent of the gross proceeds of Seasoned Equity Offerings(SEOs) in the united states in 2000, according to Chaplinsky and Haushalter (2003). To reduce the issue costs and times are the main aim of this type of private equity investment (Ferreira and Brooks 2000; Anson 2001). Major financial services firms are generally the PIPE investors and the companies look for the liquidity structure of the company before investing.

### 2.2 World Debt Markets
Debt markets like equity markets play a major role in growth and development of the economy, specially the developing nations or third world countries. Debt is something that allows government,
institutions (organizations) and individuals to do things, otherwise, they may not able to, or allowed to do. In modern financial markets, debt markets have been made more institutional and the institutional players are the major contributors in growing up the debts markets by both purchasing and issuing the debts instruments. However, it is considered that too much of debts that lead the economy to ‘more unproductive’ and it will create ‘more risks appetite’ and even can lead the country to ‘bankruptcy’. Debts are often termed as a mutual elements of inflations or deflations that can help the economy in destabilizing the price level in consequence the country may suffer higher at exchange markets as the debts are highly unlikely to be defaulted by the economic entities. So, if the economic agents are highly indebted, then it can certainly threat the economy towards downsize as the high debt/GDP ratio will have high expectation on future returns and can create a bubble in stock markets.

There are various types of debts. I discussed here the study of public debt or sovereign debt and private debt. I also focused on bond markets that is a part of debt investments.

### 2.2.1 Public Debts & External Debts

Public debt, in economic term, is government debt or national debt that a country incurs in order to meet its short term and long term national budget. Public debt, as we can say, it is an indirect debt to a country’s tax payers. Public debt can be of both internal and external, internal when a government owes to its lenders within its country, while a country owes outside can be termed as external debt.

External debt is a support to a country to mobilize its domestic resources. However, excess external debt can be a cause of failure in economic stability and have several adverse effects in a country’s growth. While International organizations often urged that a country should maintain its sustainable debt level but it is very hard to achieve. World bank and IMF hold that ‘a country can be said to achieve external debt sustainability if it can meet its current and future external debt service obligations in full, without recourse to debt rescheduling or the accumulation of arrears and without compromising growth’.

Since nineteenth century, Developing countries have experienced repeated episodes of rapidly increasing external indebtedness and debt-service burdens that have brought slower growth or recession and eventually produced renegotiation and restructuring⁵. It is agreed that Heavily Indebted Poor Countries (HIPC) cannot grow unless debt relief initiative provided.

Even developed countries are to pay financial stabilization costs with the growth of public debts. The gross government debt of United States is accumulated an increase of 38% of GDP and the financial stabilization costs will be 12.7% of GDP followed by United Kingdom has to incur 9.1% of stabilization costs with an accumulated increase of 40.4% of GDP in public debts. Canada’s public debt will increase to 20.3% of GDP and stabilization costs 4.4% while Germany has to pay 3.1% of GDP as stabilization costs for an increase in 29.9% of GDP of public debt by 2010. Italy’s sovereign debt will increase from 106 to 121, 14.2% of GDP and the country has to maintain a stabilization cost of 0.9% GDP⁵.
As the above graph depicts that Government net debts have increased continuously with the increase of GDP. Italy and Japan have increased to the excess of their total GDP, whereas Canada’s general public net debt has decreased over the time.

4. World Economic and Social Survey 2005, IMF.
5. Debt to GDP data from IMF, world economic outlook 2009 and stabilization costs estimated by the IMF Fiscal affairs department. The comparision figures are with base figure of 2008 and accumulated figure of 2010 (Global Financial Stability Report 2009-48).
The above graph shows that developing Asia is ahead of all incurring external debt. External debt, both Africa and Western Hemisphere had 29% of GDP, while central and eastern Europe incurred 27% of GDP in 1980. Developing Asia and Middle-East paid 14 and 15 per cent of GDP respectively in 1980. As of 2009, both Africa and western hemisphere have decreased their external debt to 25% and 24% respectively. Central and eastern Europe have increased its external debt 67% of GDP while developing Asia has reached in its external debt level to 16% and 27% of GDP respectively.

2.2.2 Private Debts
Private debt as defined by OECD is “Private debt is debt from a loan by a private entity, such as a bank or an exporter. It may be guaranteed by the official sector. If it is rescheduled by the official sector it is reclassified as other official flow (OOF) debt”. Private debt accounts for approximately 65% of overall debt burden of developing countries.6

Middle income and low income countries are paying a substantial part of their revenues to bondholders and commercials banks and other creditors to maintain their debt levels every year. Private debt issuance by developing countries have increased more than doubled between 2002 to 20077. Developed countries have generally issued higher debts.8

6. World Bank Debt Reporting Systems
8. IFSL Research, UK, 2009
2.2.2. World Bond Markets

It cannot be ignored that investors confidence have been affected by the recent global economic crisis. In general, Bond means, a debt investment by the bondholders or investors who lend to borrower upon obligation to pay at a future date. No doubt, bonds play a vital role in building up a country’s financial system and it is has been a safe tool to meet the government’s growing financial needs. When the markets experience fall in liquidity, the importance of bond market increases and become an importance of source of finance for companies and government.

*External public syndicated issuance, excluding bilateral deals.
Figure 1.6, Source: IMF, Global Financial Stability Report, April 2009.

The above exhibit clearly shows, there was a sharp down fall in bond issuance due global financial meltdown. In 2004, the total bond issuance by emerging market was US $128,247mn and it increased 31% in 2008. The financial crisis put pressure on bond markets to fall down by 43%

* Includes bonds, notes and money market instruments
Figure 1.7, Source: IFSL Research Bond Markets 2009.
between 2008 to 2009. The outstanding amount of global bond markets increased 6% in 2008 to US $83 trillion. The financial crisis resulted increasing amount outstanding to 43% both in US and UK followed by Japan 16%9.

This above graph shows that amount outstanding on world bond market on 1998 was 87% on domestic level while 13% on international level whereas by 2008 it was increased and domestic outstanding was 71% and rest 29% on international markets.

3. Relationship between Equity and Debt: Is it Inverse? Why??
The study of relationship between equity and debt is very interesting topic in international capital flows. The duo(debt and equity) in investment world play a significant role combining world of wealth into two different segments. It is often mentioned that the relationship between the equity and debt is nothing but an inverse.

3.1 A brief Literature Review
The literature review of equity and debt relationship can show a deeper insight about what the financial economists’ market study and their mythology related to debt and equity financing in the world capital market. The relationship can be assessed in three different categories.

3.1.1 Financing choice of firms
As concerned to the firm’s choice between equity and/or debt financing; Greenworld, Stiglitz, and Weiss(GSW) (1984) point out that first, when a firm is equity-financed, it can face intensive incentive problems, managers in firms are less restricted (laissez-faire policies) in diversion of profits for their private use with equity than debt capital. But, in case of debt-finance, firms can work rationale to woo the faith of their lenders who often can threaten to withdraw funds in case of mismanaged. Also, large bankruptcy costs may raise an alarm to manage effectively of funds. Second, signaling effects may limit a firm’s access to equity capital. A superior firm can less be affected by Bankruptcy risk and marginal increase in bankruptcy risk with extended debt levels than an ‘inferior’ firms. So, greater reliability on debt capital by superior firms implies that inferior firms are ease to issue equity capital. GSW argue that firms issue equity capital only when its operative earnings are not adequate to prevent bankruptcy. If operating earnings are large enough to sustain debt related payment, debt is preferred form of financing.

The informational problem face by lenders doesn’t know the investment pattern of a firm they invest in. Stiglitz-Weiss(1983) shows that an increase in interest rate affect the borrows will and increase the average riskiness of that project that the firms intend to finance.

3.1.2 Investment choice of investors
The financing choice(equity vs. debt) model(Hart and Moore, 1998) relies on asymmetries information and believe that it is the result of the inability of investors to verify certain actions or outcomes. Investors cannot earn higher returns without taking on greater risk, and the greater the risk, the greater the possibility of loss. (Harry M. Markowitz, 1952).

9. IFSL Research, UK, 2009
3.1.3 Risk - return trade off and debt –equity substitution and swap
Risk and return can shape the decision making choice. A higher debt-to-equity ratio leads to a higher required return on equity, because of the higher risk involved for equity-holders in a company with debt (Modigliani-Miller Theorem, 1958).

Paul R. Krugman (1995) pointed out that a reducing debt is not the same as reducing external obligations. When equity is substituted for debt, foreigners relinquish their claim on a future stream of debt services in return for a claim on future stream of repatriated earnings.

Krugman further argued, that when a debt-equity swap fails to generate additional equity investment, the swap degenerates into a re-purchase of debt using domestically generated resources.

The substitution of one asset(debt) with another(equity) is a portfolio choice, that is , the response of demand for debt to changes in expected return on equity, and vice versa. (Benjamin M. Friedman, 1983).

It is oblivious that people or firms, when they intend to invest, they either choose equity or debt as there is a adverse relationship relating to their returns, investment patterns and risk associated with. Firms also do finance either through equity or debt, but incase both equity and debt, then, there is a debt-equity ratio which may not be of equal ratio.

3.2 Equity and debt: Risk and returns
To define the relationship between equity and debt, their risks and returns can give us a possible insight and can depict us how the investors and firms are dealing to invest. It described the relationship simply an inverse unless otherwise.

Investor’s problem is to choose a portfolio. Investment decision on equity and debt depends on risk and return. Either a firm or an individual makes choice whether to make equity investment or debt investment. Apart from some variables, the most important aspects are risk and return. “Equity returns have averaged more than 10% while debt returns average between 6% to 7%. Equity earn much higher return but with higher risk.” (Markowitz- Sharpe- Miller, 1990).

The future return can be expressed mathematically using CAPM - 
\[ \lambda (R) = R_f + \beta \times (R_t - R_r) \]  
**……………..(1.0)**

\[ \lambda (R) = R_f + \beta E_p^m \]  
**……………..(1.1)**

Where \( \lambda (R) \) return on future market value, \( R_f \) is risk free rate, \( R_r \) real return, \( \beta \) market risk and \( E_p^m \) equity premium .

3.2.1 Equity Markets Returns
Historical data gives a true figure of market returns over a century and it clearly shows that stock equity returns considerably higher than those for treasury bills. Mehra - Prescott (1985) studied that
the average annual real return on US equity from 1889-2000 was 8.06, while, on the same period, a relatively riskless equity gave a return of 1.14%. So, the difference of these two returns was 6.92, i.e., premium equity. The below graphs shows the equity returns of important world markets.

Figure 1.8, Source: Mehra-Prescott(1985), UK data from Seigel (1998), Other data from Campbell(2001)

The return on Indian equity from the period 1979 to 2005, inclusive of dividends was 18%. The GDP deflator over the same period yielded an inflation estimate of 8%, which suggests that the equity returns in real term were 10%. (Ajay Shah, 2005).

3.2.2 Government Bond and Debt Markets Returns

Eichengreen and Portes(1986) tacked the payment of US dollar bond and found that the nominal rate of return for the sterling bonds was 5.4% on average while the dollar bonds were 3.3% owed or guaranteed by the government.(see appendix for graph illustration)

The main point is that these return rates are closed to government bond rates that contain virtually no default risks. The above graph shows that the debt returns are increasing with time and have low risks.

The below graph illustrates that the interest rates are increasing and the debt markets across the world have a positive return. In the first 60 years from 1880 to 1939, the average growth was 3.52 whereas the next 50 years (1940 -89) saw a significant growth in interest rates average 6.23. The risk segments were quiet low compare to high risk high equity returns.
3.2.3 Market Risk

Market risk is the most important factor for an investor. Every investor wants to accumulate curiously the market risk before making an investment. Though it has been noticed investors are able to put off money to invest in a more riskier capital in order to yield more return, but the credit goes to only some few investors. Mehra-Prescott(1985) pointed out that the difference in returns (frequently called the equity premium) implies a very high degree of risk aversion on the part of investors. The high historical equity risk premium is specially challenging compared to the very low historical rate of return on Treasury securities.
The above graph illustrates that the market risks become parallel to the common equity stock Market. A high risk led to high return all the times. Whereas, with minimum risk, treasury bills earned low returns.

Market risk can be written as

\[ R^m_{t+1} = I_t \cdot (R_m - R_f)^{t+1} + \alpha \] ................................. (1.2)

\( R^m_{t+1} \) stands for total market value of risk, \( I_t \) is the total market value of investment, \( R_m \) market rate of return, \( R_f \) risk free return, \( t \) stands for time and \( \alpha \) = the risk-adjusted excess return of the portfolio.

### 3.2.4 Costs, capital structures and financing choices

As both equity and debt are the main elements of the capital structure, and their relationships depend on costs. It is obvious that tax and financial crisis add a heavy weight to the capital structure of both equity and debt.

Jensen and Meckling (1976) documented that agency cost is costly. They identified two different types of costs; between managers and outside shareholders (agency cost of equity) and between shareholders and debt holders (agency cost of debt). The agency costs are caused because of asymmetric information between managers of the firms and its shareholders.

These costs make the firms and investors to invest more than the real value of capital. The cost obviously shapes the capital structure and financing option of a firm. So, firms generally take decision whether to issue equity or opt for debt financing because of the inverse relationship between the both.

The cost of the debt can be described:

\[ \lambda (d) = R_f + R_d + A_{c_d} + (1 - T) \] .......................... (1.3)

\( R_f \) = Risk free rate, \( R_d \) stands for default risk, \( A_{c_d} \) is agency cost and \( T \) is corporate tax rate.

Whereas, the cost of equity can be

\[ \lambda (e) = R_f + R_e + A_{c_e} \] ................................. (1.4)

\( R_e \) stands for calculated risk and \( A_{c_e} \) agency cost of equity.

### 3.2.5 Analysis of Correlation between Equity and Debt Investments

It is necessary to analyze the correlation between equity and debt in order to find their relationship. Let’s do the correlation analysis both mathematically and statistically. Suppose, equity is \( x_e \) and debt is \( y_d \) and are two variables.
We can calculate correlation between equity and debt statistically using correlation analysis. We can take into consideration of risk, and/or return of equity and debt in order to find out correlation. We can assume that Equity return is \( x_e \) and Debt return is \( y_d \). The correlation can be written –

\[
R_{x_e, y_d} = \frac{\sum_{i=1}^{n} \left( x_{ei} - \mu_{x_e} \right) \left( y_{di} - \mu_{y_d} \right)}{\left( 1 - n \right) \left( \sigma_{x_e} \cdot \sigma_{y_d} \right)}
\]  

where \( \mu_{x_e} \) stands for the sample mean of \( x_e \) and \( \sigma_{x_e} \) denotes the sample standard deviation of \( x_e \), whereas \( \mu_{y_d} \) and \( \sigma_{y_d} \) denote the sample mean and the sample standard deviation respectively for the variable \( y_d \).

Now, let’s assume that a perfect linear relationship exists between the variables \( x_e \) and \( y_d \), then the linear regression model can be written :

\[
y_{di} = \alpha + \beta x_{ei} + \epsilon \]

Where \( i = 1, 2, \ldots, n \) and \( \sum_{i=1}^{n} \epsilon \equiv 0 \)

Now we can verify using the mean and the variance in linear regression model as

\[
\mu_{y_d} = \alpha + \beta \mu_{x_e}
\]

\[
\beta = R_{x_e, y_d} \left( \frac{\sigma_{x_e}}{\sigma_{y_d}} \right)
\]

\[
\sigma_{y_d} = | \alpha | \sigma_{x_e}
\]

\[
| \alpha | = \frac{\sigma_{y_d}}{\sigma_{x_e}}
\]

This implies from Eq. 1.5 that \( R_{x_e, y_d} = 1 \) if \( \alpha > 0 \) and \( R_{x_e, y_d} = -1 \) if \( \alpha < 0 \). The case \( R_{x_e, y_d} = 1 \) corresponds a maximum possible positive correlation between \( X_e(\text{equity}) \) and \( Y_d(\text{debt}) \), meaning that all the data points will lie exactly on a straight line of positive slope. Similarly, \( R_{x_e, y_d} = -1 \) implies to the maximum possible negative association between the \( X_e(\text{equity}) \) and \( Y_d(\text{debt}) \). In general, \(-1 \leq R_{x_e, y_d} \leq 1\), the signs and magnitudes show both strength and direction of the linear relationship between the both \( X_e(\text{equity}) \) and \( Y_d(\text{debt}) \). In case, \( R_{x_e, y_d} = 0 \), that corresponds that there is no linear correlation nor weak linear correlation. A value near to 0 means , the correlation is random and non-linear. If a \( R_{x_e, y_d} \geq 0.8 \), then it’s a strong positive Correlation, whereas, \( R_{x_e, y_d} \leq 0.5 \) describe weak correlation between the two variables.

In order to get the correlation using the above methodology, I have studied USA market for the period 1802 to 2002(200 years).For the given data in table-9(see appendix pp.VI - XII,figure 1.11 ).
I find that the correlation between common stock (commonly equity) and treasury bill (typically debt) is -0.56 corresponds there is a strong negative correlation (inverse relationship) between them. I further, find that the correlation between common stock and risk premium (typically risk : \( R_m - R_f \)) is 0.97 shows a stronger positive relation (risk is inevitable and always consists with equity), whereas, treasury bill and risk premium have shown a stronger negative relationship resulting -0.74, as, there is almost little market risk (guaranteed by the US government) besides the default credit risk. But, It is noted that some debt stocks (large share of debt stock invests in capital market) have also risks, provided that, their returns are co-parallel with the calculated risks and, in that case, the correlation will be positive.

4. Conclusion

In this paper I estimate the relationship between equity and debt investment. I also come to conclusion that market is volatile with risk that leads to differential returns between equity and debt investment. I use several measures of returns of both equity and debt investment using some statistical tools such as standard deviation, covariance and correlation. I have studied intensively of world equity markets, world debt markets and world bond markets.

I find very interesting with the relationship between equity and debt investment, that shows when investors intend to investment in debt markets due to low risk (some debts instruments having no risks) and the investors are in no way to burden the market volatility due to high risk with equity segment, they simply ignore the equity investment. In other case, when investors stand for to invest in equity markets due to high returns and courageously take risks, and push aside to invest in debt instrument due to low returns. The two capital instruments, equity and debt, are always shaping their directions and magnitudes with market risks.

My analysis leaves a number of issues open to further enquiry. The first issue is the computation of correlation of debt and equity investment of various capital markets like developing and developed countries categorically. We can also calculate the emerging capital markets of the world to find further new results. For comparison, we can do further study on volatile markets and nonvolatile markets. The second issue can be the study of the correlation of equity and debt investment at the time of financial crisis which pays a significant impact on investments and returns. Newer tools and methodologies related to computation of relationship between equity and debt investments can be developed using various statistical and econometrics theory. The third important aspect is to study of market impact on risk and return which plays a significant role in equity and debt correlation.
REFERENCES:


4. Friedman, Benjamin M.(1983), The Substitutability of debt and equity securities, NBER working paper No.1130


16. Mehra, Rajnish, Prescott, Edward C. (2003), The equity premium in retrospect, NBER working paper 9525


24. World Investment Prospect survey 2009-2011, UNCTD.
### Figure 1.1

#### Evolution of PE Fundraising

<table>
<thead>
<tr>
<th>Year</th>
<th>Funds raised</th>
<th>Aggregate capital raised($B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>444</td>
<td>129</td>
</tr>
<tr>
<td>1999</td>
<td>524</td>
<td>141</td>
</tr>
<tr>
<td>2000</td>
<td>724</td>
<td>228</td>
</tr>
<tr>
<td>2001</td>
<td>589</td>
<td>166</td>
</tr>
<tr>
<td>2002</td>
<td>582</td>
<td>127</td>
</tr>
<tr>
<td>2003</td>
<td>476</td>
<td>115</td>
</tr>
<tr>
<td>2004</td>
<td>635</td>
<td>165</td>
</tr>
<tr>
<td>2005</td>
<td>825</td>
<td>352</td>
</tr>
<tr>
<td>2006</td>
<td>926</td>
<td>538</td>
</tr>
<tr>
<td>2007</td>
<td>836</td>
<td>587</td>
</tr>
<tr>
<td>2008</td>
<td>437</td>
<td>341</td>
</tr>
</tbody>
</table>

Table – 1, Source: Private Equity Council, 2009.

### Figure 1.3

#### General Government Net Debt as % of GDP

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>31</td>
<td>41</td>
<td>42</td>
<td>55</td>
<td>59</td>
<td>66</td>
<td>79</td>
<td>87</td>
</tr>
<tr>
<td>Canada</td>
<td>46</td>
<td>67</td>
<td>75</td>
<td>102</td>
<td>81</td>
<td>70</td>
<td>75</td>
<td>77</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>46</td>
<td>46</td>
<td>33</td>
<td>46</td>
<td>41</td>
<td>42</td>
<td>63</td>
<td>73</td>
</tr>
<tr>
<td>United States</td>
<td>44</td>
<td>56</td>
<td>64</td>
<td>71</td>
<td>55</td>
<td>63</td>
<td>87</td>
<td>97</td>
</tr>
<tr>
<td>France</td>
<td>20</td>
<td>31</td>
<td>35</td>
<td>55</td>
<td>57</td>
<td>66</td>
<td>75</td>
<td>80</td>
</tr>
<tr>
<td>Italy</td>
<td>56</td>
<td>80</td>
<td>95</td>
<td>121</td>
<td>109</td>
<td>106</td>
<td>115</td>
<td>121</td>
</tr>
<tr>
<td>Japan</td>
<td>52</td>
<td>69</td>
<td>69</td>
<td>93</td>
<td>142</td>
<td>192</td>
<td>217</td>
<td>227</td>
</tr>
</tbody>
</table>

Table – 2, Source: International Monetary Fund, World Economic Outlook Database, April 2009

### Figure 1.4

#### External Debts(in US $ Bn)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Africa</td>
<td>103</td>
<td>155</td>
<td>237</td>
<td>286</td>
<td>278</td>
<td>283</td>
<td>277</td>
<td>295</td>
</tr>
<tr>
<td>Central and eastern Europe</td>
<td>84</td>
<td>110</td>
<td>151</td>
<td>183</td>
<td>267</td>
<td>515</td>
<td>981</td>
<td>990</td>
</tr>
<tr>
<td>Developing Asia</td>
<td>110</td>
<td>190</td>
<td>332</td>
<td>573</td>
<td>653</td>
<td>816</td>
<td>1223</td>
<td>1323</td>
</tr>
<tr>
<td>Middle East</td>
<td>63</td>
<td>83</td>
<td>117</td>
<td>151</td>
<td>176</td>
<td>236</td>
<td>432</td>
<td>260</td>
</tr>
<tr>
<td>Western Hemisphere</td>
<td>232</td>
<td>370</td>
<td>452</td>
<td>622</td>
<td>764</td>
<td>795</td>
<td>860</td>
<td>893</td>
</tr>
</tbody>
</table>

Table – 3, International Monetary Fund, World Economic Outlook Database, April 2009
Figure 1.5

<table>
<thead>
<tr>
<th>Private Debt flow to developing countries (2003-07)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Russian Federation</td>
<td>19,7</td>
</tr>
<tr>
<td>Mexico</td>
<td>10,7</td>
</tr>
<tr>
<td>Brazil</td>
<td>10,1</td>
</tr>
<tr>
<td>Turkey</td>
<td>7,3</td>
</tr>
<tr>
<td>China</td>
<td>6</td>
</tr>
<tr>
<td>India</td>
<td>4,7</td>
</tr>
<tr>
<td>Poland</td>
<td>3,7</td>
</tr>
<tr>
<td>South Africa</td>
<td>3,5</td>
</tr>
<tr>
<td>Kazakhstan</td>
<td>3,4</td>
</tr>
<tr>
<td>Malaysia</td>
<td>2,7</td>
</tr>
<tr>
<td>BRICs</td>
<td>40,5</td>
</tr>
<tr>
<td>Low income</td>
<td>0,7</td>
</tr>
<tr>
<td>Lower middle income</td>
<td>27,9</td>
</tr>
<tr>
<td>Upper middle income</td>
<td>71,4</td>
</tr>
</tbody>
</table>

Table - 4, Sources: World Bank Debt Reporting System(Data from WB Policy Research working paper 4829, Year: 2009)

Figure 1.6

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Countinents</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Africa</td>
<td>2250</td>
<td>3170</td>
<td>4899</td>
<td>13243</td>
<td>1533</td>
</tr>
<tr>
<td>MEGA</td>
<td>14783</td>
<td>18577</td>
<td>35156</td>
<td>25327</td>
<td>12811</td>
</tr>
<tr>
<td>Latin America</td>
<td>33729</td>
<td>60967</td>
<td>30175</td>
<td>38540</td>
<td>17562</td>
</tr>
<tr>
<td>Asia</td>
<td>44567</td>
<td>44502</td>
<td>41705</td>
<td>47324</td>
<td>28285</td>
</tr>
<tr>
<td>Europe</td>
<td>33017</td>
<td>52291</td>
<td>50650</td>
<td>60476</td>
<td>45821</td>
</tr>
</tbody>
</table>

External public syndicated issuance, excluding bilateral deals.
Figure 1.7
World Bond Market

<table>
<thead>
<tr>
<th>Year</th>
<th>Domestic</th>
<th>International</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>28199</td>
<td>4284</td>
</tr>
<tr>
<td></td>
<td>29961</td>
<td>5353</td>
</tr>
<tr>
<td>2000</td>
<td>29901</td>
<td>6363</td>
</tr>
<tr>
<td></td>
<td>30404</td>
<td>7502</td>
</tr>
<tr>
<td>2002</td>
<td>34141</td>
<td>9190</td>
</tr>
<tr>
<td></td>
<td>39873</td>
<td>11662</td>
</tr>
<tr>
<td>2004</td>
<td>44467</td>
<td>13940</td>
</tr>
<tr>
<td></td>
<td>45402</td>
<td>12509</td>
</tr>
<tr>
<td>2006</td>
<td>49735</td>
<td>18419</td>
</tr>
<tr>
<td></td>
<td>56211</td>
<td>22708</td>
</tr>
<tr>
<td>2008</td>
<td>59666</td>
<td>23863</td>
</tr>
</tbody>
</table>

* Includes bonds, notes and money market instruments

Table – 6, Source: IFSL Research Bond Markets 2009.
Original Source: Bank for International Settlements

Figure 1.8
Equity Market returns

<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>% Real return on a market Index</th>
<th>% Real return on relatively riskless security</th>
<th>% Equity premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>1889-2000</td>
<td>8,06</td>
<td>1.14</td>
<td>6.92</td>
</tr>
<tr>
<td>UK</td>
<td>1947-1999</td>
<td>5,70</td>
<td>1.10</td>
<td>4.6</td>
</tr>
<tr>
<td>Japan</td>
<td>1970-1997</td>
<td>4.7</td>
<td>1.4</td>
<td>3.3</td>
</tr>
<tr>
<td>Germany</td>
<td>1978-1997</td>
<td>9.8</td>
<td>3.2</td>
<td>6.6</td>
</tr>
<tr>
<td>France</td>
<td>1973-1998</td>
<td>9.0</td>
<td>2.7</td>
<td>6.3</td>
</tr>
</tbody>
</table>

Table-7, Source: Mehra-Prescott(1985), UK data from Seigel (1998), Other data from Campbell(2001)

Figure 1.9
Average Yields of Various Debt Markets

<table>
<thead>
<tr>
<th>Period</th>
<th>American Commercial Paper</th>
<th>English Open Market</th>
<th>French Open Market &amp; Day to day money</th>
<th>American Corporate Bonds</th>
<th>English Consols</th>
<th>French 3% rentes to 1950, 5% rentes thereafter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1880-1939</td>
<td>4.24</td>
<td>2.84</td>
<td>3.06</td>
<td>3.91</td>
<td>3.22</td>
<td>3.89</td>
</tr>
<tr>
<td>1940-1989</td>
<td>4.96</td>
<td>6.17</td>
<td>5.93</td>
<td>6.09</td>
<td>7.22</td>
<td>7.02</td>
</tr>
</tbody>
</table>

Table-8, Source: Sidney Homer and Richard Sylla 1991
Figure 1.10  
**Historical Market Risk Premium**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1802-1870</td>
<td>8.1</td>
<td>5.2</td>
<td>2.9</td>
</tr>
<tr>
<td>1871-1925</td>
<td>8.4</td>
<td>3.8</td>
<td>4.6</td>
</tr>
<tr>
<td>1926-2002</td>
<td>12.2</td>
<td>3.8</td>
<td>8.4</td>
</tr>
</tbody>
</table>

Table -9, Source: Sidney Homer and Richard Sylla 1991

Figure 1.11

From the above data, we can calculate the correlation between

1. Common Stock and Treasury Bill
2. Common Stock and Risk Premium
3. Treasury Bills and Risk Premium
1. Correlation between Common Stock and Treasury Bill:

Let common stock \( x_e \) and treasury bill \( y_d \) are two variables.

<table>
<thead>
<tr>
<th>Common Stock((x_e))</th>
<th>Treasury Bills((y_d))</th>
<th>(x_e - \mu x_e)</th>
<th>((x_e - \mu x_e)^2)</th>
<th>(y_d - \mu y_d)</th>
<th>((y_d - \mu y_d)^2)</th>
<th>((x_e - \mu x_e)(y_d - \mu y_d))</th>
</tr>
</thead>
<tbody>
<tr>
<td>8,10</td>
<td>5,20</td>
<td>-1,47</td>
<td>2,15</td>
<td>0,93</td>
<td>0,87</td>
<td>-1,37</td>
</tr>
<tr>
<td>8,40</td>
<td>3,80</td>
<td>-1,17</td>
<td>1,36</td>
<td>-0,47</td>
<td>0,22</td>
<td>0,54</td>
</tr>
<tr>
<td>12,20</td>
<td>3,80</td>
<td>2,63</td>
<td>6,93</td>
<td>-0,47</td>
<td>0,22</td>
<td>-1,23</td>
</tr>
<tr>
<td>28,70</td>
<td>12,80</td>
<td>10,45</td>
<td>1,31</td>
<td>-2,05</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
R_{x_e y_d} = \sum_{i=1}^{n} \frac{(x_{ei} - \mu x_e)(y_{di} - \mu y_d)}{\sigma x_e \cdot \sigma y_d \cdot (1 - n)}
\]

To find out the correlation, first I have to find out mean, and then standard deviation. Using the result in Table 10, the mean return on the common stock\((x_e)\) is:

\[
\mu x_e = \frac{\sum_{i=1}^{n} x_{ei}}{n} \quad \Rightarrow \quad \mu x_e = \frac{28,70}{3} = 9,57
\]

The mean return on treasury bill\((y_d)\) is:

\[
\mu y_d = \frac{\sum_{i=1}^{n} y_{di}}{n} \quad \Rightarrow \quad \mu y_d = \frac{12,80}{3} = 4,27
\]

Using result in fourth column from the above table, we can compute the variance of common stock\((x_e)\):

\[
\sigma x_e^2 = \frac{1}{n} \sum_{i=1}^{n} (x_{ei} - \mu x_e)^2 \quad \Rightarrow \quad \sigma x_e^2 = \frac{10,45}{3} = 3,48
\]
Using result in sixth column from the above table, we can compute the variance of treasury bill ($y_d$)

$$\sigma_{y_d}^2 = \frac{1}{n} \sum_{i=1}^{n} ( y_{di} - \mu_{y_d} )^2 \quad \sigma_{y_d}^2 = \frac{1.31}{3} = 0.44$$

We can compute standard deviation with using the above results

$$\sigma_{x_e} = \sqrt{\frac{\left( x_{ei} - \mu_{x_e} \right)^2}{n}} \quad \sigma_{x_e} = \sqrt{\frac{10.45}{3}} = 1.87$$

$$\sigma_{y_d} = \sqrt{\frac{\left( y_{di} - \mu_{y_d} \right)^2}{n}} \quad \sigma_{y_d} = \sqrt{\frac{1.31}{3}} = 0.66$$

Now, with the above result, we can compute the covariance between common stock($x_e$) and treasury bill($y_d$). By computing covariance, we can find that $-1 \leq \text{Cov}(x_e,y_d) \leq 1$, the negative and positive correlation between these two variable, unless $\text{Cov}(x_e,y_d) = 0$ which implies the two variables are uncorrelated.

$$\text{Cov}(x_e,y_d) = \frac{1}{n} \sum_{i=1}^{n} ( x_{ei} - \mu_{x_e} )( y_{di} - \mu_{y_d} ) \quad \text{Cov}(x_e,y_d) = \frac{-2.05}{3} = -0.684$$

The covariance -0.684 implies that common stock($x_e$) and treasury bill($y_d$) are negatively correlated. Now, we can compute the correlation to find out the exact degree the correlation between them.

$$R_{x_e,y_d} = \frac{\text{Cov}(x_e,y_d)}{\sigma_{x_e} \cdot \sigma_{y_d}} = \frac{-0.684}{1.87 \times 0.66} = -0.56$$

We can re-write the correlation using both covariance and standard deviation:

$$R_{x_e,y_d} = \frac{\text{Cov}(x_e,y_d)}{\sigma_{x_e} \cdot \sigma_{y_d}}$$

The result shows that the correlation between common stock($x_e$) and treasury bill($y_d$) is negative. They are negatively associated with each other and their relationship is inverse.

VIII
2. Correlation between Common Stock and Risk Premium:

We can assume common stock \(x_e\) and risk premium \(y_d\) are two variables.

Table - 11

<table>
<thead>
<tr>
<th>Common Stock (x_e)</th>
<th>Risk Premium (y_d)</th>
<th>(x_e - \mu x_e)</th>
<th>((x_e - \mu x_e)^2)</th>
<th>(y_d - \mu y_d)</th>
<th>((y_d - \mu y_d)^2)</th>
<th>((x_e - \mu x_e)(y_d - \mu y_d))</th>
</tr>
</thead>
<tbody>
<tr>
<td>8,10</td>
<td>2,9</td>
<td>-1,47</td>
<td>2,15</td>
<td>-2,40</td>
<td>5,76</td>
<td>3,52</td>
</tr>
<tr>
<td>8,40</td>
<td>4,6</td>
<td>-1,17</td>
<td>1,36</td>
<td>-0,70</td>
<td>0,49</td>
<td>0,82</td>
</tr>
<tr>
<td>12,20</td>
<td>8,4</td>
<td>2,63</td>
<td>6,93</td>
<td>3,10</td>
<td>9,61</td>
<td>8,16</td>
</tr>
<tr>
<td><strong>28,70</strong></td>
<td><strong>15,90</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>15,86</strong></td>
</tr>
<tr>
<td><strong>10,45</strong></td>
<td><strong>12,05</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
R_{x_e y_d} = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{x_{ei} - \mu x_e}{\sigma x_e} \right) \left( \frac{y_{di} - \mu y_d}{\sigma y_d} \right)
\]

To find out the correlation, first I have to find out mean, and then standard deviation. Using the result in Table 10, the mean return on the common stock \(x_e\) is:

\[
\mu x_e = \frac{1}{n} \sum_{i=1}^{n} x_{ei} = \frac{28,70}{3} = 9,57
\]

The mean return on risk premium \(y_d\) is:

\[
\mu y_d = \frac{1}{n} \sum_{i=1}^{n} y_{di} = \frac{15,90}{3} = 5,3
\]

Using result in fourth column from the above table, we can compute the variance of common stock \(x_e\):

\[
\sigma x_e^2 = \frac{1}{n} \sum_{i=1}^{n} (x_{ei} - \mu x_e)^2 = \frac{10,45}{3} = 3,48
\]

Using result in sixth column from the above table, we can compute the variance of risk premium \(y_d\):

\[
\sigma y_d^2 = \frac{1}{n} \sum_{i=1}^{n} (y_{di} - \mu y_d)^2 = \frac{15,86}{3} = 5,29
\]
We can compute standard deviation with using the above results

\[
\alpha_e = \sum_{i=1}^{n} \sqrt{\frac{(x_e - \mu_e)^2}{n}} \quad \sigma_e = \sqrt{\frac{10.45}{3}} = 1.87
\]

\[
\sigma_d = \sum_{i=1}^{n} \sqrt{\frac{(y_d - \mu_d)^2}{n}} \quad \sigma_d = \sqrt{\frac{15.86}{3}} = 2.30
\]

Now, with the above result, we can compute the covariance between common stock(x_e) and risk premium(y_d). By computing covariance, we can find that \(-1 \leq \text{Cov}(x_e, y_d) \leq 1\), the negative and positive correlation between these two variable, unless \(\text{Cov}(x_e, y_d) = 0\) which implies the two variables are uncorrelated.

\[
\text{Cov}(x_e, y_d) = \sum_{i=1}^{n} \frac{(x_{ei} - \mu_{xe})(y_{di} - \mu_{yd})}{n} = \frac{12.50}{3} = 4.17
\]

The covariance 4.17 implies that common stock(x_e) and risk premium(y_d) are positively correlated. Now, we can compute the correlation to find out the exact degree the correlation between them.

\[
R_{x_e, y_d} = \sum_{i=1}^{n} \frac{(x_{ei} - \mu_{xe})(y_{di} - \mu_{yd})}{(1 - n) \sigma_{xe} \cdot \sigma_{yd}} = \frac{4.17}{1.87 \times 2.30} = \frac{-0.684}{4.29} = 0.97
\]

We can re-write the correlation using both covariance and standard deviation:

\[
R_{x_e, y_d} = \frac{\text{Cov}(x_e, y_d)}{\sigma_{xe} \cdot \sigma_{yd}}
\]

The result shows that the correlation between common stock(x_e) and risk premium(y_d) is positive. They are positively associated with each and their relationship is positive as both are associated with risk, as risks are increasing, their returns are increasing.
3. Correlation between Treasury Bill and Risk Premium:

We can assume treasury bill \( x_e \) and risk premium \( y_d \) are two variables.

Table - 12

<table>
<thead>
<tr>
<th>Treasury Bill ((x_e))</th>
<th>Risk Premium ((y_d))</th>
<th>( x_e - \mu_{x_e} )</th>
<th>( (x_e - \mu_{x_e})^2 )</th>
<th>( y_d - \mu_{y_d} )</th>
<th>( (y_d - \mu_{y_d})^2 )</th>
<th>( (x_e - \mu_{x_e})(y_d - \mu_{y_d}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,20</td>
<td>2,9</td>
<td>0,93</td>
<td>0,87</td>
<td>-2,40</td>
<td>5,76</td>
<td>-2,24</td>
</tr>
<tr>
<td>3,80</td>
<td>4,6</td>
<td>-0,47</td>
<td>0,22</td>
<td>-0,70</td>
<td>0,49</td>
<td>0,33</td>
</tr>
<tr>
<td>3,80</td>
<td>8,4</td>
<td>-0,47</td>
<td>0,22</td>
<td>3,10</td>
<td>9,61</td>
<td>-1,45</td>
</tr>
<tr>
<td>12,80</td>
<td>15,90</td>
<td>1,31</td>
<td>15,86</td>
<td>-3,36</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
R_{x_e y_d} = \frac{\sum_{i=1}^{n} (x_{ei} - \mu_{x_e})(y_{di} - \mu_{y_d})}{\left(1 - \frac{1}{n}\right)\sigma_{x_e} \cdot \sigma_{y_d}}
\]

To find out the correlation, first I have to find out mean, and then standard deviation. Using the result in Table 10, the mean return on the treasury bill \((x_e)\) is:

\[
\mu_{x_e} = \frac{\sum_{i=1}^{n} x_{ei}}{n} = 4,27
\]

The mean return on risk premium \((y_d)\) is:

\[
\mu_{y_d} = \frac{\sum_{i=1}^{n} y_{di}}{n} = 5,30
\]

Using result in fourth column from the above table, we can compute the variance of treasury bill \((x_e)\):

\[
\sigma_{x_e}^2 = \frac{1}{n} \sum_{i=1}^{n} (x_{ei} - \mu_{x_e})^2 = 0,44
\]

Using result in sixth column from the above table, we can compute the variance of risk premium \((y_d)\):

\[
\sigma_{y_d}^2 = \frac{1}{n} \sum_{i=1}^{n} (y_{di} - \mu_{y_d})^2 = 5,29
\]

XII
We can compute standard deviation with using the above results

$$\sigma_{x_e} = \sqrt{\frac{\sum_{i=1}^{n} (x_{ei} - \mu_{x_e})^2}{n}} \quad \sigma_{x_e} = \sqrt{\frac{1.31}{3}} = 0.66$$

$$\sigma_{y_d} = \sqrt{\frac{\sum_{i=1}^{n} (y_{di} - \mu_{y_d})^2}{n}} \quad \sigma_{y_d} = \sqrt{\frac{15.86}{3}} = 2.30$$

Now, with the above result, we can compute the covariance between treasury bill \((x_e)\) and risk premium\((y_d)\). By computing covariance, we can find that \(-1 \leq \text{Cov}(x_e, y_d) \leq 1\), the negative and positive correlation between these two variable, unless \(\text{Cov}(x_e, y_d) = 0\) which implies the two variables are uncorrelated.

$$\text{Cov}(x_e, y_d) = \frac{\sum_{i=1}^{n} (x_{ei} - \mu_{x_e})(y_{di} - \mu_{y_d})}{n} \quad \text{Cov}(x_e, y_d) = \frac{-3.36}{3} = -1.12$$

The covariance -1.12 implies that treasury bill \((x_e)\) and risk premium\((y_d)\) are negatively correlated. Now, we can compute the correlation to find out the exact degree the correlation between them.

$$R_{x_e, y_d} = \sum_{i=1}^{n} \frac{(x_{ei} - \mu_{x_e})(y_{di} - \mu_{y_d})}{\sigma_{x_e} \cdot \sigma_{y_d} \sqrt{(1 - n)}}$$

We can re-write the correlation using both covariance and standard deviation:

$$R_{x_e, y_d} = \frac{\text{Cov}(x_e, y_d)}{\sigma_{x_e} \cdot \sigma_{y_d}} \quad R_{x_e, y_d} = \frac{-1.12}{0.66 \times 2.30} = \frac{-1.12}{1.52} = -0.74$$

The result shows that the correlation between treasury bill \((x_e)\) and risk premium\((y_d)\) is negative. They are positively associated with each and their relationship is negative as both are associated with almost no market risk besides the default credit risk, as treasury bills are guaranteed by the federal government.
ADDITIONAL DATA

Figure 1.12

<table>
<thead>
<tr>
<th>Period</th>
<th>Italy</th>
<th>UK</th>
<th>USA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1920-1929</td>
<td>5.68</td>
<td>4.63</td>
<td>4.09</td>
</tr>
<tr>
<td>1930-1939</td>
<td>4.74</td>
<td>3.54</td>
<td>3.34</td>
</tr>
<tr>
<td>1940-1949</td>
<td>4.16</td>
<td>3.06</td>
<td>2.31</td>
</tr>
<tr>
<td>1950-1959</td>
<td>6.12</td>
<td>4.31</td>
<td>2.99</td>
</tr>
<tr>
<td>1960</td>
<td>5.24</td>
<td>5.4</td>
<td>4.01</td>
</tr>
<tr>
<td>1960-1969</td>
<td>5.7</td>
<td>6.53</td>
<td>4.51</td>
</tr>
<tr>
<td>1970-1979</td>
<td>10.91</td>
<td>11.77</td>
<td>6.87</td>
</tr>
<tr>
<td>1980-1989</td>
<td>14.42</td>
<td>10.42</td>
<td>10.4</td>
</tr>
</tbody>
</table>

Table -10, Source: Sidney Homer and Richard Sylla 1991

Figure 1.12

Source: Sidney Homer and Richard Sylla 1991
In Equity market, the risk is always higher compare to debt market. The risk line R is more straight than R₁. As cost is concerned, equity always pays higher cost, whereas, debt pays comparable less cost. In above graph in equity market, as the amount of investment increases, so the amount of risk also increases and same effect is also on cost of equity with the increasing amount of investment. While debt market is less volatile and the returns are low but equity investment associates with high volatility and the returns are always up and down as shown in graph. In equity market, as shown in graph, Q line shows large variance(high risk), whereas, Q₁ in debt market presents small variance (low risk).