A note on the inefficiency of bargaining over the price of a share

Athanassoglou, Stergios and Brams, Steven J. and Sethuraman, Jay

1 August 2010

A Note on the Inefficiency of Bidding over the Price of a Share

Stergios Athanassoglou [*] Steven J. Brams [†] Jay Sethuraman[‡]

May 2010; revised August 2010

Abstract

We study the problem of dissolving a partnership when agents have unequal endowments. Agents bid on the price of the entire partnership. The highest bidder is awarded the partnership and buys out her partners' shares at a per-unit price that is a function of the two highest bids. We show that there exists no price-setting mechanism satisfying certain mild regularity properties that is ex-post efficient, for any common prior of valuations. This result sharply contrasts the equal-endowment case in which efficient dissolution of the sort we are examining is possible through a simple k-double auction, as suggested by Cramton, Gibbons, and Klemperer [3].

Key Words: Partnership Dissolution; Double Auction; Ex-Post Efficiency

JEL Classification: C72, C78

[*]The Earth Institute, Columbia University, New York, NY; email: sa2164@columbia.edu

[†]Department of Politics, New York University, New York, NY; email: steven.brams@nyu.edu.

[‡]IEOR Department, Columbia University, New York, NY; email: jay@ieor.columbia.edu. We thank two anonymous referees and an associate editor for their helpful comments. We are especially grateful to one of the two referees for pointing out a few typos and omissions in the original proof of Proposition 1. His/her close scrutiny of our derivation has resulted in a substantial improvement of our work. We thank Vikram Manjunath and Petros Milionis for providing comments and suggestions on an earlier draft of the manuscript.

1 Introduction

We consider a class of natural bidding mechanisms for dissolving a partnership. Agents submit sealed bids, which determine the price per unit share of the partnership. In every mechanism in the class we consider, the price per unit share depends only on the first and second-highest bids, and is (weakly) increasing in each of these components; moreover, whenever there is a tie for highest bid, the price per unit share is this bid. The mechanism always awards the partnership to an agent submitting the highest bid (ties are broken arbitrarily), whom we call the *winner*; if p is the price per unit share, any losing agent who initially owned r shares is paid rp by the winning agent. A mechanism is *ex-post efficient* if it always awards the partnership to an agent who values it the most. Our main result is that *no mechanism* in this class is ex-post efficient if agents own unequal shares initially.

The partnership-dissolution problem has a rich history in economic theory and has been extensively studied from the mechanism-design point of view. Myerson and Satterthwaite [13] focus on the case of complete ownership asymmetry (i.e., a buyer-seller framework) and characterize all incentive-compatible and interim individually rational (i.e., affording positive expected profit in the interim stage) mechanisms, showing these two properties to be incompatible with ex-post efficiency. An important generalization of the Myerson-Satterthwaite model, due to Cramton, Gibbons and Klemperer [3], considers n agents with agent i owning a fraction r_i of the partnership.[1] They characterize the set of all incentive-compatible, individually rational, and ex-post efficient mechanisms for dissolving a partnership. They show that such a dissolution is possible if and only if $(r_1, r_2, \ldots, r_n)$ is sufficiently close to the equal-endowment vector, and that such a dissolution is impossible for extreme cases of ownership asymmetry. Several authors have extended the literature to address interdependent valuations (Fieseler et al. [5], Kittsteiner [10], Galavotti et al. [8]), fairness (Morgan [12]), as well as the explicit modeling of popularly observed "shootout" clauses (de Frutos and Kittsteiner [7], Brooks et al. [1]).

Our paper is motivated by the k-double auction[2] of Cramton et al. [3, §6]. Suppose there are n agents, with agent i owning a fraction r_i of the partnership. In a k-double auction, the agents submit sealed bids, and the partnership is awarded to the highest bidder who pays each of the $n-1$ other agents an amount

$$\frac{1}{n}\left[(1-k)b_{(1)} + kb_{(2)}\right], \tag{1}$$

where $k \in [0,1]$ and $b_{(1)}$ and $b_{(2)}$ denote the highest and second-highest bids, respectively. If

[1] The Myerson-Satterthwaite model is the special case of $n = 2$, $r_1 = 1$, and $r_2 = 0$.

[2] They refer to it as a $k+1$-price auction.

$r_i = 1/n$ for all agents i, then the k-double auction is ex-post efficient and interim individually rational [3]. Furthermore, these properties continue to hold as long as the initial ownership vector is sufficiently close to the equal-endowment vector [3, Proposition 6]. The k-double auction has been extensively studied since then: notable contributions include the work of McAfee [11], who considers risk-aversion, of de Frutos [6], who considers agents with asymmetric priors, and of Kittsteiner [10], who models interdependent valuations. All of these papers focus on symmetric environments with agents owning identical shares of the partnership.

In spite of its desirable properties, the k-double auction is not realistic in the case of asymmetric ownership. This is because the payment received by any agent who does not win is insensitive to the relative fraction of the partnership that he owns (see Cramton [3, Proposition 5]). As an example, consider $n = 3$, $r_1 = r_2 = 0.49$, $r_3 = 0.02$. In this setting, the price of Eq. (1) is strange—If agent 1 outbids 2 and 3, why should agents 2 and 3 receive the same monetary payment even though they sell very different fractions of the partnership to agent 1? More generally, it turns out that the equilibrium strategies for the agents, determined by Cramton et al., are independent of the shares they own, so an agent will, in expectation, pay or receive the exact same amount from her opponent *regardless of her endowment.* Agents' initial shares come into play only in the verification of interim individual rationality.[3]

The salience of this point is echoed by the legal process of partnership dissolution in which an exogenous (in the form of an independent audit) or endogenous (in the form of inter-agent bargaining) procedure is used to determine the value of the entire partnership. Once this quantity has been set, it is multiplied by agents' individual shares to yield their respective monetary claims. As an example, consider the recent, highly controversial acquisition of Bear Stearns by J.P. Morgan. This dissolution involved many asymmetric stakeholders (see Wright[4] for a list of Bear Stearns' 15 biggest shareholders as of December 2007); in the end the government brokered an agreement that set a per-share price on Bear Stearns, and shareholders were compensated according to their individual stakes. For all these reasons, it is natural to examine a class of mechanisms that fix a *price per unit share* based on the bids submitted by the agents.

[3]An exception is found in Bulow et al. [2], who assume different ownership shares and study special cases of the k-double auction under the assumption of uniform priors. In their model, the bidding is over the per-unit price of the partnership, and agents' claims are then adjusted accordingly. Using a special profit function, they explicitly calculate equilibrium strategies and show that they are unique in the cases of $k = 0$ and $k = 1$. On a related note, Engelbrecht-Wiggans [4] studies the implications of giving equal-endowment agents a variable share in the proceeds of a first- and second-price auction.

[4]W. Wright (March 17, 2008), "Employees lose 5bn on Bear Stearns After Knock-Down Sale to JP Morgan," Financial News Online.

In this environment *ex-post efficiency* proves to be an elusive goal, unless agents have equal endowments. The proof of this result (appearing in Section 2) requires some analysis, but the intuition is clear: When bidders are even the slightest bit asymmetric, their marginal bidding incentives are different. Thus, their bidding strategies cannot be identical, and inefficiency cannot be ruled out. This negative result holds for all possible common priors on agents' valuations.

2 Main Results

Suppose there are n agents with agent i owning r_i shares of the partnership. We normalize the total number of shares to 1, so that r_i is interpreted as the fraction of the partnership that agent i owns. We assume that $(r_1, r_2, \ldots, r_n) \neq (1/n, 1/n, \ldots, 1/n)$; this implies that there exists (at least) one pair of agents i and j with $r_i \neq r_j$. Agent i values the partnership at v_i, which is private information, but it is common knowledge that the valuations are drawn independently from a distribution F with positive, continuous density f in the interval $[0, 1]$. Our goal is to find a natural bidding mechanism that is *ex-post efficient*, i.e., one that always awards the partnership to an agent with the highest valuation.

The focus of the paper is on a class of mechanisms in which the agents submit bids for the partnership. Every mechanism in the class we consider awards the partnership to the highest bidder, breaking ties arbitrarily. The mechanism also computes $p(b_1, b_2, \ldots, b_n)$, which is the *price* per unit share of the partnership if the bids submitted are $(b_1, b_2, \ldots, b_n)$. Any agent j who is not awarded the partnership receives a monetary payment of $r_j p(b_1, b_2, \ldots, b_n)$ from the agent who wins the partnership (thus, the mechanism is *budget-balanced*). We assume that agents have quasi-linear preferences so that an agent with valuation v, who owns a share r of the partnership and has money m, has a utility of $rv + m$. Thus, if agent i submits the highest bid and is awarded the partnership, her utility is v_i, resulting in a profit of $(v_i - p)(1 - r_i)$; for every other agent $j \neq i$, the utility is pr_j, resulting in a profit of $(p - v_j)r_j$.

Restriction to price-function class $\mathcal{P}$. To specify a mechanism, we need to describe how the price per unit share $p(b_1, b_2, \ldots, b_n)$ is computed, given a bid-vector. We consider a class of mechanisms in which the price function depends only on the highest and second-highest bid; in particular, for the mechanisms we consider, the price per unit share is independent of the *identity* of the bidders. Thus, if $b_{(1)}$ and $b_{(2)}$ denote the first and second highest bids in a given bid-vector, the price per unit share is denoted simply $p\big(b_{(1)}, b_{(2)}\big)$. Furthermore, we assume that the function $p(\cdot, \cdot)$ is weakly increasing in its two arguments, and that $p(b, b) = b$. Let $\mathcal{P}$ denote the class of price functions satisfying these three properties. (Note that Groves [9] mechanisms imply per-unit price functions that are not anonymous, and therefore are not in $\mathcal{P}$.)

Our first result shows that *any* ex-post efficient mechanism with a price function $p \in \mathcal{P}$ must induce a symmetric and strictly increasing equilibrium that is unique and in which agents bid their valuations. Furthermore, the price per unit-share should be *insensitive* to changes in bids almost everywhere.

Proposition 1 *Suppose* $(r_1, r_2, \ldots, r_n) \neq (1/n, 1/n, \ldots, 1/n)$ *and consider the bidding mechanism with a price function* $p \in \mathcal{P}$*. Suppose the mechanism awards the partnership to the highest bidder at a price per unit share* p*. If the mechanism is ex-post efficient, then*

(a) it induces a symmetric and strictly increasing equilibrium that is unique and in which all agents bid their valuations,[5] *and*

(b) p *must satisfy* $\frac{\partial p}{\partial b_{(1)}} = \frac{\partial p}{\partial b_{(2)}} = 0$ *almost everywhere.*

Proof. Fix a mechanism with a price-function $p \in \mathcal{P}$, and assume that agents' valuations are i.i.d. random variables with a cdf F. By ex-post efficiency, there must exist **at least one** symmetric and strictly increasing equilibrium bidding strategy. In what follows, we will show that this strategy must be unique, and that it must consist of agents truthfully bidding their valuations.

To this end, pick a symmetric, strictly increasing strategy $h(v)$ and suppose agent i conjectures that all other agents bid according to it. Let $\pi_i(v_i, b_i)$ be her expected profit, given a valuation of v_i and a bid of b_i. Denote by G the distribution of the maximum of the other $n-1$ valuations, so that $G(u) = F(u)^{n-1}$. In addition denote by $Z(\cdot|u)$ the distribution of the second-highest of the other $n-1$ valuations, given a highest of u among them, so that $Z(y|u) = [F(y)/F(u)]^{n-2}$ for $y \leq u$. We may write

$$\begin{aligned} \pi_i(v_i, b_i) &= \int_0^{h^{-1}(b_i)} (1 - r_i)(v_i - p(b_i, h(u)))dG(u) + \int_{h^{-1}(b_i)}^1 \int_0^{h^{-1}(b_i)} r_i(p(h(u), b_i) - v_i)dZ(y|u)dG(u) \\ &\quad + \int_{h^{-1}(b_i)}^1 \int_{h^{-1}(b_i)}^u r_i(p(h(u), h(y)) - v_i)dZ(y|u)dG(u). \end{aligned} \tag{2}$$

[5] Note that the uniqueness result extends only to symmetric strictly increasing equilibria. In particular, it does not preclude the existence of asymmetric equilibria. But these are of no interest since they would immediately fail ex-post efficiency, which requires symmetric and strictly increasing bidding strategies.

For the strategy h to be an ex-post efficient equilibrium, it must satisfy the necessary first-order conditions of the associated optimization problem. Thus, we differentiate Eq. (2) with respect to b_i and set the derivative equal to 0 at $b_i = h(v_i)$. Recalling that $p(b,b) = b$ for all b obtains

$$\int_0^{v_i} -(1-r_i)\frac{\partial}{\partial b_{(1)}} p(h(v_i), h(u))dG(u) + \frac{(1-r_i)(v_i - h(v_i))g(v_i)}{h'(v_i)} + \\ r_i \frac{d}{db_i}\Bigg[\int_{h^{-1}(b_i)}^{1} \int_0^{h^{-1}(b_i)} (p(h(u), b_i) - v_i)dZ(y|u)dG(u) + \\ \int_{h^{-1}(b_i)}^{1} \int_{h^{-1}(b_i)}^{u} (p(h(u), h(y)) - v_i)dZ(y|u)dG(u)\Bigg]_{b_i = h(v_i)} = 0. \tag{3}$$

Now suppose $v_i = v$ for all i, so that $b_i = b = h(v)$ for all i. Consequently, we apply Eq. (3) to all agents and then add all of the resulting equalities. Using the fact that $p \in \mathcal{P}$, and therefore does not depend on the identity of the two highest bidders, we obtain

$$-(n-1)\int_0^{v} \frac{\partial}{\partial b_{(1)}} p(h(v), h(u))dG(u) + \frac{(n-1)(v - h(v))g(v)}{h'(v)} + \\ \frac{d}{db}\Bigg[\int_{h^{-1}(b)}^{1} \int_0^{h^{-1}(b)} (p(h(u), b) - v)dZ(y|u)dG(u) + \\ \int_{h^{-1}(b)}^{1} \int_{h^{-1}(b)}^{u} (p(h(u), h(y)) - v)dZ(y|u)dG(u)\Bigg]_{b = h(v)} = 0. \tag{4}$$

On the other hand, focusing on two agents i and j such that $r_i \neq r_j$, we subtract their respective Eqs. (3) when $v_i = v_j = v$. Taking into account that $r_i \neq r_j$ obtains

$$\int_0^{v} \frac{\partial}{\partial b_{(1)}} p(h(v), h(u))dG(u) - \frac{(v - h(v))g(v)}{h'(v)} + \frac{d}{db}\Bigg[\int_{h^{-1}(b)}^{1} \int_0^{h^{-1}(b)} (p(h(u), b) - v)dZ(y|u)dG(u) + \\ \int_{h^{-1}(b)}^{1} \int_{h^{-1}(b)}^{u} (p(h(u), h(y)) - v)dZ(y|u)dG(u)\Bigg]_{b = h(v)} = 0. \tag{5}$$

Subtracting Eq. (5) from Eq. (4) yields

$$\int_0^{v} \frac{\partial}{\partial b_{(1)}} p(h(v), h(u))dG(u) = \frac{(v - h(v))g(v)}{h'(v)}. \tag{6}$$

Since $g(v) > 0$, $h'(v) > 0$, and $p \in \mathcal{P}$ (which implies that $\partial p/\partial b_{(1)} \geq 0$), we apply the above argument to all v and conclude that

$$h(v) \leq v, \;\; \forall v \in [0,1]. \tag{7}$$

We turn to proving the other side of the inequality. First, we multiply Eq. (5) by $n-1$, and add it to Eq. (4) to obtain

$$\frac{d}{db}\Bigg[\int_{h^{-1}(b)}^{1}\int_{0}^{h^{-1}(b)}(p(h(u),b)-v)dZ(y|u)dG(u)+ \\ \int_{h^{-1}(b)}^{1}\int_{h^{-1}(b)}^{u}(p(h(u),h(y))-v)dZ(y|u)dG(u)\Bigg]_{b=h(v)}=0. \tag{8}$$

Eq. (8) can be simplified to

$$-v\frac{d}{db}\Big[1-G(h^{-1}(b))\Big]_{b=h(v)}+\frac{d}{db}\Bigg[\int_{h^{-1}(b)}^{1}p(h(u),b)F(h^{-1}(b))^{n-2}(n-1)f(u)du\Bigg]_{b=h(v)}+ \\ \frac{d}{db}\Bigg[\int_{h^{-1}(b)}^{1}\int_{h^{-1}(b)}^{u}p(h(u),h(y))dZ(y|u)dG(u)\Bigg]_{b=h(v)}=0. \tag{9}$$

Performing the necessary differentiations obtains

$$-v\frac{d}{db}\Big[1-G(h^{-1}(b))\Big]_{b=h(v)}=\frac{v(n-1)F(v)^{n-2}f(v)}{h'(v)}, \tag{10}$$

$$\frac{d}{db}\Bigg[\int_{h^{-1}(b)}^{1}p(h(u),b)F(h^{-1}(b))^{n-2}(n-1)f(u)du\Bigg]_{b=h(v)}=-\frac{h(v)(n-1)F(v)^{n-2}f(v)}{h'(v)}+ \\ (n-1)\int_{v}^{1}\left[\frac{\partial}{\partial b_{(2)}}p(h(u),h(v))F(v)^{n-2}+p(h(u),h(v))(n-2)F(v)^{n-3}\cdot\frac{f(v)}{h'(v)}\right]f(u)du, \tag{11}$$

and (by performing a double application of the Leibniz rule)

$$\frac{d}{db}\Bigg[\int_{h^{-1}(b)}^{1}\int_{h^{-1}(b)}^{u}p(h(u),h(y))dZ(y|u)dG(u)\Bigg]_{b=h(v)}= \\ -\Bigg[\int_{h^{-1}(b)}^{1}p(h(u),b)dZ(h^{-1}(b)|u)dG(u)\cdot\frac{dh^{-1}}{db}\Bigg]_{b=h(v)}= \\ -\int_{v}^{1}p(h(u),h(v))(n-2)F(v)^{n-3}(n-1)f(u)du\cdot\frac{f(v)}{h'(v)}. \tag{12}$$

Combining Eqs. (10), (11), and (12) into Eq. (9) and dividing by $n-1$ yields

$$-\int_{v}^{1}\frac{\partial}{\partial b_{(2)}}p(h(u),h(v))F(v)^{n-2}f(u)du=\frac{(v-h(v))F(v)^{n-2}f(v)}{h'(v)}. \tag{13}$$

Identical reasoning as before necessitates

$$h(v)\geq v,\ \ \forall v\in[0,1]. \tag{14}$$

Combining inequalities (7) and (14) establishes that

$$h(v) = v, \ \ \forall v \in [0, 1].$$

The above argument shows that $h(v) = v$ is the **unique** symmetric and strictly increasing bidding strategy that could satisfy the first-order conditions for an ex-post efficient equilibrium that are exhibited by Eq. (3). Therefore, since p is assumed to be ex-post efficient, $h(v) = v$ must be the unique ex-post efficient equilibrium induced by p. This concludes the proof of part (a).

We turn to proving part (b). By part (a), we must have

$$h(v) = v, \ \ \forall v \in [0, 1].$$

Thus, Eqs. (6) and (13) obtain

$$\int_v^1 \frac{\partial}{\partial b_{(2)}} p(u, v) F(v)^{n-2} f(u) du = \int_0^v \frac{\partial}{\partial b_{(1)}} p(v, u) F(u)^{n-2} f(u) du = 0, \ \ \forall v \in [0, 1]. \tag{15}$$

Eq. (15) can hold for all $v \in [0, 1]$ only if

$$\frac{\partial p}{\partial b_{(1)}} = \frac{\partial p}{\partial b_{(2)}} = 0 \text{ almost everywhere.}$$

■

Proposition 1 implies the following interesting fact. If we could find an ex-post efficient mechanism with a price function belonging in $\mathcal{P}$, then this same mechanism would be ex-post individually rational. That is, this mechanism would always result in non-negative profits for all players, regardless of their valuations. This is a consequence of two things. First, since p belongs to $\mathcal{P}$, the price it gives rise to always lies between the two bids. Given this fact, the necessary truthfulness of part (a) ensures that ex-post profits are always positive. Such a result would be remarkably strong; it should therefore come as no surprise that ex-post efficiency is not possible within our framework. Finally, notice how the assumption of unequal shares is critical: Proposition 1 is not true under symmetric ownership since the k-double auction belongs in $\mathcal{P}$ and induces a non-truthful symmetric and strictly increasing equilibrium (Proposition 5 in [3]). It also clearly violates part (b).

Theorem 1 *Suppose* $(r_1, r_2, \ldots, r_n) \neq (1/n, 1/n, \ldots, 1/n)$ *and consider the bidding mechanism with a price function* $p \in \mathcal{P}$. *Suppose the mechanism awards the partnership to the highest bidder at a price per unit-share* p. *This mechanism cannot be ex-post efficient for any common prior.*

Proof. Fix a common prior F and assume that a mechanism $p \in \mathcal{P}$ is ex-post efficient. By part (a) of Proposition 1, p must also be incentive compatible. Suppose agent i has a valuation of v. For an agent i the equilibrium expected transfer function that the mechanism p gives rise to is the following

$$-\int_0^v (1-r_i)p(v,u)dG(u) + \int_v^1 \int_0^v r_i p(u,v)dZ(y|u)dG(u) + \int_v^1 \int_v^u r_i p(u,y)dZ(y|u)dG(u).$$

Since this transfer scheme is incentive-compatible and ex-post efficient, Lemma 1 in Cramton et al. [3] applied to the appropriate ex-post efficient share function implies that

$$\begin{aligned}
&-\int_0^{v_1} (1-r_i)p(v_1,u)dG(u) + \int_{v_1}^1 \int_0^{v_1} r_i p(u,v_1)dZ(y|u)dG(u) + \int_{v_1}^1 \int_{v_1}^u r_i p(u,y)dZ(y|u)dG(u)\\
&+\int_0^{v_2} (1-r_i)p(v_2,u)dG(u) - \int_{v_2}^1 \int_0^{v_2} r_i p(u,v_2)dZ(y|u)dG(u) - \int_{v_2}^1 \int_{v_2}^u r_i p(u,y)dZ(y|u)dG(u) =\\
&\int_{v_1}^{v_2} u dG(u), \quad \forall v_1, v_2 \in [0,1]. \qquad (16)
\end{aligned}$$

Focusing on the case when $v_1 = 1$ and $v_2 = 0$, we differentiate both sides of Eq. (16) with respect to r_i to obtain

$$\begin{aligned}
&\int_0^1 p(1,u)dG(u) - \int_0^1 \int_0^u p(u,y)dZ(y|u)dG(u) = 0\\
\Rightarrow \quad &\int_0^1 \int_0^u (p(1,u) - p(u,y))dZ(y|u)dG(u) = 0. \qquad (17)
\end{aligned}$$

Since $p \in \mathcal{P}$ we must have $p(1,u) \geq p(u,y)$ for all $u \in [0,1], \;\; y \leq u$. Thus, Eq. (17) necessitates

$$p(1,u) = p(u,y), \;\; \text{for almost all } \; u \in [0,1], \;\; y \leq u. \qquad (18)$$

Now, again because $p \in \mathcal{P}$ we can deduce

$$\begin{aligned}
p(1,u) \quad &\geq u \qquad \forall u \in [0,1], \qquad (19)\\
p(u,y) \quad &\leq u \qquad \forall u \in [0,1], \;\; y \leq u. \qquad (20)
\end{aligned}$$

Expressions (18), (19), and (20) ensure that

$$p(u,y) = u, \;\; \text{for almost all } \; u \in [0,1], \;\; y \leq u. \qquad (21)$$

Eq. (21) contradicts part (b) of Proposition 1. ■

Theorem 1 establishes that ex-post efficiency is unattainable in the class of per-unit prices $\mathcal{P}$. The key property driving the impossibility result is anonymity. If anonymity is relaxed so that the price per share paid by an agent is allowed to depend on his endowment, the impossibility result no longer holds: the classical double auction, which satisfies all the other properties that define the class $\mathcal{P}$, is an ex-post efficient mechanism.

3 Conclusion

In this paper we re-examine the partnership-dissolution problem and the bidding procedures that it gives rise to. In contrast to the existing literature, we assume that agents' bids determine the per-unit price of the partnership. We find that ownership asymmetry complicates the quest for economic efficiency: Given relatively mild assumptions on the mechanism's price function, we show that ex-post efficiency is unattainable when endowments are unequal.

References

[1] R.R. Brooks, C. Landeo, and K.E. Spier (2009), "Trigger Happy or Gun Shy? Dissolving Common-Value Partnerships with Texas Shootouts," University of Alberta, *mimeo.*

[2] J. Bulow, M. Huang, and P. Klemperer (1999), "Toeholds and Takeovers," *Journal of Political Economy*, 107, 427–454.

[3] P. Cramton, R. Gibbons, and P. Klemperer (1987), "Dissolving a Partnership Efficiently," *Econometrica*, 55, 615–632.

[4] R. Engelbrecht-Wiggans (1994), "Auctions with Price-Proportional Benefits to Bidders," *Games and Economic Behavior*, 6, 339–346.

[5] K. T. Fieseler, T. Kittsteiner, and B. Moldovanu (2003), "Partnerships, Lemons, and Efficient Trade," *Journal of Economic Theory*, 113, 223–234.

[6] M. A. de Frutos (2000), "Asymmetric Price-Benefit Auctions," *Games and Economic Behavior*, 33, 48-71.

[7] M. A. de Frutos and T. Kittsteiner (2008), "Efficient partnership dissolution under buy/sell clauses," *Rand Journal of Economics*, 39, 184–198.

[8] S. Galavotti, N. Muto, and D. Oyama (2008), "On Efficient Partnership Dissolution under Ex-Post Individual Rationality," *Economic Theory*, forthcoming.

[9] T. Groves (1973), "Incentives in Teams," *Econometrica*, 617-631.

[10] T. Kittsteiner (2003), "Partnerships and Double Auctions with Interdependent Valuations," *Games and Economic Behavior* 44, 54–76.

[11] R. P. McAfee (1992), "Amicable Divorce: Dissolving a Partnership with Simple Mechanisms," *Journal of Economic Theory*, 56, 266–293.

[12] J. Morgan (2004), "Dissolving a Partnership (Un)Fairly," *Economic Theory*, 23, 909–923.

[13] R. Myerson and M. Satterthwaite (1983), "Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory*, 29, 265–281.