Financial Development and Amplification

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Abstract

Does financial development exacerbate or dampen financial amplification? This paper develops a macroeconomic model with the borrowing constraint and heterogeneous agents to answer this question. In our framework, financial development produces two competing forces. One is the effect which accelerates amplification by strengthening balance sheet effects. The other is the effect which reduces it, we call shock cushioning effects. Whether financial development exacerbates or dampens amplification depends on the balance of two effects. We find that the relation between financial development and amplification is non-monotone: amplification initially increases with financial development and later falls down.

Key Words: Non-Monotonicity, Balance sheet effects, Shock cushioning effects, the borrowing constraint, heterogeneous agents

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1 Introduction

What are the effects of the development of financial markets on amplification over the business cycle? Traditional wisdom suggests that financial

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development stabilizes the economy by providing various channels for risk diversification. According to this view, financial innovation not only promotes long-run economic growth by enhancing efficiency in resource allocation, but also it helps to cushion consumers and producers from the effects of economic shocks.[1] This classical view seems to have been widely accepted. Indeed, several empirical and quantitative studies support the positive role of financial development in reducing volatility (See Cecchetti et al, 2006; Dynan et al, 2006; Jerman and Quadrini, 2008).

However, the situation has begun to change dramatically since the outbreak of the credit crisis of 2007-08. A new perspective has emerged: financial development destabilizes the economy by accelerating financial amplification. Before the crisis, it was often pointed out that thanks to financial innovation, the leverage of borrowers increased, and this high leverage generated economic booms. However, once the credit crisis occurred, people began to state that such a high leverage could lead to significant damages in borrowers' balance sheets, and eventually in the financial system as a whole. Financial development is suddenly blamed for increasing volatility. Indeed, IMF (2006, 2008) supports this new view by presenting empirical evidences that in more-advanced financial systems, the shock propagation effects become stronger.[2]

Motivated by these conflicting views, this paper theoretically investigates whether financial development accelerates or dampens financial amplification (macroeconomic volatility). To do so, we propose a macroeconomic model with the borrowing constraint and heterogeneous agents. In our model, financial development produces two competing forces. One is the effect which accelerates amplification by strengthening *balance sheet effects.*[3] The other is the effect which dampens amplification, we call *shock cushioning effects*. Depending on which of these dominates, whether financial development exacerbates or weakens financial propagation is determined. Moreover, the balance between these two conflicting effects changes according to the degree of financial development.

[1]Levine (1997), Beck et al. (2000) show empirically that financial development causes long-run economic growth. Castro et al. (2004) and Khan and Ravikumar (2001) examine the impact of financial development including investor protection and risk-sharing on growth theoretically as well as empirically or numerically.

[2]IMF reports argue that the sensitivity of real GDP growth rate, corporate investment, household consumption, and residential investment response to equity busts, or business cycles, is increasing in more market-based financial systems.

[3]See Bernanke et al. (1996) for balance sheet effects.

Our main result shows that in a low level of financial development, while *shock cushioning effects* do not work well, financial development enhances *balance sheet effects* through raising leverage, thereby accelerating financial amplification. However, once the level of development passes a certain degree, *shock cushioning effects are generated* through an adjustment of the interest rate, which in turn weakens *balance sheet effects*, thereby dampening financial amplification. Hence, the relation between financial development and financial amplification is non-monotone: financial amplification initially increases with financial development and later falls down.

This paper is related to a number of researches on business cycle theory which emphasize the role of credit market imperfections. Following the seminal work by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), some researchers put financial factors a central role in accounting for business fluctuations (See Holmstrom and Tirole, 1997; Kiyotaki 1998, ; Bernanke et al., 1999; Kocherlakota, 2000; Cordoba and Ripoll, 2004). These studies demonstrate how shocks are amplified through *balance sheet effects*, assuming a fixed degree of financial development. The contribution of our paper is to examine how the sensitivity to the shocks changes as the degree of financial development changes.

Our paper is also related to Cooley et al. (2004), Rajan (2006), and Shin (2009) with regard to the effects of financial development on amplification (volatility). Cooley et al. emphasize a negative relation between the degree of contract enforceability, which corresponds to the degree of financial development in our paper, and aggregate volatility. They show that economies in which contracts are less enforceable display greater volatility of output than economies with stronger enforceability of contracts. The paper generates only a monotone relation. Our paper, however, generates a non-monotone dependence of volatility from financial development. Rajan argues that financial development has made the world better off, however it can accentuate real fluctuations, and economies may be more exposed to financial-sector-induced turmoil than in the past. However, Rajan does not necessarily propose a formal model of how financial development accelerates financial amplification. Shin presents a theoretical model where securitization by itself may not enhance financial stability. Our study shows within one framework that financial development initially accelerates amplification and later reduces it.

Concerning this non-monotone relation between financial development and amplification, Aghion et al. (1999) and Matsuyama (2007, 2008) are related to ours. Aghion et al. show that volatility is low when the development level is low or high. High volatility (cycles in their paper) occurs when the level has an intermediated value. Our paper also shows that volatility is high when financial development is an intermediated level. However, the source of high volatility is different from their paper. In their model, a change in the interest rate has a role in increasing volatility while in our model, it has a role in reducing volatility.[4] In our model, high volatility is caused by *balance sheet effects* with high leverage. Matsuyama develops a model of the borrowing constraint with various types of heterogeneities in an overlapping generations framework, and shows how it leads to a wide range of non-monotone phenomena. In Matsuyama's model, the source of non-monotonicity lies in the investment projects which do not produce capital goods. Matsuyama shows that a better credit market might be more prone to financing those investment projects, and such a change in credit allocation generates non-monotonicity. In our paper, the source of non-monotonicity lies in the adjustment of the interest rate which yields *shock cushioning effects*.

The remainder of the paper is organized as follows. Section 2 presents the model. We analyze the dynamics and derive implications for the relationship between financial development and financial amplification. Section 3 presents conclusion.

2 The Model

The model is an extension of Kiyotaki (1998). Consider a discrete-time economy with two types of goods, consumption goods and capital goods and two types of agents, entrepreneurs and workers. Let us start with the entrepreneurs, who are the central actors in the paper. At date t, a typical entrepreneur has expected discounted utility:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \log c_t \right], \tag{1}$$

[4] In Aghion et al., a rise (decline) in the interest rate during booms (recessions) increases (reduces) debts repayment, which in turn produces recessions (booms). In this way, endogenous cycles with high volatility occur.

where c_t is the consumption at date t, and $\beta \in (0,1)$ is the subjective discount factor, and $E_0[x]$ is the expected value of x conditional on information at date 0.

Each entrepreneur can access investment projects to produce capital. Every entrepreneur can access low profitable investment projects, but only some of the entrepreneurs, called H-entrepreneurs can access high profitable investment prejects. The rest of the entrepreneurs we call L-entrepreneurs. The investment technology follows

$$k_{t+1} = \alpha^i z_t, \tag{2}$$

where z_t is investment of goods at date t. α is the marginal productivity of investment, and $i \in \{H, L\}$ is the index for the marginal productivity of high and low profitable investment, respectively. k_{t+1} is capital produced at date $t+1$. We assume that capital depreciates fully after production. We also assume $\alpha^H > \alpha^L$.

Each type of investment projects is associated with agency problems (Hart and Moore (1994), Tirole (2006)). The entrepreneurs who undertake high (low) profitable investment projects can pledge only a fraction θ^H (θ^L) of future returns from the investment. This fraction θ^H or θ^L can be collateral in borrowing. We assume that θ^H is less than θ^L. That is, the degree of agency frictions is less severe in low profitable investment.

In addition, each entrepreneur knows his/her own type at date t of whether or not he/she has high profitable investment projects, but only knows it with probability after date $t+1$. That is, each entrepreneur shifts stochastically between two states according to a Markov process: the state with high profitable investment or the state without it. Specifically, an entrepreneur who has high (low) profitable investment at date t may have high profitable at date $t+1$ with probability p ($X(1-p)$). This probability is exogenous, and independent across entrepreneurs and over time. Assuming that the initial ratio of the entrepreneurs who have high and only low profitable investment is $X : 1$, the population ratio is constant over time. We assume that the probability is not too large:

$$\text{Assumption}: p > X(1-p). \tag{3}$$

This assumption implies that there is a positive correlation between the present period and the next period. That is, the entrepreneur who has high profitable investment in the current period continues to have it next period with higher probability than the one who has only low profitable investment in the current period.

The entrepreneur's flow of funds constraint is given by

$$c_t + z_t = q_t k_t - r_{t-1} b_{t-1} + b_t, \tag{4}$$

where r_{t-1} and b_t are the gross real interest rate, and the amount of borrowing at date $t-1$ and t, respectively. q_t is the relative price of capital to consumption goods. The left hand side of (4) is expenditure: consumption and investment. The right hand side is financing: the returns from investment in the previous period minus debts repayment, which we call net worth in this paper, and the amount of borrowing.

Because of the agency problems concerning the investment projects, the entrepreneur faces the borrowing constraint.[5] In such a situation, in order for debt contracts to be credible, debts repayment does not exceed the value of collateral. That is, the borrowing constraint becomes

$$r_t b_t \leq \theta^i q_{t+1} \alpha^i z_t. \tag{5}$$

Here, without loss of generality, we assume that θ^L is equal to one, and define θ^H to be θ. The parameter θ partly reflects the legal structure and the transaction costs in the liquidation of investment. In this sense, θ provides a simple measure of financial development. In this paper, we define an increase in θ as a financial development.

Each entrepreneur chooses consumption, investment, capital, and borrowing $\{c_t, z_t, k_{t+1}, b_t\}$ to maximize the expected discounted utility (1) subject to (2), (4), and (5).

Now, let's turn to the workers. There is only one type of workers. Each worker is endowed with one unit of labor each period, and supplies it inelastically in the labor market. Workers do not have investment project to produce capital, and therefore, do not have any collateral asset in order to borrow. At date t, a typical worker has expected discounted utility:

$$E_0 \left[\sum_{t=0}^{\infty} \beta'^t \log c'_t \right], \tag{6}$$

[5] As Matsuyama (2007, 2008) points out, there are several causes to justify the borrowing constraints from microeconomic literature (see Tirole 2006). Here, we do not get into the details about which ones are more appropriate.

where c'_t is consumption of workers at date t, and β' is the subjective discount factor of workers. We assume $\beta' < \beta$. This assumption ensures that in equilibrium workers will not choose to lend.

Each worker chooses consumption, and the amount of borrowing to maximize (6) subject to the flow of funds constraint and the borrowing constraint.

$$c'_t = w_t - r_{t-1} b'_{t-1} + b'_t, \tag{7}$$

$$r_t b'_t \leq 0, \tag{8}$$

where w_t and b'_t are the wage rate and the borrowing of the worker at date t.

There is a competitive final goods market. Production function of a representative firm is

$$Y_t = A K_t^{\prime,\sigma} N_t^{1-\sigma} \bar{k}_t^{1-\sigma}, \tag{9}$$

where A is productivity, and Y_t is output of the representative firm at date t.[6] K'_t and N_t are capital and labor inputs of the firm at date t. $\bar{k}_t$ is per-labor capital of this economy at date t, capturing the positive externality in the sense of Romer (1986).

Each firm chooses capital and labor inputs to maximize its profit, given the relative price of capital to consumption goods, q_t, the wage rate, w_t, and the externality, $\bar{k}_t$. Considering the equilibrium of $k'_t = \bar{k}_t$, we obtain $y_t = Ak'_t$, where k'_t, and y_t are per-labor capital and output of the firm. Because the worker's population is one, the aggregate capital input and output equal per-labor capital and output. Competitive factor prices produce

$$q_t = \sigma A, \qquad w_t = A(1-\sigma)k'_t. \tag{10}$$

Let us denote aggregate consumption of H-entrepreneurs, L-entrepreneurs, and workers at date t as C_t^H, C_t^L, and C'_t. Similarly, let Z_t^H, Z_t^L, B_t^H, B_t^L, and B'_t be aggregate investment, and the amount of borrowing of each type. Then, the market clearing for goods, credit, and capital are

$$C_t^H + C_t^L + C'_t + Z_t^H + Z_t^L = Y_t, \tag{11}$$

[6] Here, we suppose that each firm is operated by workers. Since the net profit of each firm is zero in equilibrium, the flow of funds constraint of the workers does not change, and is the same as (7).

$$B_t^H + B_t^L + B_t' = 0, \tag{12}$$

$$k_t' = K_t, \tag{13}$$

where K_t is the aggregate capital stock produced by the entrepreneurs at date t.

2.1 Equilibrium

The competitive equilibrium is defined as a set of prices $\{r_t, q_t, w_t\}_{t=0}^{\infty}$ and quantities $\left\{c_t, c_t', b_t, b_t', z_t, C_t^H, C_t^L, C_t', B_t^H, B_t^L, B_t', Z_t^H, Z_t^L, K_t', K_t, Y_t\right\}_{t=0}^{\infty}$ which satisfies the conditions that (i) each entrepreneur and worker maximizes utility, and each firm maximizes its profit, and (ii) the market for goods, labor, credit, and capital all clear. Because there is no shock except for the idiosyncratic shocks to the state of the entrepreneurs, there is no aggregate uncertainty, and the agents have perfect foresight about future prices and aggregate quantities in the equilibrium.

We are now in a position to characterize equilibrium behavior of entrepreneurs. Let us consider the case where θ is lower than θ_1 (θ_1 is defined later in Proposition 1. We use a method of guess-and verify here.). If θ is lower than θ_1, in the neighborhood of the steady state, the real interest rate equals the rate of return on low profitable investment (This can be verified in Proposition 1.). That is, we have

$$r_t = q\alpha^L. \tag{14}$$

And so, H-entrepreneurs prefer high profitable investment with maximum leverage. The borrowing constraint of H-entrepreneurs binds because the rate of return on their investment is greater than the real interest rate. Since the utility function is log, they consume a fraction $(1-\beta)$ of the net worth, $c_t = (1-\beta)(qk_t - r_{t-1}b_{t-1})$. Then, by using (4), and (5), the investment function of H-entrepreneurs becomes

$$z_t = \frac{\beta(qk_t - r_{t-1}b_{t-1})}{1 - \dfrac{q\theta\alpha^H}{r_t}}. \tag{15}$$

The numerator of (15) is the required down payment for unit investment.

From (15), we see that the investment equals the leverage, $1/\left[1-(q\theta\alpha^H/r_t)\right]$ times savings, $\beta(qk_t - r_{t-1}b_{t-1})$. The leverage is greater than one, and increases with θ. This implies that when θ is large, H-entrepreneurs can finance more investment with smaller net worth. We also see that the sensitivity of investment response to a change in the net worth becomes higher with θ. This implies that even a small decline (increase) in the net worth can have a large negative (positive) effect on the investment.

Concerning workers, in the neighborhood of the steady state, the borrowing constraint binds (This can be verified later in footnote 8.). Thus, they consume all the income at every date, $c_t' = w_t$. From this behavior of workers, credit market equilibrium, (12) becomes

$$B_t^H + B_t^L = 0. \tag{16}$$

To L-entrepreneurs, they are indifferent between lending and investing by themselves because the real interest rate is the same as the return on their investment. Their saving rate is also a fraction β of their net worth. Then, the aggregate lending and investment of them are determined by goods market clearing condition, (11).

Since consumption, debt and investment are linear functions of the net worth, we can aggregate across agents to find the law of motion of the aggregate capital:

$$\begin{aligned} K_{t+1} &= K_{t+1}^H + K_{t+1}^L = \alpha^H \frac{\beta E_t^H}{1-\dfrac{q\theta\alpha^H}{r_t}} + \alpha^L\left(\beta\sigma Y_t - \frac{\beta E_t^H}{1-\dfrac{q\theta\alpha^H}{r_t}}\right) \\ &= \left[1+\left(\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}\right)s_t\right]A\beta\sigma\alpha^L K_t, \end{aligned} \tag{17}$$

where K_{t+1}^H and K_{t+1}^L are the aggregate capital stock produced by H-entrepreneurs and L-entrepreneurs at date $t+1$, respectively. E_t^H is the aggregate net worth of H-entrepreneurs, and $s_t \equiv E_t^H/\sigma Y_t$ is their net worth share against the aggregate net worth of all entrepreneurs. Since $Y_t = AK_t$ holds in equilibrium, and from (17), economic growth rate becomes

$$g_{t+1} \equiv \frac{Y_{t+1}}{Y_t} = \left[1+\left(\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}\right)s_t\right]A\beta\sigma\alpha^L. \tag{18}$$

From (18), once s_t is determined, economic growth rate is also determined. (18) implies that economic growth rate increases with financial development. Intuitively, when financial development improves, the borrowing constraint of H-entrepreneurs becomes relaxed. In the credit market, more credit can be allocated to high profitable investment projects, which promotes capital accumulation, and eventually economic growth. As in a traditional endogenous growth setting, capital accumulation is the engine of economic growth.

The movement of the aggregate net worth of H-entrepreneurs evolves according to

$$E_t^H = p(q_t K_t^H - r_{t-1} B_{t-1}^H) + X(1-p)(q_t K_t^L - r_{t-1} B_{t-1}^L). \tag{19}$$

The first term of (19) represents the aggregate net worth of the entrepreneurs who continue to have high profitable investment from the previous period. The second term represents the aggregate net worth of the entrepreneurs who switch from the state of having only low profitable investment to the state of having high profitable investment. By using (18) and (19), we can derive the law of motion of the net worth share of H-entrepreneurs:

$$s_{t+1} = \frac{p\dfrac{\alpha^H(1-\theta)}{\alpha^L - \theta\alpha^H} s_t + X(1-p)(1-s_t)}{1 + \dfrac{\alpha^H - \alpha^L}{\alpha^L - \theta\alpha^H} s_t} \equiv \Phi(s_t, \theta). \tag{20}$$

The dynamic evolution of the economy is characterized by the recursive equilibrium: $(w_t, K_{t+1}, Y_{t+1}, g_{t+1}, s_{t+1},)$ that satisfies (10), (13), (17), (18), and (20) as functions of the state variables (K_t, Y_t, s_t).

2.2 Steady State Equilibrium

The stationary equilibrium of this economy depends upon the degree of financial development. That is, we have the following proposition (Proof is in Appendix 1).

Proposition 1 *There are three stages of financial development, corresponding to three different values of θ. The characteristics of each region are as follows:*

(a) Region 1: $0 \leq \theta < \theta_1 \equiv (1-p)/\left[\alpha^H/\alpha^L - p + X(1-p)\right]$. Since the real interest rate equals the rate of return on low profitable investment, the

borrowing constraint of H-entrepreneurs binds. Both H-and L-entrepreneurs produce capital. The steady state values of g^*, s^*, and r^* satisfy

$$g^* = \left[1 + \left(\frac{\alpha^H - \alpha^L}{\alpha^L - \theta\alpha^H}\right) s^*\right] A\beta\sigma\alpha^L,\ s^* = \Phi(s^*, \theta),\ r^* = \sigma A\alpha^L. \tag{21}$$

(b) Region 2: $\theta_1 \leq \theta < \theta_2 \equiv 1/(1+X)$. Since the real interest rate takes the value of $r^* \in \left[\sigma A\alpha^L, \sigma A\alpha^H\right)$, the borrowing constraint of H-entrepreneurs binds, and they produce capital. However, L-entrepreneurs do not produce capital because the real interest rate is greater than the rate of return on their investment. The steady state values satisfy

$$g^* = A\beta\sigma\alpha^H,\ s^* = p(1-\theta) + X(1-p)\theta,\ r^* = \frac{\sigma A\alpha^H}{(1-p)/\theta + p - X(1-p)}. \tag{22}$$

(c) Region 3: $\theta_2 \leq \theta \leq 1$. Since the real interest equals the rate of return on high profitable investment, the borrowing constraint of H-entrepreneurs does not bind. Only H-entrepreneurs produce capital. The steady state values satisfy

$$g^* = A\beta\sigma\alpha^H,\ s^* = \frac{X}{1+X},\ r^* = \sigma A\alpha^H. \tag{23}$$

In region 1 where financial development is relatively low, the financial system can not transfer enough savings to high profitable investment because of agency problems. In the credit markets, some of the savings flow to low profitable investment because they are not subject to agency frictions. In this region, as financial development improves, more credit is allocated to high profitable investment. This improvement of credit allocation promotes capital accumulation, the wage rate, and economic growth. However, in this region the real interest rate is unchanged. This property is similar to Stiglitz and Weiss (1981) model. In their model, when information asymmetry is large, the real interest rate is insensitive, and becomes constant where the bank's profit is maximized. Similarly, in our model, when financial development is low, the real interest rate is sticky.

In region 2 where financial development is high, but not so high, the situation changes. As financial markets develop, the real interest rate starts rising because of the tightness in the credit market, and all the savings are allocated to high profitable investment, even though the borrowing constraint still binds for H-entrepreneurs. Only H-entrepreneurs produce capital. As a result, the growth rate of the economy becomes constant, and independent of θ. This implies that once the financial system is developed to some degree, it can transfer enough purchasing power to the entrepreneurs who have high profitable investment from the entrepreneurs who have only low profitable investment. In addition, in region 1 and 2, since the interest rate is lower than the rate of return on H-entrepreneurs' investment, income distribution is different between H-and L-entrepreneurs.

When financial markets grow further, and reaches region 3, the real interest rate becomes equal to the rate of return on high profitable investment. Therefore, the borrowing constraint for H-entrepreneurs no longer binds. As in region 2, the financial system can allocate all the savings to only high profitable investment. Moreover, since H-and L-entrepreneurs earn the same rate of return, there is no difference in income distribution.[7]

2.3 Dynamics

Now, let us look at how this economy responds to an unexpected shock to productivity. Suppose that at date $\tau - 1$ the economy is in region 1, and in the steady state: $g_{\tau-1} = g^*, s_{\tau-1} = s^*$ and $r_{\tau-1} = r^*$. There is then an unexpected shock to productivity at date τ: A declines by ε, and becomes $A_\tau = A(1-\varepsilon)$. However, the shock is known to be temporary. The productivity at date $\tau+1$ and thereafter returns to A. Here since we consider a negative shock, we set ε to be positive.

Following Kocherlakota (2000), we measure financial amplification (volatility) of a downward shock ε to be how far economic growth rate from τ to $\tau+1$ jumps down from the steady-state growth rate through the borrowing constraint. Considering $q_\tau = \sigma A(1-\varepsilon)$ and $A_\tau = A(1-\varepsilon)$, from (18) and (19), we obtain

$$\text{Amplification} \equiv \frac{dg_{\tau+1}}{d\varepsilon}|_{\varepsilon=0} = \left(\frac{\alpha^H - \alpha^L}{\alpha^L - \theta\alpha^H}\right) \underbrace{\frac{ds_\tau}{d\varepsilon}|_{\varepsilon=0}}_{\ominus} A\beta\sigma\alpha^L < 0. \tag{24}$$

[7]In our model, in the neighborhood of the steady state equilibrium, the borrowing constraint of workers binds in all three regions because $\beta' r_t/g_{t+1} < 1$ holds. This can be verified by embedding (21), (22), and (23) into the inequality.

Since H-entrepreneurs have a net debt in the aggregate, and debts repayment does not change by this shock, the net worth share of H-entrepreneurs decreases at date τ ($\frac{ds_\tau}{d\varepsilon} < 0$). Because the adjustment of the real interest rate does not work well in region 1, their borrowing constraint becomes tightened. As a result, they are forced to cut back on their investment. Moreover, these balance sheet effects cause more credit to flow to the investment with less agency frictions. What is called "flight to quality" occurs. Through these effects, less capital is produced at date $\tau + 1$, so that economic growth rate at date $\tau + 1$ jumps down from the steady state growth rate.

Now, we are in a position to examine whether financial development accelerates or dampens these financial amplification effects.

First, let's check region 1. By differentiating (24) with respect to θ, we obtain

$$\frac{\partial^2 g_{\tau+1}}{\partial\theta\partial\varepsilon}|_{\varepsilon=0} = \underbrace{\frac{\partial}{\partial\theta}\left(\frac{\alpha^H - \alpha^L}{\alpha^L - \theta\alpha^H}\right)}_{\oplus}\underbrace{\frac{\partial s_\tau}{\partial\varepsilon}}_{\ominus}|_{\varepsilon=0}A\beta\sigma\alpha^L + \left(\frac{\alpha^H - \alpha^L}{\alpha^L - \theta\alpha^H}\right)\underbrace{\frac{\partial^2 s_\tau}{\partial\theta\partial\varepsilon}}_{\ominus}|_{\varepsilon=0}A\beta\sigma\alpha^L < 0. \tag{25}$$

The first term represents the sensitivity of the H-entrepreneurs' investment response to a change in the net worth share. Since it becomes higher with θ, with even a small decline in the net worth share, H-entrepreneurs are forced to reduce their investment substantially. The second term represents the degree of a decline in the net worth share. It says that the decline by itself becomes larger with θ (See Appendix 2). This implies that when θ is high, the leverage and debt/asset ratios of H-entrepreneurs also rise. In such a situation, even a small negative productivity shock can cause a large decline in the net worth share. Taken together, H-entrepreneurs have to make deeper cuts in their investment. Moreover, this causes a substantial credit shift from the investment with agency frictions to the one with less agency frictions. That is, balance sheet effects and flight to quality are significant. Hence, in region 1, financial development accelerates financial amplification effects, thereby leading to increased macroeconomic volatility.

Once the economy enters region 2, the situation changes dramatically. The shock absorbing effects start operating through the adjustment of the real interest rate. This weakens the balance sheet effects, and prevents flight to quality.

In order to clarify this point, let's look at how the real interest rate responds to this shock. In region 2, the equilibrium in the credit market at date τ becomes

$$\frac{s_\tau}{1-\dfrac{q_{\tau+1}\theta\alpha^H}{r_\tau}}=1. \tag{26}$$

And then, the economic growth rate from τ to $\tau+1$ is

$$g_{\tau+1}=\frac{Y_{\tau+1}}{Y_\tau}=A\alpha^H\frac{\beta s_\tau}{1-\dfrac{q_{\tau+1}\theta\alpha^H}{r_\tau}}\sigma. \tag{27}$$

By embeding (26) into (27), we obtain

$$g_{\tau+1}=\sigma A\alpha^H\beta. \tag{28}$$

From (28), we see that the economic growth rate from τ to $\tau+1$ is not affected by the shock. Why is the growth rate independent of the shock? In order to make this point clear, let's look at how the net worth share of H-entrepreneurs changes by this shock.

The aggregate net worth of H-entrepreneurs and the aggregate output at date τ follow

$$E_\tau^H=p\left[\sigma A(1-\varepsilon)K_\tau^H-r_{\tau-1}B_{\tau-1}^H\right])+X(1-p)r_{\tau-1}B_{\tau-1}^H, \tag{29}$$

$$Y_\tau=A(1-\varepsilon)\alpha^H\beta\sigma Y_{\tau-1}. \tag{30}$$

From (29) and (30), the net worth share of H-entrepreneurs at date τ follows

$$s_\tau=\frac{p(1-\theta-\varepsilon)+X(1-p)\theta}{1-\varepsilon}. \tag{31}$$

And so, by using (26) and (31), we obtain an expression for the equilibrium interest rate at date τ:

$$r_\tau=\frac{\sigma A\theta\alpha^H(1-\varepsilon)}{(1-p)(1-\varepsilon)+\left[p-X(1-p)\right]\theta}. \tag{32}$$

From (32), we observe that the real interest rate declines at the time of the shock. Intuitively, following the shock, the borrowing constraint becomes tightened as in region 1. And then, the investment function is shifted to the left. However, in region 2, together with this shift, the real interest rate goes down in the credit market. This decline in the real interest rate in turn relaxes the borrowing constraint, thereby weakening the balance sheet effects and preventing flight to quality. As a result, financial amplification is dampened. This implies that once financial development passes a certain degree, the adjustment of the real interest rate recovers, so that even if the shock hit the economy, all the credit flow only to high profitable investment. Therefore, the shock does not get amplified. Financial development leads to macroeconomic stability.

When financial development reaches region 3, even with the shock, the financial system can transfer enough purchasing power to those who have high productive investment from those who have only low profitable investment without the adjustment of the real interest rate. The real interest rate at date τ, $\sigma A\alpha^H$ and the growth rate from τ to $\tau+1$, $\sigma A\alpha^H\beta$ are independent of the shock. So, no financial amplification occurs.

Here we should add remarks about the difference between Kiyotaki (1998) and ours. Although Kiyotaki does not explicitly mention it, Kiyotaki's analysis implicitly assumes a certain low θ, which is within region 1 in this paper, and then, keeping the θ fixed, Kiyotaki examines how amplification occurs. On the other hand, our paper analyzes whether or not the size of amplification by itself increases or decreases together with θ not only in low θ region, but also high θ region.

The following proposition summarizes the results.

Proposition 2 *The relationship between financial development and financial amplification is non-monotone: financial amplification initially increases with financial development (in region 1) and later falls down (in region 2 and 3).*

This non-monotonicity is consistent with empirical studies. For example, Easterly et al. (2000) demonstrate that the relation between financial development and growth volatility is non-monotone. They show that while developed financial systems offer oppurtunities for stabilization, they may also imply higher leverage of firms and thus more risks and less stability. A recent study by Kunieda (2008) also show empirically that the relation is hump-shaped, i.e., in early stages of financial development, as the financial sector develops in an economy, it becomes highly volatile. However, as the financial sector matures further, the volatility starts to reduce once again.

Based on the above analysis, we might be able to explain why we observe two conflicting views. The traditional view might discuss region 1-2 or 2 where financial markets are well developed. Indeed, in Arrow-Debreu economy where there is no agency friction in the credit market, θ is equal to one. On the other hand, the new view might discuss region 1-1 where financial development is not so high, and there are agency frictions to some degree in financial markets. In this sense, the discrepancy between two views might arise from the difference in the degree of financial development.[8] We depict this situation in Figure 1. In the Figure 1, we take θ in horizontal axis, and in vertical axis, we take the magnitude of amplification. It is shown that the relation between θ and the magnitude is non-monotone.

This non-monotonicity has impliciations for the relation between growth and macroeconomic volatility. That is, in region 1-1, financial development causes economic growth. However, once negative productivity shocks hit the economy, downward amplification is significant since the economy is highly leveraged. In this sense, there is a trade-off between higher economic growth and macroeconomic stability. But, once financial development reaches region 1-2 or 2, both go together.

Moreover, our model may also have implications for asymmetric movements of business fluctuations. As Kocherlakota (2000) emphasizes, macroeconomics looks for an asymmetric amplification and propagation mechanism that can turn small shocks to the economy into the business cycle fluctuations. Our model might deliver this. For example, if the economy is around θ_2, to positive productivity shocks, even though the borrowing constraint is binding for the entrepreneurs, the economy will not respond upwardly because the interest rate will go up in the credit market, which generates shock cushioning effects. On the other hand, to negative productivity shocks, it

[8] You may wonder why large downward amplification occurs repeatedly in the real economy where financial development keeps increasing over time, even though our model suggests that financial amplification eventually becomes small in high θ region. Here is one interpretation from this model. In this model, the important factor which affects the size of financial amplification is θ, which is put on higher profitable investment, not on the storage technology with low returns. Considering this point, think about the case where the existing low technology with ρ disappper, and new investment opportunities with higher profitability than the existing α come into the economy. In such a situation, the θ which is put on those new investment projects matters. If the θ is low, the economy will get into region 1 again even if it was in region 2 or 3 before. In the real economy, this process might repeats itself.

will react downwardly because the interest rate does not adjust.[9]

Proposition 3 *if the level of financial development is around θ_1, business fluctuations are asymmetric.*

3 Conclusion

This paper develops a macroeconomic model of credit market imperfections with heterogeneous agents in order to investigate whether financial development exacerbates or dampens financial amplification. In our framework, financial development produces two competing forces. One is the effect which accelerates amplification by strengthening *balance sheet effects.* The other is the effect which dampens amplification, we call *shock cushioning effects.* Depending on which of these dominates, whether financial development exacerbates or weakens financial propagation is determined. Moreover, the balance between these two conflicting effects changes according to the level of financial development.

We show that in a low level of financial development, while shock cushioning effects do not work well, financial development enhances balance sheet effects through raising leverage, thereby accelerating financial amplification. However, once the level of development passes a certain degree, financial development generates shock cushioning effects, which in turn weakens balance sheet effects, thereby dampening financial amplification. Hence, the relation between financial development and financial amplification is non-monotone: financial amplification initially increases with financial development and later falls down.

As future research, the next step would be that we want to develop quantitative assessment into the relation between the development of financial markets and volatility of the economy. Another step would be to consider the welfare cost of volatility in a heterogeneous agents model with aggregate uncertainty. These directions will be promising.

[9] Here we consider small shocks. However, if we think about relatively large productivity shocks, business fluctuations may become asymmetric, even if the economy is far from θ_2. In the case with relatively large positive shocks, positive propagation occurs, but the degree of it is weakened because the adjustment of the interest rate works. However, to the negative shocks, because the adjustment does not work, the economy experiences large downward propagation.

Appendix 1

Proof of Proposition 1

In order to verify that (14) holds in equilibrium, we only need to check that the entrepreneurs with low profit investment invest positive amounts of goods:

$$Z_t^L = \beta\sigma Y_t \left(1 - \frac{s_t}{1 - \dfrac{\theta\alpha^H}{\alpha^L}} \right). \tag{33}$$

Using (20), we find that (33) becomes positive in the neighborhood of the steady state if, and only if θ is lower than θ_1.

Moreover, from (22), if $\theta < 1/(1+X)$, then $r^* < \sigma A\alpha^H$. That is, the real interest rate is lower than the marginal productivity of the entrepreneurs with high profit investment. Thus, the borrowing constraint for H-entrepreneurs binds. For L-entrepreneurs, since the real interest rate is greater than the rate of return on their investment, they would prefer lending to investing by themselves.

We also see that if $\theta = 1/(1+X)$, then $r^* = \sigma A\alpha^H$. Thus, the borrowing constraint for H-entrepreneurs no longer binds. Furthermore, If θ is greater than $1/(1+X)$, then for the credit market to clear, the real interest rate has to equal $\sigma A\alpha^H$ (If the real interest rate is greater than $\sigma A\alpha^H$, nobody is willing to borrow in the credit markets. This can not be an equilibrium.).

Appendix 2

By embedding $q_\tau = \sigma A(1-\varepsilon)$ and $A_\tau = A(1-\varepsilon)$ into (18) and (19), and differentiating s_τ with respect to ε, we obtain

$$\frac{\partial s_\tau}{\partial \epsilon}|_{\varepsilon=0} = [p - X(1-p)] \frac{-\theta\alpha^H s^*}{\alpha^L - \theta\alpha^H + (\alpha^H - \alpha^L)s^*} < 0. \tag{34}$$

And then, by using (34), we have

$$\frac{\partial^2 s_\tau}{\partial\theta\partial\epsilon}|_{\varepsilon=0} = \left[p - X(1-p)\right]\alpha^H \frac{-\theta\frac{\partial s^*}{\partial\theta}(\alpha^L - \theta\alpha^H) - \alpha^L s^* - (\alpha^H - \alpha^L)s^{*2}}{\left[\alpha^L - \theta\alpha^H + (\alpha^H - \alpha^L)s^*\right]^2} < 0. \tag{35}$$

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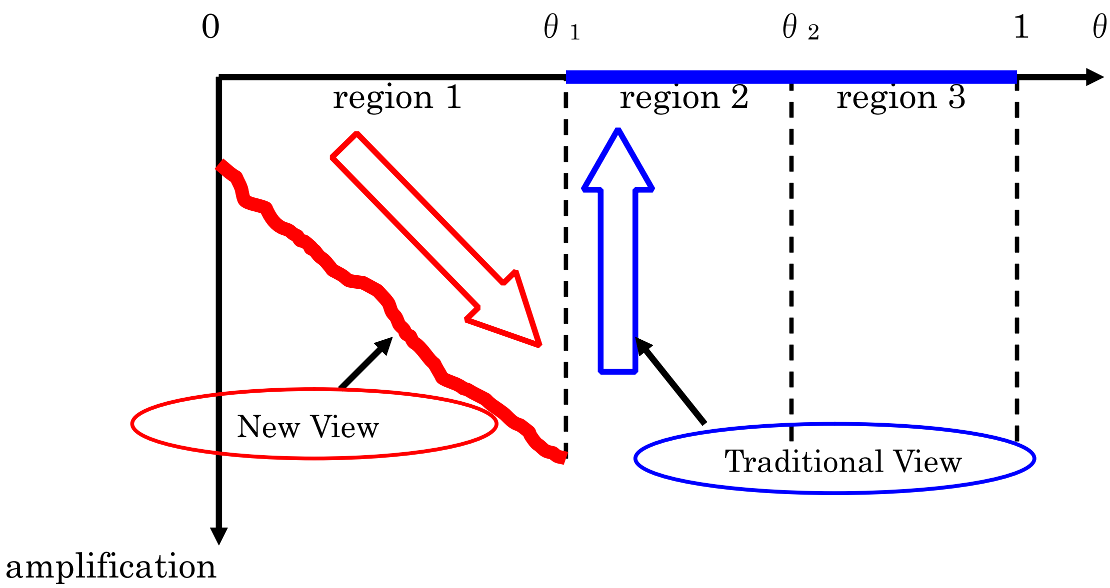

Figure 1: relation between θ and amplification