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# Modeling Long-Term Memory Effect in Stock Prices: A Comparative Analysis with GPH Test and Daubechies Wavelets

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## Abstract

Long-term memory effect in stock prices might be captured, if any, with alternative models. Though Geweke and Porter-Hudak (1983) test model the long memory with the OLS estimator, a new approach based on wavelets analysis provide WOLS estimator for the memory effect. This article examines the long-term memory of the Istanbul Stock Index with the Daubechies-20, Daubechies-12, the Daubechies-4 and the Haar wavelets and compares the results of the WOLS estimators with that of OLS estimator based on the Geweke and Porter-Hudak test. While the results of the GPH test imply that the stock returns are memoryless, fractional integration parameters based on the Daubechies wavelets display that there is an explicit long-memory effect in the stock returns. The research results have both methodological and practical crucial conclusions. On the theoretical side, the wavelet based OLS estimator is superior in modeling the behaviours of the stock returns in emerging markets where nonlinearities and high volatility exist due to their chaotic natures. For practical aims, on the other hand, the results show that the Istanbul Stock Exchange is not in the weak-form efficient because the prices have memories that are not reflected in the prices, yet.

**Key words:** *Long-term memory, Wavelets, Stock prices, GPH test*

**JEL classification:** *C4, G12, G14*

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## 1. Motivation

This article aims to test the long-term partial stationary memory effect in stock prices by wavelet analysis. Finding out the long-term memory parameter in accordance with timescales might be useful for the emerging markets in which high volatility, financial turbulences and chaotic behaviours are observed. By using wavelets in detecting long-term memory effect provides information on memories of the stocks in case f shocks.

Timescaling capital asset pricing model and variance as a risk measure provide an alternative approach for the portfolio theory by giving information on change of risk amount in accordance with hold period and frequency.

The empirical evidence on time-varying systematic risk supports the arguments that the systematic risk might change on frequency. On the other hand, scale based long-term memory

or partial stability give financial information that the shock effects might be maintained in long-term and the current prices might depend on the historical price patterns.

Long-term memory indicates the correlation structure of a series at long lags. If a series shows long-term memory, (or the biased random walk), there is persistent temporal dependence even between distant observations. Those time series are characterized by distinct but non-periodic cyclical patterns (Barkoulas and Balum, 2000).

Maheswaran (1990) underlies that the presence of long-term memory in stock returns has important implications for many of the paradigms in financial economics. For example, optimal consumption /savings and portfolio decisions might become extremely sensitive to the investment period if the returns were long-range dependent. There might be also fundamental problems in the pricing of derivatives with martingale methods, since the continuous time stochastic processes most commonly employed are inconsistent with long-term memory.

The presence of long-term memory volatility in stock returns has important implications for pricing contingent claims in emerging markets. High volatility in the capital markets of Turkey as similar to other emerging equity markets creates non-linear behaviours in the stock returns, which might have long-term effect in their time series. Though the markets are expected to be efficient in terms of information flow, the chaotic climate in the emerging stock markets might block the investors to reflect their expectations on the their portfolio decisions or create a long-term effect in the markets especially in case of negative shocks. The negative shocks lead capital markets to affect long-term risk appetite reflected in the stock prices.

The importance of the test results in terms of financial theory, on the other hand, comes from the fact that existence of long-term memory effect in the stock returns implies that the market is not weak form efficient. According to LeRoy (1989), the CAPM and the APT are not valid in the current financial markets because the usual forms of statistical inference do not apply to time series exhibiting such persistence; and tests of efficient markets hypotheses also hang precariously on the presence of long-term memory in the returns. Therefore, this research should also be considered as the examination of the weak-form efficiency in the Istanbul Stock Exchange-30 Index and 100 Index.

As much as we know, this article includes the first empirical research on the long-term memory effect in the share prices at the Turkish capital markets. The long-term memory effect in the Istanbul Stock Exchange-30 Index, and randomly selected ten companies within the ISE-30 Index since 02.01.2002 are examined by using alternative tests, namely Geweke and Porter-Hudak (1983) tests, Hurst Exponent test and wavelet tests (Jensen, 1999; and Tkacz, 2001).

The paper is organized as follow. In the next part, a recent literature on the long-term memory effect in the stock prices and application of wavelets theory in financial time series are reviewed. The methodologies of Geweke and Porter-Hudak (1983) tests, Hurst Exponent test and wavelet test (Jensen, 1999; and Tkacz, 2001) are presented in a cointegrated perspective in the third part. After discussing the empirical results in terms of both methodological and empirical points of view in the fourth part, the paper ends with some methodological suggestions for the future research on the modeling long-term memory in the financial assets.

## 2. Literature Review

Wavelet based integration and causality analysis are applied to the finance and economics by Jensen (1999, 2000), Jensen and Whitcher (2000) and Whitcher and Jensen (2000) who use the initial works of Wornell and Oppenheim (1992), Tefvik and Kim (1992), McCoy and Walden (1996). Percival and Walden (2000), Tkacz (2001), Gencay, Selcuk and Whitcher (2002) and Vourenmaa (2005) show that wavelet based long-term memory effect model performs better than Geweke and Porter-Hudak (1983) and fractional integration and causality tests. Jin, Elder and Koo (2006) show that wavelet based models solve the abnormal results of the alternative models dealing with long-term memory in the stock prices.

Sowell (1992) suggests an exact maximum-likelihood estimator for long-memory models possessing both short and long-memory parameters, namely the autoregressive, fractionally integrated, moving-average model. However, the assumption of a constant long-memory in the returns might not be reliable in all case. Though Bayraktar et al (2003) argue a solution for the time-varying long-memory by segmenting the data before its prediction, the solution is not always sufficient in that Whitcher and Jensen (2000) argue that the capacity of estimating local behavior by applying a partitioning model to a broad estimating process is not enough when compared with an estimator constructed to model time-varying characteristics. For that reason, in this paper, we use the MODWT-based methodology to estimate long-memory effect.

Wavelets employ Fourier analysis to decompose the variance of time series on different frequencies. The scales contributing the most to the variance of the series are related to those coefficients with the largest variance. Devore, Jawerth, and Popov (1992) state that the capacity of wavelet to localize the time series in multi-scale space causes the computational efficiency of the wavelet representation of an  $N \times N$  matrix operator by allowing the  $N$  largest elements of the wavelet-represented operator to represent the matrix operator. This feature enables dense matrices to have sparse representation when transformed by wavelets.

Empirical evidences show that though the GPH-estimator based on the ordinary least squares regression of the log-periodogram for frequencies close to zero is consistent under Gaussian distribution, the normality assumption in the distributions are not realistic with financial data which have generally abnormally distributed volatility. In that framework, Jensen (1999) shows that a wavelet based OLS-estimator is consistent with the real financial dynamics, when the sample variance of the wavelet coefficients is used in the regression.

The wavelets methodology is theoretically well-examined in the works by Schleicher (2002) and Crowley (2005). Usage of wavelets analysis in statistical time series can be followed on Nason and Von-Sachs (1999). Mathematical concerns on the wavelets are discussed within the work of Hardle et al (1998).

In this paper, the predictive performance of wavelets is compared to those of the alternatives, namely GPH and Hurst Exponent test for capturing long-term memory in the stock returns at the Turkish capital markets. In that context, the article contributes the literature comparative empirical evidence.

### 3. GPH Test For OLS Estimator vs. Wavelet Based OLS Estimator

In the methodology part, we discuss the wavelet based OLS estimation and GPH test for detecting the long-term memory effect in the financial time series. [Jensen \(1999\)](#) argues that the wavelets are superior in estimation because of their strength to simultaneously localize a process in time and scale. Wavelets are based on Fourier analysis where any covariance-stationary process  $x_t$  is expressed as a linear combination of sine and cosine functions in the frequency domain. However, since few financial time series show smooth rhythmic cycles suggested by sine and cosine functions, an alternative transformation model, namely wavelet transforms where function  $f(x)$  on  $[0,1]$  interval is expressed in the wavelet domain as

$$f(x) = c_0 \sum_{j=0}^{\infty} \sum_{k=0}^{2^j-1} c_{jk} \Psi(2^j x - k) \quad (1)$$

On the formula cited from [Tkacz \(2001\)](#),  $\Psi(x)$  is mother wavelet, as the mother to all dilations and translations of  $\Psi$  in above equation. A mother wavelet can be shown as ([Tkacz, 2001](#))

$$\Psi(x) = \begin{cases} 1, & \text{if } 0 \leq x < 1/2 \\ 1, & \text{if } 1/2 \leq x < 1 \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

The functions of  $\Psi_{jk}(x) = \Psi(2^j x - k)$  for  $j \geq 0$  and  $0 \leq k < 2^j$  are orthogonal, and they create a basis in the space of all square-integrable functions  $L_2$  along the  $[0,1]$  interval.  $j$  is the dilation index compressing the function  $\Psi(x)$ , and  $k$  is the translation index shifting the function  $\Psi(x)$ . Any such basis in  $L_2(\mathbb{R})$  is a wavelet, and the Equation 2 is the Haar wavelet ([Tkacz, 2001](#)).

Among several wavelets models in the literature we use [Daubechies model \(1988\)](#), which has supported wavelets whereby each wavelet represents different degrees of smoothing of the step function, namely the Equation 2. By following [Tkacz \(2001\)](#), we use different Daubechies and the Haar wavelets to verify the robustness of the results to different degrees of smoothing. The Daubechies-20 wavelet is used as the smoothest wavelet, followed by the Daubechies-12 and the Daubechies-4, with the Haar wavelet being the least smooth.

To test the long-term memory effect with wavelet based OLS estimator, we use [Jensen \(1999\)](#) and [Tkacz \(2001\)](#) technique. The matlab codes used by [Tkacz \(2001\)](#) are restructured for the empirical analysis. The WOLS estimator can be formulized as follow:

Let define  $x_t$  as a random process,

$$(1-L)^d x_t = e_t \quad (3)$$

where  $L$  is the lag operator,  $e_t$  is i.i.d. normal with zero mean and constant variance  $\sigma^2$ , and  $d$  is a differencing parameter. The wavelet OLS estimator of  $d$  comes from the smooth decay of long-memory processes. When  $d=0$ , the random process  $x_t$  equals to  $e_t$  and therefore,  $x_t \sim (N(0, \sigma^2))$ , or  $x_t \sim I(0)$ . On the other hand, when  $d=1$ ,  $x_t$  follows a unit root process with a zero mean and infinite variance ([Tkacz, 2001](#)).

If  $d$  is noninteger, the random  $x_t$  process becomes fractionally integrated, namely ARFIMA process as shown on the Equation 3. [Hosking \(1981\)](#) shows that when  $0 < d < 1/2$  the autocovariance function of the random process declines hyperbolically to zero as a long-memory process. When  $1/2 \leq d < 1$ , the random process takes an infinite variance but still reverts to its trend in the very long run ([Tkacz, 2001](#)). On the Table 1, the features of the fractional integration parameter ( $d$ ) are summarized.

**Table 1: Memory Features With Respect to the Fractional Integration Parameter Values**

$D$	Variance	Shock Duration	Stationarity
$d=0$	Finite	Short Memory	Stationary
$0 < d < 0.5$	Finite	Long Memory	Stationary
$0.5 \leq d < 1$	Infinite	Long Memory -Finite Impact-Reflect	Nonstationary
$d=1$	Infinite	Finite-Not Revert to Its Mean	Nonstationary
$d > 1$	Infinite	Finite-Not Revert to Its Mean	Nonstationary

*Source:* [Tkacz, G. \(2001\)](#), *Estimating the Fractional Order of Integration of Interest Rates Using a Wavelet OLS Estimator*, *Studies in Nonlinear Dynamics & Econometrics*, 5(1): pp. 23.

[Tevfik and Kim \(1992\)](#), [McCoy and Walden \(1996\)](#), [Jensen \(1999\)](#) and [Tkacz \(2001\)](#) empirically show that for an  $I(d)$  process  $x_t$  with  $|d| < 1/2$ , the autocovariance function shows that the wavelet coefficients  $c_{jk}$  in Equation 1 are distributed as  $N(0; \sigma^2 2^{-2jd})$ . By taking logarithms, an estimate of  $d$  can be reached using ordinary least squares from

$$\ln R(j) = \ln \sigma^2 - d \ln 2^{2j} \quad (4)$$

where  $R_j$  denotes the wavelet coefficient's variance at scale  $j$  ([Tkacz, 2001](#)). In that framework, [Tkacz \(2001\)](#) states that the wavelets are used only in the estimation of the  $d$  consistent with the observed autocovariance function. What's more, the number of observations for the random process should be a factor of two due to the form of the wavelet expansion 1.

To detect the long-term memory effect in stock returns, [Geweke and Porter-Hudak \(1983\)](#) test, Hurst Exponent test and wavelets are used. Long-memory was firstly developed by [Hurst \(1951\)](#) and adopted into the econometrics by [Granger and Joyeux \(1980\)](#) and [Geweke and Porter-Budak \(1983\)](#).

The GPH test starts with the spectral density function of  $x_i$  ([Kasman, Kirbas and Turgutlu, 2005](#)):

$$f_x(\lambda) = (\sigma^2 / 2\pi) \{4 \sin^2(\lambda / 2)\}^{-d} f_u(\lambda) \quad (5)$$

$f_u(\lambda)$ , is the spectral density function of  $u_i$  at  $\lambda$  frequency. After taking logarithmics and including  $I(\lambda_j)$ , at  $j$  ordination, the Equation 5 is reached ([Chambers ve Cifter, 2006](#)).

$$\ln\{I(\lambda_j)\} = \ln\{\sigma^2 f_u(0) / 2\pi\} - d \ln\{4 \sin^2(\lambda / 2)\} + \ln\{I(\lambda_j) / f_x(\lambda_j)\} + \ln\{f_u(\lambda_j) / f_u(0)\} \quad (6)$$

When  $\lambda_j$  has low frequency,  $\ln\{f_u(\lambda_j)/f_u(0)\}$  becomes inefficient. GPH regression can be formulated as

$$\ln[I(\lambda_j)] = \beta_0 + \beta_1 \ln[4 \sin^2(\lambda_j/2)] + u_j \quad (7)$$

The constant term,  $\beta_0$ , is  $(\ln\{\sigma^2 f_u(0)/2\pi\})$ ,  $\beta_1 = -d$ ,  $u_j = \ln\{I(\lambda_j)/f_z(\lambda_j)\}$  as the error term.

If  $\lim_{T \rightarrow \infty} g(T)^2 / g(T) = 0$ , then asymptotic distribution of  $\hat{d}_{GPH}$  is formulized by the Equation 8.

$$\hat{d}_{GPH} \rightarrow N\left(d, \pi^2 / 6 \sum_{i=1}^n (x_i - \bar{x})\right) \quad (8)$$

$x_i = \ln[4 \sin^2(\lambda_j/2)]$ . In the GPH test, if  $\hat{d} < 0$ ,  $\hat{d}_{GPH}$  provides statistically acceptable and asymptotic normality.

#### 4. Data

The data is obtained from 10 common stocks namely AKBNK, AEFES, AKGRT, ARCLK, EREGL, KCHOL, KRDM, TCELL, TUPRS and YKBANK, all of which are included in ISE-30 index. These shares have been selected randomly and they represent 33% sample rate (10/30). Further, ISE 100 index has also been considered as an analysis subject. In this sense, the empirical study provides a wide range comprising ISE 30 and ISE 100 indexes along with the randomly selected shares from ISE 30. The volatility of the returns of the selected stocks depending on the size, systematic risk and long-term memory parameter are determined by using wavelet analysis. The statistical features of the selected stocks are available in Table 2. All stocks can be accepted in normal distribution according to Jarque-Bera normality test, and their depressiveness and distortion features are different from those of each other's.

**Table 2: Descriptive Statistics**

Index/Equity	Min.	Maks.	Std.Dev.	Skewness	Kurtosis	Jarque-Bera
AKBNK	14786	135103	29218.5	1.08972	3.49509	221.864
AEFES	93607	497321	97053.8	0.975298	2.94809	169.117
AKGRT	14680	145396	26896.7	1.64071	5.84979	838.987
ARCLK	21185	130137	23953.9	0.371356	2.7581	27.1002
EREGL	12199	97200	24496.7	0.708779	2.30727	110.568
KCHOL	25991	82246	13387.7	0.307466	2.16677	47.6328
KRDM	0.0299	0.7452	0.225812	0.352008	1.44914	128.845
TCELL	16126	102214	23572.3	0.534852	1.84816	109.754
TUPRS	44665	303477	66169	1.05593	2.81388	199.636
YKBANK	10195	79864	17992.2	0.394087	2.1502	59.6686
ISE100	8627.42	47728.5	10039.8	1.01437	3.1919	184.445
ISE 30	10880.5	60772.1	12882.6	0.978913	3.11851	170.877

In the determination of both volatility and the long-term memory effect parameters, first level logarithmic differences of series are considered. Taking logarithmic differences in data analysis of high frequency daily data is widely used in the literature. The first level logarithmic differences of all series have been taken.

In the Table 3, the stability values of the stocks are given according to Lo's RS test, KPSS test, P-P test, Augmented Dickey Fuller test and Robinson D test. All series are stable to logarithmic difference.

**Table 3: Unit Root Test**

Index/Equity	Lo's RS test of I(0)	KPSS test of I(0)	P-P test of I(1)	Aug. D-F test of I(1)	Robinson's d
AKBNK	6.00128 {<0.005}	20.0126 {<0.01}	0.638136 {<1}	0.504895 {<0.99}	0.49484
AEFES	6.44612 {<0.005}	20.456 {<0.01}	0.389534 {<0.99}	0.563084 {<0.99}	0.4957
AKGRT	5.69271 {<0.005}	17.512 {<0.01}	-1.44333 {<0.9}	-1.46362 {<0.9}	0.496279
ARCLK	6.41797 {<0.005}	20.4979 {<0.01}	-0.218052 {<0.95}	-0.393636 {<0.95}	0.493426
EREGL	6.5642 {<0.005}	21.6616 {<0.01}	-0.000165 {<0.975}	-0.0483108 {<0.975}	0.496771
KCHOL	6.48821 {<0.005}	18.6793 {<0.01}	-0.686503 {<0.9}	-0.838449 {<0.9}	0.493957
KRDMD	7.21092 {<0.005}	20.7315 {<0.01}	0.0472024 {<0.975}	0.0981312 {<0.975}	0.496121
TCELL	6.84462 {<0.005}	21.8022 {<0.01}	-0.096907 {<0.95}	-0.0878089 {<0.95}	0.49652
TUPRS	6.37948 {<0.005}	19.7428 {<0.01}	1.52696 {<1}	1.60709 {<1}	0.495828
YKBNK	6.36436 {<0.005}	16.8746 {<0.01}	0.231628 {<0.99}	0.0857086 {<0.975}	0.497552
ISE100	5.2.619 {<0.005}	20.2774 {<0.01}	1.57173 {<1}	1.63275 {<1}	0.496025
ISE30	5.2.6284 {<0.005}	20.3486 {<0.01}	1.35768 {<1}	1.39816 {<1}	0.496063

## 5. Empirical Evidence

In Table 4, there are stability test results according to Geweke and Porter-Hudak method. The long-term memory parameter (d) changes between 0.98138 and 1.08555. The series are not stable and the shock period is infinite (non-returnable in the average of series), since the long term memory parameter is closing to 1, in the long-term. Owing to the long-term memory parameters of KCHOL and KRDMD are smaller than 1, these series are not stable and have long term memory. According to GPH method, it is evidenced that the stocks other than KCHOL and KRDMD are not stable, and no-returning is available in the averages of the series.

**Table 4 : Fractional Integration Parameters With Geweke/Porter-Hudak Method \***

Index/Equity	d	T(d)	Bias Test: ChiSq(1)
AKBNK	1.07727	14.583	0.273943 {0.601}
AEFES	1.06873	14.468	0.443364 {0.506}
AKGRT	1.08555	14.695	0.00479251 {0.945}
ARCLK	1.01197	13.699	0.351684 {0.553}
EREGL	1.07203	14.512	0.500397 {0.479}
KCHOL	0.98672	13.357	1.24923 {0.264}

KRDMD	0.98138	13.285	0.327224 {0.567}
TCELL	1.0949	14.822	0.334297 {0.563}
TUPRS	1.01841	13.787	1.17552 {0.278}
YKBNK	1.08551	14.695	2.52501 {0.112}
ISE 100	1.06538	14.422	0.162058 {0.687}
ISE 30	1.07107	14.499	0.150576 {0.698}

\* Bandwidth = 90 (=  $T^{0.65}$ )

All series are not stable and shock period is infinite according to Fractal Dimension – Hurst Exponent. In other words, the series do not return to their averages. In case of taking the difference of the series, all series are stable and the shock period is permanent (Table 5).

**Table 5 : Fractal Dimension – Hurst Exponent**

Index/Equity	Series			Difference Series		
	H	Standart Dev. (H)	R <sup>2</sup>	H	Standart Dev. (H)	R <sup>2</sup>
AKBNK	1.0216	0.002054	0.9948	0.8141	0.006859	0.9915
AEFES	1.0137	0.001837	0.9995	0.8055	0.006702	0.9916
AKGRT	1.0249	0.001866	0.9995	0.8081	0.006830	0.9911
ARCLK	1.0230	0.002052	0.9995	0.8118	0.006781	0.9913
EREGL	1.0208	0.002164	0.9994	0.8113	0.005877	0.9936
KCHOL	1.0271	0.001817	0.9996	0.8246	0.006629	0.9921
KRDMD	1.0078	0.001994	0.9994	0.7706	0.006956	0.9904
TCELL	1.0163	0.001578	0.9996	0.8217	0.006401	0.9925
TUPRS	1.0093	0.002565	0.9991	0.7923	0.006644	0.9917
YKBNK	1.0310	0.002317	0.9993	0.8118	0.003744	0.9970
ISE 100	1.0190	0.001391	0.9997	0.7590	0.005861	0.9928
ISE 30	1.0144	0.001236	0.9998	0.7563	0.005890	0.9927

\* Bandwidth = 90 (=  $T^{0.65}$ )

The partial stability test results of 10 stocks from ISE-30 according to Daubechies-4, Daubechies-20 and Daubechies-20 wavelet are available in the table 3.16.

**Table 6: Fractional Integration With Wavelets**

Index/Equity	Haar	Daubechies-4	Daubechies-12	Daubechies-20
AKBNK	1.0792 (0.0297)	0.9433 (0.0134)	0.9575 (0.0204)	0.9453 (0.0196)
AEFES	1.0358 (0.0415)	0.9401 (0.0204)	0.9518 (0.0199)	0.9421 (0.0226)
AKGRT	1.0870 (0.0337)	0.9348 (0.0195)	0.9490 (0.0259)	0.9361 (0.0267)
ARCLK	1.0011 (0.0245)	0.8923 (0.0260)	0.9084 (0.0295)	0.8994 (0.0226)
EREGL	1.0721 (0.0177)	0.9834 (0.0129)	0.9945 (0.0203)	0.9801 (0.0247)
KCHOL	0.9397 (0.0413)	0.9157 (0.0241)	0.9276 (0.0261)	0.9170 (0.0254)
KRDMD	0.9776 (0.0809)	0.9722 (0.0121)	0.9850 (0.0157)	0.9612 (0.0279)

TCELL	1.0316 (0.0372)	0.9726 (0.0119)	0.9825 (0.0200)	0.9570 (0.0368)
TUPRS	1.0548 (0.0368)	0.9414 (0.0125)	0.9458 (0.0180)	0.9312 (0.0203)
YKBNK	1.0125 (0.0372)	0.9543 (0.0541)	0.9682 (0.0382)	0.9548 (0.0349)
ISE 100	1.1104 (0.0279)	0.9544 (0.0102)	0.9678 (0.0196)	0.9531 (0.0237)
ISE 30	1.1002 (0.0274)	0.9561 (0.0097)	0.9691 (0.0186)	0.9540 (0.0232)

Note: The values in the paranthesis are standart deviation.

The findings found by using Harr wavelet are in parallel with GPH method. By performing Daubechies 4, 12 and 20 days wavelet analysis, it is found that all series can return to their averages where as they are not stable completely ( $0.5 \leq d < 1$ ), and series has finite impact-response weight. Parameter d's being within the range of 0,89-0,98 and closing of the memory parameter in the long-run show that prediction models based on past period volatility can be effective in the long run for the stocks included in ISE 30.

## 6. Concluding Remarks

Detecting long-term memory effect in stock prices has fundamental results for both methodological concerns and portfolio management. Efficient market hypothesis states that the information comes into the markets is reflected in the prices immediately, thus the historical time series does not present any clues for the future prices. In other words, the markets are memoryless.

On the other hand, interdisciplinary applications in the literature give opportunity to construct the advanced methodological models for the financial economics. In this paper, wavelets used in the quantum physics are applied for the fractional integration parameter detecting the long-term memory in the stock returns. Due to relatively high volatility and turbulences in the emerging capital markets, wavelets are proper to use in detecting long-term behaviours in the returns by filtering short-term biased movements arising from the asymmetric information flows.

This paper comparatively examines the alternative methods to model the long-term memory effect in the stock returns. The empirical findings show that methodological concerns create crucial differences, which might affect the decisions of the investors. While the results of OLS estimators based on the GPH tests imply that the markets are in the weak-form efficiency, the wavelets based OLS estimators display the fact that the prices have memory stating that analyzing the time series might be still effective in estimating the future behaviours in the returns.

In the long-term memory analysis, despite the long term parameter of GPH and Hust Test is in the level of 1. The d is calculated within the range of 0.89-0,98 according to Daubechies 4,12 and 20-day wavelets. The closing of long-term memory parameter in the long run suggest that prediction models base on the past period volatility can be effective.

Empirical evidences show that Daubechies wavelet analysis providing the accurate determination for the long-term memory in the selection of volatility model also can increase

the prediction performance. As a result of the study, it is concluded that wavelet method can be applied to develop strategy in the different frequencies (scales) in finance engineering and portfolio management.

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