Consolidated-Budget Rules and Macroeconomic Stability with Income-Tax and Finance Constraints

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Abstract

In some Business-Cycle models a fiscal policy that sets income taxes counter cyclically can cause macroeconomic instability by giving rise to multiple equilibria and as a result to fluctuations caused by self fulfilling expectations. This paper shows that consolidated budget rules with endogenous income-tax rates can be stabilizing if they exhibit monetary dominance, where monetary policy manages expectations by implementing an active interest rate rule. This result is robust for plausible degrees of externalities in production. The size of the government, however, plays a key role in the degree of activeness that the monetary authority should exhibit in order to stabilize the economy. If government spending are not too large relative to private consumption, a neutral monetary policy [such that the real rate of interest is constant in and off the steady state] is also stabilizing

Key words: Fiscal Policy; Capital-Income Tax; Monetary Policy; Macroeconomic Stabilization; Finance Constraint; Arbitrage Channel; Investment-Based Channel; Consumption-Based Channel;

Introduction

The problem of macroeconomic stability where governments implement balanced budget rules has drawn much attention in the past several years. One important issue concerning this literature is that conclusions can be very different, even opposite, if government spending is set so as to balance income taxes generated by fixed rates, or if income taxes are set so as to balance a budget with fixed wasteful government spending. Schmitt-Grohe and Uribe (1997) explore the interrelations between local stability of equilibria and a balanced budget rule whereby constant government expenditures are financed by proportional taxation on labor and capital income. They use a one-sector infinite-horizon representative agent model with perfectly competitive markets and a constant returns to scale technology. It turns out that for empirically plausible values of labor and capital income tax rates, the economy can exhibit an indeterminate steady state and a continuum of stationary sunspot equilibria. Under this type of balanced budget constraint, when agents become optimistic about the future of the economy and decide to work harder and invest more, the government is forced to lower the tax rate as total output rises. The countercyclical tax policy will help fulfill agents' initial optimistic expectations, thus leading to indeterminacy of equilibria and endogenous business cycle fluctuations. Guo and Harrison (2008) extend Schmitt-Grohe and Uribe's (1997) analysis by the inclusion of useful government spending. They show that Schmitt-Grohe and Uribe's (1997) indeterminacy results are robust to incorporating useful government purchases of goods and services, regardless

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of how they are introduced to the model. Under a balanced-budget specification, fixed government spending act simply as a scaling constant in either firms' production or households' utility function. It follows that none of the model's stability analysis is affected by allowing for productive or utility-generating government expenditure. This robustness finding highlights the importance of an alternative fiscal policy specification under an exogenous public-spending regime.

The present paper contributes to the literature by introducing a consolidated budget setup where a fiscal authority taxes income and a monetary authority finance the primary deficit via seniorage. We follow Leeper (1991) and assume that the size of seniorage and its composition (bonds and money) are set by the monetary authority alone before the size of primary deficit is revealed. Only then the fiscal authority sets the rate of income tax so as to balance the consolidated budget. Schmitt-Grohe and Uribe (2007) provide evidence using a numerical model calibrated to the U.S. economy that this type of policy prescription is stabilizing. Here a formal proof is provided.

The rest of the paper is organized as follows: section 2 illustrates a model where a consolidated government runs a balanced budget. The composition of the budget and the restriction imposed on the two authorities (fiscal and monetary) are thoroughly described. The optimal program of a representative household is then scrutinized and local stability analysis of equilibrium is performed. it turns out that a policy such that imposes via financial markets an increase in the expected real rate of interest during booms is sufficient to overcome the indeterminacies reported in Schmitt-Grohe and Uribe's (1997) and in Guo and Harrison (2008). Results slightly change where government spending are not too large in the sense of the ratio between government expenditure and private consumption relative to a multiple of the elasticity of production with respect to capital and the intertemporal elasticity of consumption substitution. In that case a policy that induces a constant expected real rate of interest in and off steady state is also stabilizing. Section 3 extends the analysis to economies that exhibit production externalities associated with per capita capital. Results in section 3 show that the prescribed policy rule is robust to production externalities. Section 4 concludes.

1 Consolidated Balanced Budget with Income Tax and Finance Constraints

In the present context we assume that the government is comprised of a fiscal authority and a monetary authority, and that the government runs a consolidated balanced budget. Hence, assuming a monetary authority we implicitly assume the existence of money. Accordingly, money enters the economy via a cash-in-advance constraint on all transactions. To avoid steady state multiplicity, the analysis is restricted, following Benhabib et. al (2002), to steady states where the nominal rate of interest is strictly positive. Finally, it is assumed throughout that the economy is perfectly competitive.

1.1 The Economic Environment

1.1.1 The Government

The balanced budget rule under scrutiby is in the spirit of Schmitt-Grohe and Uribe (2007) who derive their determinacy results numerically. It is assumed that the consolidated government prints money, M_t, issues nominal risk free bond, B_t, collects taxes in the amount of T_t and faces an exogenous fixed stream of expenditure $\overline{g}$. Its instantaneous dollar denominated budget constraint is given by

$$R_t B_t + P_t \overline{g} = \dot{M}_t + \dot{B}_t + P_t T_t$$

where P_t is the level of nominal prices. It is assumed throughout that the fiscal regime is passive in the terminology of Leeper (1991). The central bank implements an interest rate feedback rule. It imposes a desired interest rate, R_t, by controlling the price of riskless nominal bonds and exchanging money for bonds at any quantities demanded at that price. In that sense, the nominal rate of interest is exogenous and M_t, B_t are endogenous.

The fiscal authority is then constrained to set T_t so as to balance the budget. It is assumed throughout the paper that $T_t = \tau_t k_t \left[r_t - \delta q_t\right]$ where τ_t denotes an income tax rate and it can vary with time, $r_t k_t$ is total income in the economy, where k_t, r_t denote the stock of capital and the rent on capital, respectively. The term $\delta q_t k_t$ represents a depratiation tax allowance where δ is a constant rate of capital depreciation and q_t denotes the market price of one unit of installed capital [1] . Accordingly, the nominal consolidated budget constraint is

[1] In general, total tax revenues consist of lump sum taxation, revenues from labor

given by $R_t B_t + P_t \overline{g} = \dot{M}_t + \dot{B}_t + P_t \tau_t k_t \left[r_t - \delta q_t \right].$

let $m_t \equiv \frac{M_t}{P_t}$ and $b_t \equiv \frac{B_t}{P_t}$denote real money holdings and real bonds holdings, respectively. Also let $a_t \equiv b_t + m_t$ denote a measure of financial wealth, which consists of government liabilities, denominated in real goods. Dividing both sides of the nominal instanteneous budget constraint by P_t and rearranging, yield that the real financial wealth evolves according to:

$$\dot{a}_t = (R_t - \pi_t) a_t - R_t m_t + \left[\overline{g} - \tau_t k_t (r_t - \delta q_t) \right]$$

Where $\pi_t \equiv \frac{\dot{P}_t}{P_t}$ is the rate of change of nominal prices i.e. the rate of inflation.

In this economy, printing money to finance the primary deficit gives rise to inflation. As inflation erodes real liabilities it can be viewed as a source of revenue. Inflation therefore plays a role similar to that of a lump sum tax. The type of fiscal policy considered in this paper is such that given an exogenous stream of (real) expenditure $\overline{g}$ and the nature of monetary policy, to be specified in following sections, the fiscal authority sets the income tax rate, τ_t, so as to balance the instantaneous budget of the consolidated government.

It is assumed that monetary policy takes the form of an interest-rate feedback rule whereby the nominal interest rate is set as an increasing function of instantaneous inflation. Specifically, it is assumed that

$R_t = R(\pi_t)$ where π_t can be interpreted as expected-inflation. $R(\cdot)$ is

income taxation, revenues from capital income taxation, and revenues from firms' profits taxation. However, in our model, under perfect competition firms' profits are zero. Also, in this model, we can ignore lump sum taxation without loss of generality as we know that it is not a source of indeterminacies.

continuous, non-decreasing and strictly positive, and there exists at least one steady-state, π^*, such that $R(\pi^*) = \rho + \pi^*$ where ρ denotes the rate of time preference of a representative household and π^* is a desired inflation target. It is further assumed that the monetary authority reacts to an increase (decrease) in the rate of inflation by increasing (decreasing) the nominal rate of interest.

Dupor (2001) and Benhabib et al. (2001) discuss the issue of monetary-fiscal regimes and determination of equilibrium in a continuous time model where the monetary authority sets a nominal interest rate as a function of the instantaneous rate of inflation. The policy considered here follows this line and is also in one line with the forward-looking policy considered by Carlstrom and Fuerst (2005) in their discrete-time model. As we know, the instantaneous rate of inflation in a continuous-time setting is the right-derivative of the logged price level and thus, the discrete-time counterpart of a countinuous-time policy rule that sets the interest rate in response to the instanteneous rate of inflation is characterized by forward-looking policy that responds to expected future inflation.

1.1.2 Households

The model is a continuous time, flexible price version of Benhabib et. al. (2001). The economy is populated by a continuum of identical infinitely long-lived households, with measure one. It is assumed that consumption and money balances are Edgeworth complements. In Benhabib et. al. (2001) money enters the utility function, and Edgeworth complementarity between consumption and money balances is achieved by assuming a positive cross derivative of money and consumption. Here, in order to keep the analysis simple, we impose a cash-in-advance constraint on consumption and so money enters the liquidity constraint [2]. The representative household's lifetime utility function is given by

$$U = \int_0^\infty e^{-\rho t} u(c_t)dt$$

where $\rho > 0$ denotes the rate of time preference, c_t denotes consumption, $u(\cdot)$ is twice differentiable, strictly increasing, and strictly concave. The household's budget constraint is:

$$c_t + I_t + \dot{b}_t + \dot{m}_t = (R_t - \pi_t)b_t - \pi_t m_t + (1 - \tau_t)r_t k_t + \tau_t k_t \delta q_t$$

where I_t is the flow of investment. Finally it is assumed that the production function, $f(k)$, is twice differentiable, strictly increasing and strictly concave. By considering a_t as the real value of non-capital wealth, the household's budget constraint becomes:

$$\dot{a}_t = (R_t - \pi_t)a_t - R_t m_t + (1 - \tau_t)f(k_t) - c_t - I_t + \tau_t k_t \delta q_t$$

[2] Feenstra (1986) demonstrates that a using real money as an argument of the utility function is functionally equivalent to entering money into a liquidity constraint. Specifically, he argues that cash-in-advance constraints can be viewed as a special case of a utility function that includes real balances with the crucial feature of a zero elasticity of substitution between goods and money. Feenstra (1986) argues that the zero elasticity of substitution of the cash-in-advance specification means that it is approximated by utility functions with a positive cross derivative between goods and money regardless of how concave u(c,m) may be. In what follows, the optimal program, specified by equations (2)-(5), demonstrate that the liquidity constraint is observably equivalent to Benhabib et. al.'s (2002) MIU specification where money and consumption are Edgeworth complements.

Competitive equilibrium in the goods market at the closed economy implies $c_t + I_t + \overline{g} = f(k_t) = r_t k_t$, it is thus straightforward to show that the evolution of goverment liabilities due to the fiscal-monetary rule coinside with the evolution of the households' financial wealth.

Money enters the economy via a liquidity constraint on all transactions: $\int\limits_t^{t+\Gamma} [c(s) + I(s)]\, ds \leq m_t$ that can be linearly approximated as[3]:

$\Gamma(c_t + I_t) \leq m_t$

Finally, and without loss of generality, Γ is normalized to 1 and the household's lifetime maximization problem becomes

$$Max \int\limits_0^{\infty} e^{-\rho t} u(c_t) dt$$

$$s.t.$$

$$\dot{a_t} = (R_t - \pi_t)a_t - R_t m_t + (1 - \tau_t) f(k_t) - c_t - I_t + \tau_t k_t \delta q_t \qquad (1)$$

$$\dot{k_t} = I_t - \delta k_t$$

$$c_t + I_t \leq m_t$$

With the following no-Ponzi-game condition $Lim_{t\to\infty} e^{-\int\limits_0^t [R(s)-\pi(s)]ds} [a_t + k_t] = 0$. The household's problem suggests that capital accumulation entails an opportunity cost due to a finance constraint. This specification is similar to Woodford (1984). In general, macroeconomic continuous time modeling could

[3] This version of cash-in-advance is similar to Rebelo and Xie (1999) and Feenstra (1985). a Taylor series expansion gives $\int\limits_t^{t+\Gamma} [c(s)+I(s)]ds = \Gamma[c(s)+I(s)] + \frac{1}{2}\Gamma^2[\dot{c}(t) + \dot{I}(t)] + \cdots$ and so $\Gamma(c + I) \leq m$ can be interpreted as a first-order approximation.

be misleading in the sense that it does not correctly approximate the behavior of the discrete time model of arbitrarily small periods. Therefore, special care should be taken with assumptions of the model that are not realistic for small period length. Carlstrom and Fuesrt (2005) point out that modeling policy issues in continuous time could end up with conclusions that are opposite to the conclusions drawn from a discrete-time counterpart of the model. They attribute the opposite conclusions to the difference in timing in the no-arbitrage condition of investing in bonds and capital between the two settings: while the continuous-time setting entails a contemporaneous no-arbitrage condition, a similar no-arbitrage condition in the discrete-time setting involves only future variables which bring a zero eigenvalue into the linearized dynamic system. Gliksberg (2009) shows that introducing finance constraints as in Woodford (1984) is one way to overcome implausible contemporaneous features of no-arbitrage in continuous time macroeconomic models that enter at the "back door" as the period length gets shorter.

1.1.3 ***The optimal program***

Households choose sequences of $\{c_t, I_t, m_t\}$ so as to maximize lifetime utility, taking as given the initial stock of capital k_0, and the time path $\{\tau_t, R_t, \pi_t\}_{t=0}^{\infty}$ which is exogeneous from the view point of a household. The necessary conditions for an interior maximum of the household's problem are

$$u'(c_t) = \lambda_t + \zeta_t \tag{2}$$

$$\frac{q_t}{\lambda_t} = 1 + R_t \tag{3}$$

$$\zeta_t = R_t \lambda_t \tag{4}$$

$$\zeta_t(m_t - c_t - I_t) = 0; \zeta_t \geq 0 \tag{5}$$

Where λ_t and q_t are time-dependent co-state variables interpreted as the marginal valuation of financial assets and installed capital, respectively. ζ_t is a time-dependent Lagrange multiplier associated with the liquidity constraint and equation (5) is the corresponding Kuhn-Tucker condition. Second, the co-state variables must evolve according to the law

$$\dot{\lambda_t} = \lambda_t \left[\rho + \pi_t - R_t\right] \tag{6}$$

$$\dot{q_t} = -\lambda_t \left[(1 - \tau_t) f'(k_t) + \tau_t \delta q_t\right] + (\rho + \delta) q_t \tag{7}$$

where equation (6) is the euler equations and equation (7) describes the evolution of the market price of an installed unit of productive capital.

Following Benhabib et. Al. (2002) attention is restricted to steady states with non negative inflation targets which in turn imply that the nominal rate of interest is positive. As a result, equation (4) implies that ζ_t, the Lagrange multiplier associated with the liquidity constraint is non zero. It then follows from (5) that $m_t = c_t + I_t$. The economic intuition is simple: near a steady state with positive nominal interest rate holding money entails opportunity costs, and minimizing the opportunity cost of holding money implies that the liquidity constraint is binding. It then follows from equations (2),(4)-(5) that near this steady state $u'(c_t) = \lambda_t(1 + R_t)$.

Consequently, the law of motion for the real value of financial assets becomes

$$\dot{a}_t = (R_t - \pi_t)a_t + (1 - \tau_t)f(k_t) - (c_t + I_t)(1 + R_t) + \tau_t k_t \delta q_t \tag{8}$$

and the law of motion for capital is

$$\dot{k}_t = f(k_t) - c_t - \overline{g} - \delta k_t \tag{9}$$

Following much of the recent literature, the baseline model developed here attaches a very limited role for money. This is demonstrated by equation (8). Seeing that near a staedy state where nominal interest rate is positive the equilibrium stock of (real) money equals output, the only explicit role played by money is to serve as a unit of account. This issue is extensively emphasized in Woodford (2003) and in Gali (2008).

Note that in this setup productive capital and financial assets are perfect substitutes in the private level. Let $\eta_t \equiv \frac{q_t}{\lambda_t}$ represent the ratio between the marginal valuations of the two saving devices. Then, η_t evolves according to:

$$\dot{\eta}_t = -(1-\tau_t)f'(k_t) - \lambda_t \eta_t \tau_t \delta + \eta_t(R_t - \pi_t + \delta) \tag{10}$$

Thus, equations (6), (9) – (10) fully describe the optimal program of a representative household as it takes the time path $\{\tau_t, R_t, \pi_t\}_{t=0}^{\infty}$ as (exogenously) given. Finally, as we study equilibria close to the steady state the transversality condition holds.

1.2 General Equilibrium

In equilibrium, the goods market clear

$$f(k_t) = c_t + I_t + \overline{g} \tag{11}$$

The rate of investment is set so as to equate the ratio between marginal valuations of perfect saving substitutes [bonds and capital] to the gross rate of interest which is the opportunity cost of investing in capital due to the finance constraint

$$\eta_t = 1 + R_t \tag{12}$$

Assets market clears so as to equate the marginal utility of consumption to the marginal valuation of wealth

$$u'(c_t) = \lambda_t(1 + R_t) \tag{13}$$

and the motion equations (6), (8) – (10) display the evolution of $\{\lambda_t, \eta_t, k_t, a_t\}_{t=0}^{\infty}$

.

1.2.1 Equilibrium Dynamics

Conjecture that equilibrium in the underlying economy is a mapping of (λ, η, k).[from this point on the time notation is omitted for simplicity] In this section we will characterize the monetary-fiscal policy that induce a unique equilibrium. Note that equation (12) and the type of interst rate rule imply that

$$\eta = 1 + R(\pi) \tag{14}$$

it then follows that $\pi = \pi(\eta); \pi_\lambda = \pi_k = 0; \pi_\eta = \frac{1}{R'(\pi)}$ where subscripts denote partial derivatives and $R'(\pi)$ is the increment in percentage points to the nominal interest in response to a one percent increase in the rate of inflation relative to the target. Also, equations (12)-(13) imply that $u'(c) = \lambda\eta$ and therefore $c_\lambda = \frac{\eta}{u''(c)}; c_\eta = \frac{\lambda}{u''(c)}; c_k = 0;$

Thus, the dynamics of all the variables in the economy is a mapping in the (λ, η, k) space and the evolution of (λ, η, k) can be described by: $\dot{\lambda} = F(\lambda, \eta, k), \dot{\eta} = G(\lambda, \eta, k), \dot{k} = H(\lambda, \eta, k)$

where

$$
\begin{aligned}
F(\lambda, \eta, k) &\equiv \lambda\left[\rho + \pi(\eta) - R(\pi(\eta))\right] && (15)\\
G(\lambda, \eta, k) &\equiv -(1-\tau)f'(k) - \lambda\eta\tau\delta + \eta\left[R(\pi(\eta)) - \pi(\eta) + \delta\right] && (16)\\
H(\lambda, \eta, k) &\equiv f(k) - c(\lambda, \eta) - \overline{g} - \delta k && (17)
\end{aligned}
$$

and the transversality condition is $Lim_{t\to\infty} e^{-\int\limits_0^t [R(\pi(\eta))-\pi(\eta)]ds} \left[a(\lambda, \eta, k) + k\right] = 0$

1.2.2 Equilibrium and Local Real Determinacy (LRD)

Following Evans and Guesnerie (2005) I consider only saddle-path stable solutions as macroeconomically stable.

Definition 1 *Equilibrium displays Local-Real-Determinacy (LRD) if there exists a Saddle-Path stable solution in the* (λ, η, k) *space. Otherwise equilibrium is non-LRD.*

Local-Real-Determinacy

Equations (13), (15)–(17) imply that in the steady state $R^* = \rho + \pi^*, \eta^* = 1 + R^*, \lambda^* = \frac{u'(c^*)}{1+R^*}, f'(k^*) = \frac{(1+R^*)[\rho+\delta(1-\lambda^*\tau^*)]}{1-\tau^*}$. Linear approximation to the set of equations (15)–(17) near the steady state is obtained through the system

$$
\begin{bmatrix} \dot{\lambda} \\ \dot{\eta} \\ \dot{k} \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & \lambda^* \frac{1-R'(\pi^*)}{R'(\pi^*)} & 0 \\ -(1+R^*)\delta\tau^* & \rho + \delta(1-\tau^*\lambda^*) + (1+R^*)\frac{R'(\pi^*)-1}{R'(\pi^*)} & -(1-\tau^*)f''(k^*) \\ -\frac{1+R^*}{u''(c^*)} & -\frac{\lambda^*}{u''(c^*)} & f'(k^*) - \delta \end{bmatrix}}^{A} \begin{bmatrix} \lambda - \lambda^* \\ \eta - \eta^* \\ k - k^* \end{bmatrix} \tag{18}
$$

where $R'(\pi^*)$ is the increment in percentage points to the nominal interest in response to a one percent increase in the rate of inflation relative to the target and τ^* is the rate of income tax that balances the consolidated budget in the steady state. Specifically, τ^* is the solution to $0 = \rho a^* - R^* m^* + \overline{g} - \tau^* k^* \left[f'(k^*) - \delta q^* \right]$

Let α_i *(i=1,2,3)* denote the eigenvalues of matrix A, then,

$$
\alpha_1\alpha_2\alpha_3 = \tag{19}
$$
$$
-\frac{u'(c^*)}{u''(c^*)} \left[(1-\tau^*)f''(k^*) + (f'(k^*) - \delta)\delta\tau^* u''(c^*) \right] \frac{R'(\pi^*) - 1}{R'(\pi^*)}
$$
$$
\alpha_1 + \alpha_2 + \alpha_3 = \rho - \delta\tau^*\lambda^* + f'(k^*) + (1+R^*)\frac{R'(\pi^*) - 1}{R'(\pi^*)} \tag{20}
$$

Proposition 2 $R'(\pi^*) > 1 \Rightarrow$*Equilibrium is LRD*

$R'(\pi^*) < 1 \Rightarrow$Equilibrium is non-LRD

(Proof in Appendix A)

Under Schmitt-Grohe and Uribe's (1997) balanced budget constraint, when agents become optimistic about the future of the economy and decide to work harder and invest more, the government is forced to lower the tax rate as total output rises. The countercyclical tax policy will help fulfill agents' initial optimistic expectations, thus leading to indeterminacy of equilibria and endogenous business cycle fluctuations.

This paper offers a slighly differnt approach by considering the a consolidated balanced budget constraint. Here, the monetary authority controls for the real interst rate via financial markets. Suppose that the economy shifts away from the steady state as a result of a positive shock to expected productivity. In terms of the model, the stock of capital is now below its steady state level, and the marginal product of capital is higher than its steady state level. The nominal interest rate would consequently rise because initially, the real interest rate has increased. In order to finance the increase in payments following the rise of the real interest rate, inflation tax revenues must increase which in turn further increases the nominal interest rate.

At the next instant, the stance of the monetary authority is carried out in the open market. Under the active stance, the monetary authority increases the rate of bond creation relative to the rate effective prior to the shock, thus driving the real interest rate above its steady state level. This policy effects households' allocation between investment and consumption via an arbitrage channel.

Note that λ measures the marginal utility of consumption distorted by the nominal interest-rate. Under the active stance the real interest rate is above its steady state level, and according to the euler equation (13) an active stance induces a negative growth rate in the marginal utility of consumption. Thus implying an increase in consumption. Accordingly, when households become optimistic about the future of the economy this type of consolidated budget rule motivates the households to eat away their capital stock, so as to allow consumption to increase, which further distances the economy from the steady state. It is this mechanism that prevents optimism that is not anchored in fundamentals from becoming self fulfilling.

Under the neutral monetary policy stance, where $R'(\pi^*) = 1$, the real interest

rate remains constant in and off the steady state and equals ρ.According to equation (19) this policy introduces a zero eigenvalue and the type of equilibrium stability becomes sensitive to the type of fiscal policy.

Proposition 3 *Under a neutral monetary policy stance equilibrium is LRD iff fiscal policy is such that* $\frac{\bar{g}}{c^*} < \sigma(c^*)\varphi(k^*) - 1$ *where* $\sigma(c^*) \equiv -\frac{u'(c^*)}{c^* u''(c^*)}, \varphi(k^*) \equiv -k^* \frac{f''(k^*)}{f'(k^*)}$

(Proof in Appendix A)

$\sigma(c^*)$ and $\varphi(k^*)$ measure the intertemporal elasticity of consumption substitution and the elasticity of marginal product of capital near the steady state, respectively. Consider for example an economy where $u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$ and where production technology exhibits constant returns to scale with respect to capital and is of the form $f(k_t) = Ak_t^{\alpha}$. Then, the elasticities of intertemporal substitution and marginal productivity are constant and equal $\frac{1}{\gamma}, 1-\alpha$, respectively. Thus, for this economy, a neutral monetary policy stance is stabilizing only if $\frac{\bar{g}}{c^*} < \frac{1}{\gamma}(1-\alpha) - 1$

2 Equilibria with Externalities

Benhabib et. al. (2000) show that a small divergence between the social and private returns in multisector growth model is sufficient for multiple equilibria. In the previous section a consolidated-budget rule with monetary dominance was found to induce LRD in a single sector growth model where income taxes distort private returns. In view of Benhabib et. al.'s (2000) upshot for multi-sector models it is worthwhile to establish the robustness of the consolidated-budget rule for single sector models. In this section I will therefore assume that the production technology exhibits an externality associated with per capita capital [4]. Production externalities enter the model economy as in Kehoe et al. (1992) and Rebelo and Xie (1999). Suppose that the production function, $f(k_{p,t}, k_{a,t})$, exhibits a positive externality where k_p, k_a are private capital stock and per capita capital stock in the entire economy, respectively. $f(\cdot,\cdot)$ is strictly increasing in both arguments and concave in k_p and continuously differentiable. The representative household's optimal program given the initial stock of private capital $k_{p,0}$, the per capita stock of capital $k_{a,0}$, and the time paths of $\{\tau, R, \pi\}$ maximizes the current value hamiltonian $H \equiv u(c) + \lambda\left[(R-\pi)a + (1-\tau)f(k_p, k_a) - (c+I)(1+R) + \tau\delta q k\right] + q\left[I - \delta k\right]$; hence, the optimality conditions associated with the household's problem are:

[4] Guo and Harrison (2008) extend Schmitt-Grohe and Uribe's (1997) analysis by the inclusion of useful government spending. They show that fixed government spending act simply as a scaling constant in either firms' production or households' utility function. Thus, we should look for external effects in production that do not derive from goverment purchases.

$$\lambda = \frac{u'(c)}{1+R} \tag{21}$$
$$\eta = 1 + R \tag{22}$$
$$\dot{\lambda} = \lambda \left[\rho + \pi - R\right] \tag{23}$$
$$\dot{\eta} = -(1-\tau) f_1(k,k) - \lambda\eta\tau\delta + \eta \left[R - \pi + \delta\right] \tag{24}$$
$$\dot{k} = f(k,k) - c - \overline{g} - \delta k \tag{25}$$
$$\dot{a} = (R-\pi)a + (1-\tau) f(k,k) - (c+I)(1+R) + \tau\delta q k \tag{26}$$

and the transversality condition is $Lim_{t\to\infty} e^{-\int\limits_0^t [R(s)-\pi(s)]ds} [a+k] = 0$

Where subscripts denote partial derivatives, $\eta \equiv \frac{q}{\lambda}$ denotes the ratio between the co-state variables, and the condition for a symmetric equilibrium, $k_p = k_a = k$, is substituted into equation (24) only after the derivative of H with respect to k_p is taken.

2.1 The Government

The real value of the government's liabilities evolves according to $\dot{a} = [R-\pi]\, a - Rm + [g - \tau\,[f(k,k) - \delta q k]]$ whereas the interest rate feedback rule is of the form $R = R\,[\pi(\lambda, \eta, k)]$ .The government, unlike the households sector, internalizes the externality.

2.2 General Equilibrium

Consumption per capita and the rate of inflation are set in general equilibrium. Also, the fiscal and monetary policy are set so as to obtain a solution to the central planner's problem . Hence, these magnitudes are derived as if the central planner internalizes the externality. In equilibrium, the goods market clear

$$f(k,k) = c + I + \overline{g} \tag{27}$$

Also, in equilibrium the rate of investment is set so as to equate the ratio between marginal valuations of financial wealth and productive capital to the opportunity cost which is the gross rate of interest, and assets market clears so as to equate the marginal utility of consumption to the marginal valuation of financial wealth. Thus, equation (12) - (13) hold and the motion equations (23) – (26) display the evolution of $\{\lambda, \eta, k, a\}_{t=0}^{\infty}$.

2.2.1 Equilibrium Dynamics

Conjecture that equilibrium in the underlying economy is a mapping of (λ, η, k). In this section we will characterize the monetary-fiscal policy that induce an LRD equilibrium. Note that equation (23) and the type of interst rate rule imply that

$$\eta = 1 + R(\pi) \tag{28}$$

it then follows that $\pi = \pi(\eta); \pi_\lambda = \pi_k = 0; \pi_\eta = \frac{1}{R'(\pi)}$ where subscripts denote partial derivatives and $R'(\pi)$ is the increment in percentage points of the nominal interest to a one percent increase in the rate of inflation relative to the target. Also, equations (21)-(22) imply that $u'(c) = \lambda\eta$ and therefore $c_\lambda = \frac{\eta}{u''(c)}; c_\eta = \frac{\lambda}{u''(c)}; c_k = 0;$

The dynamics of all the variables in the economy can thus be described by (λ, η, k) and the evolution of (λ, η, k) can be described by: $\dot{\lambda} = F(\lambda, \eta, k), \dot{\eta} = G(\lambda, \eta, k), \dot{k} = H(\lambda, \eta, k)$

where

$$\begin{aligned}
F(\lambda, \eta, k) &\equiv \lambda\left[\rho + \pi(\eta) - R(\pi(\eta))\right] && (29)\\
G(\lambda, \eta, k) &\equiv -(1-\tau)f_1(k,k) - \lambda\eta\tau\delta + \eta\left[R(\pi(\eta)) - \pi(\eta) + \delta\right] && (30)\\
H(\lambda, \eta, k) &\equiv f(k,k) - c(\lambda, \eta) - \overline{g} - \delta k && (31)
\end{aligned}$$

and the transversality condition is $Lim_{t\to\infty} e^{-\int_0^t [R(\pi(\eta)) - \pi(\eta)]ds} \left[a(\lambda, \eta, k) + k\right] = 0$

2.2.2 Equilibrium and Local Real Determinacy (LRD)

Local-Real-Determinacy

Equations (13), (29)–(31) imply that in the steady state $R^* = \rho + \pi^*, \eta^* = 1 + R^*, \lambda^* = \frac{u'(c^*)}{1+R^*}, f_1(k^*, k^*) = \frac{(1+R^*)[\rho + \delta(1-\lambda^*\tau^*)]}{1-\tau^*}$. Linear approximation to the set of equations (29)–(31) near the steady state is obtained through the system

$$\begin{bmatrix} \dot{\lambda} \\ \\ \dot{\eta} \\ \\ \dot{k} \end{bmatrix} = B \times \begin{bmatrix} \lambda - \lambda^* \\ \\ \eta - \eta^* \\ \\ k - k^* \end{bmatrix}$$

where $B \equiv$

$$\begin{bmatrix} 0 & \lambda^* \frac{1-R'(\pi^*)}{R'(\pi^*)} & 0 \\ \\ -(1+R^*)\delta\tau^* & \rho + \delta(1-\tau^*\lambda^*) + (1+R^*)\frac{R'(\pi^*)-1}{R'(\pi^*)} & -(1-\tau^*)\left[f_{11}(k^*,k^*) + f_{12}(k^*,k^*)\right] \\ \\ -\frac{1+R^*}{u''(c^*)} & -\frac{\lambda^*}{u''(c^*)} & f_1(k^*,k^*) + f_2(k^*,k^*) - \delta \end{bmatrix}$$

Let β_i *(i=1,2,3)* denote the eigenvalues of matrix B, thus,

$$\beta_1\beta_2\beta_3 =$$
$$-\frac{u'(c^*)}{u''(c^*)}\left[(1-\tau^*)\left[f_{11}(k^*,k^*) + f_{12}(k^*,k^*)\right] + (f_1(k^*,k^*) + f_2(k^*,k^*) - \delta)\delta\tau^* u''(c^*)\right]\frac{R'(\pi^*)-1}{R'(\pi^*)}$$
$$\beta_1 + \beta_2 + \beta_3 =$$
$$f_1(k^*,k^*) + f_2(k^*,k^*) + \rho - \delta\tau^*\lambda^* + (1+R^*)\frac{R'(\pi^*)-1}{R'(\pi^*)}$$

Proposition 4 *In an economy where marginal product of capital is non-increasing in the social level, an active monetary policy stance within a consolidated-budget rule induces an LRD equilibrium.*

(Proof in Appendix A)

3 Conclusion

Schmitt-Grohé and Uribe (1997) show that the exact formulation of a balanced-budget fiscal policy plays an important role in affecting the determination of real allocation in an otherwise standard one-sector real business cycle model with wasteful government purchases. Specifically, they show that when a balanced budget rule consists of fixed goverment spending and proportional taxation on total income, the economy may exhibit an indeterminate steady state and a continuum of stationary sunspot equilibria. Furthermore, Guo and Harrison (2008) show that the indeterminacy result obtained in Schmitt-Grohé and Uribe (1997) remain unchanged by the inclusion of useful government spending, regardless of wheter it has an external effect on firms' production or on households' utility. In this paper I maintain the assumption of exogenous government spending within a balanced-budget requirement. However, I augment the balanced-budget requirement so as to incorporate the interaction between fisacl policy and monetary policy within the requirement that the instantaneous budget of the consolidated government is balanced.

Following Leeper (1991) I focus on rules that exhibit monetary dominance. Under this set of rules it is assumed that the monetary authority implements its policy stance regardless of the fiscal authority. The monetary authority trade bonds for money in the open market so as to impose over the economy a desired nominal interest rate. By doing so, the monetary authority affects the size of primary deficit. Only then, given the size of government purchases and the size of interest payments over outstanding debt, the fiscal authority sets the rate of income tax so as to balance the consolidated budget. Results show that a consolidated budget rule that exhibits monetary dominance whereby the monetary authority sets the nominal interest so as to increase the real rate of interest during booms induces a determinate equilibrium. This result is consistent with the celebrated Taylor principle and with the results obtained in Schmitt-Grohé and Uribe (2007). Unlike in previous literature, results show that with high degrees of intertemporal substitution, a small government [in the sense of $\frac{\bar{g}}{c^*}$] can stabilize the economy by assuming a neutral stance such that induces a constant real interest rate in and off the steady state.

Finally, it should be noted that it is not straightforward to conclude that the policies prescribed in this paper can also eliminate indeterminacies in multi sector models. I think that this issue deserves further research.

Appendix A

Proof of Proposition 1

Consider an active stance, i.e. $R'(\pi^*) > 1$: Note the right hand side of equation **??**. When monetary policy is active the product of eigenvalues is negative, which imply that either there is one negative eigenvalue and two eigenvalues with positive real parts, or all three eigenvalues are negative. Note equation **??**. Under an active stance the sum of eigenvalues is positive which rules out the possibility that all the eigenvalues are negative. With one negative eigenvalue and one predetermined state variable the equilibrium is saddle-path stable.

Consider the passive stance: The passive policy implies the product of eigenvalues is positive which implies that either two eigenvalue are stable and one is unstable, or that all three eigenvalues are unstable. And in this case equilibrium is non-LRD. QED.

Proof of Proposition 2

Consider a neutral monetary policy stance, i.e. $R'(\pi^*) = 1$: under this policy stance the real interest rate is constant in and off the steady state and equals ρ.Consequently, as we can see from the euler equation 29, consumption is constant and the time path of all the variables in the economy is spanned by $\{\eta, k\}$. Accordingly, the evolution of $\{\lambda, \eta, k\}$ is

$$\dot{\lambda} = 0$$

$$\dot{\eta} = -(1-\tau)f'(k) - \lambda\eta\tau\delta + \eta\left[\rho + \delta\right]$$

$$\dot{k} = f(k) - c\left(\lambda, \eta\right) - \overline{g} - \delta k$$

equilibrium dynamics also implies that $\pi_k = 0; \pi_\eta = 1$ and $c_\eta = \frac{\lambda}{u''(c)}; c_k = 0;$

Linear approximation near the steady state is obtained through

$$\begin{bmatrix} \dot{\eta} \\ \\ \dot{k} \end{bmatrix} = \overbrace{\begin{bmatrix} \rho + \delta(1-\tau^*\lambda^*) & -(1-\tau^*)f''(k^*) \\ \\ -\frac{\lambda^*}{u''(c^*)} & f'(k^*) - \delta \end{bmatrix}}^{A} \begin{bmatrix} \eta - \eta^* \\ \\ k - k^* \end{bmatrix}$$

and accordingly, equilibrium is LRD iff the product of eigenvalues, denotes as $\alpha_1\alpha_2$, is negative.

Note that $\alpha_1\alpha_2 = \left[\rho + \delta(1-\tau^*\lambda^*)\right]\left[f'(k^*) - \delta\right] - \left[(1-\tau^*)f''(k^*)\right]\frac{\lambda^*}{u''(c^*)}$

and in the steady state $\frac{(1-\tau^*)f'(k^*)}{(1+R^*)} = \rho + \delta\left(1 - \lambda^*\tau^*\right), \eta^* = 1 + R^*, \lambda^* = \frac{u'(c^*)}{1+R^*},$ Thus

$\alpha_1\alpha_2 = \frac{(1-\tau^*)f'(k^*)}{(1+R^*)}\left[f'(k^*)-\delta\right] - \left[(1-\tau^*)f''(k^*)\right]\frac{u'(c^*)}{u''(c^*)(1+R^*)} = \frac{(1-\tau^*)f'(k^*)}{(1+R^*)}\left[\left[f'(k^*)-\delta\right] - \frac{f''(k^*)}{f'(k^*)}\frac{u'(c^*)}{u''(c^*)}\right]$

$= \frac{(1-\tau^*)f'(k^*)}{(1+R^*)}\frac{c^*}{k^*}\left[\frac{k^*}{c^*}\left[f'(k^*)-\delta\right] - \frac{k^*f''(k^*)}{f'(k^*)}\frac{u'(c^*)}{u''(c^*)c^*}\right]$

finally $\frac{k^*}{c^*}\left[f'(k^*)-\delta\right] = \frac{1}{c^*}\left[f(k^*)-I^*\right] = \frac{1}{c^*}\left[c^*+\bar{g}\right] = 1+\frac{\bar{g}}{c^*}$ and hence $\alpha_1\alpha_2 < 0 \Longleftrightarrow 1+\frac{\bar{g}}{c^*} - \frac{k^*f''(k^*)}{f'(k^*)}\frac{u'(c^*)}{u''(c^*)c^*} < 0$

QED.

Proof of Proposition 3

Marginal product of capital at the social level is non increasing iff $f_{11}(k^*,k^*) + f_{12}(k^*,k^*) \leq 0$.Consider an active stance, i.e. $R'(\pi^*) > 1$. When monetary policy is active the product of eigenvalues is negative, which imply that either there is one negative eigenvalue and two eigenvalues with positive real parts, or all three eigenvalues are negative. Under an active stance the sum of eigenvalues is positive which rules out the possibility that all the eigenvalues are negative. Thus, equilibrium is saddle-path stable.

Consider the passive stance: The passive policy implies the product of eigenvalues is positive which implies that either two eigenvalue are stable and one is unstable, or that all three eigenvalues are unstable. And in this case equilibrium is non-LRD. QED.

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